Neural Network Reduced Models for Plasma Turbulence

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Background and Motivation:

- Plasma turbulence plays a central role in transport.
- Direct numerical simulations remain expensive.
- Closure models/reduced models needed.
- Machine learning, particularly physics-informed neural networks (PINNs) provides promising tools for this task

Physical model: 2D Hasegawa-Wakatani (HW)[1]

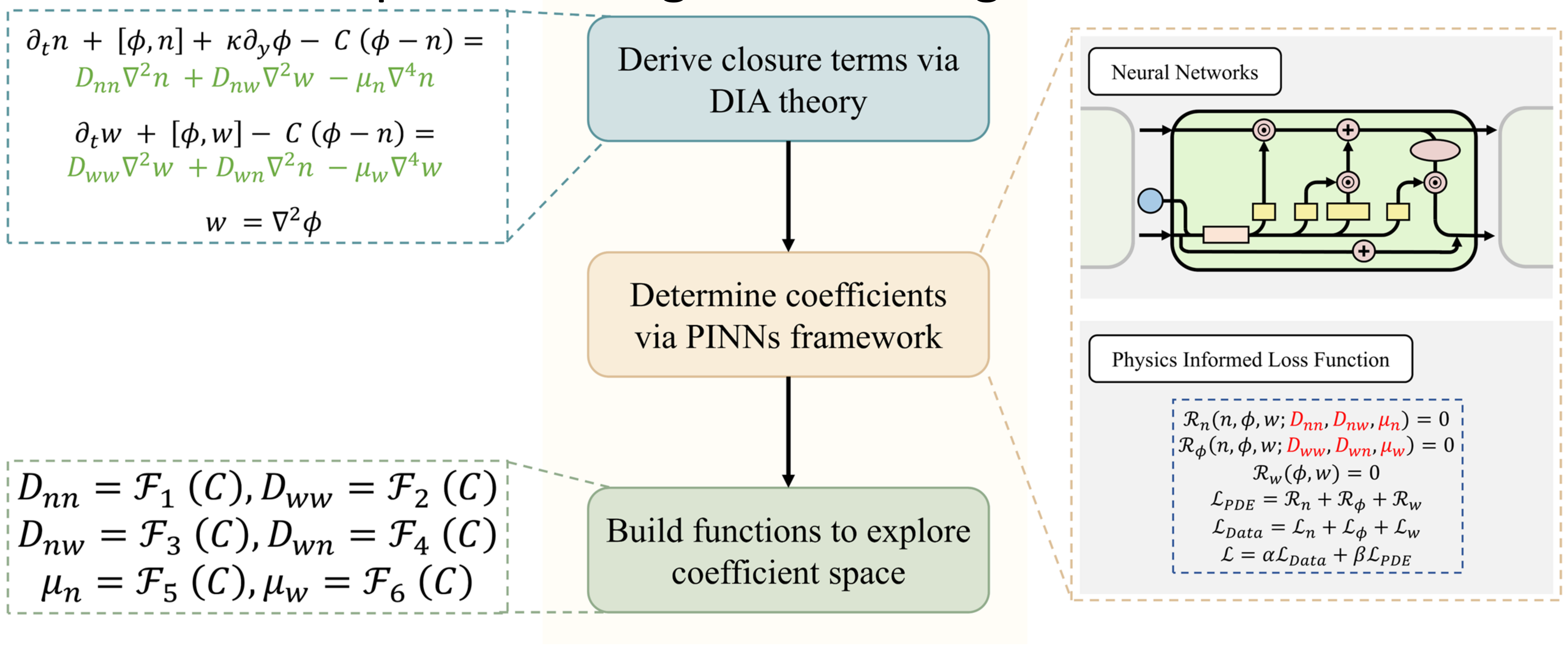
$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} = \alpha(\phi - n) - D\nabla^4 \omega - \nu\langle\omega\rangle_y,$$
$$\frac{\partial n}{\partial t} + \{\phi, n\} = \alpha(\phi - n) - \kappa\frac{\partial \phi}{\partial y} - D\nabla^4 n,$$

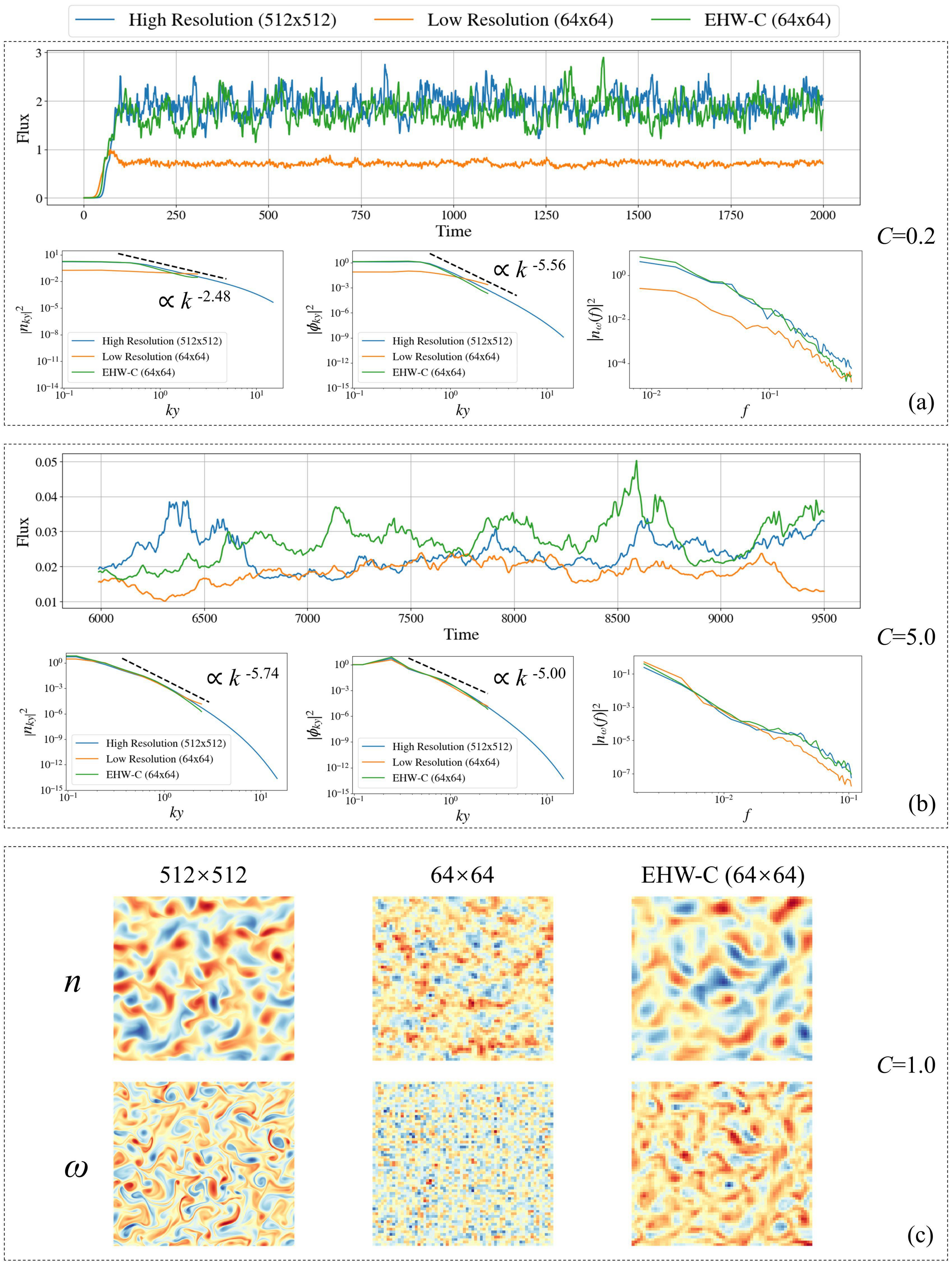
Radial:x, Poloidal: y, Density: n, Potential: ϕ, Vorticity $\omega = \nabla^2\phi$, Poisson bracket: {}, Adiabatic parameter (conductivity): α, (hyper)viscosity: D, Zonal flow damping: ν, Equilibrium density gradient length κ

Simulation with the TOKAM2D[2] code (GPU-enabled)

With closure terms, 64*64 simulation is as accurate as 512*512, saving >10x resources[3]

- Candidate LES (large-eddy simulation) terms proposed by DIA[4] (direct interaction approximation).
- Coefficients identified by ConvLSTM[5]+PINN.
- Flux and spectrum as good as fine-grid simulations.





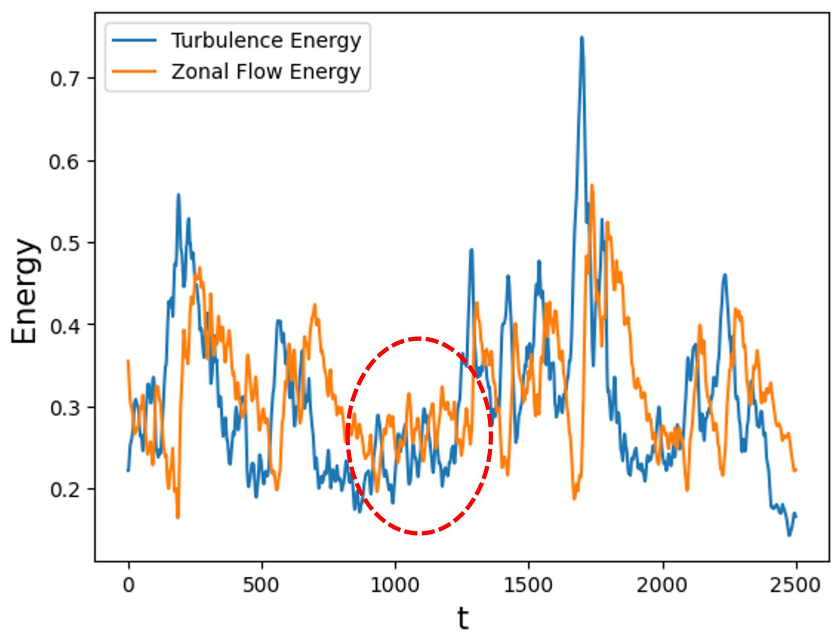
First direct identification of the governing equation of turbulence-zonal flow interaction from data[6]

- Modified HW, $(\phi - n) \rightarrow (\tilde{\phi} - \tilde{n})$, stronger zonal flow

$$\tilde{\phi} = \phi - \langle\phi\rangle_y, \quad \tilde{n} = n - \langle n\rangle_y.$$

- **Can one get from simulation data $\frac{dE}{dt} = ?, \frac{dU}{dt} = ?$**

Turbulence energy E, zonal flow energy U



YES, but only with the stochasticity of turbulence.

Normal predator-prey[7]

$$\frac{dE}{dt} = \alpha E - \beta EU$$
$$\frac{dU}{dt} = -\gamma U + \sigma EU$$

✗ Dynamics too simple
✗ No randomness
✗ Directly use e.g. SINDY gives unmeaningful results.

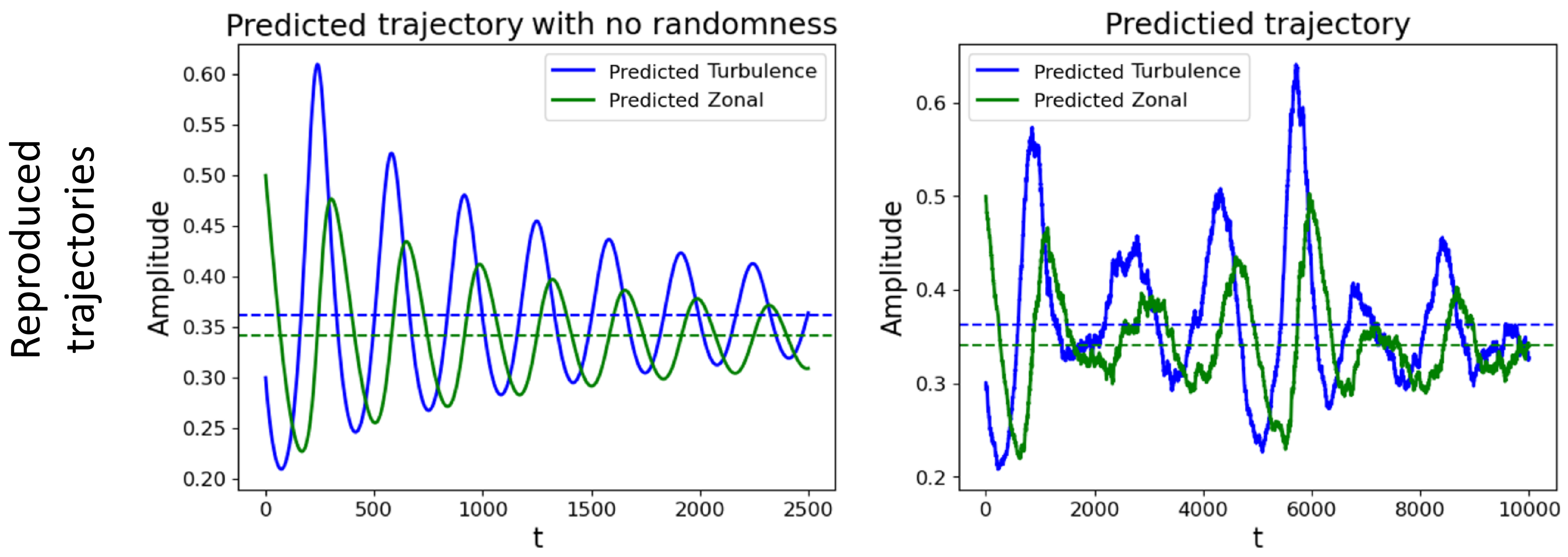
Stochastic predator-prey with NN[8]

$$dE = g_{11}(E,U)dt + g_{21}(E,U)dw$$
$$dU = g_{12}(E,U)dt + g_{22}(E,U)dw$$

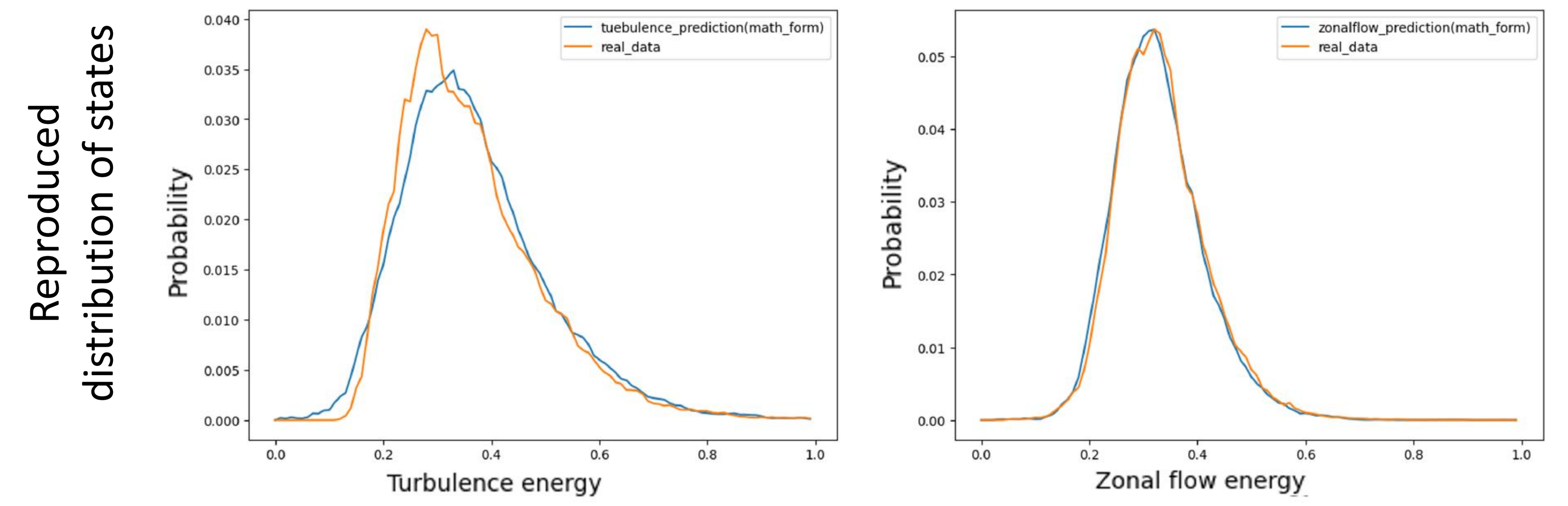
NN functions: $g_{11}, g_{12}, g_{21}, g_{22}$
Brownian motions: dw

✓ More general dynamics
✓ With randomness of turbulence
✓ Good statistical match with data, can reproduce similar curves

Reproduced trajectories



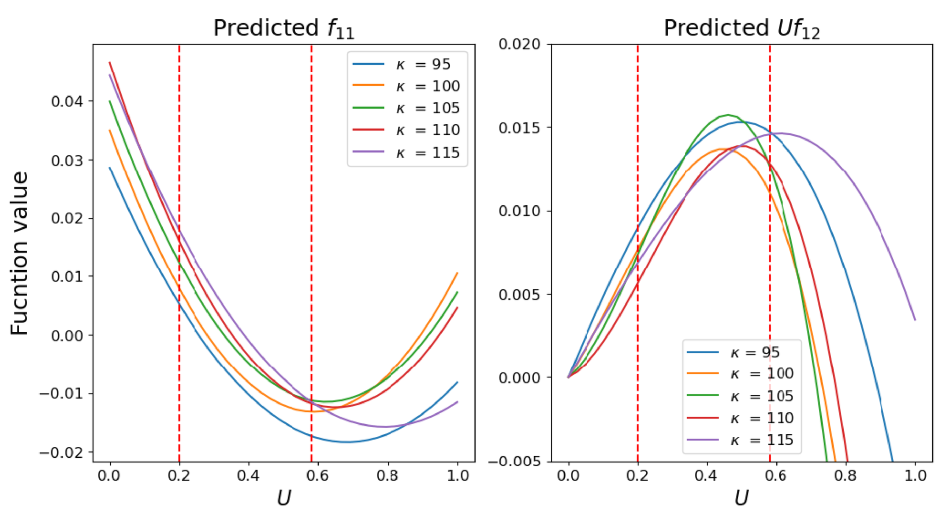
Reproduced distribution of states



Best result with assumptions:

$$g_{11}(E, U, \theta_1) = Ef_{11}(U, \theta_1) + AE^2,$$
$$g_{12}(E, U, \theta_2) = -BU + EUf_{12}(U, \theta_2),$$

Recovers: 1. Turbulence suppression due to zonal flow
2. Effect reduces at high flow



Conclusion: Two neural network–based reduced models were introduced for plasma turbulence, addressing sub-grid closure and turbulence–zonal flow dynamics. **Highlight the potential of machine learning in constructing reduced turbulence models.**

Future work: extend these methods to more complicated fluid or gyrokinetic systems.

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References

[1] A. Hasegawa, M. Wakatani, Physical Review Letters 50, 682 (1983).
[2] P. Ghendrih, et al., Journal of Physics: Conference Series 1125, 012011 (2018).
[3] K. Li, et al., arxiv:2509.11576 (2025).
[4] R. Kraichnan, Journal of Fluid Mechanics 5, 497 (1959).
[5] X. Shi, et al., Advances in neural information processing systems 28 (2015).
[6] J. C. Huang, et al., arxiv:2508.10408 (2025).
[7] P. H. Diamond, S.-I. Itoh, K. Itoh, T. S. Hahm, Plasma Physics and Controlled Fusion 47, R35 (2005).
[8] E.-j. Kim, R. Hollerbach, Phys. Rev. Res. 2, 023077 (2020).