Average magnetic drift model for ITG in Tokamaks

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Background

- Ion temperature gradient (ITG) mode is recognized as major driver of anomalous ion transport (Tang 1986)
- Toroidal effect (magnetic drift) has significant impact on ITG resonance condition (Sun et al. 2022; Jia et al. 2025)
- Reversed magnetic shear q-profile can reduce ITG turbulent transport, which may be beneficial for the formation of internal transport barriers (Connor et al. 2004)
- Ballooning mode representation fails in reversed shear case, requiring the consideration of generalized translational invariance
- Extending the reversed shear effect, previously investigated in slab geometry (Dong, Horton, and Kishimoto 2001), to toroidal geometry presents significant computational challenges arising from magnetic drift.
- Using average magnetic drift approximation, a reduced gyrokinetic model applicable to normal and reversed magnetic shear cases is presented.

Average Magnetic Drift Model

- Gyrokinetic equation: $\left(\frac{iv_\parallel}{qR_0}\partial_\eta+\omega-\omega_{dj}\right)h_j=\frac{q_jF_{Mj}}{T_j}\left(\omega-\omega_{*j}^T\right)J_0\left(k_\perp\alpha_j\right)\delta\phi\left(\eta\right)$
- Kinetic integral equation: $(1+1/\tau)\,\delta\phi\left(\eta\right)=\int_{-\infty}^{\infty}d\eta' K\left(\eta,\eta'\right)\delta\phi\left(\eta'\right)$
- Local approximation: $\omega_d\left(\eta\right)\approx\omega_d\left(0\right)=\bar{\omega}_{dj}\left(v_\parallel^2+v_\perp^2/2\right)/2v_{tj}^2$
- Average approximation: $\omega_d\approx\bar{\omega}_{dj}\left\langle\cos\left(\eta\right)+\hat{s}\eta\sin\left(\eta\right)\right\rangle_{-\eta_s}^{\eta_s}\left(v_\parallel^2+v_\perp^2/2\right)/2v_{tj}^2$
- Truncated approximation: $\int_0^\eta\omega_d\left(\chi\right)d\chi\approx$

$$\left(\eta+\frac{2\hat{s}-1}{6}\eta^3+\frac{(1-4\hat{s})}{120}\eta^5\right)\mathrm{U}\left(\eta\right)\left(v_\parallel^2+v_\perp^2/2\right)/2v_{tj}^2;\ \mathrm{U}\left(\eta\right)=\begin{cases}1 & -\pi\le\eta\le\pi\\ 0 & \eta<-\pi\text{ or }\eta>\pi\end{cases}$$

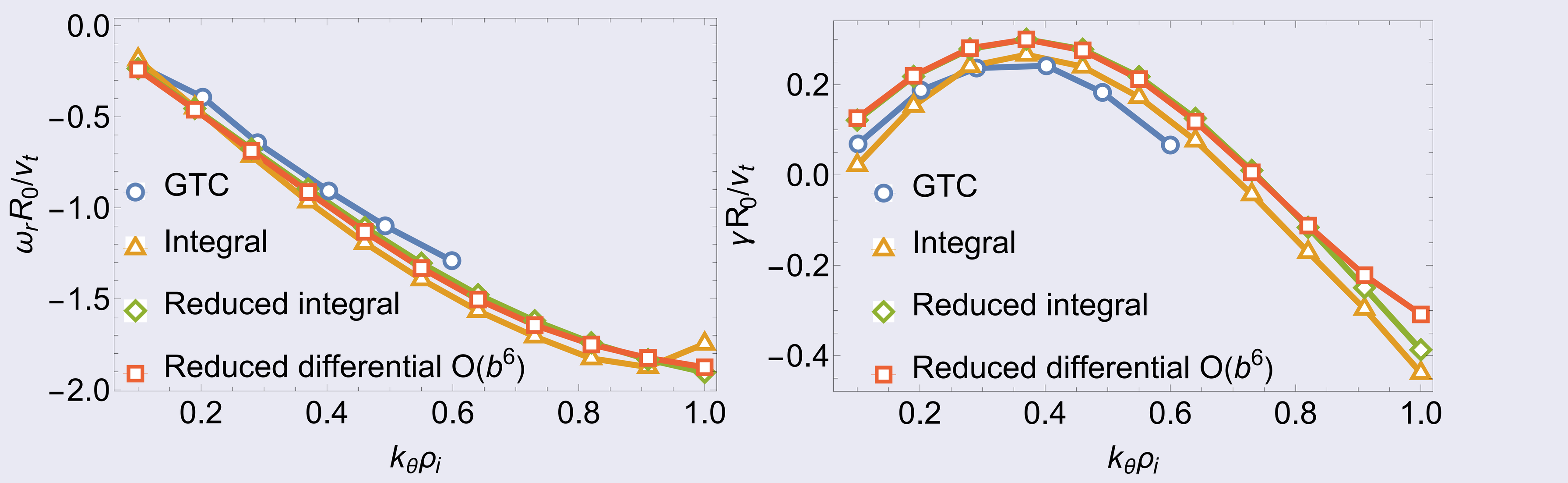

Figure: ω_r and γ vary with $k_\theta\rho_i$

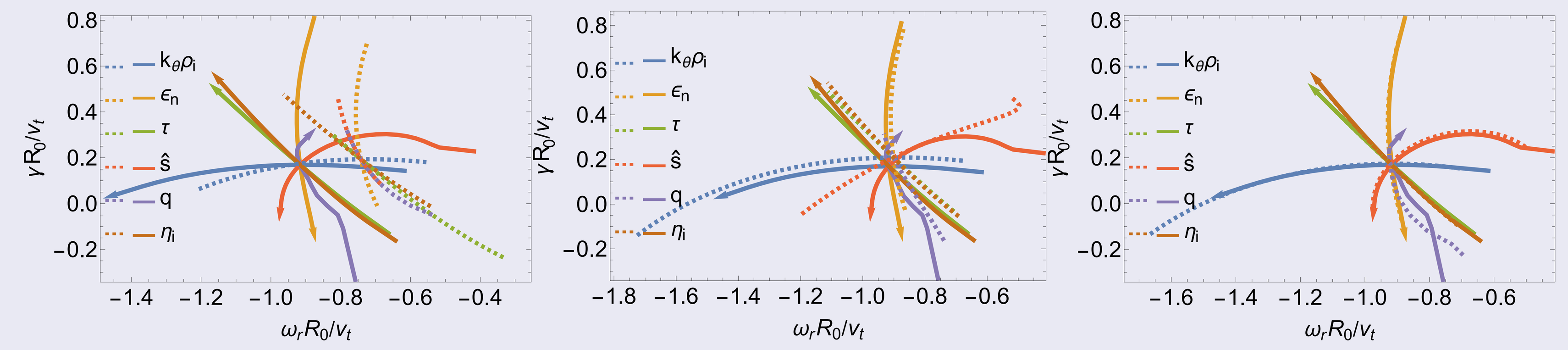

Figure: Eigenvalue ($\omega_r+i\gamma$) trajectories in the complex plane varing $k_\theta\rho_i$, ϵ_n, τ, $\hat{s}$, q and η_i.

Radial Eigenvalue Equation and its Schrödinger Form

Radial eigenvalue equation in $z=qRk_\parallel$ space

$$\left(1+\frac{1}{\tau}-\frac{2}{\sqrt{\pi}}\int\int dx J_0^2\left(\sqrt{2b}\right)\frac{\exp\left(-x^2\right)\left(\omega-\omega_{*in}\left(1+\eta_i\left(x^2-\frac{3}{2}\right)\right)\right)}{\omega-\frac{\sqrt{2}zv_{ti}x_\parallel}{qR}-\bar{\omega}_{di}f\left(\hat{s}\right)\left(\frac{x_\perp^2}{2}+x_\parallel^2\right)}\right)\delta\hat{\phi}\left(z\right)=0$$

$$J_0^2\left(\sqrt{b}\right)=\sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^nJ_0^2(k_\perp\alpha_i)}{d(k_\perp\alpha_i)^n}\bigg|_0\left(k_\perp\alpha_j\right)^n=\sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^nJ_0^2(k_\perp\alpha_i)}{d(k_\perp\alpha_i)^n}\bigg|_{k_\theta\alpha_j}\left(-i\alpha_j\partial_r\right)^n$$

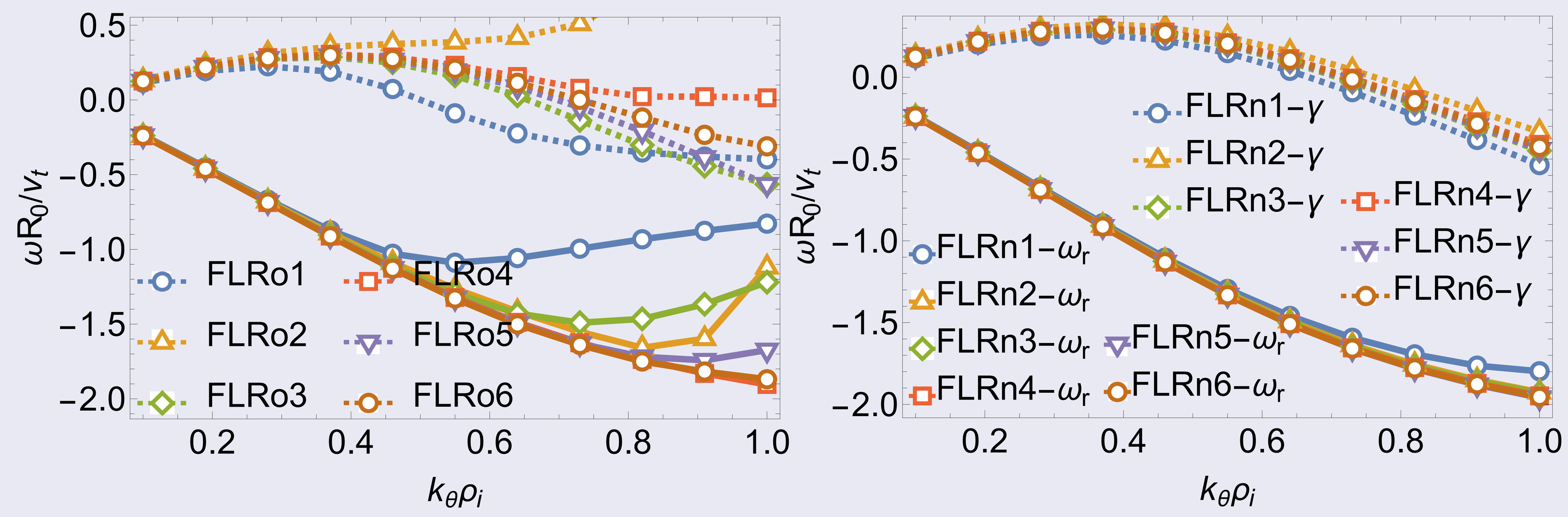

Figure: ω , γ vary with $k_\theta\rho_i$ for different order FLR effects

1st FLR gives Schrödinger form equation $\left(\frac{\partial^2}{\partial r_\kappa^2}+\frac{\bar{\omega}_{di}f\left(\hat{s}\right)\left(1+\frac{1}{\tau}\right)+\mathcal{K}_0}{\sqrt{2b_\theta}\mathcal{K}_1}\right)\delta\phi\left(r_\kappa\right)=0$, where

$\mathcal{K}_l=\left[\omega-\omega_{*i}\left(1-\frac{3}{2}\eta_i\right)\right]\mathcal{M}_{l10}-\eta_i\omega_{*i}\left(\mathcal{M}_{l30}+\mathcal{M}_{l12}\right)$

and $\mathcal{M}_{(l,n,m)}=\frac{2}{\sqrt{\pi}}\int_0^\infty dx_\perp\int_{-\infty}^{\infty}dx_\parallel\exp\left(-x^2\right)\frac{x_\perp^nx_\parallel^mJ_0\left(\sqrt{2b}x_\perp\right)J_l\left(\sqrt{2b_\theta}x_\perp\right)}{x_\parallel^2+x_\perp^2/2+\zeta_\alpha-\zeta_\beta x_\parallel}$.

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Translational Invariance and Reversed Shear Case

- Assuming generalized translational invariance, the Schrödinger form equation is also valid in reversed shear case using coordinate $r_\kappa=k_\theta\left(r-r_0\right)$ and q profile $nq-m=\delta_{A,m}+\hat{s}r_\kappa+\frac{s_2^2}{2n}r_\kappa^2=q_0R_0k_\parallel$ (Zonca et al. 2002), in which $\hat{s}=\frac{r_0q'(r_0)}{q_0}$, $s_2^2=\frac{q''r_0^2}{q_0^2}$, $q_0=\frac{m}{n}$, $r_\kappa=k_\theta\left(r-r_0\right)$, $\delta_{A,m}=n\left(q\left(r_0\right)-q_0\right)$.

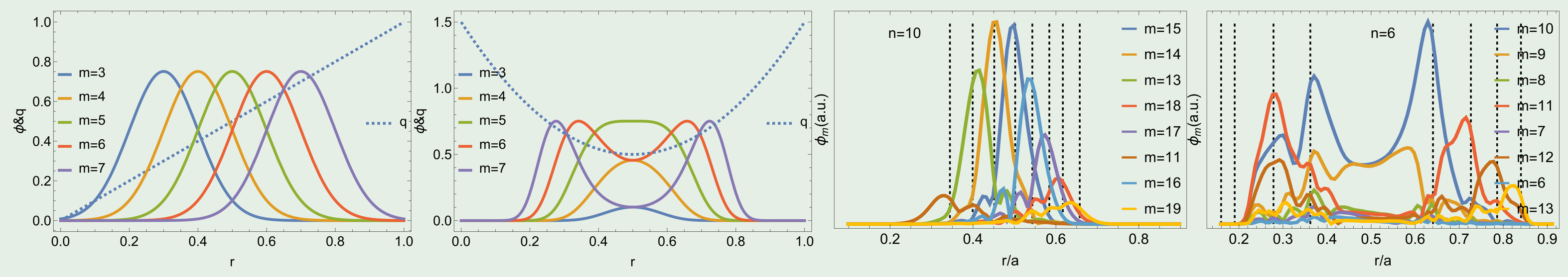

Figure: Demostration of translational invariance in normal and reversed shear cases

- This model is benchmarked with GTC in normal and reversed shear profile in CBC case $\eta_i=3.13$, $\epsilon_n=0.45$, $\tau=1$, $n=10$, $k_\theta\rho_i=0.4$, $\delta_{A,m}=0$, with $\hat{s}=0.78$, $s_2=0$ and $\hat{s}=0$, $s_2=1.78$ for normal and reversed shear, respectively.
- FLR expansion converge and $\omega_r\approx\epsilon\omega_d$ for both reversed and normal shear.

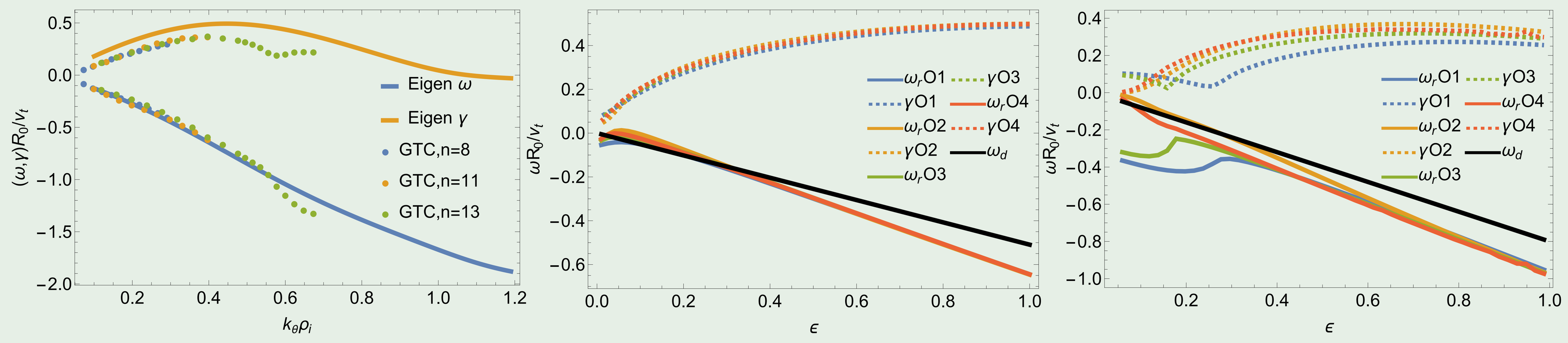

Figure: ω_r and γ of ITG varying $k_\theta\rho_i$ (1st figure) and artificial multiplier ϵ (2nd and 3rd)

- Reversed magnetic shear creates double well potential structure, leading to the degeneracy of the even and odd states when $\delta_{A,m}\ll 1$

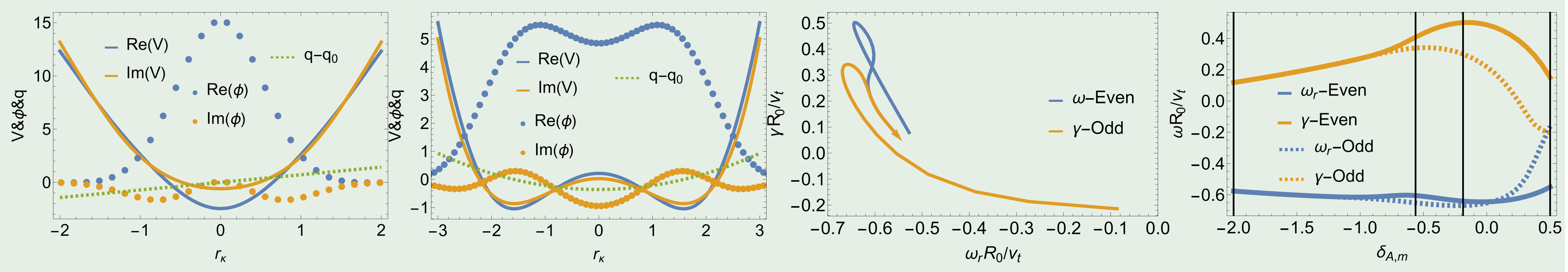

Figure: (a): Potential wells of normal and reversed magnetic shear q profile (1st and 2nd subfigures), Eigenvalue vary with $\delta_{A,m}$ for even and odd modes (3th and 4th subfigures)

- As rational surfaces separate (or equivalently, as $\delta_{A,m}$ decreases), eigenmode peaks separate and even/odd modes progressively degenerate. ($\delta_{A,m}=0.5,-0.1875,-0.5625$ and -2 corresponds to the no rational surface, most unstable, mode structure partially and well separated cases)

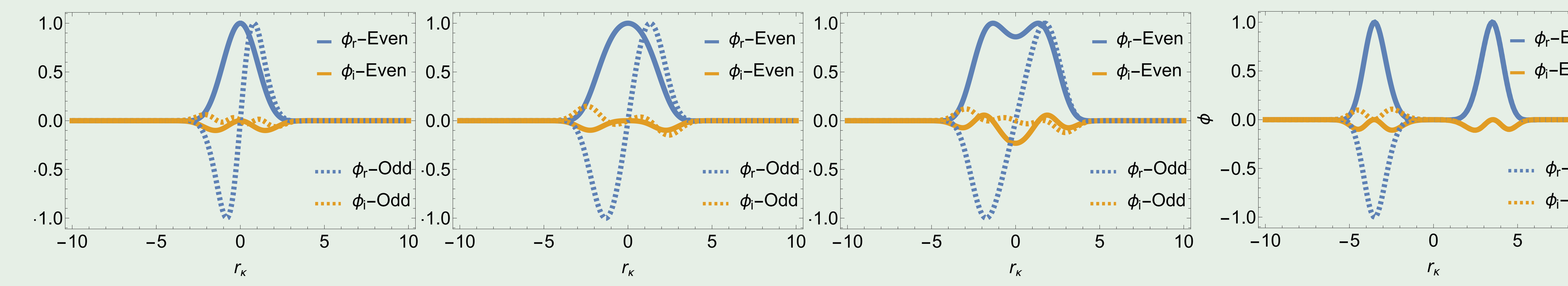

Figure: Eigenfunctions of even (solid lines) and odd (dashed lines) eigenstates.

- Eigenvalue trajectories for $\delta_{A,m}=0.5$, -0.1875, -0.5625, -2, demonstrate similar parameter dependencies throughout the scanning of $\delta_{A,m}$.

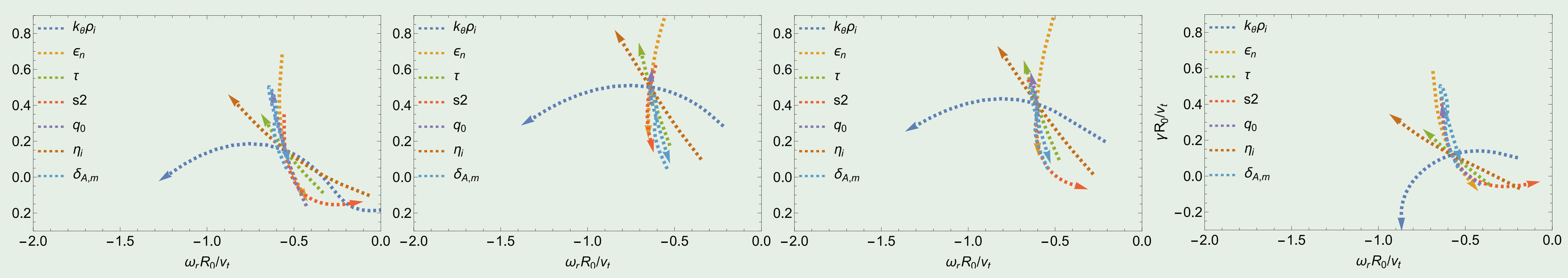

Figure: Eigenvalue trajectories scanning $k_\theta\rho_i$, ϵ_n, τ, s_2, q_0, η_i

Summary

- We presented a reduced kinetic model for ITG in toroidal configurations, which is applicable to both normal and reversed magnetic shear cases.
- Through quantitative comparison with GTC, we demonstrated that the reduced kinetic model is reliable across relevant experimental parameters.
- The analysis highlighted that the potential and mode structures are primarily governed by the safety factor profile.
- Reversed magnetic shear profile introduces double-well potential structure, which induces degeneracy of lowest-order even mode and first-order odd mode when the mode structure peaks are sufficiently separated.
- ITG frequency was found to resonate with the magnetic drift frequency in both normal and reversed shear cases.
- Parameter dependence of the model in different reversed magnetic shear cases are also demonstrated, It's shown that they are qualitatively agree with each other.
- In reversed shear case, ITG is most unstable when two rational surfaces are slightly separated. A quantitative criterion can be established based on the radial mode width and separation between the double rational surfaces.