SINGLE MODE EVOLUTION IN WAVE-PARTICLE INTERACTIONS IN TOKAMAKS

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Wave-particle interactions occur ubiquitously whenever there are waves present in tokamak plasmas [1-5]. The interactions enhance particle, momentum, and energy losses. An Eulerian approach has been developed to solve drift kinetic equation to treat the interactions analytically for waves with frequencies less than the gyro-frequency [6-13]. The equation is relevant to fusion grade tokamak plasmas because the width of a typical gyro-radius is about one thousandth of the physical dimensions of the device and the significance of the effects of the finite gyro-radius is lessened. An unconventional set of independent variables is chosen to expunge the poloidal mode coupling terms in the equation. In particular, the toroidal component of the canonical momentum p_ζ is chosen to be the radial coordinate. The coefficients of particle kinetics do not depend on the angle variables in the chosen independent variables in the equation. Consequently, the resonant condition for a single (l,n) mode can be stated clearly, and they are, respectively, $\omega + l\omega_b = n\omega_d$, for the equilibrium trapped particles, and $\omega + \sigma\left[l - nq\left(p_\zeta\right)\right]\omega_t = n\omega_d$, for circulating particles. Here, l is the poloidal mode number, n is the toroidal mode number, ω is the wave frequency, ω_b is the bounce frequency of the equilibrium trapped particles, ω_t is the transit frequency of the circulating particles, ω_d is the bounce or transit averaged toroidal drift frequency, σ is the sign of the parallel particle speed to the equilibrium magnetic field, and q is the safety factor evaluated at p_ζ. Realistic collision operator can be employed in the approach. In particular, the collision frequency can be *effectively* enhanced several orders of magnitude over its nominal value for small amplitudes waves when nonlinear trapping occurs. The enhancement of the collision frequency cannot be obtained if a Krook model is used, which is the only collision operator that can be employed in solving the kinetic equation by the method of integration along the unperturbed orbit. Thus, the merit of the Eulerian approach becomes transparent. Because the equilibrium magnetic field has a gradient due to the toroidal magnetic field strength, the nonlinear particle trapping by the wave in the direction of the magnetic field line is more difficult in tokamaks. Nonlinear trapping occurs in the (E, p_ζ) phase space instead for waves with frequency lower than the gyro-frequency. Here, $E = \mathrm{v}^2/2 + e\phi/M$ is the particle energy, v is the particle speed, e is the charge, M is the mass, and ϕ is the electrostatic potential including both the equilibrium and the wave potential. In the nonlinear trapping regime [10], the distribution function flattens in the vicinity of the resonance. The energy exchange rate between the wave and particles follows the flux-power relation. When nonlinear trapping occurs, the growth rate of the wave depends on the wave amplitude and collision frequency. In the resonant plateau regime [12], the growth rate does not depend on the collision frequency even though the resonant layer width does. The mirror force term that is responsible for the flattening of the distribution is subdominant in this regime. The physics of the resonant plateau regime corresponds to that of the linear theory of Landau damping and the nonlinear trapping regime to that of the nonlinear theory of Landau damping.
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The theory is extended to include the compressible component of the perturbed magnetic field. The flux-power relation that relates transport flux driven by the structure of the wave that breaks toroidal symmetry in tokamaks to the power transfer between the wave and particles [10,12] is still valid, and the distribution function is flattened in the nonlinear trapping regime as well. Particle and energy transport losses are also enhanced. The results of the theory are compared with those in the bounce-transit and drift resonance regime in the theory of neoclassical toroidal plasmas viscosity [11]. There is a critical value of the normalized magnitude of the perturbed magnetic field strength over which the transport losses of energetic alpha particles will be greater than those of the neoclassical theory. It imposes a limit on the perturbed magnetic field strength in devices such as ITER to confine energetic alpha particles.

The nonlinear evolution of a single (l,n) mode is examined based on the theory. When the wave is linearly unstable excited by the unfavorable slopes of the distribution function in the (E, p_ζ) phase space—this should be compared with the theory of Landau damping in which only the slope of the distribution function in the velocity space matters for instability—its amplitude grows exponentially till nonlinear trapping occurs. The mirror force term that is responsible for the nonlinear trapping of the particles is subdominant in the exponential growth phase. After that, the wave amplitude grows algebraically because wave-particle energy transfer rate depends on the wave amplitude in the nonlinear trapping regime. This transition from the exponential to algebraic growth rate as the wave amplitude increases is similar to the nonlinear evolution of a single tearing mode. When the width of the island is wider than that of the resistive layer, the growth rate of the island becomes algebraic [14]. The formation of the island-like structure either in the phase space or in the real space is the basic mechanism for the algebraic growth. During the evolution, particles transfer part of their energy to the wave that results in the change of resonance parameters, and the frequency may also evolve [15-17]. The rate of the frequency change is either exponential or algebraic. The results could shed some light on the phenomena of nonlinear wave-particle interactions in tokamaks.

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