DEVELOPMENT OF A FAMILY OF RAYS TRACING CODE BASED ON A NON-COMMUTATIVE KINETIC RAY SYSTEM

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Improving the predictions for electron cyclotron resonance heating and current drive is important for realizing high-performance fusion plasma. While the locality of the driven current is significant for the suppression of the neoclassical tearing mode, conventional models of electron cyclotron (EC) wave propagation, based on geometrical optics, often underestimate the spatial profile of wave beams compared to experiments. In this study, a new ray tracing model in "ray phase space" is introduced, using techniques from quantum theory, such as the Wigner-Weyl transform and Moyal product, to describe evolution of non-localized wave energy more accurately than quasioprical model, where an optical axis has to be defined. This model overcomes the "caustic," where ray crossings lead to unphysical divergences of wave energy, and suits for parallel processing since each ray is computed independently in ray phase space. Newly developed code based on this model reproduces more realistic results for broader propagation and absorption of EC wave beams in toroidal plasmas.

Improving the predictions for electron cyclotron (EC) resonance heating and current drive is important for the design and control of high-performance fusion plasmas in future devices, where those are expected to play a more significant role as actuators than current devices, i.e., it is well known that, to efficiently suppress the neoclassical tearing mode, it is required to drive the current sufficiently localized than the size of the magnetic island [1]. Therefore, the behavior of radio-frequency wave beams in the EC frequency range has been extensively studied both experimentally and theoretically. Throughout the long history of study on the propagation and absorption of EC wave beams in laboratory toroidal plasmas, it has been known that experimental measurements often show broader spatial profiles than those predicted by numerical simulations based on geometrical optics [2]. Although this discrepancy might be reduced by improving the accuracy of experimental measurements, an alternative approach, that is, improving the geometric optics model used in the calculations, has also been pursued to fully resolve the discrepancy [3].

The family of geometrical optics rays tracing has long been employed to describe EC wave beams in inhomogeneous plasmas [4]. In this multiple rays approach, the initial positions of rays are distributed in real space, and each ray trajectory traced towards an initial direction set for each corresponding initial position. Recently, as one of the extensions of geometrical optics, quasioptical ray tracing, which accounts for higher-order effects such as diffraction and scattering, has been developed and is steadily used [3]. In this quasioptical framework, instead of restricting ray trajectories to a single trajectory corresponding to the optical axis of the EC wave beam, a complex wave field localized at the vicinity of the beam axis is introduced as a series expansion. This technique provides a more accurate description of the diffusive behaviour of the wave energy profile along the propagation of well-behaved EC wave beams. However, this quasioptical extension is not well-suited for describing wave fields that are not spatially localized, where the “optical axis” cannot be clearly defined. There is a dilemma: the more the energy profile of the EC wave beam is diffused due to the higher-order effects in the series expansion around the beam axis, the less accurate the description of the wave field becomes. Thus, in this study, we reconsider the family of rays tracing, not in conventional real space, but in a “ray phase space” that combines both real space and its dual, wave number space. By exploring the behaviour of a family of rays in a ray phase space, we aim to develop an EC wave beam model that does not degrade in accuracy even as the wave field broadens.

We focus on the analogy between ray theory and quantum theory in developing a family of rays tracing model in the ray phase space [5]. Specifically, we emphasize the fact that both systems are described by non-commutative dual operators: coordinate operators and wave number (or momentum) operators. To construct the new ray model, we employ the Wigner-Weyl transform (WWT) and the Moyal product, which are mathematical tools developed alongside the evolution of quantum theory. The WWT maps any operator in a Hilbert space to a symbol in phase space. The Moyal product translates the non-commutative multiplication rule from Hilbert space to phase space without losing its non-commutativity. By using these techniques, a general linear wave equation in a Hilbert space can be mapped to a von-Neumann type equation in ray phase space. As a first-order approximation of this, the Liouville type equation governing the phase space evolution of the wave energy profile is derived. In this model, the phase space profile of the wave energy is represented by the Wigner function, which, in quantum theory, is interpreted as a probability density profile. Specific Wigner function on this model can be understood as the ray phase space spectrum computed from the (spatial) autocorrelation of launched wave field $\psi(x)$ from the antenna. As shown in Fig.1, (a) the wave energy profile $W(x,k)$ is divided into microscopic phase space volume elements $\delta W(x_i,k_j)$, and (b) each energy element is assigned to each ray corresponding to the center of each element. Then, (c) these energies are delivered along the ray trajectories $(\dot{x}_i,\dot{k}_j)$ that do not cross each other in phase space, according to the Liouville theorem. This model extends the conventional family of rays tracing, which is limited to real space, to the ray phase space, and describes the evolution of wave energy profiles by computing many independent rays, similar to the particle method used in kinetic theory to trace the evolution of distribution function. Conventional real-space models will encounter the caustic problem, where wave energy diverges unphysically at ray crossing points. However, in the higher-dimensional ray phase space, rays do not cross (in the first-order approximation), and thus, the caustic problem does not arise. Additionally, the divergence term of the group velocity, which describes ray interactions in real space models, is decomposed in the wave number direction in the phase space model. This independence of each ray facilitates a parallel processing.

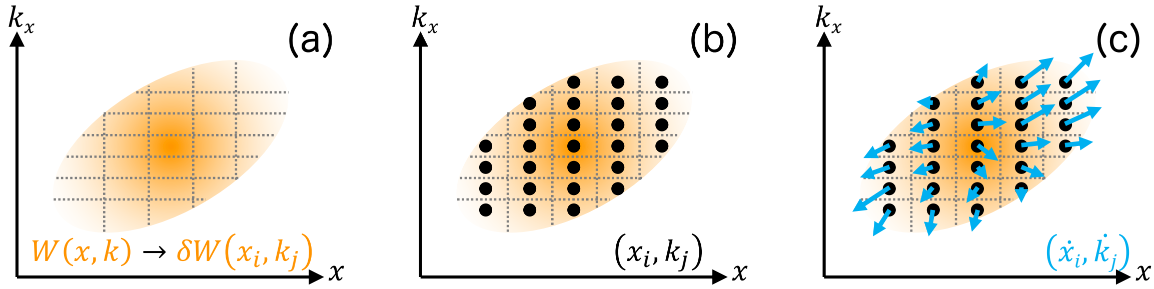

Figure 1. Schematics of a family of rays tracing procedure on the ray phase space.

We have newly developed a family of rays tracing code, based on the Liouville type ray phase space system mentioned above. As a reference, we present the results from a test run of this new code on JT-60SA tokamak in Fig. 2. Figure 2(a) illustrates how rays spread out in multiple directions, rather than in a conventional single direction, from each initial point. Along these rays, the wave energy, divided into microscopic elements, is transported. Figure 2(b) shows the phase space profile of the wave energy of the Gaussian beam, which was used as the initial condition. It is evident that the spectrum in the wave number direction, which is often inadequately accounted for in conventional family of rays models, is produced at each spatial point. By applying this code, which is not restricted to the vicinity of the optical axis, to EC analysis, we anticipate more realistic results for broader propagation and absorption of EC wave beams.

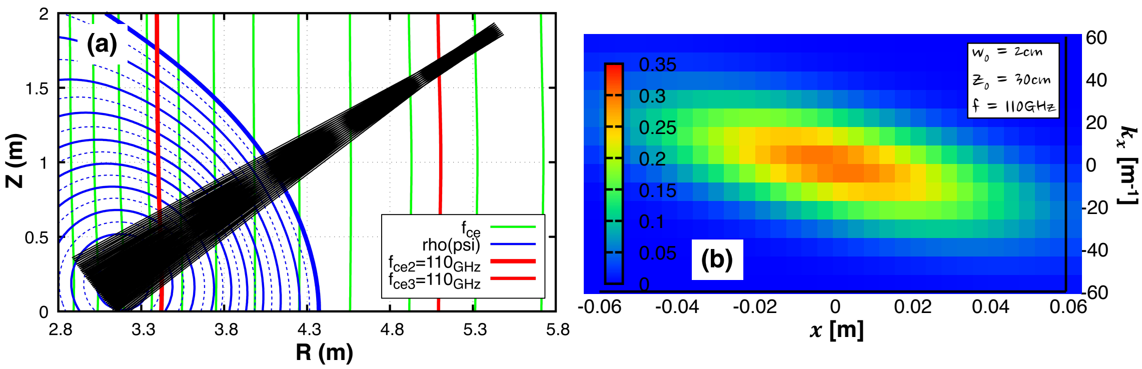

Figure 2. Results from a test simulation of newly developed code for Gaussian EC wave beam on JT-60SA. (a) Family of ray trajectories (black lines) spreading out from each distributed initial position. Thick and thin red lines are 2nd and 3rd order EC resonances. Magnetic surfaces are represented as blue, and magnetic field strength contour is as green. (b) Ray phase space projection of injected Gaussian wave beam energy as Wigner function used as an initial condition, where x is a beam-cross direction. w_0 is a beam waist, Z_0 is a focal length, and f is a frequency of injected Gaussian beam.

References

[1] N. Hayashi, T. Ozeki, K. Hamamatsu, T. Takizuka, Nucl. Fusion **44** (2004), 477.

[2] K. Yanagihara, S. Kubo, I. Y. Dodin, the LHD experiment Group, Nucl. Fusion **61** (2021), 106012.

[3] K. Yanagihara, I. Y. Dodin, S. Kubo, Phys. Plasmas **26** (2019), 072112.

[4] T. I. Tsujimura, S. Kubo, et. al., Nucl. Fusion **55** (2015), 123019.

[5] E. R. Tracy, A. J. Brizard, A. S. Richardson, and A. N. Kaufman, Cambridge: Cambridge University Press (2014).