AVERAGE MAGNETIC DRIFT MODEL FOR ION TEMPERATURE GRADIENT DRIVEN INSTABILITY IN TOKAMAKS

[1]B. Jia, Q. Zhong, Y. Xiao, [2]Y. Li

1. ABSTRACT

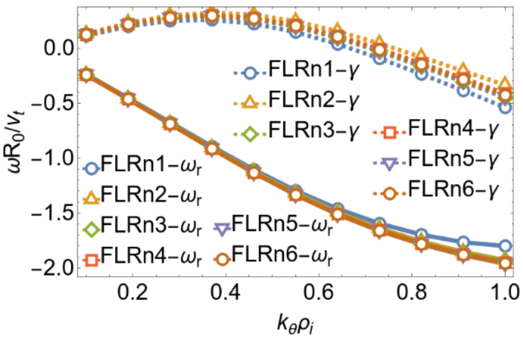
We present an average magnetic drift model for studying ion temperature gradient (ITG) mode in tokamaks. Using partial expansion method and the generalized plasma dispersion function, taking wave-particle resonance into account, we develop a reduced differential model for the ITG mode, demonstrating good consistence with the gyrokinetic particle simulation using Gyrokinetic Toroidal Code (GTC). Through comparison between Gyrokinetic Toroidal Code (GTC) and the average magnetic drift model, it is discovered that the slab branch rather than the toroidal branch dominates the ITG mode in Cyclone Base Case (CBC) parameters. This reduced differential model has been employed to confirm the crucial role of perpendicular magnetic drift resonance in the ITG mode.

2. AVERAGE MAGNETIC DRIFT APPROXIMATION, PARTIAL FLR EXPANSION SCHEME AND DIFFERENTIAL KINETIC ITG MODEL

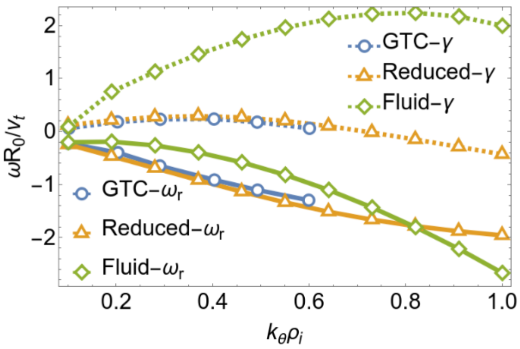
We explore an average magnetic drift model which approximates the magnetic drift frequency by a constant along the ballooning angle, and takes the form of an average over the bad curvature region $\eta \in [-\eta_s, \eta_s]$: $\omega_d \approx \overline{\omega}_d \frac{v_\parallel^2 + v_\perp^2/2}{2v_{tj}^2} \langle \cos(\eta) + \hat{s}\eta\sin(\eta) \rangle_{-\eta_s}^{\eta_s}$. Only with this constant magnetic drift approximation, we can describe kinetic toroidal ITG using a differential equation in radial z (normalized by the rational surface distance) space. To give the differential model, we have to expand the finite Larmor radius effect $J_0^2(k_\perp \rho_i)$ to sixth order at $k_\perp \rho_i = 0$. However, employing the newly developed partial expansion method which expand $J_0^2(k_\perp \rho_i)$ at $k_\perp = k_\theta$: $J_0^2\left(\sqrt{2b}x_\perp\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n J_0^2(\sqrt{2b}x_\perp)}{db^n}|_{b=b_\theta} \left(-b_\theta \widehat{s^2} \frac{\partial^2}{\partial z^2}\right)^n$, we find that the first order expansion is good enough for ITG mode, which is shown in Fig.1(a) by checking the convergence of the expansion order from 1 to 6. The first-order partial expansion enables us to derive a reduced Schrödinger type equation for studying the ITG mode:

$$\left\{\frac{\partial^2}{\partial z^2} + \frac{\overline{\omega}_d f(\hat{s})(1+1/\tau) + \{[\omega - \omega_{*i}(1-3\eta_i/2)]\mathcal{M}_{(1,0)} - \eta_i \omega_{*i}(\mathcal{M}_{(3,0)} + \mathcal{M}_{(1,2)})\}}{\sqrt{2b_\theta \hat{s}^2}\{[\omega - \omega_{*i}(1-3\eta_i/2)]\mathcal{N}_{(2,0)} - \eta_i \omega_{*i}(\mathcal{N}_{(4,0)} + \mathcal{N}_{(2,2)})\}}\right\}\delta\phi(z) = 0,$$

in which the integrals $M_{(n,m)}$ and $N_{(n,m)}$ represent 2d velocity integrations [1]. This differential equation enables straightforward analysis of the ITG eigen system through potential well structures. For a normal shear case, the potential well can be approximated by a harmonic oscillator and the differential model degenerates to a weber equation. However, in a reverse shear case, the potential well has typically double potential well structures and may exhibit very different behaviors such as radial oscillation. We compare the ITG dispersion relation for the CBC case from the new reduced differential kinetic model, the fluid model and the initial value gyrokinetic code GTC in Fig.1(b). We find that the differential model offers a significantly better approximation for moderate and short wavelengths than the fluid model does.



(a)



(b)

Fig. 1: ITG dispersion relation in CBC (cyclone base case) parameters (a) using different expansion orders for $J_0(k_\perp \rho_i)$ at $k_\perp = k_\theta$; (b) using different models including: reduced differential model (orange open triangles), fluid model (green open diamonds) and GTC simulation (blue open circles). Real frequency ω_r and growth rate γ markers are connected by solid and dashed lines, respectively.

3. SLAB-BRANCH ITG, PERPENDICULAR AND PARALLEL RESONANCE

Since ITG toroidal-branch eigenfunctions are induced by the periodic potential well due to ω_d, and are characterized by a fast variation over the connection length scale [2], the toroidal branch is thus suppressed in the average magnetic drift approximation where the periodic ω_d are replaced by a constant. So, the reasonable accuracy of the differential model (shown in Fig.1(b)) which utilize the average magnetic drift approximation, suggests that the slab branch is likely to play a dominant role in the CBC parameters.

We find that perpendicular magnetic drift resonance plays a crucial role in the toroidal ITG mode. We plot ω_r and $\overline{\omega}_d$ as a function of $k_\theta\rho_i$ in Fig.2(a). It's shown that $\omega_r \approx \overline{\omega}_d$ across the entire range of $k_\theta\rho_i$, which shows the importance of perpendicular magnetic drift resonance in these cases [3]. Parallel resonance primarily contributes a stabilizing effect. We compare the kinetic integral model with the local long wavelength model which ignores the parallel resonance in Fig.2(b). Since $k_\parallel \sim \partial_\eta / qR$, it is found that the eigenvalue trajectory of the kinetic integral model enlarging the safety factor q converges to the local long wavelength model. Out of the limit $q \gg 5$, the full kinetic integral model is always more stable than long wavelength model.

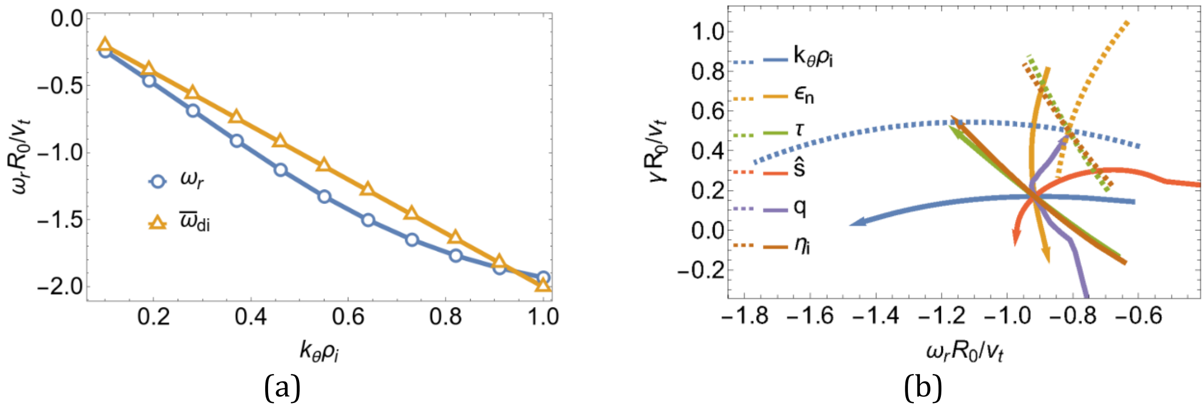


Fig. 2: (a) Real frequency ω_r (blue open circles) and ion magnetic drift frequency $\overline{\omega}_d$ (orange open triangles) vary with $k_\theta\rho_i$. (b) Eigenvalue (Real frequency ω_r and growth rate γ) trajectories in the complex plane with respect to different parameter scans. Solid lines represent the kinetic integral model. Dashed lines represent local long wavelength approximation which ignores parallel resonance. The scanning directions are explicitly indicated by arrows.

REFERENCES

[1] Ö. D. Gürcan, “Numerical computation of the modified plasma dispersion function with curvature,” J Comput Phys, vol. 269, pp. 156–167, 2014, doi: 10.1016/j.jcp.2014.03.017.

[2] L. Chen, S. Briguglio, and F. Romanelli, “The long-wavelength limit of the ion-temperature gradient mode in tokamak plasmas,” Physics of Fluids B: Plasma Physics, vol. 3, no. 3, pp. 611–614, Mar. 1991, doi: 10.1063/1.859859.

[3] P. Terry, W. Anderson, and W. Horton, “Kinetic effects on the toroidal ion pressure gradient drift mode,” Nuclear Fusion, vol. 22, no. 4, p. 487, Apr. 1982, doi: 10.1088/0029-5515/22/4/004.