## Extrapolative Predictability of Plasma Turbulent Transport via a Multi-Fidelity Data Fusion Approach

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Building predictive models for turbulent transport is a cornerstone of performance forecasting in fusion devices. This study introduces a multi-fidelity data fusion approach to turbulent transport modeling, effectively integrating low-fidelity simulation data with high-fidelity experimental data. By leveraging the correlations between these datasets, the method achieved substantial improvements from conventional regression in extrapolative predictability for plasma turbulent transport. These findings highlight the potential of multi-fidelity modeling in advancing transport predictions crucial for the design of next-generation fusion devices.

Understanding and predicting turbulent transport is a critical challenge in magnetic fusion research. To date, surrogate models based on gyrokinetic simulation databases or experimental data have been discussed extensively. We propose applying a multi-fidelity data fusion algorithm to turbulent transport modeling by treating theoretical, simulation, and experimental data with different accuracies, sample sizes, and parameter ranges. Multi-fidelity modeling is a methodology that combines less accurate but numerous low-fidelity data with highly accurate but sparse high-fidelity data to improve the overall predictive performance. The applicability of this approach was demonstrated in several problems related to plasma turbulent transport modeling [1]. This study aims to advance this approach further for extrapolative problems.

We employ a multi-fidelity algorithm called nonlinear auto-regressive Gaussian process regression (NARGP) [2]. It is known that the kernel function design determines the model's expressive ability in Gaussian process (GP) regression. For applications to extrapolative problems, it is essential to use a complex kernel that can capture both short-term fluctuations and long-term trends. This study adopts neural kernel networks [3], allowing data-driven kernel generation and great flexibility. However, just improving the kernel design is not sufficient. We further discuss how integrating low-fidelity data can enhance extrapolative performance.

First, a two-dimensional test function was analyzed. A Bessel function was used as the high-fidelity data  $y_h = f_h(x_0, x_1)$  [Figure 1(a)], and their approximated envelope function was used as the low-fidelity data. Specifically, only high-fidelity data for  $x_0 < 0$  were used for training data. Figure 1(b) shows that conventional single GP regression fails to predict the value at the extrapolation region  $x_0 > 0$ , whereas NARGP in Figure 1(c) gives a better extrapolative prediction. NARGP expresses the high-fidelity data not as a direct function of the input space,  $y_h = f_h(x_0, x_1)$ , but as a function of an extended space involving both input ( $x_0, x_1$ ) and the low-fidelity function  $f_i(x_0, x_1)$ , namely,  $y_h = f_h(x_0, x_1) = g_h(x_0, x_1, f_i)$ . For  $x_0 > 0$ , where no high-fidelity data were available, NARGP leveraged correlations with the low-fidelity data, converting the extrapolative problem into an interpolative setting, thus significantly improving extrapolative performance. This result demonstrates the ability of NARGP to effectively utilize correlations with low-fidelity data for extrapolation problems.



Figure 1. Extrapolative prediction for a test function. (a) The two-dimensional test function  $f_h(x_0, x_1)$ . (b) Single GP prediction using only the test function data at the area surrounded by a red dashed square. (c) NARGP prediction using the high-fidelity and low-fidelity test function data.

Next, the method was applied to a plasma turbulence transport dataset [1], where multiple local plasma parameters were used as input variables to predict experimentally observed turbulent diffusion coefficients. For multi-fidelity linear growth regression, rates and wavenumbers of micro-instabilities are incorporated as low-fidelity data. The extrapolative problem was set up by training the model using data with large diffusion coefficients and predicting those with smaller coefficients. Figure 2(a) shows results from conventional that GP regression indicate good agreement within the training data range but show poor



Figure 2. Extrapolative prediction for turbulent diffusion coefficient from JET experimental dataset. Comparison of predicted and actual values for (a) single GP and (b) NARGP prediction. The blue daggers denote the training data, and the orange circles denote the test data used for validation. The red cross dots in (b) are the test data far from the training data in parameter space, measured by using Mahalanobis distance.

predictive performance in the extrapolation region. Conversely, Figure 2(b) demonstrates that NARGP improves predictive performance even in the extrapolation region. Analysis of data points with large prediction errors using the Mahalanobis distance [4] revealed that these points lie significantly outside the training data distribution. This evaluation clarified that such out-of-distribution points are inherently difficult to predict, providing valuable insights into the limitations of extrapolative predictability.

This study demonstrates that the multi-fidelity data fusion approach improves the extrapolative predictability of plasma turbulent transport using a magnetic fusion experimental dataset. By integrating theoretical, simulation, and experimental data, this approach provides a foundation for enhancing the accuracy of transport models essential for the design of next-generation fusion devices.

- [1] MAEYAMA, S., et al., Sci. Rep. 14 (2024) 28242.
- [2] PERDIKARIS, P., et al., Proc. R. Soc. A 473 (2017) 20160751.
- [3] SUN, S., et al., Proc. 35th Int. Conf. on Mach. Learn. 80 (2018) 4828.
- [4] MAHALANOBIS, P.C., Proc. Nat. Inst. Sci. India 2 (1936) 49.