FDTD SIMULATION OF THE PROPAGATION CHARACTERISTICS OF MILLIMETER-WAVE VORTEX IN MAGNETIZED PLASMA

C. WANG[1], H. NAKAMURA[1] , H. KAWAGUCHI[2] and S.KUBO[3]

[1] National Institute for Fusion Science, [2] Muroran Institute of Technology, [3]Chubu University

Email: naka-lab@nifs.ac.jp

ABSTRACT

This study investigates the propagation characteristics of millimeter-wave vortex fields in the magnetized plasma using 3D Finite-Difference Time-Domain (FDTD) method. The findings demonstrate that the hybrid mode of millimeter-wave vortex can propagate in high-density plasma, where normal plane wave cannot propagate due to plasma cut-off condition [1-2]. Additionally, it was also found that the propagation power in the magnetized plasma is highly dependent on the topological charge *l* [3]. To further explore the practical implications, the paper also analyzes Laguerre-Gaussian (LG) beams incident from free space to magnetized plasma and then investigate the dependence of topological charge on penetrated power without the waveguide boundaries. These findings provide a high potential for functional plasma heating.

BACKGROUND

Wave mode in magnetized plasma

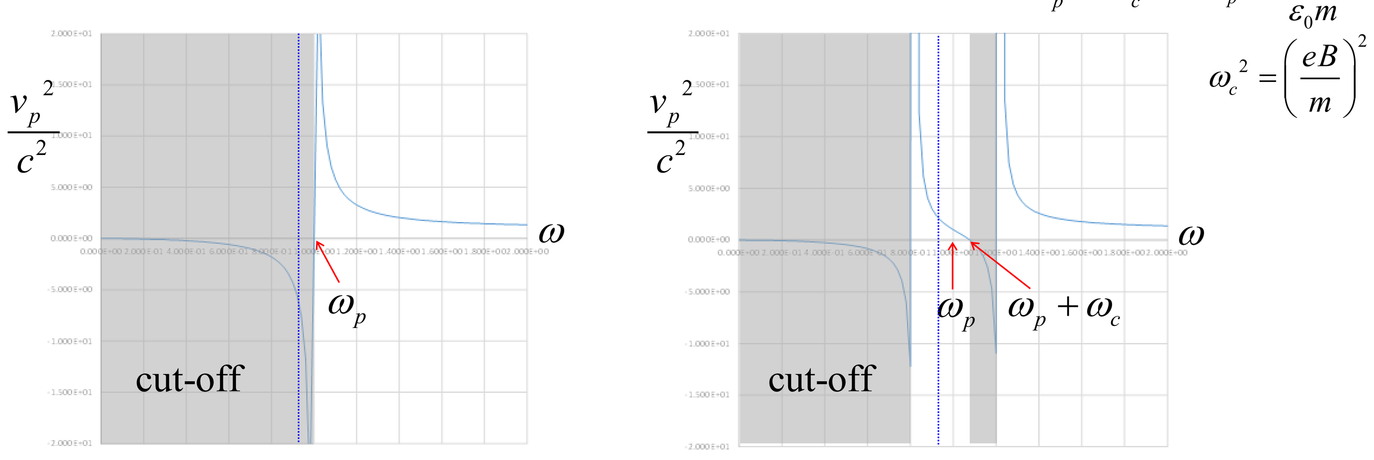
O - mode E⊥k B⊥k B∥E

$$\frac{v_p^2}{c^2}=\frac{\omega^2}{\omega^2-\omega_p^2}$$

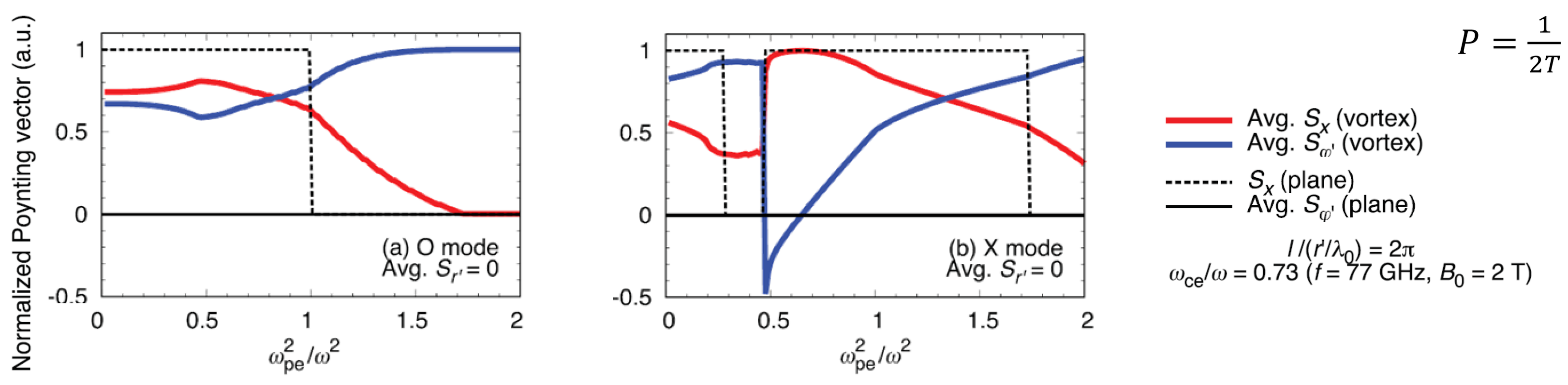
X - mode E⊥k B⊥k B⊥E

$$\frac{v_p^2}{c^2}=\frac{\omega^2}{\omega^2-\omega_p^2\dfrac{\omega^2-\omega_p^2}{\omega^2-\omega_p^2-\omega_c^2}}$$

$$\omega_h^2=\omega_p^2+\omega_c^2,\quad \omega_p^2=\frac{e^2n_e}{\varepsilon_0 m},\quad \omega_c^2=\left(\frac{eB}{m}\right)^2$$



Vortex propagation in cut-off plasma [1]



METHODOLOGY

Propagation mode of millimeter-wave vortex

***HE_{nm}* mode in corrugated waveguide**

$$E_x=-iE_{mn}\frac{\beta}{k_c}J_{m-1}(k_cr)\times e^{i(\omega t-\beta z-(m-1)\theta)}$$

$$E_y=-iE_{mn}\frac{\beta}{k_c}J_{m-1}(k_cr)\times e^{i(\omega t-\beta z-(m-1)\theta-\frac{\pi}{2})}$$

$$E_z=E_{mn}J_m(k_cr)\times e^{i(\omega t-\beta z-m\theta)}$$

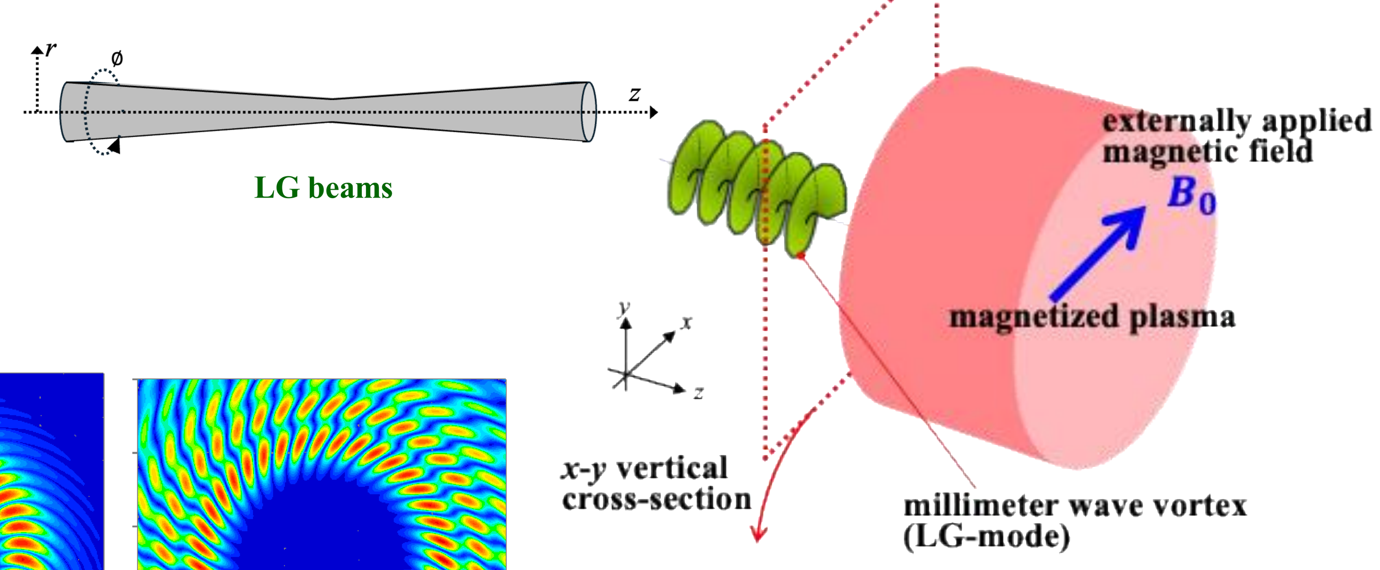
$$H_x=-i\frac{E_{mn}}{Z_0}\frac{\beta}{k_c}J_{m-1}(k_cr)\times e^{i(\omega t-\beta z-(m-1)\theta+\frac{\pi}{2})}$$

$$H_y=-i\frac{E_{mn}}{Z_0}\frac{\beta}{k_c}J_{m-1}(k_cr)\times e^{i(\omega t-\beta z-(m-1)\theta)}$$

$$H_z=i\frac{E_{mn}}{Z_0}J_m(k_cr)\times e^{i(\omega t-\beta z-m\theta)}$$

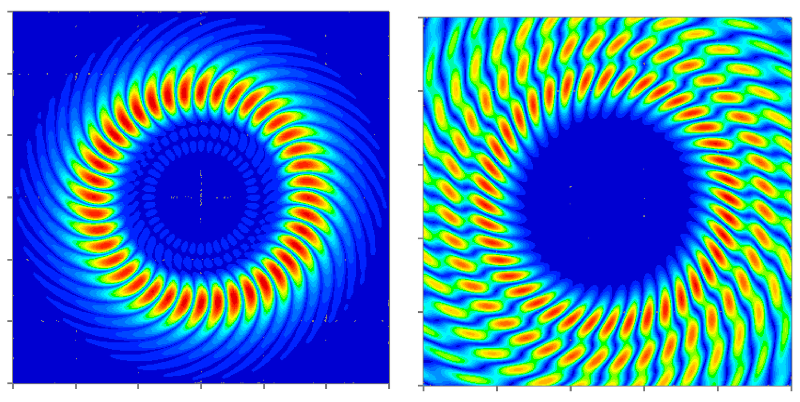
LG mode in free space

$$E(r,\emptyset,z)=\frac{C}{\left(1+\frac{z^2}{z_R^2}\right)^{\frac{1}{2}}}\left(\frac{\sqrt{2}r}{w(z)}\right)^l L_p^l\left(\frac{2r^2}{w^2(z)}\right)\exp\frac{-r^2}{w^2(z)}\cdot\exp\frac{-ikr^2z}{2(z^2+z_R^2)}\exp(-il\emptyset)\exp\left(i(2p+l+1)\tan^{-1}\frac{z}{z_R}\right)$$

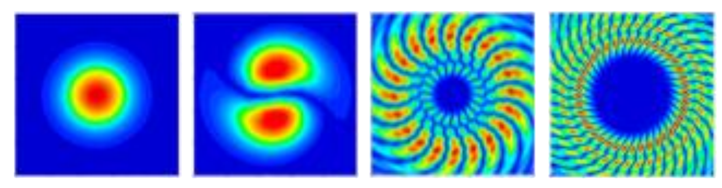


Poynting power P

$$P=\frac{1}{2T}\int_0^T dt\int_S(E\times H^*)\cdot dS$$



l = 20 (hybrid mode) *l* = 20 (LG mode)



l = 0 *l* = 1 *l* = 10 *l* = 20

FDTD FORMULATION

Lorentz model and FDTD method

$$\mathbf{J}=\frac{d\mathbf{P}}{dt}\qquad \frac{d\mathbf{J}}{dt}+\nu\mathbf{J}+\omega_0^2\mathbf{P}=\varepsilon_0\omega_p^2\left(\mathbf{E}+\frac{\mathbf{J}}{en_e}\times\mathbf{B}\right)$$

$$\frac{\mathbf{P}^{n+1}-\mathbf{P}^n}{\Delta t}=\mathbf{J}^{n+\frac{1}{2}}$$

$$\frac{\mathbf{J}^{n+\frac{1}{2}}-\mathbf{J}^{n-\frac{1}{2}}}{\Delta t}+\nu\frac{\mathbf{J}^{n+\frac{1}{2}}+\mathbf{J}^{n-\frac{1}{2}}}{2}+\omega_0^2\mathbf{P}^n=\varepsilon_0\omega_p^2\mathbf{E}^n+\frac{\varepsilon_0\omega_p^2}{en_e}\frac{\mathbf{J}^{n+\frac{1}{2}}+\mathbf{J}^{n-\frac{1}{2}}}{2}\times\mathbf{B}_0$$

$$\frac{J_x^{n+\frac{1}{2}}-J_x^{n-\frac{1}{2}}}{\Delta t}+\nu\frac{J_x^{n+\frac{1}{2}}+J_x^{n-\frac{1}{2}}}{2}+\omega_0^2P_x^n=\varepsilon_0\omega_p^2E_x^n+\frac{\varepsilon_0\omega_p^2}{en_e}\frac{J_y^{n+\frac{1}{2}}+J_y^{n-\frac{1}{2}}}{2}B_{z0}-\frac{\varepsilon_0\omega_p^2}{en_e}\frac{J_z^{n+\frac{1}{2}}+J_z^{n-\frac{1}{2}}}{2}B_{y0}$$

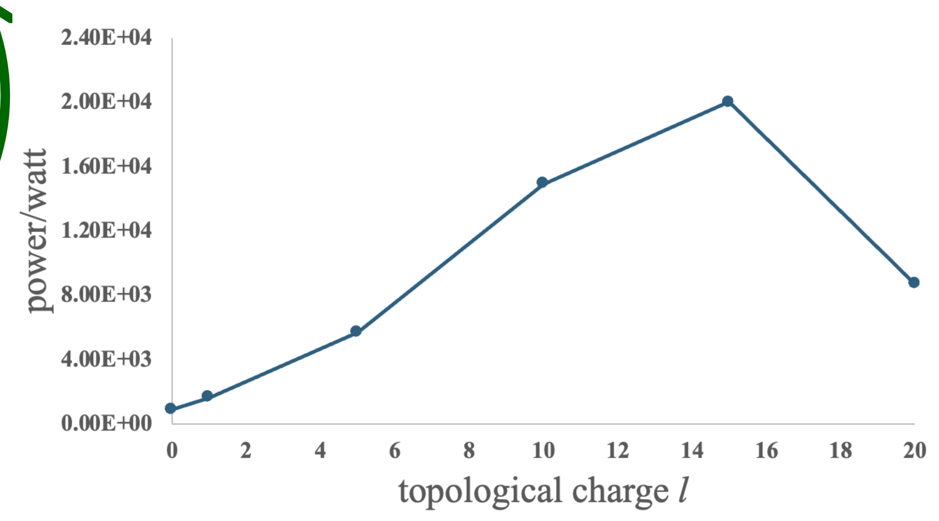
$$\frac{J_y^{n+\frac{1}{2}}-J_y^{n-\frac{1}{2}}}{\Delta t}+\nu\frac{J_y^{n+\frac{1}{2}}+J_y^{n-\frac{1}{2}}}{2}+\omega_0^2P_y^n=\varepsilon_0\omega_p^2E_y^n+\frac{\varepsilon_0\omega_p^2}{en_e}\frac{J_z^{n+\frac{1}{2}}+J_z^{n-\frac{1}{2}}}{2}B_{x0}-\frac{\varepsilon_0\omega_p^2}{en_e}\frac{J_x^{n+\frac{1}{2}}+J_x^{n-\frac{1}{2}}}{2}B_{z0}$$

$$\frac{J_z^{n+\frac{1}{2}}-J_z^{n-\frac{1}{2}}}{\Delta t}+\nu\frac{J_z^{n+\frac{1}{2}}+J_z^{n-\frac{1}{2}}}{2}+\omega_0^2P_z^n=\varepsilon_0\omega_p^2E_z^n+\frac{\varepsilon_0\omega_p^2}{en_e}\frac{J_x^{n+\frac{1}{2}}+J_x^{n-\frac{1}{2}}}{2}B_{y0}-\frac{\varepsilon_0\omega_p^2}{en_e}\frac{J_y^{n+\frac{1}{2}}+J_y^{n-\frac{1}{2}}}{2}B_{x0}$$

$$\nabla\times\mathbf{E}=-\mu\frac{\partial\mathbf{H}}{\partial t}\qquad \nabla\times\mathbf{H}=\mathbf{J}+\sigma\mathbf{E}+\varepsilon\frac{\partial\mathbf{E}}{\partial t}$$

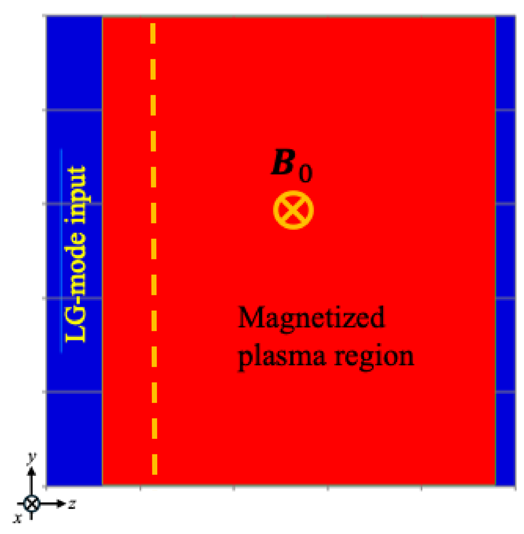
$$\mathbf{H}^{n+\frac{1}{2}}=\mathbf{H}^{n-\frac{1}{2}}-\frac{\Delta t}{\mu}\nabla\times\mathbf{E}^n$$

$$\mathbf{E}^{n+1}=\frac{\frac{\varepsilon}{\Delta t}-\frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t}+\frac{\sigma}{2}}\mathbf{E}^n+\frac{1}{\frac{\varepsilon}{\Delta t}+\frac{\sigma}{2}}\nabla\times\mathbf{H}^{n+\frac{1}{2}}-\frac{1}{\frac{\varepsilon}{\Delta t}+\frac{\sigma}{2}}\mathbf{J}^{n+\frac{1}{2}}$$

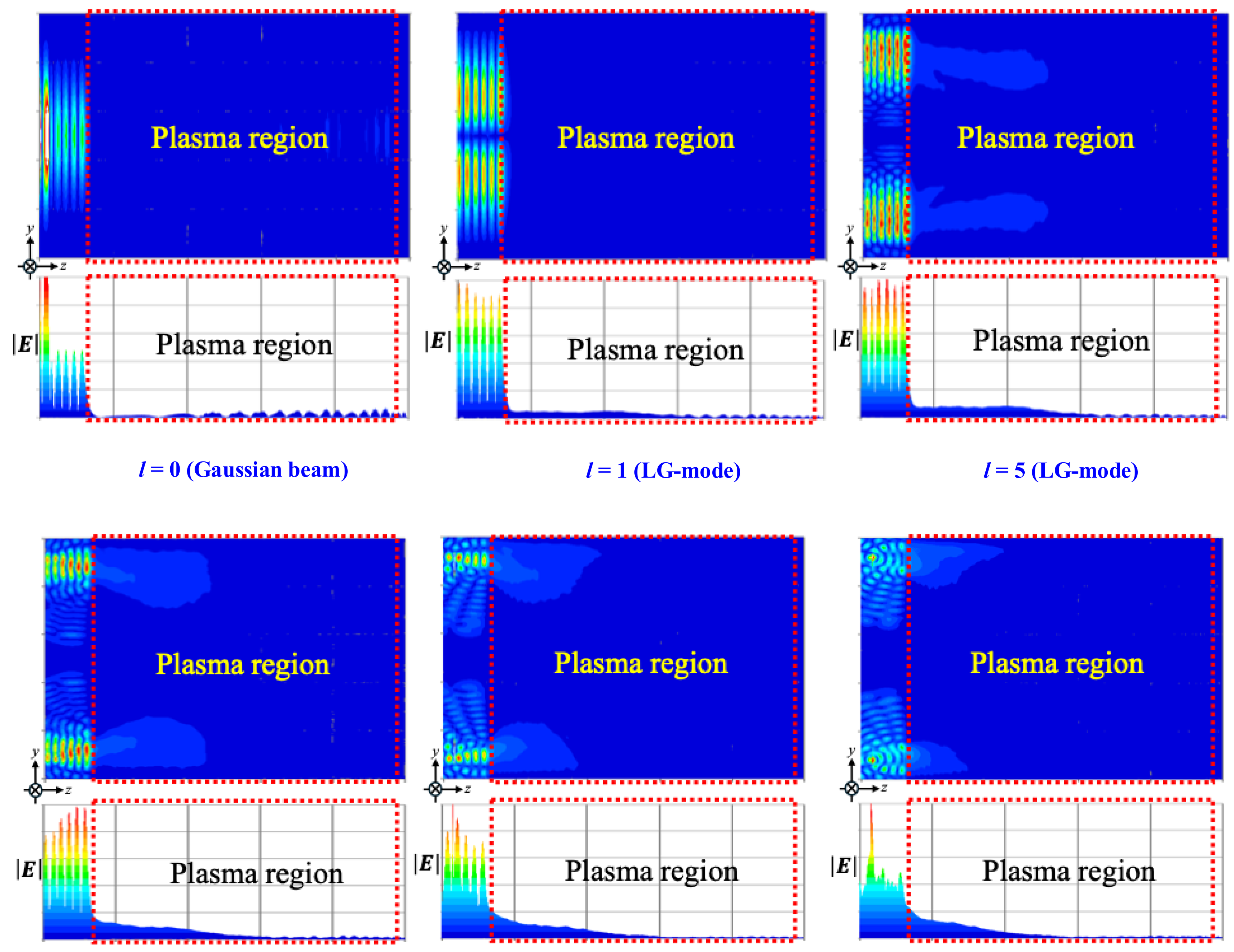


OUTCOME

Numerical model



Numerical examples



l = 0 (Gaussian beam) *l* = 1 (LG-mode) *l* = 5 (LG-mode)

l = 10 (LG-mode) *l* = 15 (LG-mode) *l* = 20 (LG-mode)

CONCLUSION

Propagation of the millimeter wave vortex is evaluated by finite-difference time-domain (FDTD) method. A LG-mode was introduced from free space into the plasma, and its penetration properties were analyzed for different topological charges. The simulations show that the penetrated power strongly depends on the topological charge *l*.

References

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Acknowledgement

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