NEOCLASSICAL THEORY ON LOW FREQUENCY DRIFT ALFVÉN WAVES

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1. INTRODUCTION

Alfvén waves and energetic particles, resulted from fusion reaction and auxiliary heating, are crucial to the performance of Tokamak devices. The theoretical research on low frequency drift Alfvén waves (LFDAW) is based on the general fishbone-like dispersion relation (GFLDR) and gyrokinetic theory [1]. Besides recovering diverse limits of the kinetic magnetohydrodynamic (MHD) energy principle, the GFLDR approach is also applicable to electromagnetic fluctuations, which exhibit a wide spectrum of spatial and temporal scales consistent with gyrokinetic descriptions of both the core and supra-thermal plasma components. Formally, the GFLDR can be written as

$$\mathrm{i}\,\Lambda = \delta W_f + \delta W_k, \tag{1}$$

where Λ is the generalized inertia, and δW_f and δW_k are, respectively, fluid and kinetic potential energy of electromagnetic fluctuations. By taking $\Lambda = 0$, we can calculate the accumulation point of the waves in the frequency continuum.

In the original theoretical works on LFDAW, ions considered in the kinetic analysis are assumed to be well circulating [3]. Later on, the kinetic analysis was extended to a neoclassical theory by including the deeply trapped ions and electrons [2]. Moreover, the researches mentioned above are all based on the $s-\alpha$ model in Tokamak plasmas with circular configuration. However, the effects of general magnetic geometry and full circulating/trapped particles are not included in previous researches. Especially, the particles near circulating/trapped separatrix are not included in the previous theoretical models. In order to obtain a better understanding of experimental observations and provide a more precise kinetic model for theoretical researches, we need to include the general magnetic geometry and full orbit effects without assuming well-circulating or deeply-trapped ions and small ion orbit width.

2. UP-DOWN SYMMETRY AND PSEUDO-ORTHOGONALITY

For the circular up-down symmetric configuration, the magnetic field $\mathrm{B} = B_0(1-\epsilon\cos\theta)$, where $\epsilon = \mathrm{r}/R_0$ is the inverse aspect ratio. In order to study the responses of trapped and circulating particles, the canonical angle can be introduced as

$$\vartheta_c = \omega_{bs}\int^{\vartheta_0}\frac{d\vartheta_0'}{\dot{\theta}_s}, \tag{2}$$

where the bouncing frequency $\omega_{\mathrm{bs}} = 2\pi/\left(\oint \mathrm{d}\vartheta_0/\dot{\theta}_s\right)$. In long wave length limit, the orbit width of both circulating and trapped particles can be neglected. And with up-down symmetry, the following pseudo-orthogonality relations for both circulating and trapped particles can be given as [4]

$$\oint \sin m\,\vartheta_0 \cos l\,\vartheta_c\,\mathrm{d}\vartheta_c = \oint \cos m\,\vartheta_0 \sin l\,\vartheta_c\,\mathrm{d}\vartheta_c = 0, \tag{3}$$

where m and l are integers. The relations above can greatly simplify the calculation for our problem. For circulating ions, we can define the weight function

$$M = \frac{1}{\pi}\oint \sin\vartheta_0 \sin\vartheta_c\, d\vartheta_c, \tag{4}$$

which is illustrated in Fig. 1 by comparing with $M' = \frac{1}{\pi}\oint \sin\vartheta_0 \sin 2\vartheta_c d\vartheta_c$. Similarly, the weight function for trapped particles can be defined as $L = \frac{1}{\pi}\oint \sin\vartheta_0 \sin\vartheta_c\, d\vartheta_c$. These functions can indicate the coupling between the trigonometric harmonics in ϑ_0 and ϑ_c , which is related to the resonant contribution of the harmonics of the bouncing frequency. And the values of M and L versus pitch angle variable λ give a rough estimate of the contribution from particles with different orbits. Thus, the value of the weight function M in Fig. 1 show that the well passing assumption is valid for the majority of passing ions. And the value of M' shows that the contribution from second harmonic in bouncing frequency resonance can be neglected. Similarly, we can conclude that the main contribution of trapped particles comes from the 'medium' trapped ones.

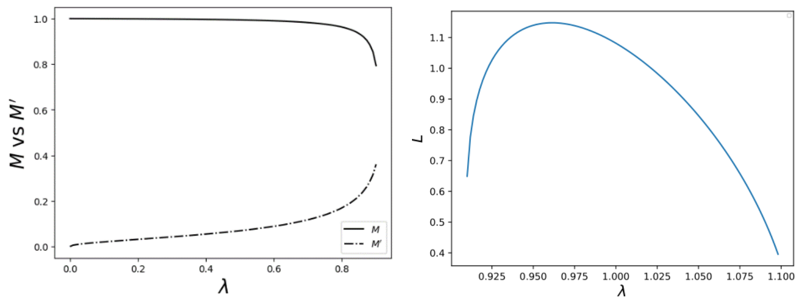

Figure 1. Value of weight function versus pitch angle variable λ for $\epsilon = 0.1$.

3. NUMERICAL RESULTS

With the weight functions defined above and the GFLDR in Eq. (1), an interpolated frequency formula of the Beta-induced Alfvén Eigenmodes (BAE) in fluid limit is given as

$$\omega_{BAE} = q\omega_{ti}\sqrt{\tau + \frac{7}{4} - 1.7\tau\sqrt{\epsilon} - 0.2\sqrt{\epsilon}}, \quad (5)$$

where q is the safety factor, ω_{ti} is the thermal transition frequency, τ is the ratio of electron temperature over ion temperature. The comparison with the numerical result is presented in Figure 2.

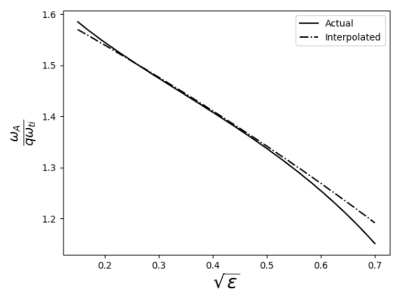

Figure 2. Neoclassical correction on the frequency of BAE for $\tau \approx 1$.

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