

Modelling of mildly relativistic runaway electrons

Development of reduced-kinetic model and validation in KSTAR ohmic startup

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Main references

[¹] H.-T. Kim et. al., Development of full electron magnetic plasma burn-through model and validation in MAST, *Nucl. Fusion*. 62, 126012 (2022)

[²] Y. Lee et. al., Kinetic modelling of start-up runaway electrons in KSTAR and ITER, *Nucl. Fusion*. 63, 106011 (2023)

[³] Y. Lee et. al., Binary Nature of Collisions Facilitates Runaway Electron Generation in Weakly Ionized Plasmas, *Phys. Rev. Lett.* 133, 175102 (2024)

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*Lay physical and engineering foundation of designing
a RE-free scenario in future fusion reactors*

Introduction

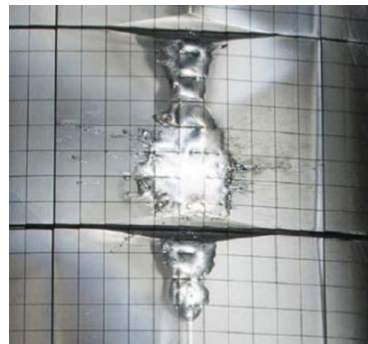
Runaway Electrons

- Runaway Electrons (REs)

- Runaway electrons are the energetic electrons that will be continuously accelerated by the electrical field due to the decrease in the Coulomb collision frequency with increasing energy [Wilson1925].

- Hazards of REs

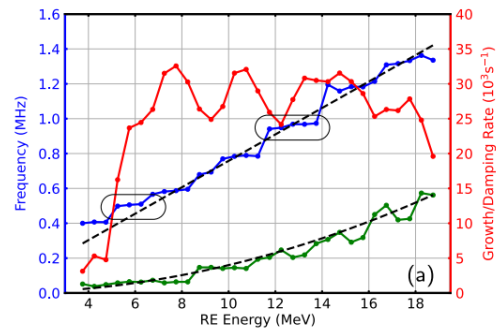
- Damage to devices [Matthews et al., 2016]
- Trigger instabilities [Liu et al., 2023]



[JET in-vessel image]

- Disruption REs

- Main interest of RE research community [Breizman et al., 2019].

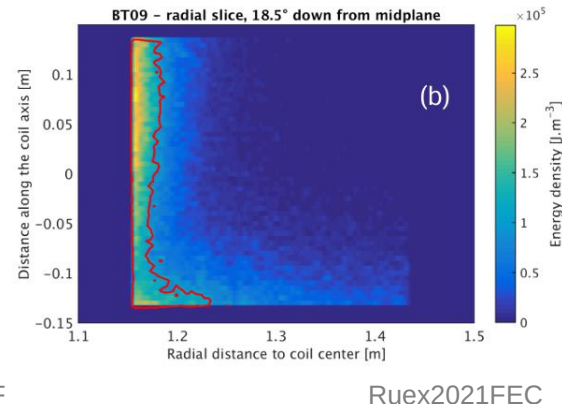
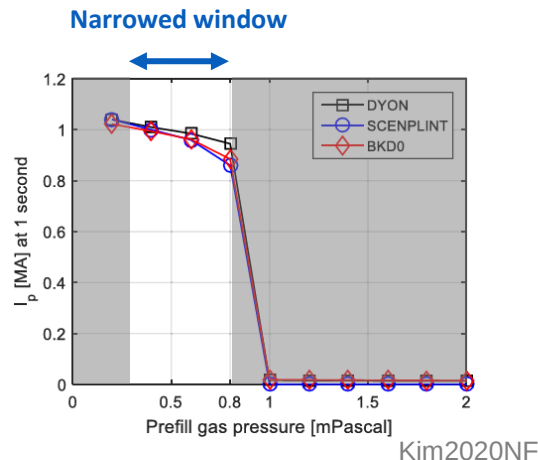


[CAW excited by REs]

Startup runaway electrons – Obstacles of ITER startup

- Strict conditions for burn-through success
 - Plasma-driven failure at high prefill pressure (≥ 0.8 mPa) Kim2020NF
 - Runaway-driven failure at low prefill pressure (≤ 0.3 mPa) Gribove2018EPS Hoppe2022JPP
- The conservative prediction suggested the failure at 0.6 mPa. Lee2024PRL

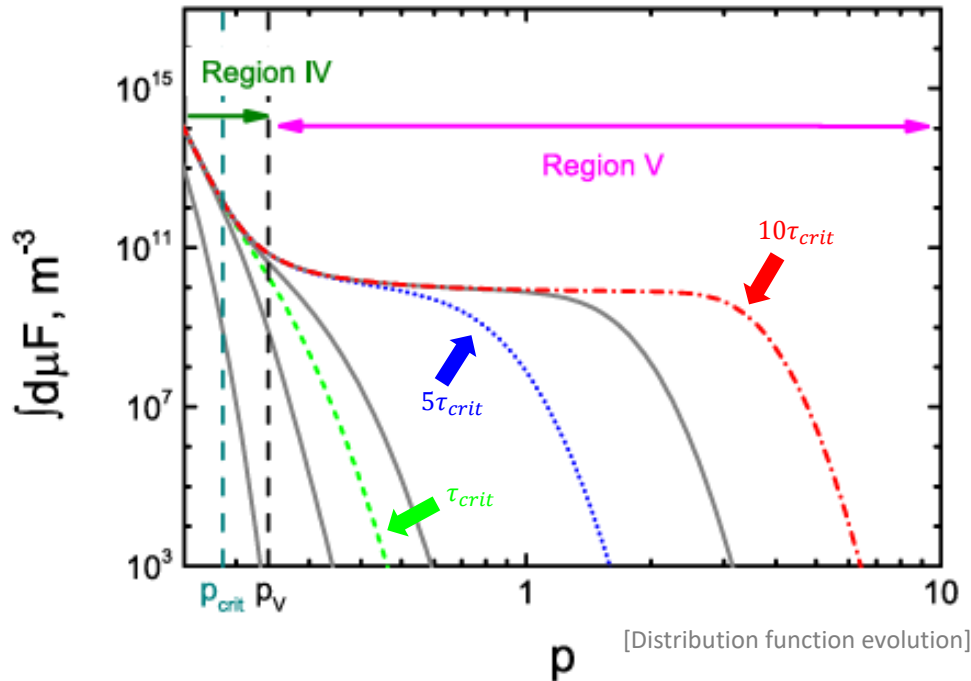
- Risk of catastrophic threat
 - Toroidal field coil quench
- Reported in WEST



Safe plasma startup in ITER requires understanding of startup REs.

Improved understanding of startup runaway physics

- Runaway electrons are born at the region-based critical momentum p_V .
 - Not the force-free momentum p_{crit} often referred to as the critical momentum.



Force-free critical momentum p_{crit}

$$F = F_{electric} - F_{fric} = 0$$

Region-based critical boundary p_V

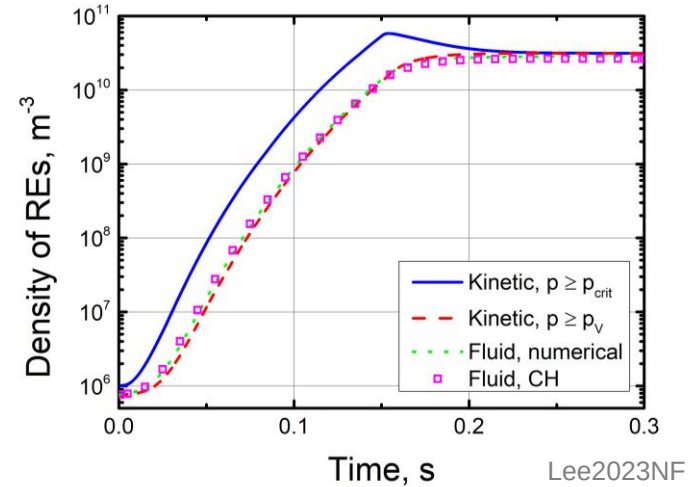
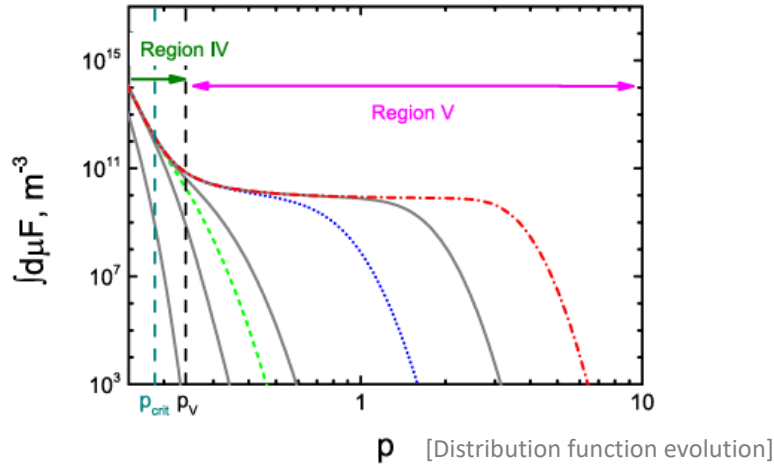
Region IV
Dominant energy diffusion
due to $F \approx 0$

Region V
Dominant acceleration
due to $F > 0$

p_V

Improved understanding of startup runaway physics

- The initial startup runaway momentum is the region-based momentum p_V .
 - Not the force-free momentum p_{crit} often referred to as the critical momentum.



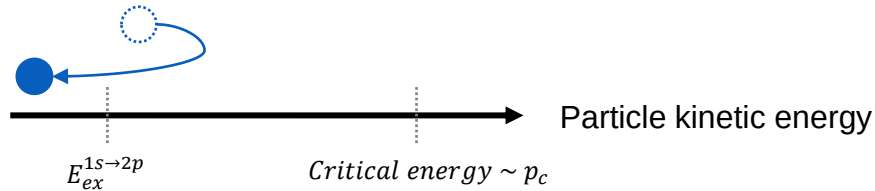
– Main Conclusions

- Fluid description of runaway density evolution is good.
- Primary runaway particles are born with initial momentum p_V .

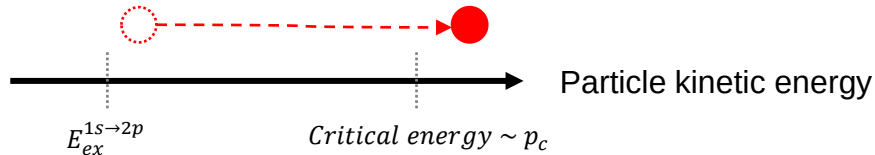
Improved understanding of startup runaway physics

- In early startup, runaway generation mechanism is non-diffusive.

Inelastic collision **prohibits**
electron acceleration

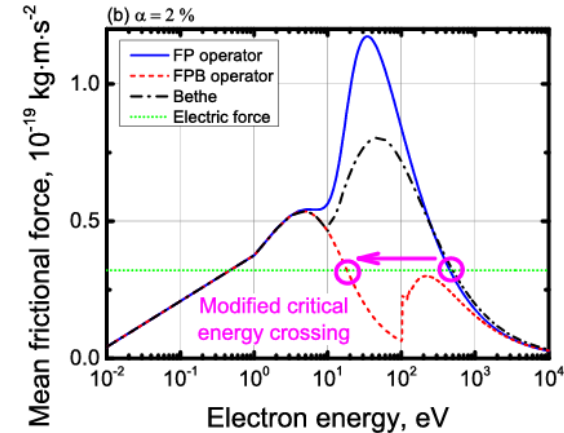
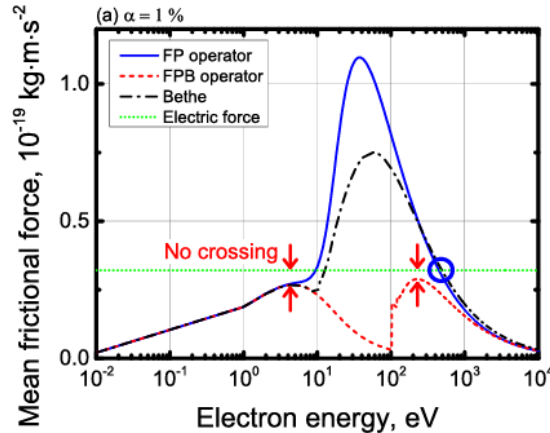
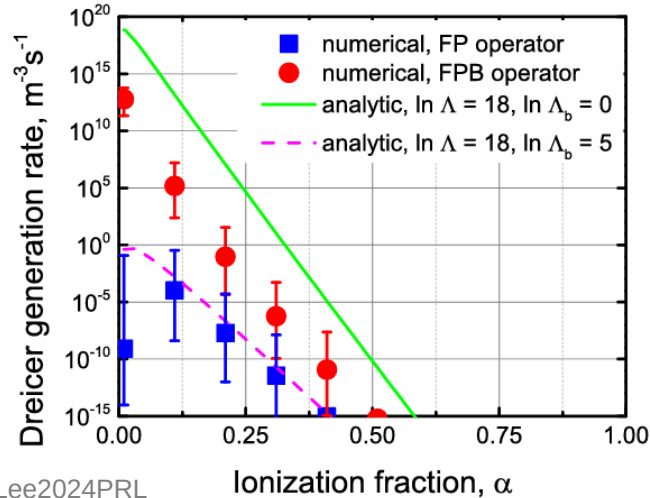


Collisionless acceleration **allows**
electron energy gain above p_c



Improved understanding of startup runaway physics

- In early startup, runaway generation mechanism is non-diffusive.
 - The Fokker-Planck description severely underestimates the generation rate.
 - The effective critical momentum can be significantly reduced or even disappear.



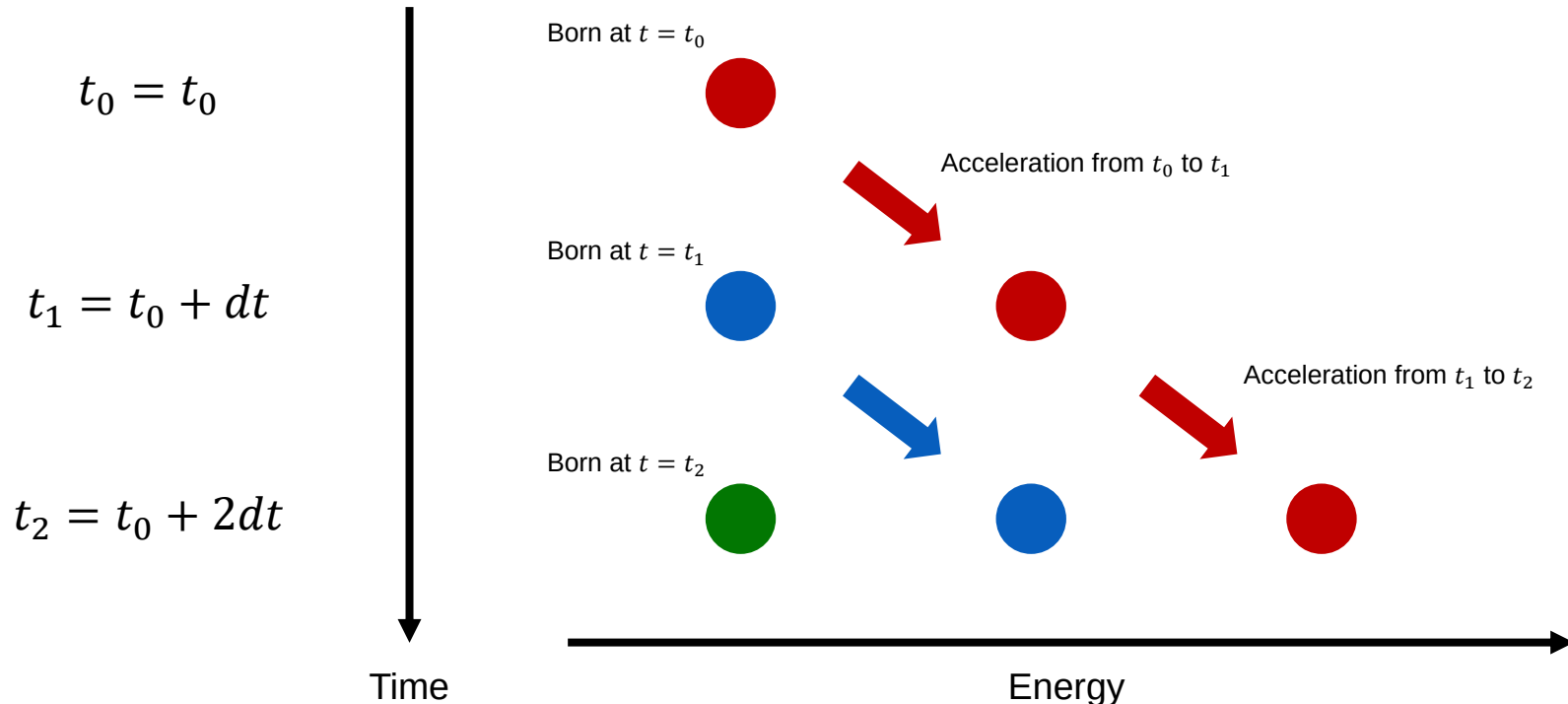
This implies $v_{RE} \ll c$ and thereby necessitates a mildly relativistic correction

Model development

Reduced kinetic model of mildly relativistic runaway electrons
Coupling with DYON

Multi fluid approximation of mildly relativistic runaway current

- Multi-fluid description of runaway electrons



Multi fluid approximation of mildly relativistic runaway current

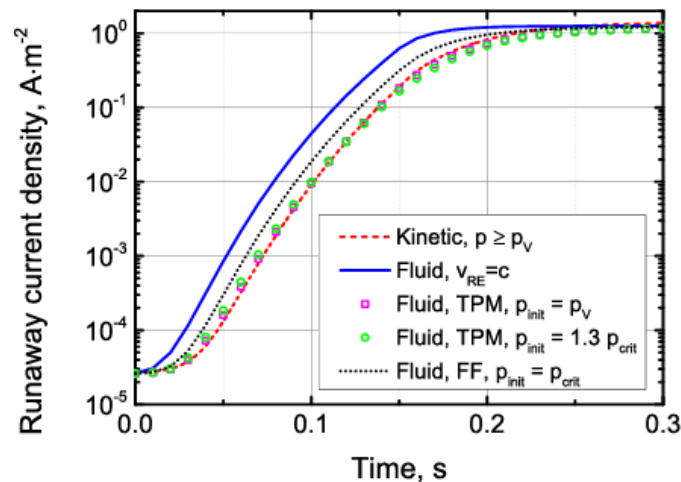
- The multi-fluid runaway model is consistent with the kinetic model.
 - The total runaway current density is “sum” of every runaway fluid currents

$$j_{RE,i+1} = ec \sum_{i' \leq i} dn_{RE,i'+\frac{1}{2}} \beta_{i'+\frac{1}{2}}(t_{i+1}),$$

– Fluid runaway velocity

$$\beta_{i'+\frac{1}{2}}(t_{i+1}) = \begin{cases} \beta_V \approx p_V & \text{if } i = i' \text{ Seed velocity} \\ \beta_{i'+\frac{1}{2}}(t_i) + \left(\frac{d\beta}{dt} \right)_{i'+\frac{1}{2}} (t_{i+1} - t_i) & \text{if } i > i' \text{ Accelerated velocity} \end{cases}$$

Test Particle Method (TPM)



Successful verification !

Single fluid approximation of mildly relativistic runaway current

- Simple idea is to introduce the mean runaway velocity $\beta_{RE,i} \equiv \frac{j_{RE,i}}{ec n_{RE,i}}$

– Runaway current density : $\frac{dj_{RE}}{dt} = ec\beta_V \frac{dn_{RE}^{seed}}{dt} + ec \frac{d\beta_{RE}}{dt} n_{RE}^{beam} + \dots$

➤ Single fluid model has comparable accuracy to multi-fluid free fall model.

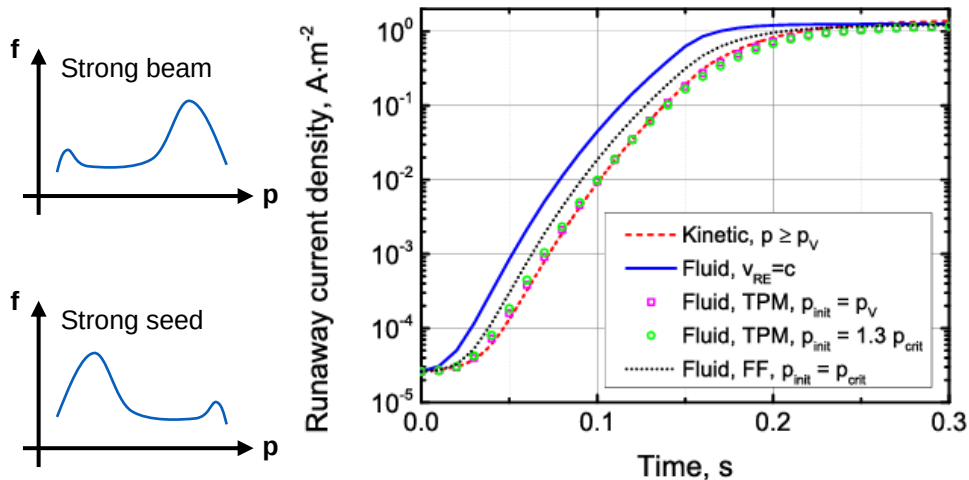
– Two critical asymptotes

➤ When runaway seeding is weak

- β_{RE} goes to runaway “beam” velocity.

➤ When runaway seeding is strong

- β_{RE} goes to runaway “seed” velocity.

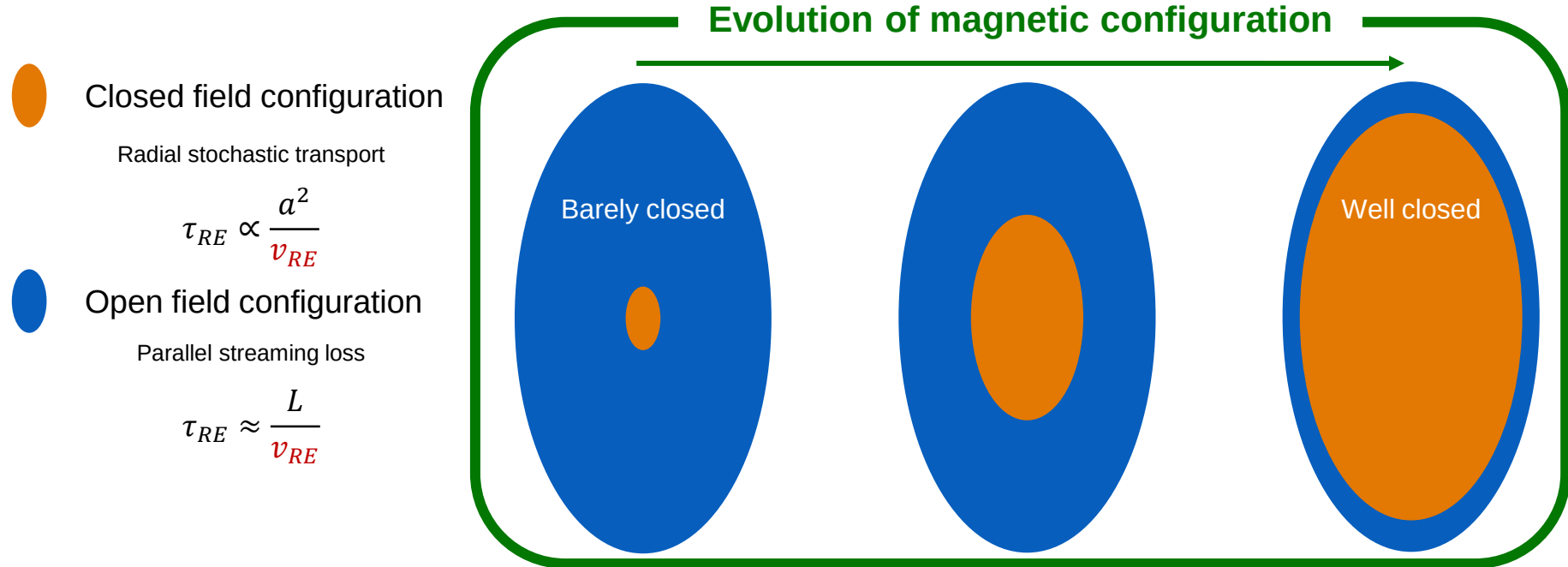


Model development

Reduced kinetic model of mildly relativistic runaway electrons
Coupling with DYON

Transport features of runaway electrons during startup

- Runaway transport mechanism relies on magnetic configuration.



Need to be coupled with mildly relativistic runaway velocity $v_{RE} \neq c$!

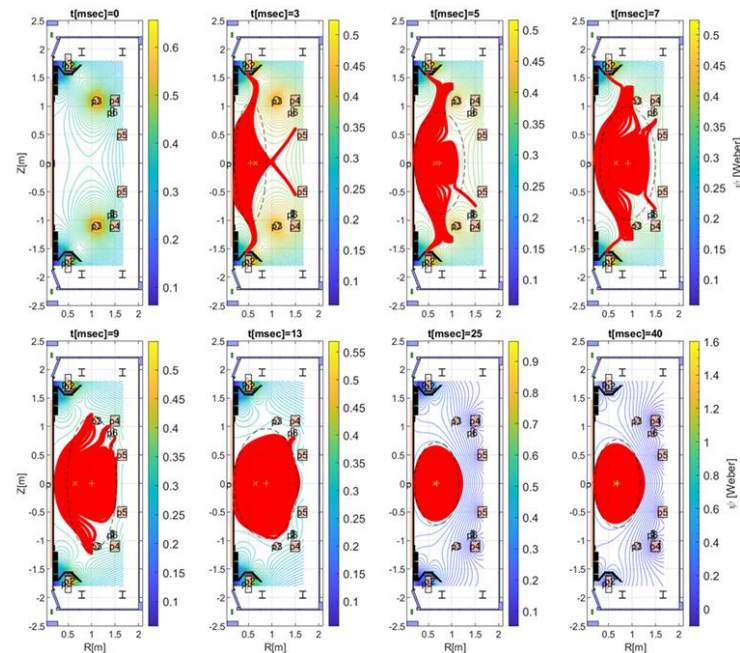
Reliable startup design by multi-machine validated code, DYON

- Self-consistent coupling with DYON
- Key code features
 - Multi-machine validated code.
 - MAST-U, DIII-D, EAST, KSTAR, VEST
 - Unique approach for *model-based* description of magnetic configuration.
 - Minimize free parameters and reduce solution non-uniqueness.
 - Critical lesson from startup RE model validation in JET [deVries2025NF].

Regular oral talk on model validation

by H.-T. Kim et al

17th Oct, 11AM



Kim2022NF

Two macroscopic populations on dual magnetic configuration

- Describe species in closed (cl) and open (op) magnetic configurations
 - The particle-conserving evolution of runaway particle density

$$\frac{dn_{RE}^{cl}}{dt} = S_p^{cl} + (\gamma_{ava}^{cl,i} - \frac{1}{\tau_{RE, \perp}^{cl}}) n_{RE}^{cl} - \boxed{n_{RE}^{cl} \frac{d}{dt} \log V_p^{cl}},$$

Particle-conserving correction

$$\frac{dn_{RE}^{op}}{dt} = S_p^{op} + (\gamma_{ava}^{op,i} - \frac{1}{\tau_{RE}^{op}}) n_{RE}^{op} + \boxed{\frac{V_p^{cl}}{V_p^{op}} \frac{n_{RE}^{cl}}{\tau_{RE, \perp}^{cl}}} - \boxed{n_{RE}^{op} \frac{d}{dt} \log V_p^{op}}.$$

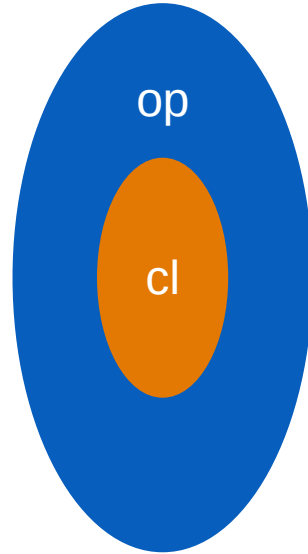
- The momentum-conserving evolution of runaway current density

$$\frac{dj_{RE}^{cl}}{dt} = ecS_p^{cl} \beta_V^{cl} + (\gamma_{ava}^{cl} - \frac{1}{\tau_{RE, \perp}^{cl}}) j_{RE}^{cl} + j_{RE}^{cl} \frac{d}{dt} \log \beta_{RE}^{cl} - \boxed{j_{RE}^{cl} \frac{d}{dt} \log V_p^{cl}},$$

Momentum-conserving correction

$$\frac{dj_{RE}^{op}}{dt} = ecS_p^{op} \beta_V^{op} + (\gamma_{ava}^{op} - \frac{1}{\tau_{RE}^{op}}) j_{RE}^{op} + j_{RE}^{op} \frac{d}{dt} \log \beta_{RE}^{op} + \boxed{\frac{V_p^{cl}}{V_p^{op}} \frac{j_{RE}^{cl}}{\tau_{RE, \perp}^{cl}}} - \boxed{j_{RE}^{op} \frac{d}{dt} \log V_p^{op}}.$$

Transport from closed to open field region



S_p : Primary generation rate
 γ_{ava} : Runaway avalanche growth rate

τ_{RE} : Runaway confinement time
 V_p : Plasma volume

Self-consistent coupling with DYON

- Self-consistent runaway current evolution

Amended circuit equations

$$\frac{dI_p}{dt} = \frac{1}{L_p} \left(V_{loop} - R_p(I_p - I_{RE}^{op} - I_{RE}^{cl}) - \frac{dL_p}{dt} I_p \right),$$

$$\frac{dI_{RE}^{cl}}{dt} = ecA^{cl} S_p^{cl} \beta_V^{cl} + I_{RE}^{cl} \frac{d}{dt} \log \beta_{RE}^{cl} + \left(\gamma_{ava}^{cl,j} - \frac{1}{\tau_{RE,\perp}^{cl}} \right) I_{RE}^{cl},$$

$$\frac{dI_{RE}^{op}}{dt} = ecA^{op} S_p^{op} \beta_V^{op} + \left(\gamma_{ava}^{op} - \frac{1}{\tau_{RE}^{op}} \right) I_{RE}^{op} + I_{RE}^{op} \frac{d}{dt} \log \beta_{RE}^{op} + \frac{R^{cl}}{R^{op}} \frac{I_{RE}^{cl}}{\tau_{RE,\perp}^{cl}}.$$

Dominant balancing

$$n_{RE}^{op} \approx S_p^{op} \tau_{RE,\parallel}^{op},$$

$$I_{RE}^{op} \approx ecA^{op} S_p^{op} \beta_V^{op} \tau_{RE,\parallel}^{op},$$

- Self-consistent runaway confinement on dual magnetic configurations

$$\tau_{RE,\parallel}^{op} \approx \frac{\langle L_{op} \rangle}{v_{RE}^{op}} \xrightarrow{\text{DYON model}}$$

$$\tau_{RE,\perp}^{cl} \approx c \frac{a_{cl}^2 \tilde{b}_r^2}{2\pi R_0 v_{RE}^{cl} B_\phi^2} \xrightarrow{\text{Runaway model}}$$

$$\tau_{RE}^{cl} \approx \tau_{RE,\perp}^{cl},$$

$$(\tau_{RE}^{op})^{-1} \approx (\tau_{RE,\perp}^{op})^{-1} + (\tau_{RE,\parallel}^{op})^{-1}$$

Model validation

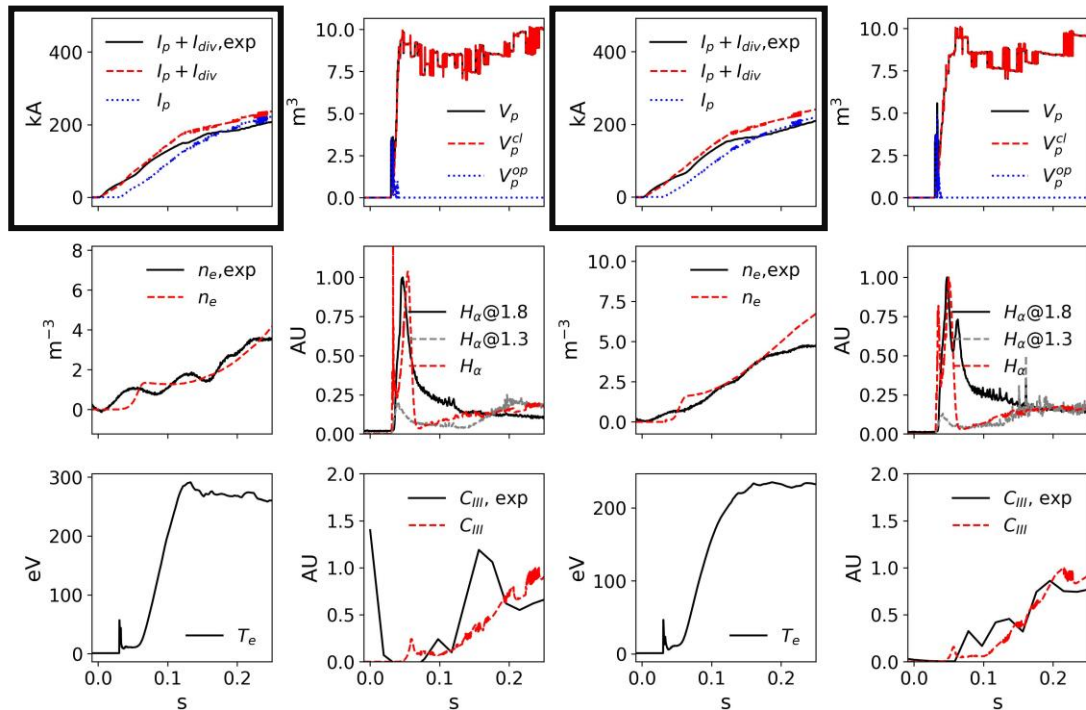
Validation in KSTAR ohmic startup

Validation of DYON-RE in KSTAR Ohmic startup

- Reliable prediction of plasma parameters by coupled DYON-RE

26031 (RE rich)

27340 (RE scarce)



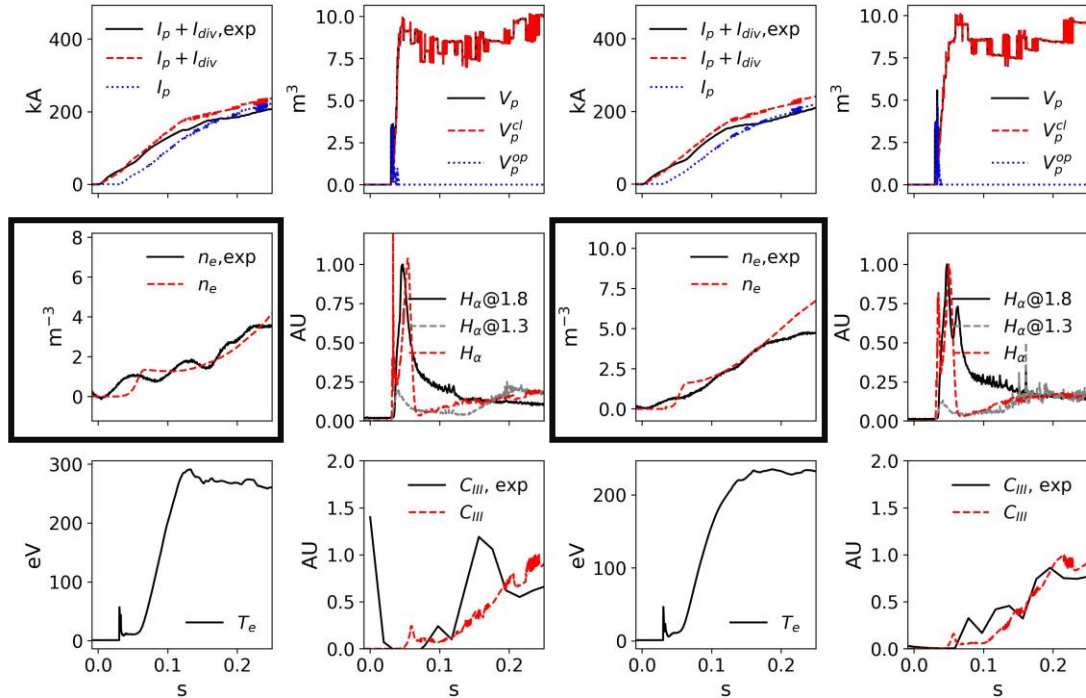
- Measured rogoski coil current agrees with synthetic signal ($I_p + I_{div}$)
 - Measured line-averaged electron density agrees with synthetic averaged electron density (n_e)
 - Measured line missions agree with synthetic line emission intensity (H_α, C_{III})
- * Indirect Te validation

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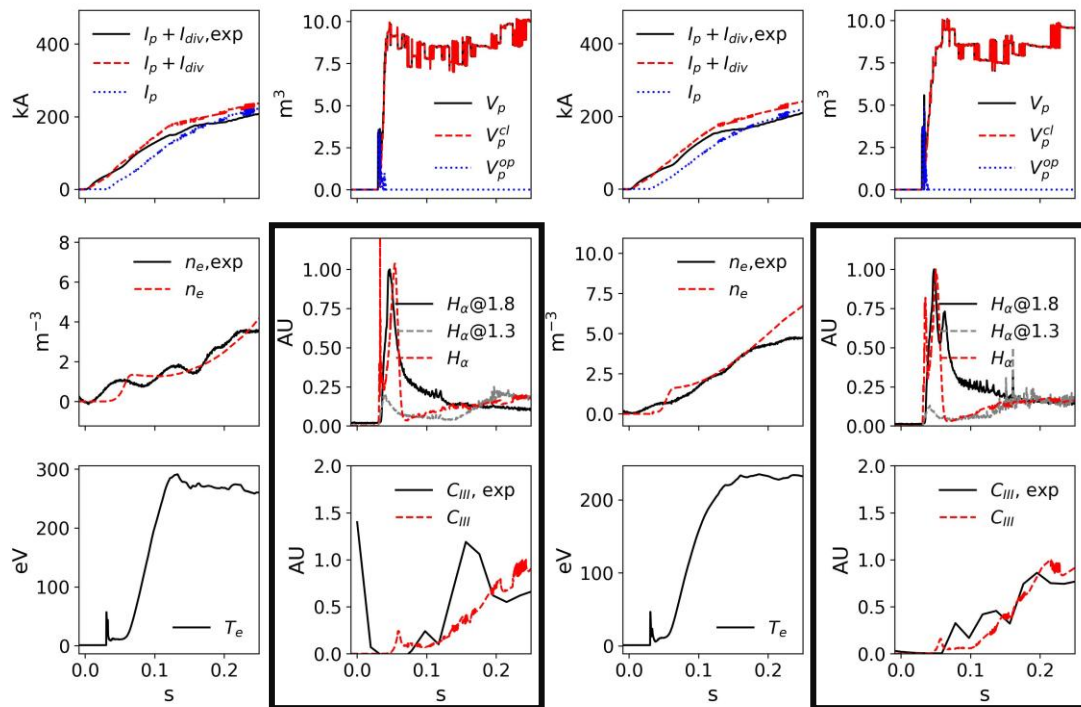
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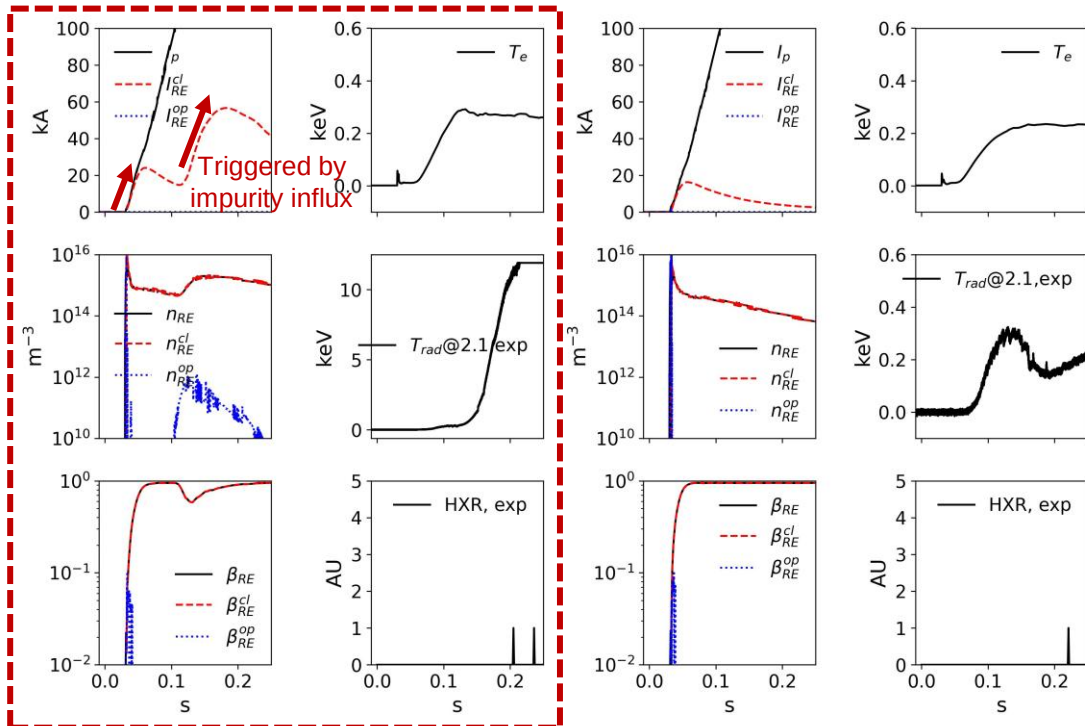
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Validation of DYON-RE in KSTAR Ohmic startup

- Qualitative validation of runaway signatures

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In RE rich discharge,

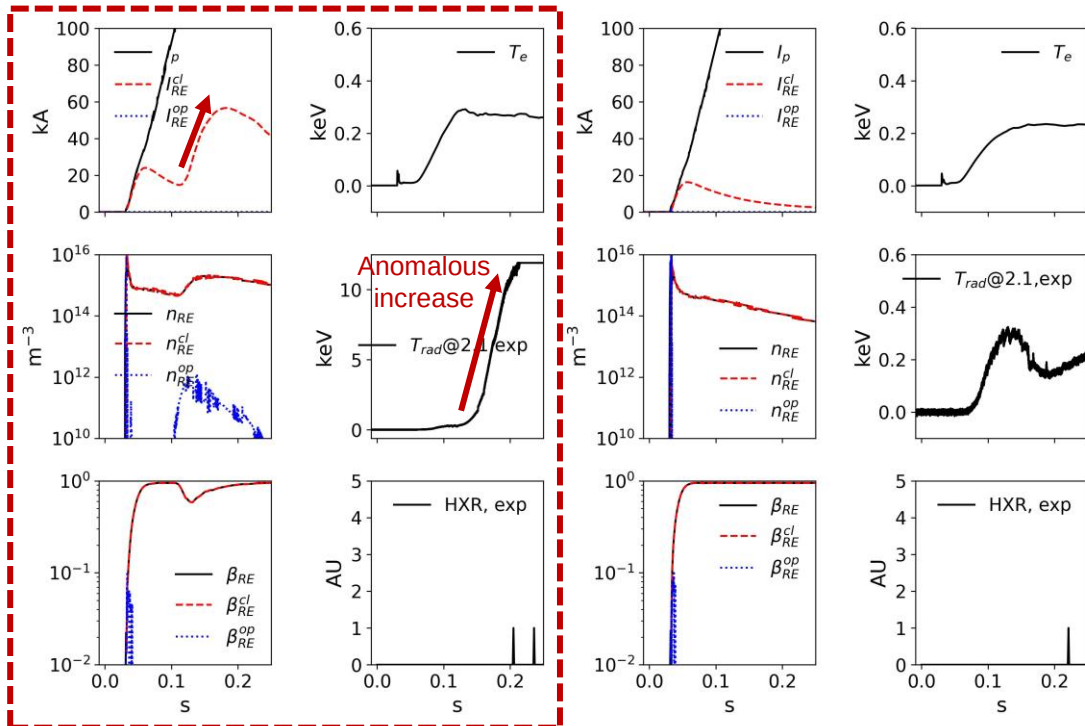
- Two-stage runaway generations are observed. 2nd stage trigger is impurity influx.
- Strong runaway seeding ($\sim t=0.1$ s) triggers anomalous increase in nonthermal ECE radiation (> 12 keV [exp] $\gg 0.25$ keV [DYON]).
- During strong runaway production, mildly relativistic correction renders $\beta < 1$.
- Note that highly relativistic runaway electrons do not satisfy the resonance condition so can't explain the anomalous increase.

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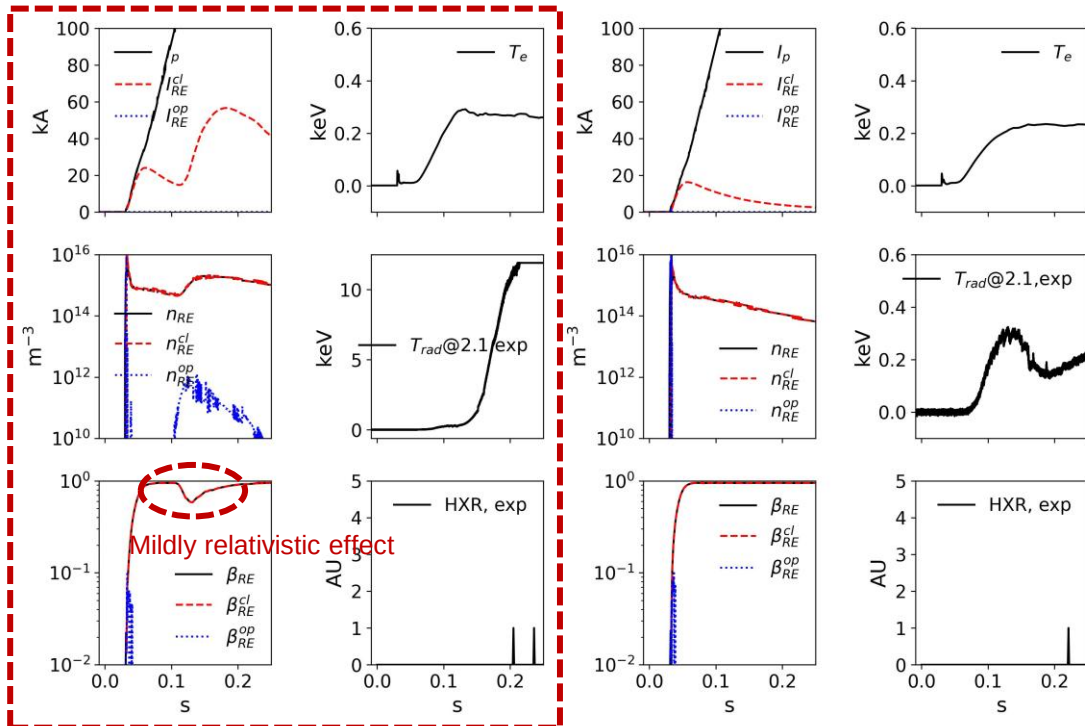
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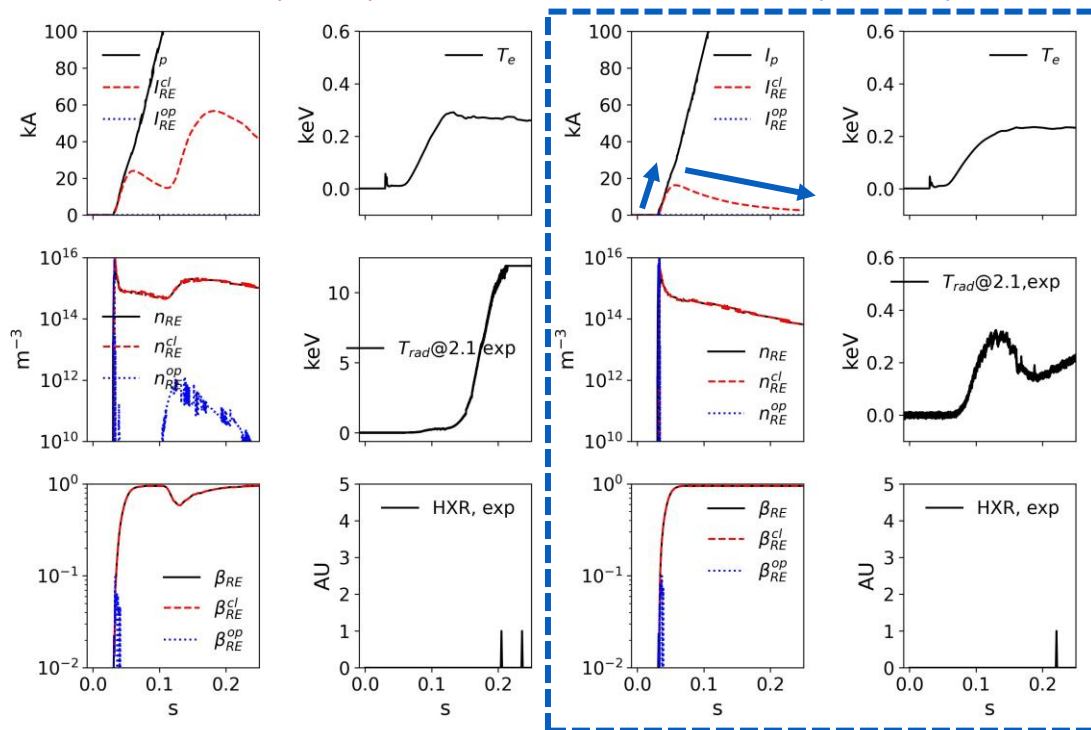
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Validation of DYON-RE in KSTAR Ohmic startup

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In RE scarce discharge,

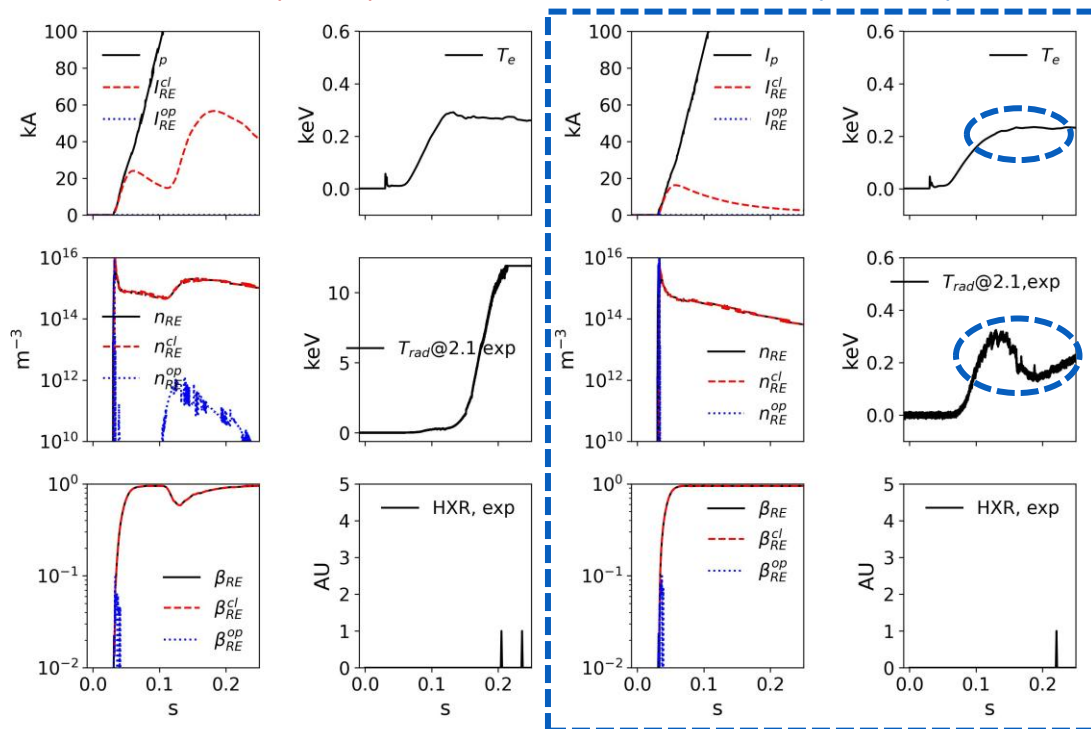
- Strong startup runaway seed forms (~ 20 kA) and decays.
- Electron temperature (~ 0.23 keV) has a comparable order to measured ECE intensity ($0.2 \sim 0.3$ keV).
- DYON prediction suggested kinetic instability is triggered by runaway beam but stabilized due to their deconfinement.

Validation of DYON-RE in KSTAR Ohmic startup

- Qualitative validation of runaway signatures

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27340 (RE scarce)



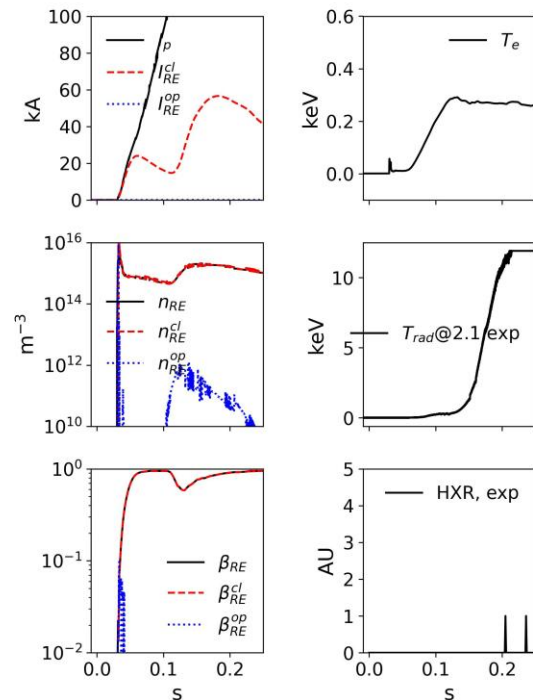
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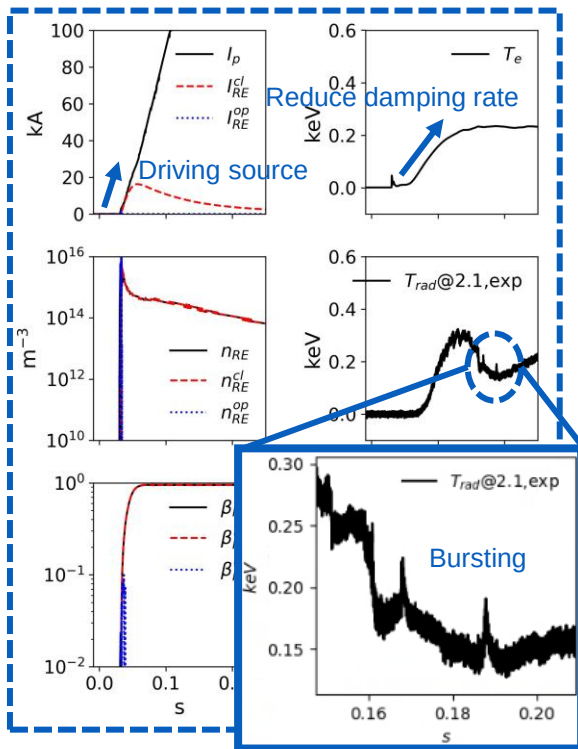
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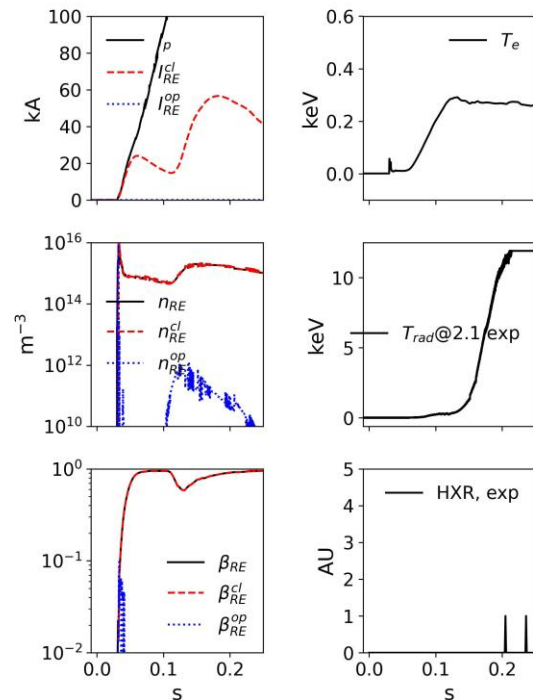
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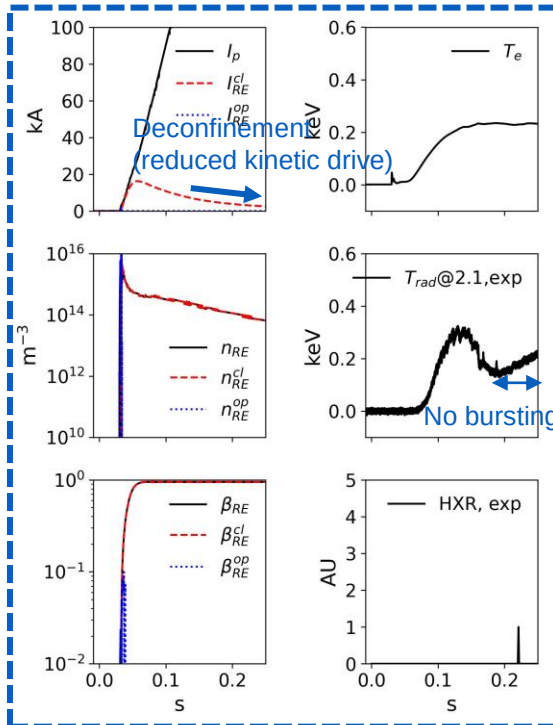
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- DYON prediction suggested kinetic instability is triggered by runaway beam but **stabilized due to their deconfinement**.

Conclusions

- Lay physical and engineering foundation of designing a RE-free scenario in future fusion reactors
 - Model development of reduced startup runaway model
 - **Single fluid model** with non-light-speed velocity $v_{RE} \neq c$
 - No overestimation in **runaway current density**
 - No underestimation in **runaway confinement time**.
 - **Self-consistent coupling** with reliable startup code DYON.
 - Model validation in KSTAR Ohmic startup
 - In RE rich shot,
 - Timing of strong runaway seed formation coincides with that of anomalous increase in non-thermal ECE intensity.
 - In RE scarce shot,
 - Measured ECE intensity is comparable to DYON's electron temperature.
 - Bursting characteristics in ECE intensity is consistent with formation and loss of runaway beam.