Drift-kinetic and fully kinetic simulations of plasma waves based on a geometric Particle-In-Cell discretization of the Vlasov-Maxwell system

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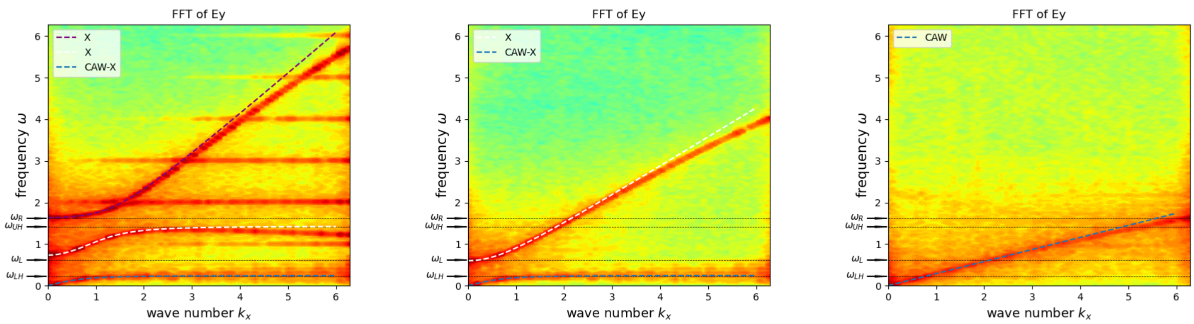
Most current gyrokinetic codes in the magnetically confined fusion community are based on the traditional scalar and vector potential formulation, typically considering only the parallel component of the vector potential. These studies are generally limited to specific branches or a few types of instabilities, such as drift-wave turbulence or Alfvénic modes. In contrast, we have developed a more comprehensive theoretical and numerical model with higher fidelity and enhanced capability to capture a wider range of physical phenomena across multiple spatio-temporal scales. This work extends the geometric Particle-In-Cell (PIC) framework on dual grids to a gauge-free drift-kinetic Vlasov-Maxwell model and its coupling with the fully kinetic model. The approach relies on a discrete action principle using only the electric and magnetic fields, avoiding the potentials typically used in drift-kinetic and gyrokinetic models. This results in a clearer physical interpretation of the numerical solution, enhancing consistency and eliminating the extra degrees of freedom from gauge choices. The resulting macroscopic Maxwell equations incorporate polarization and magnetization terms, enabling seamless coupling with a fully kinetic model for treating energetic particles and edge physics. The results from the hybrid model highlight the benefits of simplified ion equations and the ability to resolve ion-cyclotron frequency ranges [1].

Plasma physics models often exhibit a Hamiltonian structure with conserved invariants such as the Hamiltonian, Gauss's law, and div B = 0. Structure-preserving numerical methods aim to maintain these invariants for stable numerical solutions. The geometric approach discretizes the Hamiltonian structure rather than the partial differential equations, ensuring conservation of appropriately discretized invariants [2].

Beginning with the gyrokinetic Lagrangian of Burby and Brizard [3] in the Zero Larmor Radius limit, we develop a Lagrangian that couples drift-kinetic electrons with fully kinetic ions at the continuous level. We then propose a discretized version of the Lagrangian based on the Mimetic Finite Difference framework on dual grids [4] and the PIC method. From this discretized formulation, we derive the equations of motion for the PIC markers, as well as the discrete generalized Maxwell equations, which include polarization and magnetization terms arising from the drift-kinetic particles. The hybrid kinetic model integrates both kinetic and drift-kinetic contributions, providing a unified framework applicable to edge plasma physics in magnetic confinement fusion devices.

The study employs a Mimetic Finite Differences (MFD) discretization scheme on dual grids [4]. Scalar and vector potentials, electric and magnetic fields, and current densities are discretized separately on primal and dual grids. Discrete gradient, curl, and divergence operators are defined using Kronecker products, ensuring exact representation of Maxwell's equations. A low-storage Runge-Kutta scheme is used for time discretization, optimizing memory usage in large-scale PIC simulations. The scheme ensures conservation of total energy, including kinetic and electromagnetic field contributions. Verification tests confirm the accuracy and stability of the numerical method.

The dispersion relation of the drift-kinetic model is derived, demonstrating consistency with theoretical expectations. The numerical experiments validate the models by analyzing wave dispersion relations in a uniform plasma with a background magnetic field. Simulations of Langmuir waves, X-modes, and compressional Alfvén waves validate the numerical approach. To further verify the drift-kinetic model, a one-species simulation with only drift-kinetic electrons demonstrated accurate reproduction of Langmuir waves, with numerical results aligning well with analytical solutions. Later a two-species simulation compared fully kinetic, hybrid, and drift-kinetic models, revealing differences in wave dispersion properties. As shown in 1, the hybrid model captures key plasma wave phenomena absent in purely drift-kinetic or fully kinetic models, highlighting its applicability to magnetic confinement fusion research. The hybrid model, combining fully kinetic ions with drift-kinetic electrons, preserved key wave features while eliminating high-frequency electron cyclotron modes. The fully kinetic model exhibited Bernstein waves, absent in the hybrid and drift-kinetic models. These simulations illustrate how different models impact wave propagation and validate the effectiveness of the hybrid approach. The proposed geometric PIC discretization successfully extends structure-preserving methods to hybrid kinetic models.



(a) FK. (b) Hybrid. (c) DK.

Figure 1: Comparison of waves with k perpendicular to $\boldsymbol{B}_{\text{ext}}$ for different models. Left: Fully kinetic for both electrons and ions (FK). Middle: Drift-kinetic electrons with fully kinetic ions (Hybrid). Right: Drift-kinetic for both electrons and ions (DK).

Conclusion and Outlook

We have developed a new geometric PIC discretization for a gauge-free drift-kinetic model that can be seamlessly integrated with a fully kinetic model. The geometric PIC framework successfully bridges drift-kinetic and fully kinetic models, preserving structure and enabling hybrid simulations. Future work will integrate quasi-neutrality assumptions to suppress light waves and address Darwin approximations, further optimizing for edge plasma studies.

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