Nonlinear Spectrum Evolution of Lower Hybrid Waves and Density limit of Lower Hybrid Current Drive

Zhe Gao, Kunyu Chen, Zhihao Su, Zikai Huang and Long Zeng

Department of Enginneering Physics, Tsinghua University

gaozhe@tsinghua.edu.cn

ID: 2997

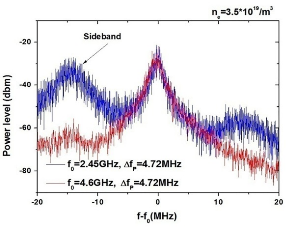
Outline

- Parametric Decay Instability (PDI) leads to the failure of lower hybrid current drive (LHCD) in high density plasma (density limit), as well as the nonlinear spectrum evolution of the lower hybrid waves (LHWs) in the scrape-off layer (SOL) region
- The theory of PDI growth and saturation encounters the difficulty of quasi-mode decay
- Renewed PDI modeling and simulation considering quasi-mode decay successfully reproduced the nonlinear spectrum evolution and density limit of LHCD in experiments
- A theoretical scaling of the density limit is obtained, suggesting LHCD remains effective in ITER / reactor regime

Background

PDI spectrum evolution

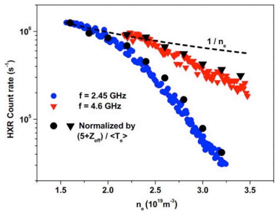
$$\omega_{\text{pump}} = \omega_{\text{sideband}} + \omega_{\text{LF}}$$
$$\boldsymbol{k}_{\text{pump}} = \boldsymbol{k}_{\text{sideband}} + \boldsymbol{k}_{\text{LF}}$$



LHCD-PDI @EAST Li 2022

Density limit of LHCD

- CD efficiency falls faster than $1/n_e$
- Together with PDI sideband
- Improved by quenching PDI



LHCD density limit @EAST Li 2019

Quasi-mode decay

In MCF plasma, PDI is typically displaying as quasi-mode decay. Quasi-mode has strong damping (Landau, cyclotron, etc) and no linear propagation, resulting in difficulties

- Asymmetry between sideband and LF → Quasilinear coupling of PDI is not enough
- Saturation of PDI relies on the wavenumber mismatch due to plasma inhomogeneity.
- The mismatch of quasi-mode PDI cannot be given by the linear dispersion relation → The absolute instability and convective instability needs to be revisited

Coupling of quasi-mode decay

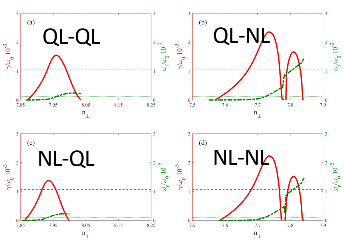
Quasilinear and nonlinear coupling

Expanding Vlasov equation

$$f_{sj} = f_{sj}^{\text{L}} + f_{sj}^{\text{NL}}$$
$$f_{sj}^{\text{L}} = \mathbb{L}(f_{\text{M}s}, \boldsymbol{E}_j)$$

Needs iteration

$$\boxed{f_{s,\text{LF}}^{\text{NL}}} = \mathbb{C}(f_{s,0}, \boldsymbol{E}_1) + \mathbb{D}(\boxed{f_{s,1}}, \boldsymbol{E}_0)$$
$$\boxed{f_{s,1}^{\text{NL}}} = \mathbb{E}(f_{s,0}, \boldsymbol{E}_{\text{LF}}) + \mathbb{F}(\boxed{f_{s,\text{LF}}}, \boldsymbol{E}_0)$$



QL-QL iteration

$$f_{s,\text{LF}}^{\text{QL}} = \mathbb{C}(f_{s,0}^{\text{L}}, \boldsymbol{E}_1) + \mathbb{D}(f_{s,1}^{\text{L}}, \boldsymbol{E}_0)$$
$$f_{s,1}^{\text{QL}} = \mathbb{E}(f_{s,0}^{\text{L}}, \boldsymbol{E}_{\text{LF}}) + \mathbb{F}(f_{s,\text{LF}}^{\text{L}}, \boldsymbol{E}_0)$$

resonant mode: $f^L \gg f^{NL}$, →QL is enough

QL-NL iteration

$$f_{s,\text{LF}}^{\text{NL}} \approx f_{s,\text{LF}}^{\text{QL}} = \mathbb{C}(f_{s,0}^{\text{L}}, \boldsymbol{E}_1) + \mathbb{D}(f_{s,1}^{\text{L}}, \boldsymbol{E}_0)$$
$$f_{s,1}^{\text{NL}} \approx \mathbb{E}(f_{s,0}^{\text{L}}, \boldsymbol{E}_{\text{LF}}) + \mathbb{F}\left(f_{s,\text{LF}}^{\text{L}} + f_{s,\text{LF}}^{\text{QL}}, \boldsymbol{E}_0\right)$$

quasi-mode mode: $f^L \sim f^{NL}$, →requires NL iteration

Saturation of quasi-mode decay

Nonlocal coupling of PDI

For electrostatic coupling dominates PDI [Gao NF 2025], we can expand the ES coupling equation

$$\epsilon_{LF}\phi_{LF} = \alpha_{1\to LF}\phi_0^*\phi_1,$$
$$\epsilon_1\phi_1 = \alpha_{LF\to 1}\phi_0\phi_{LF}$$

To obtain the nonlocal PDI equation, considering both inhomogeneous plasma and finite pump profile [Chen and Gao NF 2025]

$$\left[\underbrace{\boxed{\varepsilon_j(\omega_j, k_j)}}_{\text{Local term}} + \underbrace{\boxed{\frac{\partial\varepsilon_j}{\partial\omega_j}\cdot i\partial_t - \frac{\partial\varepsilon_j}{\partial k_j}\cdot i\partial_x}}_{\text{temporal and spatial evolution}} + \underbrace{\boxed{\frac{\partial\varepsilon_j}{\partial x}\cdot x}}_{\text{Plasma inhomogeneity to replace } \Delta k_j}\right]\phi_j(x,t) = \alpha_{j\leftarrow i}\phi_0(x,t)\phi_i(x,t)$$

Absolute and convective instability

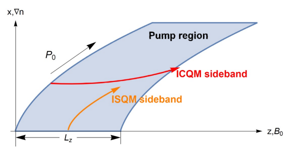
- Absolute instability has an extremely high threshold [Chen and Gao, Communications in theoretical Physics 2025]
- Convective saturation of quasi-mode PDI

$$A = \int\gamma_0 \mathrm{d}t = \int \underbrace{\boxed{\gamma_0}}_{\text{plasma inhomogeneity}} \underbrace{\boxed{\frac{\mathrm{d}x}{v_{1gx}}}}_{\text{finite pump profile}}$$

Modeling the nonlinear evolution of LHCD

LHCD induced PDI in the SOL region

- Decay channels: ISQM + ICQM
- Plasma inhomogeneity : x (radial) direction
- Finite pump profile : z (toroidal) direction



The nonlinear spectrum evolution

Energy conservation constrained by PDI

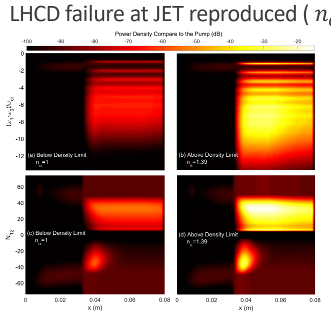
$$\begin{cases} \nabla\cdot\boldsymbol{P}_0(\boldsymbol{r}) + \sum_{\omega_1,\boldsymbol{k}_1}\left[\nabla\cdot\boldsymbol{P}_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1) - 2\gamma_{1L}(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)U_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)\right] = 0 \\ U_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1) = U_{\text{th}}(\boldsymbol{k}_{\text{LF}})\exp(2A) \end{cases}$$

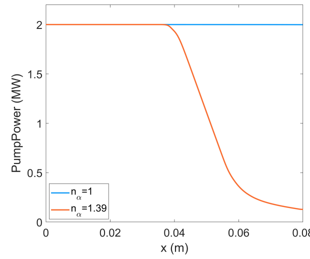
- Considering pump to sideband transfer and sideband damping
- Initial state U_{th} is the thermal noise
- Convective amplification factor is integrated along sideband trajectory

Spectrum evolution and density limit

Simulation results

LHCD failure at JET reproduced ($n_\alpha = 1$ corresponds to $\langle n_e\rangle = 3$)





Scaling of LHCD density limit

$$A \propto P_0 L_y^{-1} B_0^{-2} \omega_{LH}^3 \omega_0^{-3} T_e^{-3/2}$$
$$n_{PDI} \propto P_0^{-2/3} L_y^{2/3} B_0^{4/3} \omega_0^2 T_e$$

- For ITER, n_{PDI} is 10 times higher than JET!