BEYOND FINITE-DIFFERENCES : A LATTICE BOLTZMANN APPROACH
FOR SOLVING FOKKER-PLANCK EQUATIONS IN MAGNETIC FUSION

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Modeling fusion plasma often requires solving the high-dimensional Fokker-Planck equation (with 3 spatial, 3 velocity and 1 time dimensions). A traditional finite-difference scheme in velocity space becomes computationally impractical. In a 6D phase space, an enormous number of grid points is needed for adequate resolution and an implicit time-stepping involves solving large dense matrices for the collision operator at each spatial location. These factors make direct finite-difference solvers prohibitively expensive for 6D Fokker-Planck problems.

In this work, we suggest a completely new approach (vastly adopted in fluid dynamics), a Lattice Boltzmann Method (LBM) [1,2]. LBM uses a discrete set of velocity basis functions and local collision operations instead of global matrix solvers, greatly reducing computational cost. We formulate higher-order lattice schemes, by carefully choosing discrete velocity sets (e.g. high-order Gauss-Hermite quadrature nodes) that can capture the essential physics with far fewer velocity grid points than a brute-force finite-difference grid [3]. This efficiency, combined with the inherently parallel and local nature of LBM updates, makes it a promising approach for high-dimensional kinetic equations. In typical fusion plasma scenarios, the distribution function is close to Maxwellian (thermal equilibrium) with only perturbative deviations; thus, a lattice Boltzmann formulation – which naturally expands the distribution around a local Maxwellian equilibrium – can efficiently and accurately capture the dynamics. To incorporate self-consistent charge and field dynamics, we have extend the lattice Boltzmann solver with a Vlasov-like forcing term. This forcing term represents the influence of electromagnetic forces (e.g. electric fields from charge separation or applied fields) on the particle distribution.

This high-dimensional FP equation appears in several important case studies including Collisional Electron Transport [4], Wave-Particle Interactions (RF Heating) [5], Turbulence and Transport Modeling [6]. We show that a naive FD grid in 6D grows exponentially with resolution. For example, using 100 points in each dimension yields 100^6 ≈ 10^12 grid cells – "enormous matrices" that are intractable. In practice, FD Fokker-Planck codes reduce dimensionality (assuming symmetries or averaging) to make the grid manageable (down to 3–4D). By contrast, an LBM approach uses a discrete velocity lattice with a fixed number of velocity directions (e.g. 19 or 27 in 3D). This drastically cuts down the effective phase-space grid. Instead of a dense 3D velocity mesh, each spatial cell carries a small set of discrete velocity populations. Example: If a 3D spatial domain has Nx^3 cells and LBM uses Q discrete velocities, total points~ (Nx^3) × Q. For Nx = 100 and, say, Q = 27, that's 100^3 × 27 ≈ 5.4 × 10^7 degrees of freedom – orders of magnitude fewer than 10^12. This reduces memory and computation dramatically (e.g. memory from petabytes down to a few hundred MB).

In this work, we demonstrate the new method significantly reduces the computational time and memory required versus a traditional finite-difference scheme – by factors of tens to thousands, depending on the scenario – while maintaining acceptable accuracy. This makes high-dimensional kinetic simulations (e.g. capturing electron tail formation or multi-species collisional dynamics) much more practical. The LBM's combination of efficiency and massive parallel scalability offers a promising path to tackle fusion plasma Fokker-Planck problems that were previously beyond reach with conventional methods.

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