EXTRACTING THE NEAREST CANONICAL EQUILIBRIUM DISTRIBUTION VIA NATURAL GRADIENT DESCENT METHOD

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This paper presents an efficient method for numerically extracting the nearest canonical equilibrium distribution f_{NE} from an arbitrary axisymmetric distribution function of tokamak plasmas by formulating the problem as an optimization task for the discrete form of the gyrokinetic Vlasov equation. An iterative scheme utilizing natural gradient descent is employed to obtain f_{NE} with a specified numerical accuracy. This approach incorporates an enhancement algorithm in order to accelerate the convergence process for the phase space points near the trapped-passing boundary. It is found that the numerical accuracy of the new method is significantly higher than that of the commonly used direct orbit average method and overcomes the stiff numerical difficulties near the trapped-passing boundaries. Possible applications of this algorithm are also discussed. This extraction algorithm is crucial for several applications, including:

1. Avoiding spurious zonal flows for simulations using the full-f method or the direct δf approach [2].

2. Precise equilibrium updating during prolonged δf simulations [4].

3. PSZS related framework for the EP transport problems [3].

4. Dimension reduction down to the 3D constants of motion (CoM) space (P_ζ, E, μ) in EP diagnoses [6].

Introduction: The canonical equilibrium distribution [1] is defined to be constant along the unperturbed orbits:

$$\dot{Z}_0^i \frac{\partial}{\partial Z^i} f = 0 \qquad (1)$$

Here $\mathbf{Z} = (\boldsymbol{X}, v_\parallel, \mu)$ is the gyrocenter coordinate variables, $\dot{Z}_0^i$ is the unperturbed motion determined by equilibrium fields. In toroidal axisymmetric systems like tokamaks, any function of constants of motion $f_{CoM}(P_\zeta, E, \mu)$ satisfies equation (1). This work propose an algorithm to extract the nearest canonical equilibrium to the given distribution function f_0---a concept referred to as the "Nearest Canonical Equilibrium Distribution" (abbreviated as 'NE' for convenience).

Method: The extraction of f_{NE} is realized by formulating the problem as an optimization task utilizing natural gradient descent method. A positive weight function is multiplied to the loss function that significantly accelerate the convergence rate. Starting from given distribution function f_0, this algorithm is guaranteed to converge to its nearest canonical equilibrium f_{NE}. The algorithm is applied within the 5D gyrokinetic code Nonlinear Lie-Transform (NLT). The test function f_0 is chosen as a local Maxwellian with high profile gradient. The time consuming of this algorithm is negligible comparing to the time required for simulating transport phenomena.

Results: This algorithm has been verified to find NE with high accuracy. Fig 1 shows the normalized loss $S_{norm} = \sqrt{\frac{\int \mathcal{J} d^5 z \left(\dot{z}_0^i \frac{\partial f}{\partial z^i}\right)^2}{\int \mathcal{J} d^5 z f_0^2}}$ decreasing with iterations. This convergence result indicates that $\dot{Z}_0^i \frac{\partial}{\partial z^i} f_{NE}$ can be considered exactly zero, validating the effectiveness of the algorithm.

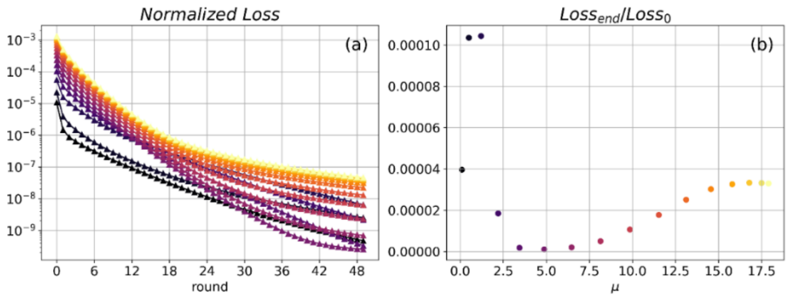

Figure 1: (a) Normalized Loss S_{norm} decreasing trend with iterations. Different colors represent different values of μ (lighter color for larger μ); (b) Relative change in loss.

Besides, the extracted NE can be proven to be consistent with an orbit average, i.e. $f_{NE} = \frac{1}{\tau_b}\oint d\tau\, f_0$. The results are compared to a direct orbit average. It is found that the numerical accuracy of the new method is significantly higher than that of the direct orbit average method and overcomes the stiff numerical difficulties near trapped passing boundaries.

Since the extracted NE is already a function of CoMs. A direct transformation from f_{NE} to a function of CoM can be done by a simple interpolation along a certain poloidal angle θ, which is shown in Figure 2.

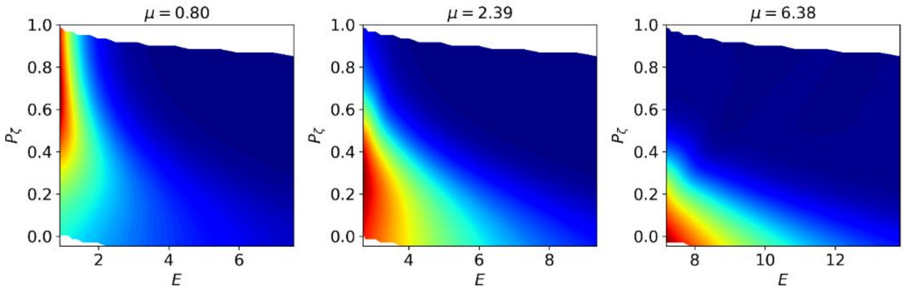

Figure 2: transformation to CoM space

REFERENCES

[1] Y. Idomura, S. Tokuda, and Y. Kishimoto, “Global gyrokinetic simulation of ion temperature gradient driven turbulence in plasmas using a canonical maxwellian distribution,” Nuclear Fusion, vol. 43, no. 4, p. 234, 2003.LETTER-WRITER, A., Organization, personal communication, Year.

[2] P. Angelino, A. Bottino, R. Hatzky, S. Jolliet, O. Sauter, T. Tran, and L. Villard, “On the definition of a kinetic equilibrium in global gyrokinetic simulations,” Physics of Plasmas, vol. 13, no. 5, 2006.CHAPTER-AUTHOR, A., “Title of chapter in sentence case”, Book Title in Title Case, Publisher, Place of Publication (Year).

[3] M. V. Falessi, L. Chen, Z. Qiu, and F. Zonca, “Nonlinear equilibria and transport processes in burning plasmas,” New Journal of Physics, vol. 25, no. 12, p. 123035, 2023..

[4] S. Wang, Z. Wang, and T. Wu, “Self-organized evolution of the internal transport barrier in ion-temperature-gradient driven gyrokinetic turbulence,” Physical Review Letters, vol. 132, no. 6, p. 065106, 2024.

[5] S. Benjamin, H. J¨ arleblad, M. Salewski, L. Stagner, M. Hole, and D. Pfefferl´e, “Dis tribution transforms for guiding center orbit coordinates in axisymmetric tokamak equilibria,” Computer Physics Communications, vol. 292, p. 108893, 2023.