RECONSTRUCTION OF QUASI-SYMMETRIC STELLARATOR GEOMETRY FROM A LOW-DIMENSIONAL PARAMETER SPACE

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In the axisymmetric tokamak geometry, the single particle motion is perfectly confined due to the conservation of toroidal angular momentum. While this is not generally true for a stellarator, it is proved that the stellarators with 'quasi-symmetric' (QS) geometries can be constructed in which the toroidal angular momentum is approximately conserved, and the single particle orbit loss is significantly reduced [1]. The specific geometry can be found by optimization algorithm with certain constraints such as major radius, aspect ratio, pressure profile, ι profile (or q profile).

For stellarator to be an attractive reactor concept, more physics optimization beyond the single particle orbit loss should be considered, such as the MHD instability and turbulent transport. Previous works have used heuristic criterion or local models to evaluate these more complicated collective modes. However, these local approximate estimations may not be reliable due to the intrinsically 3D geometries of the stellarators [2]. On the other hand, the first-principles global simulations are computationally expensive and difficult to integrate in the optimization process. A desirable alternative method is to use the fast machine learning based surrogate model [3] to replace the first-principles codes. However, the training of the surrogate model relies on the first-principles global simulation data covering the parameter space. Due to the high degree-of-freedom of the stellarator design, the parameter space can have a dimension of several hundreds. Here, we demonstrate that the QS geometries are distributed in a low-dimensional subspace. Thus, the amount of global simulation data required to train a surrogate model can be greatly reduced.

First, a variety of QS geometries are generated by the open-source optimization framework DESC [4]. In DESC, the magnetic flux surface shapes are described by the coefficients of Fourier-Zernike basis functions, $R(\psi,\vartheta,\phi)=\sum_{lmn} R_{lmn}\,\xi_{lmn}(\psi,\vartheta,\phi)$, $Z(\psi,\vartheta,\phi)=\sum_{lmn} Z_{lmn}\xi_{lmn}(\psi,\vartheta,\phi)$, where the magnetic field line is straight in (ψ,ϑ,ϕ) coordinate system, (l,m,n) stand for the orders of basis functions in radial, poloidal and toroidal direction. A variety of starting point for geometry optimization is selected in order to explore the valid quasi-symmetric design space. During the optimization, the constraints are applied such as toroidal period NFP=4, aspect ratio ε between 3 and 10, elongation κ between 1 and 3, total toroidal magnetic flux $\psi_w=0.04$ Wb, with a maximum mode number 8 in radial, poloidal and toroidal direction.

We use the auto-encoder neural network to find the low-dimension parameter space (the latent space) where the QS geometries are distributed in. The input is the stacked 1-dimensional array of R and Z coefficients, with a length of 765. The output is also an array with a length of 765. In the middle of the network a layer of size s is used, and $s \ll 765$, as shown in Fig. 1. If the output matches the input perfectly, we can conclude this data set can be described by s parameters.

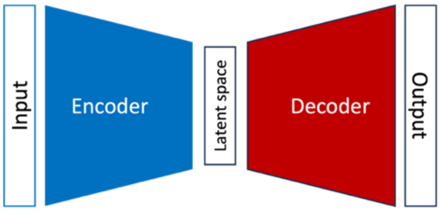

Figure 1 Auto-encoder architecture.

To improve the reconstruction accuracy, we add the distance of collocation points in the loss function. The ground truth of collocation points positions is the R, Z coordinates with different (ψ,ϑ,ϕ) using the R, Z coefficients from DESC data. While the predicted collocation points positions are calculated from the output R, Z coefficients from the auto-encoder. The final loss function we use to train the auto-encoder network is $L=MSE(output,input)+L_{points}$.

The relative error of outputs on the test dataset dependence on the latent space dimension s is shown in Fig. 2. It is shown that the reconstruction error is smaller than 1% when $s \geq 3$. In Fig. 3, we show the comparison of DESC optimized geometry and reconstructed geometry using $s=3$. The reconstructed R and Z coefficients agrees well with the input DESC result. The boundary of reconstructed geometry has only slight difference with the DESC result, and the difference becomes smaller when approaching to the magnetic axis. An analysis shows that the reconstruction becomes better for geometries with smaller QS error, which means the low dimensional parameter space can exclude the geometries with bad symmetry.

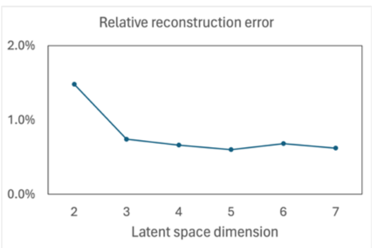

Figure 2 Reconstruction error for different latent space dimension.

To demonstrate the capability of the low dimensional parameter space beyond reconstruction, we use the 3-d space found by auto-encoder to predict the Rosenbluth-Hinton (RH) residual level of zonal flows in QH stellarators. Because of the strong suppression of turbulent transport by zonal flows, the RH level can act as an effective indicator of transport level. A new neural network is designed to predict the analytical RH level[5] in the QH geometry from the corresponding 3 coordinates of the 3-d latent space. The prediction is shown in Fig. 4. The agreement between prediction and theoretical values demonstrate that the RH level can be described simply by the 3 parameters.

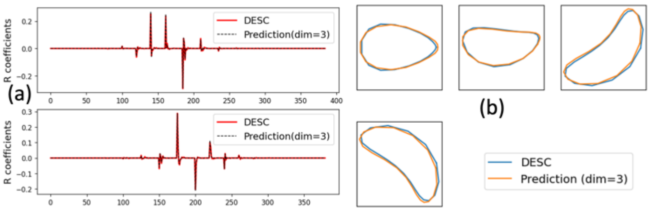

Figure 3 Comparison between DESC geometry and reconstructed geometry using the auto-encoder.

It becomes straightforward to find a QH geometry with low turbulent transport – we can just select the points in Fig. 4 with high RH levels, or select a point near the high RH level region in the 3-d space and reconstruct the geometry. We have selected two points in Fig. 4 to show the impact on transport from the RH level. Two equilibria with RH level 0.21(Case584) and RH level 0.40 (Case364) are selected. The global turbulence simulations in the two geometries are carried out using gyrokinetic code GTC, and the heat transport and zonal flow shearing rate evolution are shown in Fig. 5. Indeed, the geometry with higher RH level (Case364) has higher zonal flow shearing rate and lower transport level.

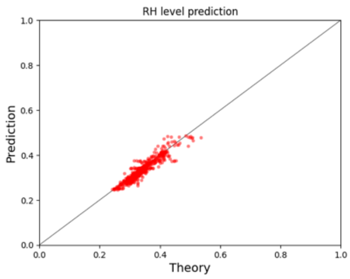

Figure 4 Comparison of theoretical and predicted RH levels.

Our results demonstrate that the QS geometry has a low dimensional nature, and the low dimension space can be found through machine learning. The low dimensional space is useful to analyze the zonal flow residual level and find the geometry with low transport level. This result enables the generation of global gyrokinetic simulation data for training surrogate models to optimize the stellarator geometry with more complex physics, like global MHD instability and turbulent transport.

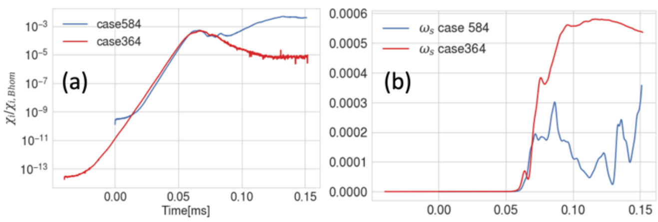

Figure 5 The evolution of heat transport coefficient (panel a) and zonal flow shearing rate (panel b) for geometries with different RH level.

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