ID: IAEA-CN-2657

NEOCLASSICAL THEORY ON LOW FREQUENCY DRIFT ALFVÉN WAVES

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Abstract

- In this work, we developed kinetic models based on general fishbone-like dispersion relations.
- By disregarding ion orbit width and approximating the magnetic geometry as circular, we introduce a simplified model that fully incorporates neoclassical effects by considering full circulating/trapped particles.
- By considering the limit of ions being well-circulating or deeply trapped, the results directly revert to those observed in earlier theoretical studies[Chavdarovski&Zonca, 2009, PPCF].
- The accumulation point frequency of Beta-induced Alfvén Eigenmode is calculated in the fluid limit.

BACKGROUND

- Alfvén waves and energetic particles, resulted from fusion reaction and auxiliary heating, are crucial to the performance of Tokamak devices.
- The theoretical research on low frequency drift Alfvén waves is based on the general fishbone-like dispersion relation (GFLDR)[Chen&Zonca, RMP, 2016] and gyrokinetic theory[Brizard&Hahm, RMP, 2007].
- In the original theoretical works on these modes, ions considered in the kinetic analysis are assumed to be well circulating[Zonca et al, 1996, ppcf].
- Later on, the kinetic analysis was extended by including the deeply trapped ions and electrons[Chavdarovski&Zonca, 2009, ppcf].
- However, , the particles near circulating/trapped separatrix are not included in the previous theoretical models.
- In this work, we include full neoclassical effects without assuming well-circulating/deeply-trapped ions.

CHALLENGES

INCORPORATING NEOCLASSCAL THEORY IN BALLOONING ANGLE REPRESENTATION

The intertial layer is mainly concerned in this work. Thus we can have separated scales in ballooning angle as $|\vartheta_1| \gg |\vartheta_0|$. Here ϑ_1 represents the singular structure of inertial layer. And ϑ_0 denotes the periodic motion of particles in the poloidal angle. In this way, the neoclassical theory of passing and trapped particles can be applied.

SIMPLIFICATION OF THE CORRELATION BETWEEN THE FOURIER SERIES IN FAST BALLOONING ANGLE AND CANONICAL ANGLE

Here we have introduced the canonical angle as

$$\vartheta_c = \omega_b \int^{\vartheta_0} \frac{d\vartheta_0'}{\dot{\theta}},$$

where $\omega_b$is the bouncing frequency. Since the magnetic configuration with up-down symmetry is considered, the canonical angle can be defined piece-wisely. First, we consider the circulating particles with $\sigma = 1$. For $0 \leq \vartheta_0 < \pi$,

$$\vartheta_c (\vartheta_0, \sigma = 1) = \vartheta^+_{circ} (\vartheta_0) = 2\pi \frac{\int_0^{\vartheta_0} d\vartheta_0'/\dot{\theta}}{\oint d\vartheta_0/\dot{\theta}},$$

For trapped particles, the canonical angle is obtained piece wisely as

$$\theta_c (\vartheta_0) = \theta^+_{tr} (\vartheta_0) = 2\pi \frac{\int_0^{\vartheta_0} d\vartheta_0'/\dot{\theta}}{\oint d\vartheta_0/\dot{\theta}}, \quad \text{for } 0 \leq \theta_0 \leq \theta_b \text{ and } \dot{\theta} > 0.$$

With the up-down symmetry, the definition of canonical angle along the rest parts of the orbit can be obtained as

$$\theta_c (\vartheta_0) = \begin{cases} \pi - \theta^+_{tr} (\theta), & \text{for } \theta_b > \theta_0 \geq 0 \text{ and } \dot{\theta} < 0, \\ \pi + \theta^+_{tr} (2\pi - \theta), & \text{for } 2\pi > \theta_0 \geq 2\pi - \theta_b \text{ and } \dot{\theta} < 0, \\ 2\pi - \theta^+_{tr} (2\pi - \theta), & \text{for } 2\pi - \theta_b < \theta_0 < 2\pi \text{ and } \dot{\theta} > 0. \end{cases}$$

With the definitions for ϑ_c, by integrating along the orbit piece-wisely, the pseudo-orthogonality relation in bouncing average for both trapped and circulating particles can be given as

$$\oint \sin m\vartheta_0 \cos l\vartheta_c d\vartheta_c = \oint \cos m\vartheta_0 \sin l\vartheta_c d\vartheta_c = 0,$$

where m and l are integers.

OUTCOME

Deep trap limit

Here we introduce weight functions for both passing and trapped particles respectively as $M(\epsilon, \lambda, \sigma) = \frac{1}{\pi} \oint \sin\vartheta_0 \sin\vartheta_c d\vartheta_c$and $L(\epsilon, \lambda) = \frac{1}{\pi} \oint \sin\vartheta_0 \sin\vartheta_c d\vartheta_c$. Moreover, the higher harmonics for passing particles $M_1 = \frac{1}{\pi} \oint \sin\vartheta_0 \sin 2\vartheta_c d\vartheta_c$ is also calculated to be compared with M. The numerical results are shown in Fig. 1. If the circulating particles are approximately taken to be well circulating, i.e. $\lambda \ll 1$, , the parallel velocity $v_\parallel$ can be treated approximately independent of ϑ_0. Then the weight term M can be given as

$$M(\sigma = 1) \simeq \frac{1}{\pi} \int_0^{2\pi} \sin\vartheta_0 \sin\vartheta_0 d\vartheta_0 = 1.$$

For the trapped particles, by assuming trapped particles are deeply trapped, we have $\theta_b \simeq 2\kappa$, $\vartheta_0 \simeq \theta_b \sin\vartheta_c$, $v_\parallel \simeq \theta_b \omega_b q R_0 \cos\vartheta_c$ and $\omega_b = \sqrt{\epsilon\mu B_0}/qR_0$. Then the weight term L can be given as

$$L = \frac{4}{\pi} \int_0^{\pi/2} \sin\vartheta_c \sin\vartheta_0 d\vartheta_c \simeq \theta_b.$$

Here the approximation $\sin\vartheta_0 \simeq \theta_b \sin\vartheta_c$ has been used.With the analysis of the weight functions, the results can go back those in the previous research[Chavdarovski & Zonca, 2009, ppcf].

Fluid limit

In kinetic analysis, we take $\delta\omega_{di} \sim \omega \sim \omega_{*Pi} \sim \omega_{bi}$ for circulating ions, $\omega_{de} \sim \omega_{di} \sim \omega \sim \omega_{bi} \sim \delta\omega_{be}$ for trapped ions and electrons, small Larmor radius$k_\perp \rho_{ti} \sim \delta$ and low energetic particle density $n_i \sim \delta^{-3} n_E$ as our orderings. By taking the fluid limit $\omega \gg \omega_{ti} \sim \omega_{*Ts} \sim \omega_{*ns} \sim \bar{\omega}_{di} \sim \bar{\omega}_{bi}$, the accumulation point of BAE can be calculated as

$$\omega_{BAE} \simeq q\omega_{ti} \sqrt{\tau + \frac{7}{4} - 1.7\tau\sqrt{\epsilon} - 0.2\sqrt{\epsilon}},$$

which is shown in Fig. 2.

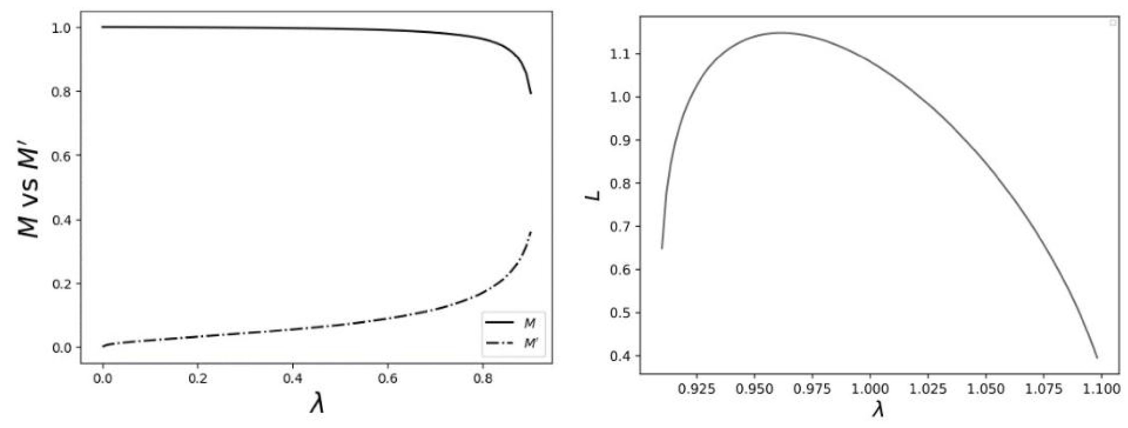

Figure 1. Value of weight function versus pitch angle variable λ for ϵ=0.1.

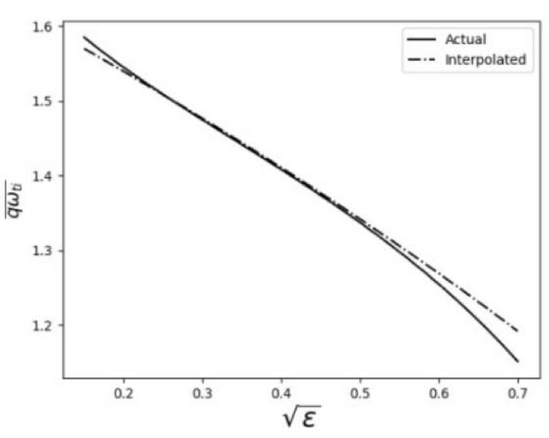

Figure 2. Neoclassical correction on the frequency of BAE for τ≈1.

CONCLUSION

- In this work, a kinetic model with full neoclassical effects is developed.
- The novel inspections on the symmetry of particle orbits are introduced to make the analytical calculation accessible.
- The results can go back to those in the previous researches by taking the deep trap limit.
- It is shown that the neoclassical effects can lower the frequency of BAE, which is absent in the deep trap limit calculation in the previous research..

ACKNOWLEDGEMENTS / REFERENCES

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- This work is conducted under the support of Innovation Program of Southwestern Institute of Physics (Grant No. 202301XWCX001).