Global linear drift-wave eigenmode structures on flux surfaces in stellarators: ion temperature gradient mode

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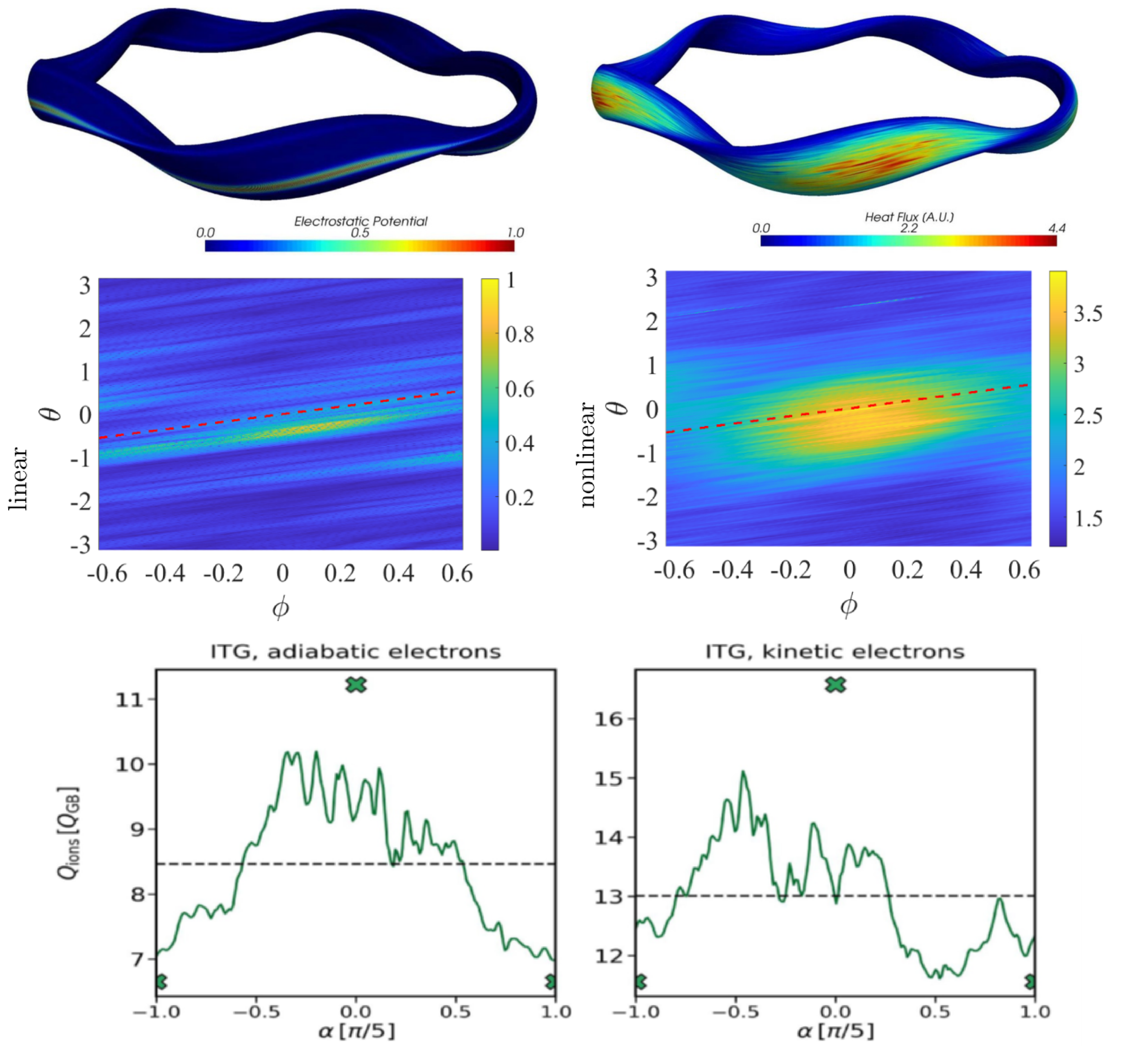
Motivation: are local simulations accurate enough for predicting turbulent transport in stellarators?

Turbulence in stellarators are non-uniform in the toroidal direction and do not obey stellarator symmetry (local and global GENE comparisons [1, 2])

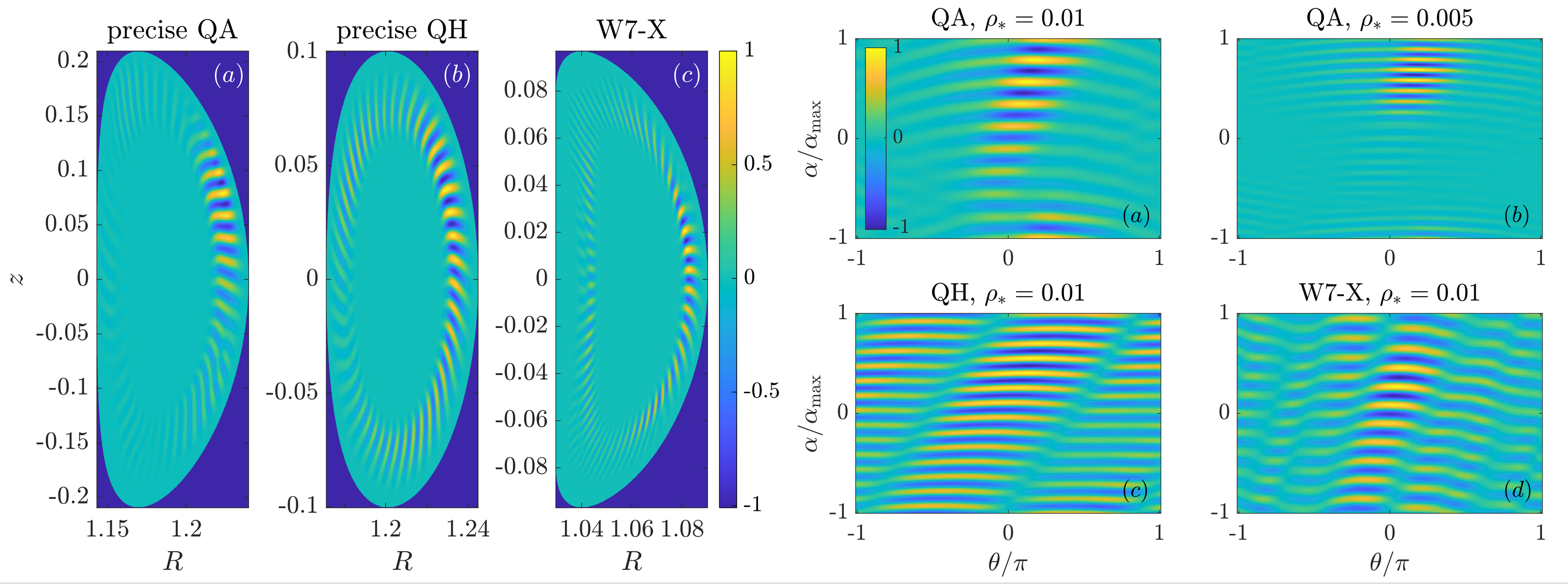
- Linear ITG eigenmode appears to be localized on a single field line.
- Nonlinear fluctuations are also localized and violates stellarator symmetry.
- Local (flux-tube) simulations could qualitatively reproduce the nonlinear fluctuation level at each field line, but quantitatively might differ.



Linear ITG eigenmodes are localized on flux surfaces in non-axisymmetric stellarator geometries

We simulate linear ITG eigenmodes using global gyrokinetic code GTC [3] for several stellarator geometries [4, 5] with $a/L_n = 0$ and $a/L_T = 2$.

- In field-line following coordinates $(\alpha = \theta - \iota\varphi, l = \theta)$, ITG eigenmodes are localized at $\alpha > 0$. The localization is more pronounces at smaller $\rho_* = \rho_i/a$.
- The cross-field-line structure in α cannot be obtained from local simulations.



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Potential topics including (but not limited to):

- Theoretical/computational study of stellerator turbulence and zonal flows.
- Developing tokamak/stellarator global edge gyrokinetic codes.
- Machine learning and local gyrokinetic simulations for transport prediction.

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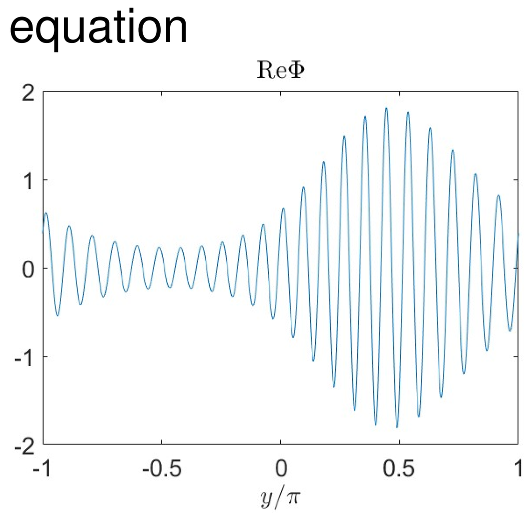
Describe the ITG localization from a simple 1D model

Zocco's model [6]: for a global eigenmode $\Phi(\alpha, l, \psi)e^{-i\omega t}$, describe the cross-field-line structure Φ from a 1D differential equation

$$\left[1 - \frac{g(y)}{4\omega}(-i\rho_*\partial_y)^3 + \frac{f(y)}{\omega^2}(-i\rho_*\partial_y)^2\right]\Phi(y) = 0.$$

- $y = \alpha/\iota$: field-line label stretched to $[-\pi, \pi]$.
- $f(y)$ and $g(y)$ depends on y, $\rho_* = \rho_i/a \ll 1$.
- Example: $\rho_* = 0.1,\ f = 1,\ g = 1 - 0.2\cos y$. The eigenmode is localized at $y > 0$.

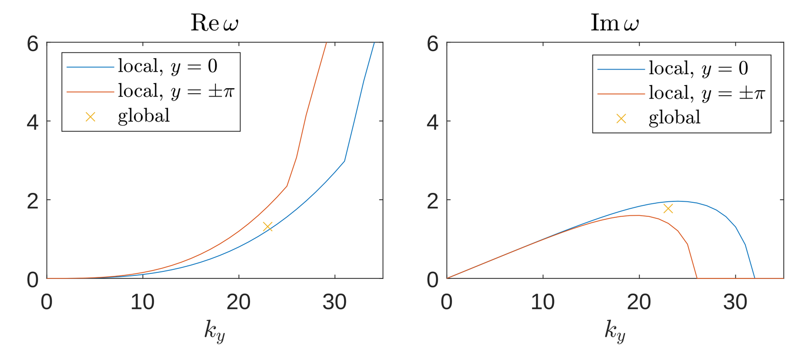


WKB analysis from [7]: let $\Phi(y) = \hat{\Phi}(y)e^{iS(y)/\rho_*}$, expand in ρ_*

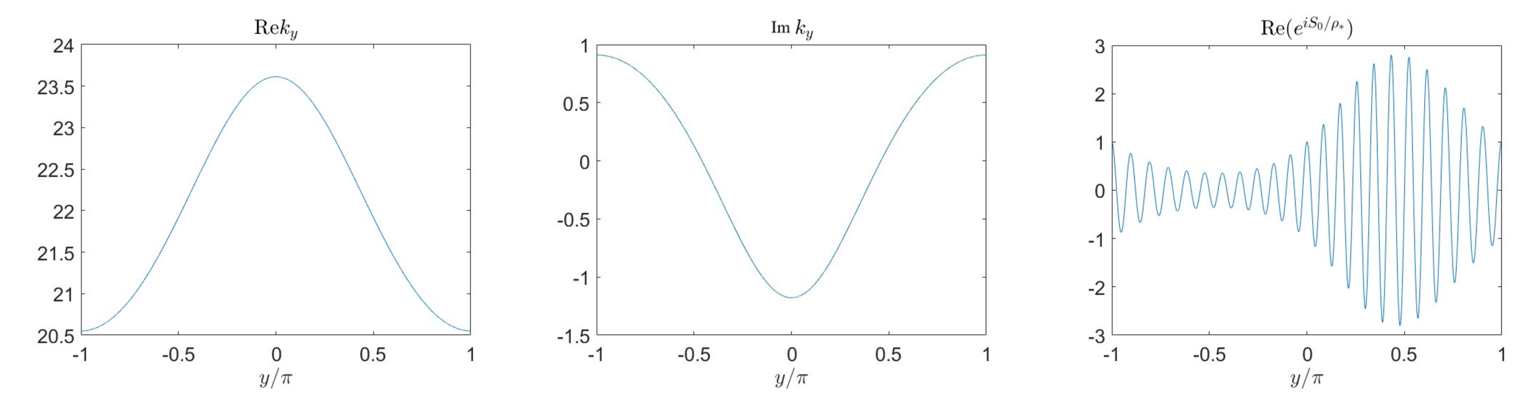
- $\mathcal{O}(\rho_*^0)$ local dispersion relation:

$$\omega^2 - \frac{1}{4}\omega g(y)k_y^3 + f(y)k_y^2 = 0, \quad k_y(y) = S'/\rho_*.$$

- $y = 0$: most unstable flux tube;
- $y = \pm\pi$: least unstable flux tubes.
- Global eigenmode: constant ω between $y = 0$ and $y = \pm\pi$.



- Since ω is constant, must invert the dispersion relation to solve for $k_y(\omega, y)$.
- We found $\mathrm{Im}k_y \propto -\cos y$, resulting in $|e^{iS/\rho_*}| \sim e^{\rho_*^{-1}\sin y}$.



Key features of ITG localization from the simple model

- $\gamma_{\rm min} < \gamma < \gamma_{\rm max}$, mode localization at $y > 0$ (already found by Zocco *et al.*).
- Eigenmode localization due to the complex poloidal wavenumber.
- Localization at the downstream direction of the ion diamagnetic drift:

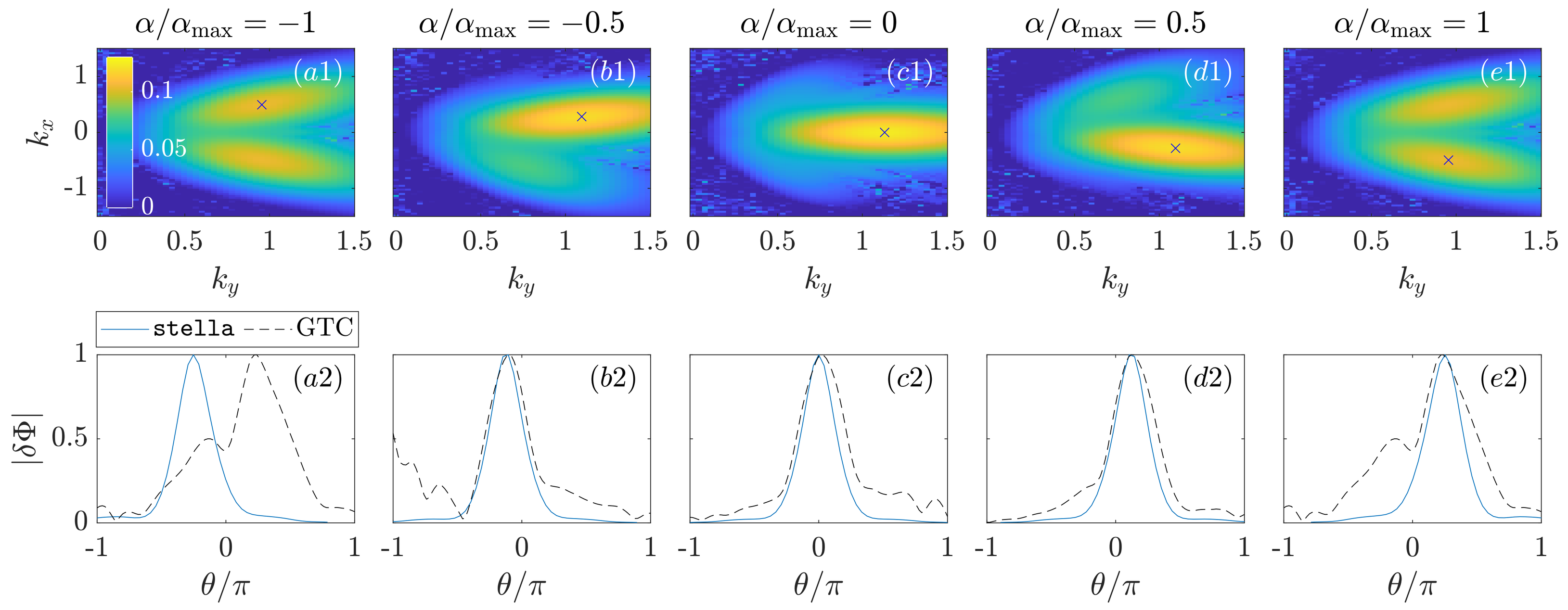
$$\Delta k_y \approx (\partial\omega/\partial k_y)^{-1}\Delta\omega \quad \Rightarrow \quad \mathrm{Im}(\Delta k_y) \approx (\partial\omega_r/\partial k_y)^{-1}\Delta\gamma < 0.$$

- Localization increases exponentially with ρ_*^{-1}: $|e^{iS/\rho_*}| \sim e^{\rho_*^{-1}\sin y}$.

Compare local and global gyrokinetic simulation results

Solve the local dispersion relation $\omega = \omega_l(\boldsymbol{k}^{\rm real}, \alpha)$

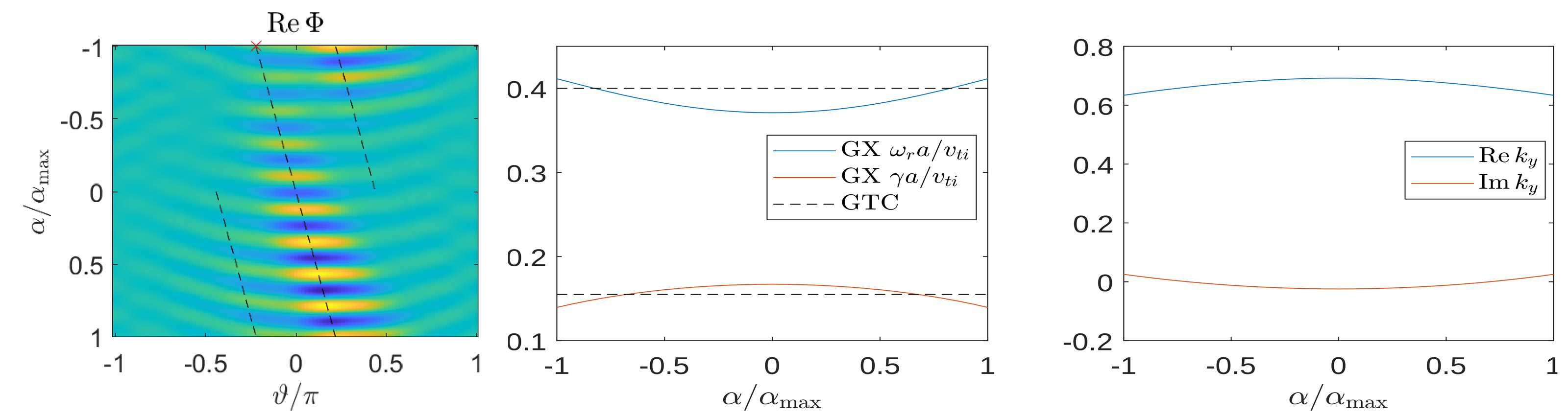
- Use GX [8] and stella [9] to simulate the local eigenmode for each field line α independently, each field line spans from $\theta = -\pi$ to $\theta = \pi$.
- The global solution roughly follows the local solution, and is periodic in α.



Invert the local dispersion relation to solve for complex $\boldsymbol{k} = \boldsymbol{k}(\omega, \alpha)$

- Approximately obtain the local complex wavenumber from Taylor expansion

$$\boldsymbol{k}^{\rm complex} \approx \boldsymbol{k}^{\rm real} + (\partial\omega/\partial\boldsymbol{k})^{-1}\Delta\omega, \quad \Delta\omega = \omega_{\rm global} - \omega_{\rm local}.$$

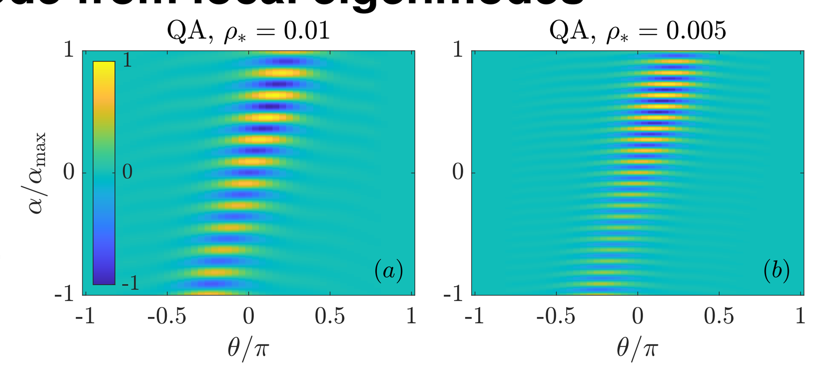


Construct a surface-global eigenmode from local eigenmodes

$$\Phi^{\rm global} = \Phi^{\rm local}(\vartheta, k_x^{\rm local}, k_y^{\rm local}, \alpha)e^{i\int k_\alpha d\alpha},$$

$$k_\alpha = k_y^{\rm complex} r/\rho_i.$$

- Only considered one period in α: periodicity not enforced.



Conclusions (details in arXiv:2506.12948)

- 3D stellarator geometry allows for complex wavenumbers for the ITG mode.
- In contrast, existing flux-tube simulations only allow for real wavenumbers.
- Linear global mode can be inferred from local results (analytic continuation).
- Can we measure the complex-wavenumber spectrum in global simulations?

References

[1] Bañòn Navarro *et al.*Plasma Phys. Control. Fusion **62**, 105005 (2020).
[2] Wilms *et al.*Nucl. Fusion **63**, 086004 (2023).
[3] https://sun.ps.uci.edu/gtc/
[4] M. Landreman and E. Paul, Phys. Rev. Lett. **128**, 035001 (2022).
[5] A. González-Jerez *et al.*J. Plasma Phys. **88**, 905880310 (2022).
[6] Zocco *et al.*, Phys. Plasmas **23**, 082516 (2016); Phys. Plasmas **27**, 022507 (2020).
[7] R. Dewar and A. Glasser, The Physics of Fluids **26**, 3038 (1983).
[8] Mandell *et al.*, J. Plasma Phys. **90**, 905900402 (2024).
[9] Barnes *et al.*J. Comp. Phys. **391**, 365 (2019).