IAEA-FEC 2025

A novel method to optimize omnigenity like quasisymmetry for stellarators

Caoxiang Zhu

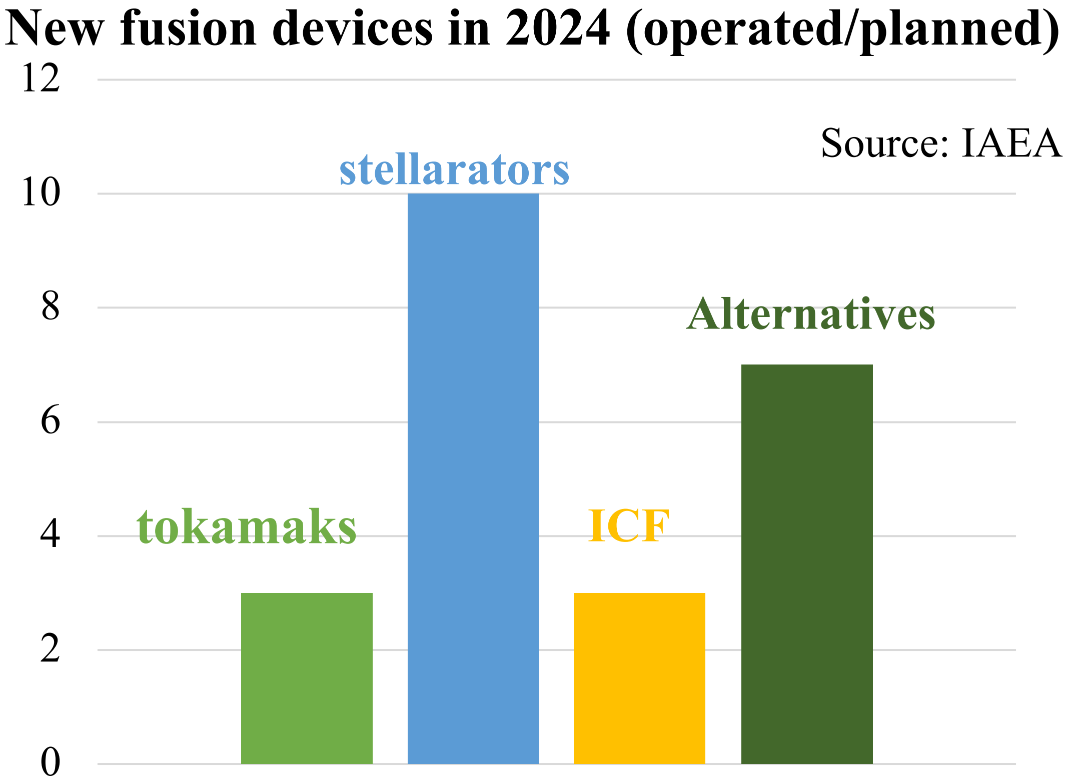
Piecewise omnigenous fields: a radically new family of optimized magnetic fields for stellarator reactors

José Luis Velasco

Introduction

Stellarators have become the rapidest growing fusion concept

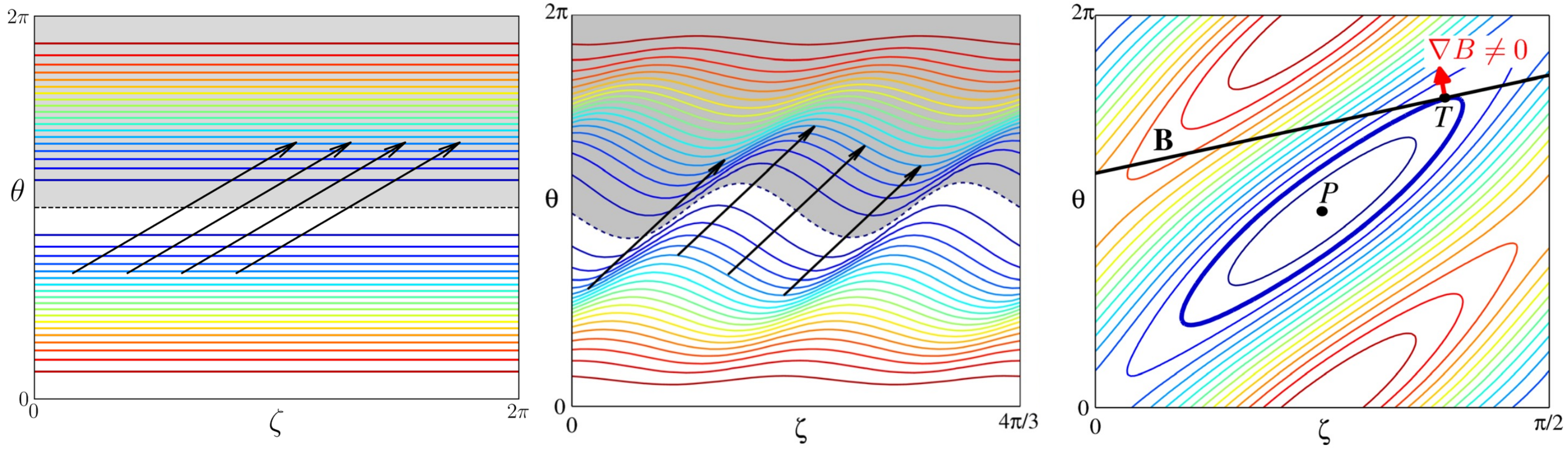
- A stellarator is a toroidal plasma confinement configuration that uses external coils to produce a non-axisymmetric magnetic field.
- Stellarators have many advantages.
    - **Steady state operation**
    - **Low recirculating power**
    - **Stable to plasma currents inducing MHD instabilities**
    - **Free of disruptions**
    - **High density operation**



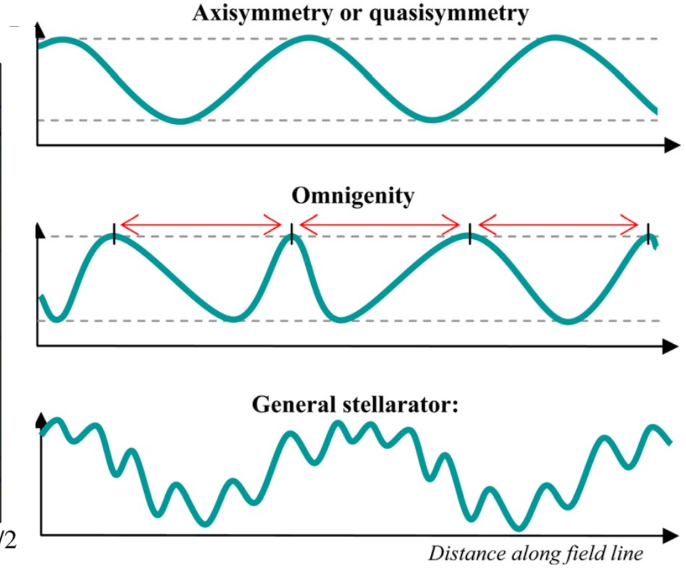
- Conventional stellarators suffered bad neoclassical transport and it can be overcome by carefully **optimized** magnetic fields [Beidler Nature 2021].

Omnigenity is a primary way to improve confinement

Axisymmetry/Quasi-axisymmetry **Omnigenity** **General stellarator**



Magnetic field strength B on the flux surface



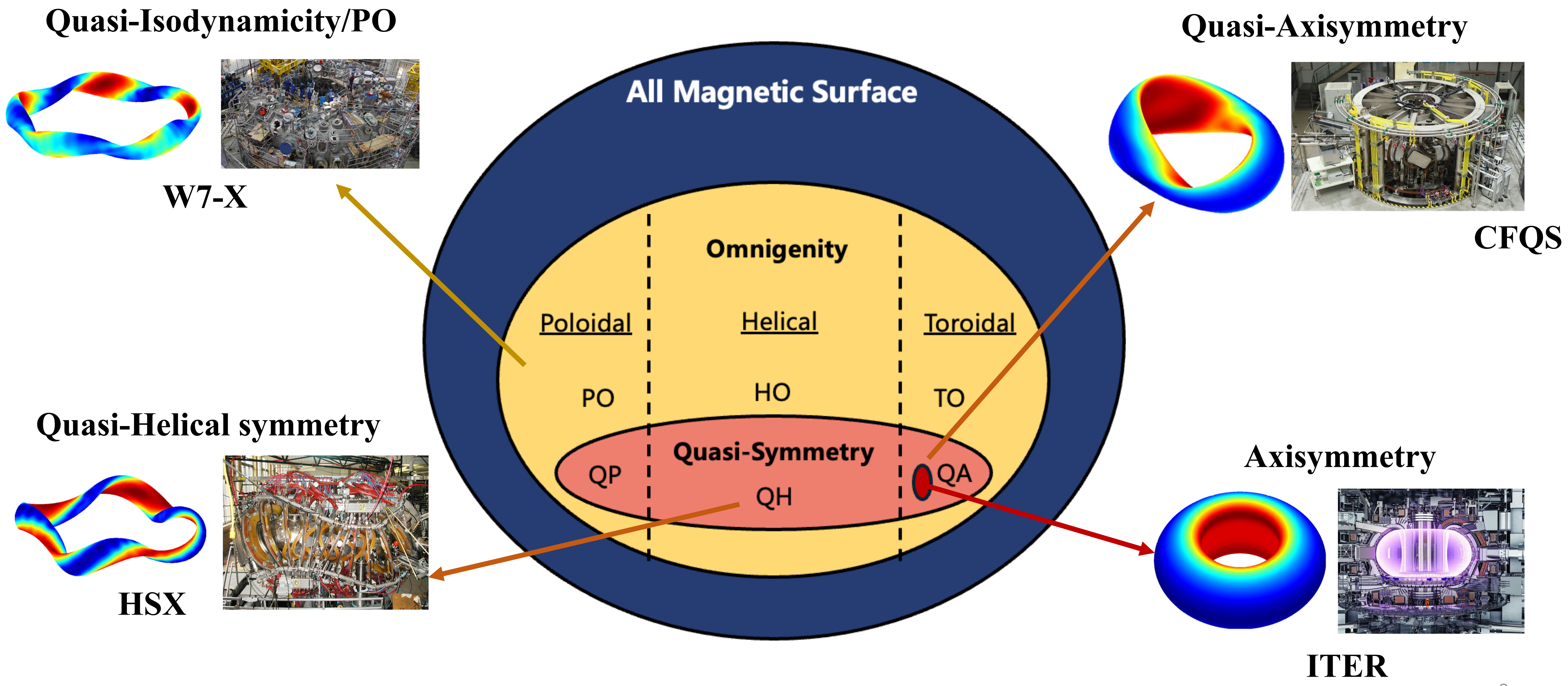
B along the field-line
[Landreman & Catto PoP 2012]

Omnigenity: time-averaged radial drift along each field line vanishes.

$$\langle \vec{v}_d \cdot \nabla\psi \rangle = 0 \iff \frac{\partial \mathcal{J}}{\partial \alpha} = 0 \qquad \mathcal{J} = m\int v_\parallel dl = \sqrt{2m(u - q\phi)} \oint \sqrt{1 - \frac{B}{B^*}}\, dl$$

- **No closed B contours**
- **Equal bounce distance**

Quasisymmetry is a subset of omnigenity



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A novel method to optimize omnigenity like quasisymmetry for stellarators

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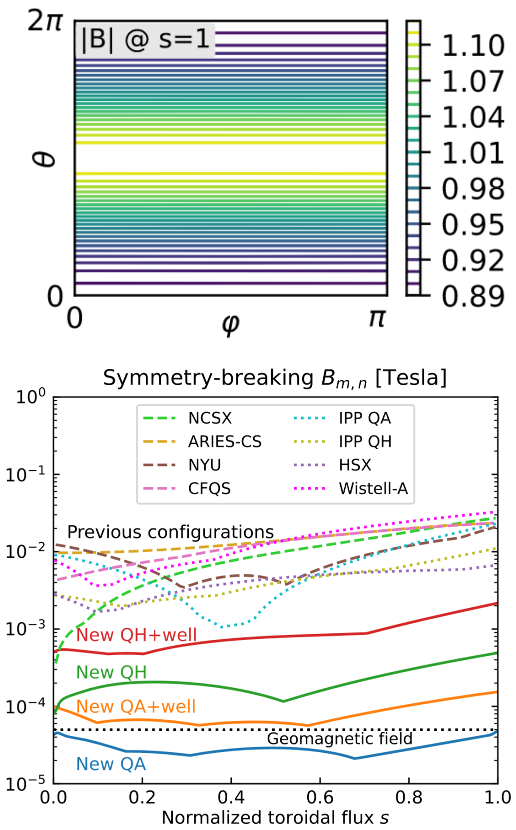
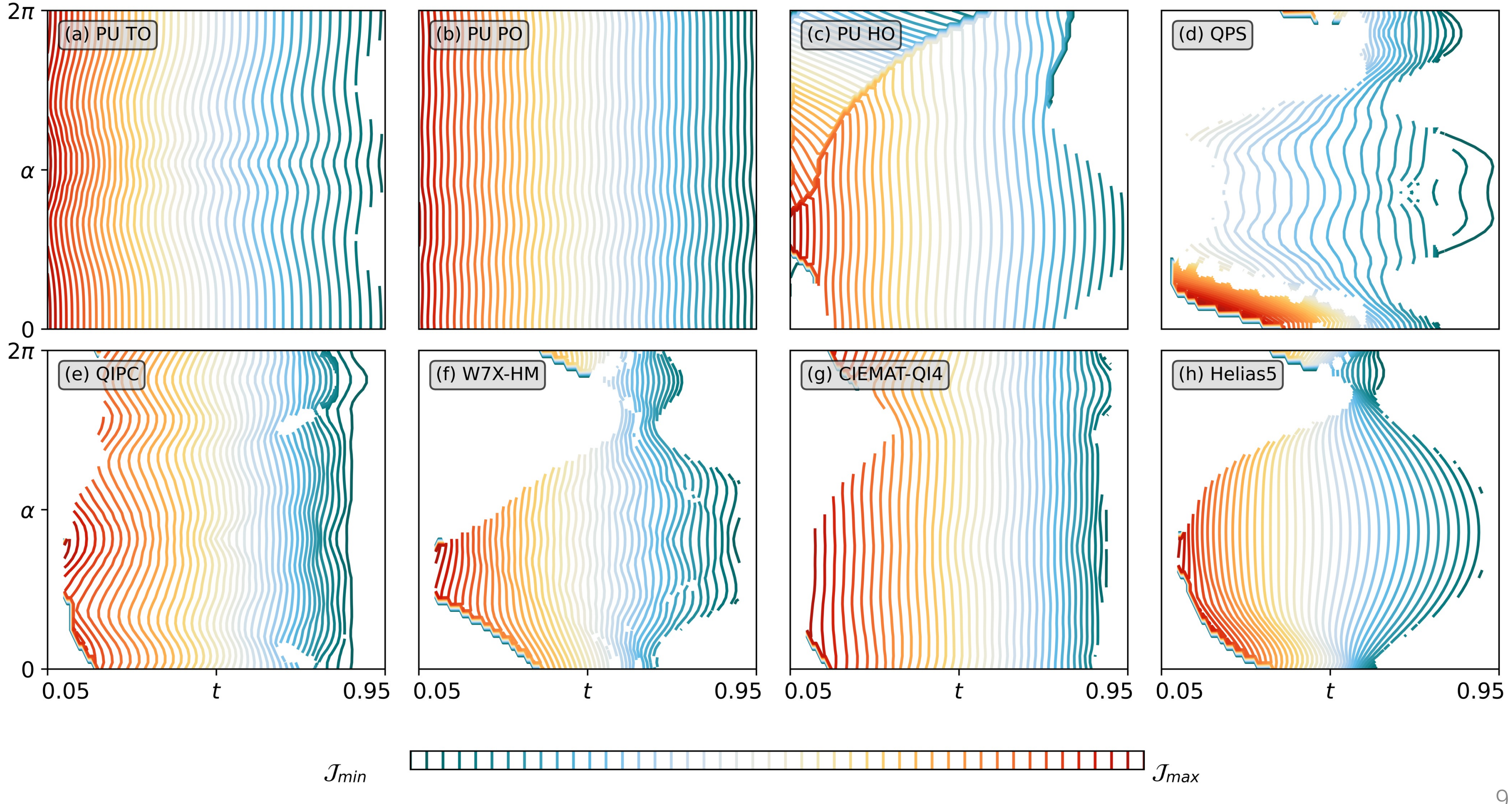
Optimizing non-QS omnigenity is not easy

- Quasisymmetry (QS) can be achieved at high accuracy by minimizing asymmetric modes.
- Non-QS omnigenity optimization is generally more difficult due to the lack of symmetry.



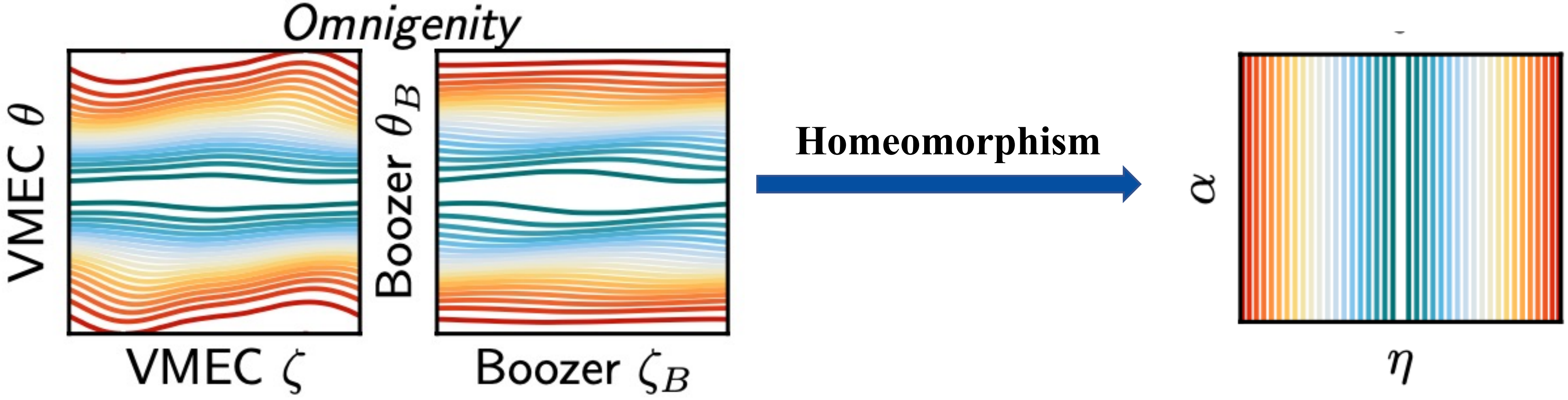
Precise QS configurations

[Landreman & Paul PRL 2021]



Homeomorphic coordinate mapping for omnigenity

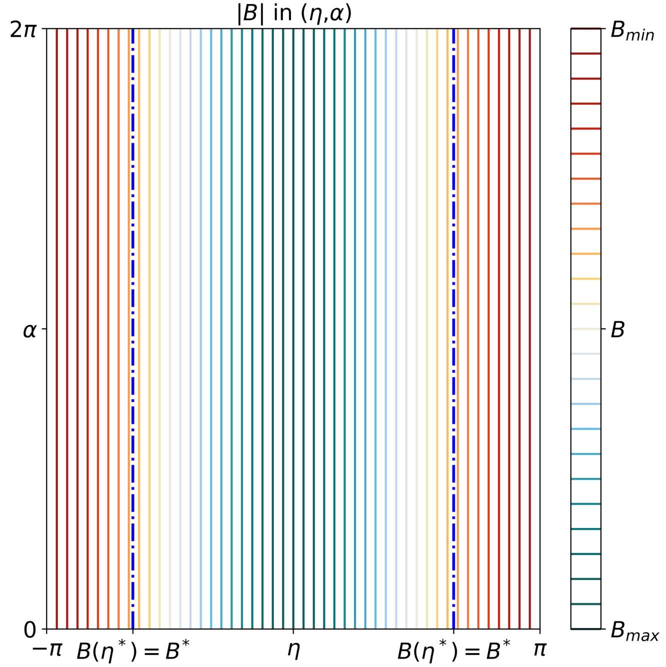
- Omnigenous fields have no locally closed contours.
- There exists homeomorphic coordinates where B contours are straight.



- For quasi-symmetry (QS), Boozer coordinates satisfy such conditions.
- For non-QS omnigenity, one have to find a new coordinate system that meets the requirements and **preserves omnigenity**.

Constructing omnigenous fields in Boozer coordinates is easier

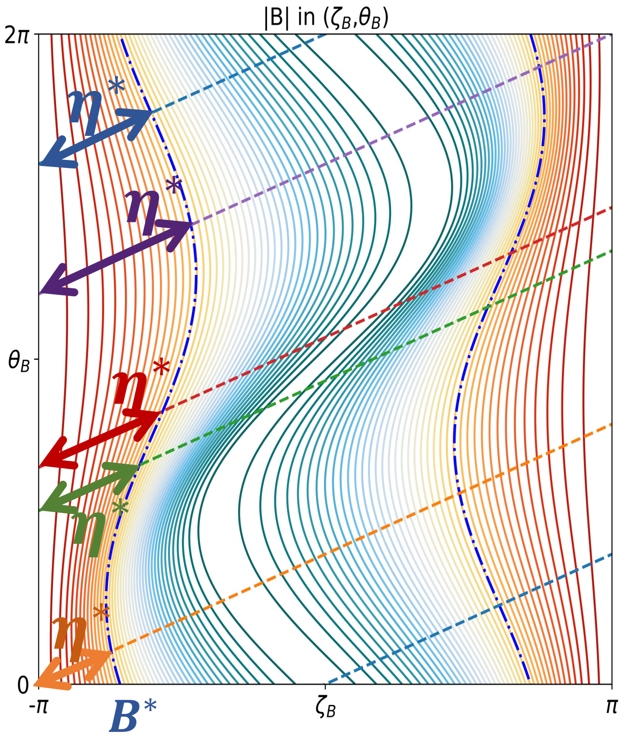
- Several mappings (Cary & Shasharina PoP 1997, Landreman & Catto PoP 2012, Dudt JPP 2024) have been proposed to construct omnigeneous fields.
  - ➢ **Omnigenity is preserved**
  - ➢ **Constant $\boldsymbol{\eta}$ is constant** B
- A similar mapping is applied here.



Preserve $\dfrac{\partial \tilde{J}(B^*, \alpha)}{\partial \alpha} = 0$

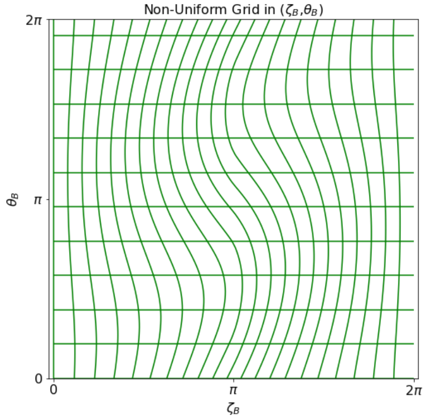
$$\tilde{\zeta}(\alpha, \eta) = \eta - D(\eta) \cdot S(\alpha, \eta)$$
$$\tilde{\theta} = \alpha$$

S controls the contour shape
D determines the bounce distance



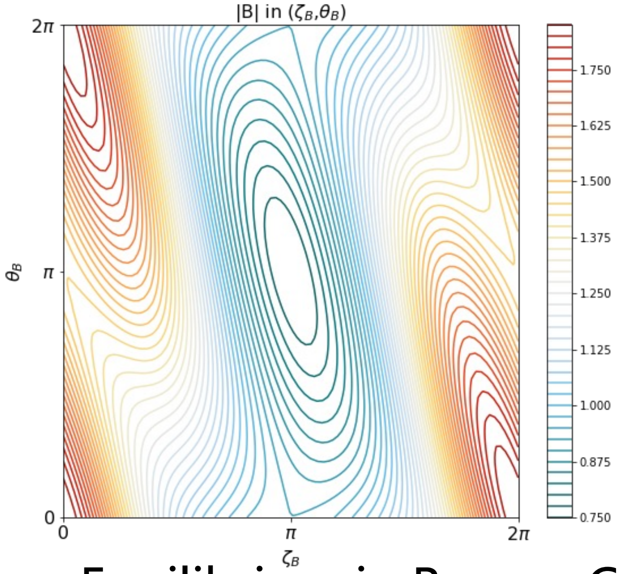
Omnigenity OPtimization like quasiSymmetry (OOPS)

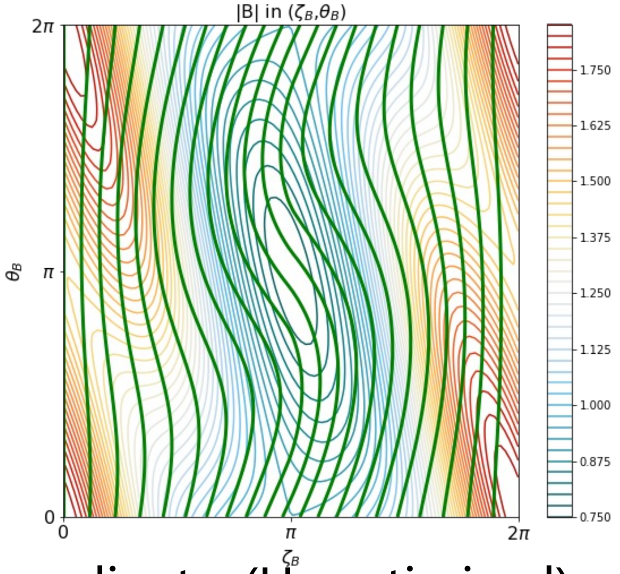
Pick up an **arbitrary** omnigenity mapping



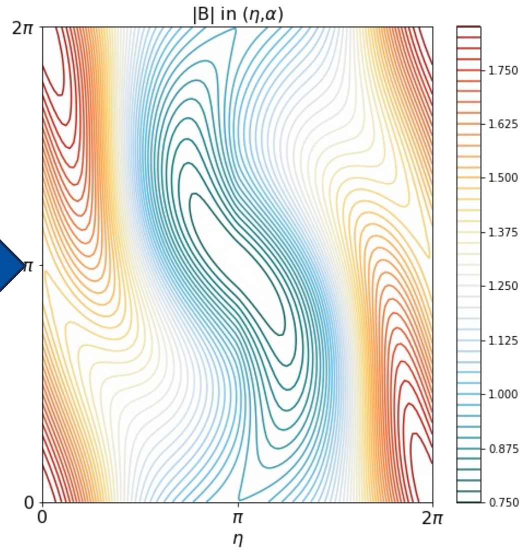
For quasisymmetry, we can use $\eta = \zeta_B, \alpha = \theta_B$

The omnigenity mapping can be varied during the iteration of optimization.
(For simplicity, we keep it fixed.)



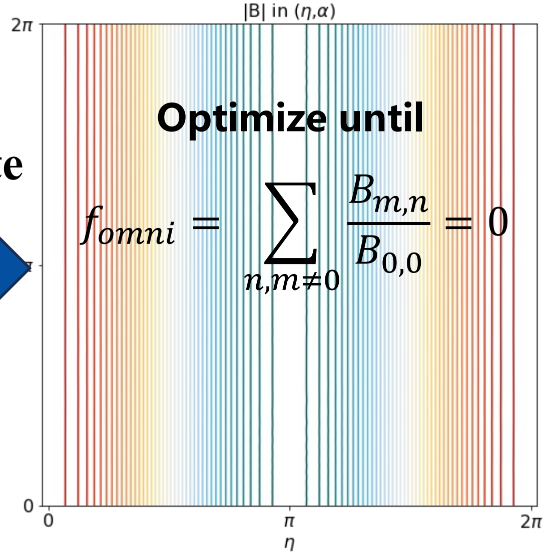


Equilibrium in Boozer Coordinates(Unoptimized)



Equilibrium in $(\boldsymbol{\eta}, \boldsymbol{\alpha})$

iterate



Optimize until

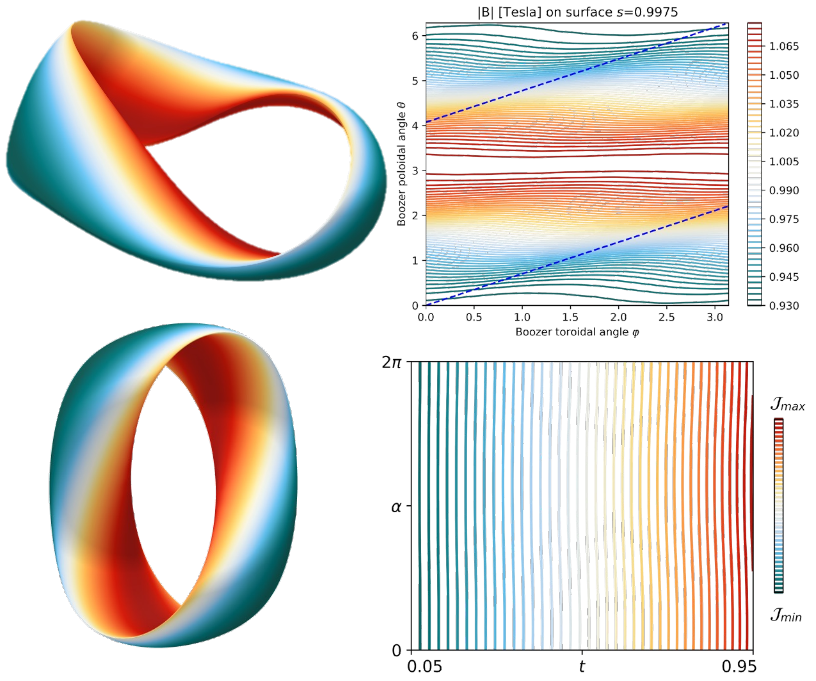
$$f_{omni} = \sum_{n,m\neq 0} \frac{B_{m,n}}{B_{0,0}} = 0$$

Target

Precise, compact omnigenous configurations have been obtained

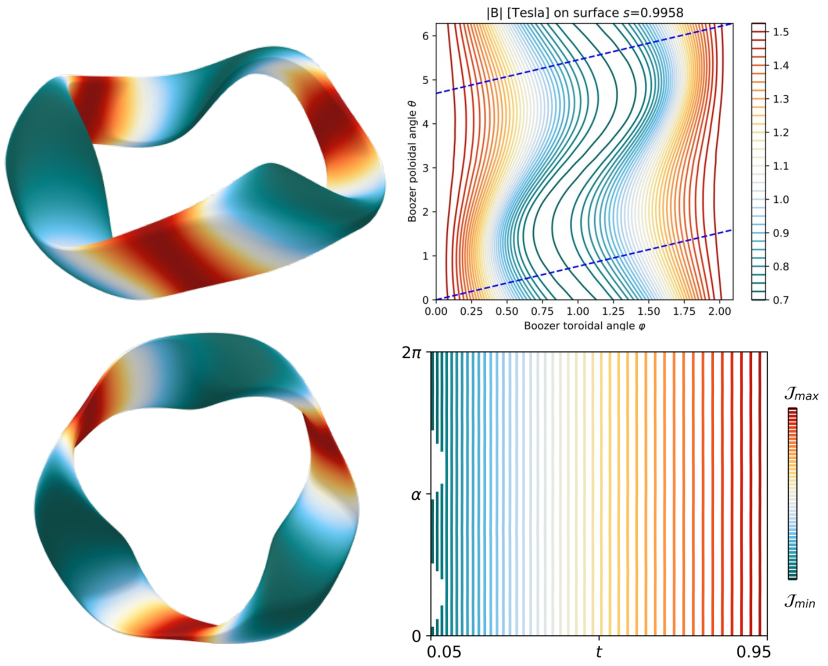
Toroidal Omnigenity (TO)

Nfp=2 Ap=6 $\iota_{edge} = 0.711$



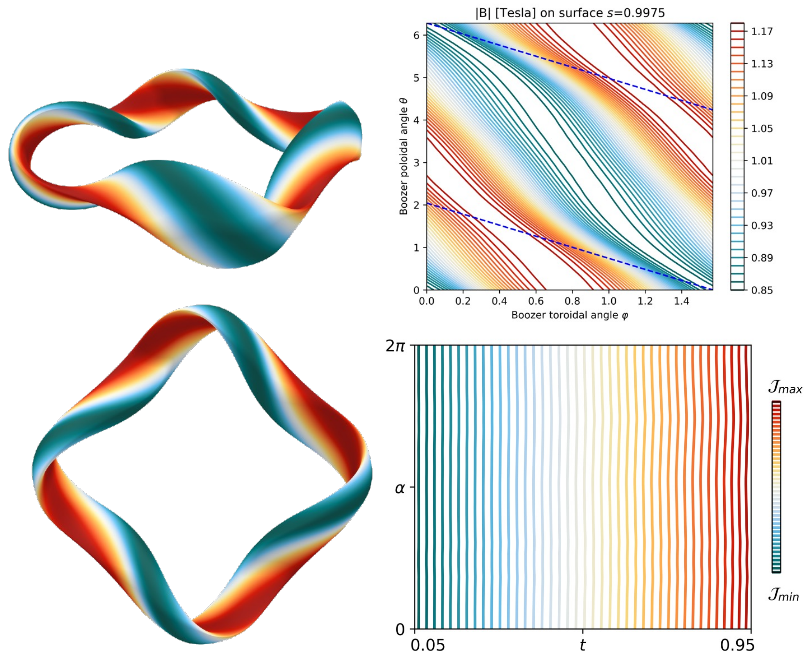
Poloidal Omnigenity (PO)

Nfp=3 Ap=6.5 $\iota_{edge} = 0.765$



Helical Omnigenity (HO)

Nfp=4 Ap=8 $\iota_{edge} = 1.326$

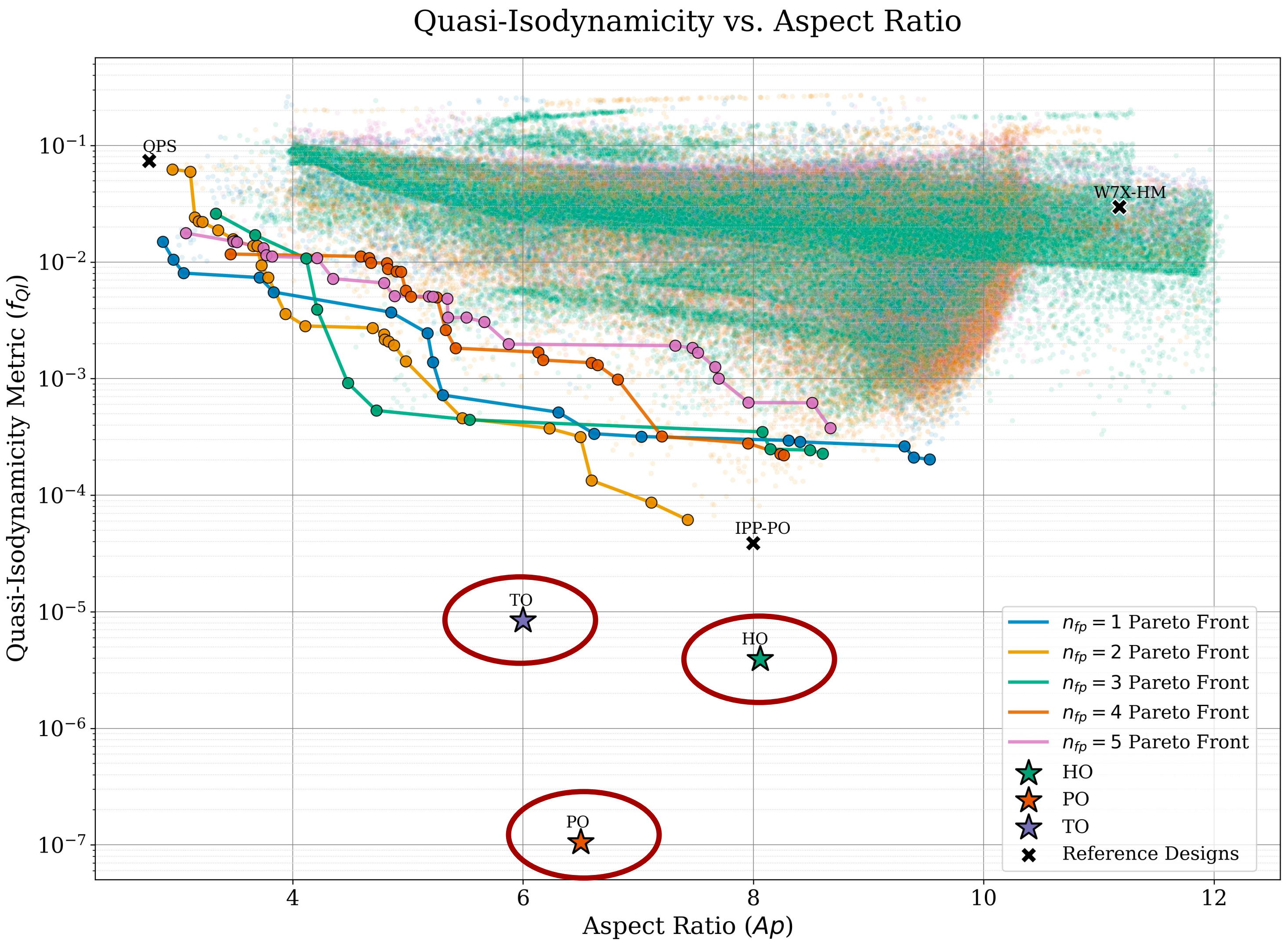


- **No warm starts needed. Initial guesses are circular torus.**
- **Simplest mapping is used. (One single Fourier coefficient)**
- **Alpha-particle losses at reactor size < 1%.**

Beyond the Pareto fronts of the "Constellaration" database

- Proxima Fusion recently released the "Constellaration" database [Cadena arXiv 2025].
- 158, 000 QI configurations were optimized using existing approaches.
- Pareto fronts of the QI-metric (f_{QI}) and aspect ratio (Ap) for each period are identified.

$$f_{QI} = \frac{1}{4\pi^2} \iint (B(\theta,\phi) - B^{QI}(\theta,\phi))^2 d\theta d\phi$$



Piecewise omnigenous fields: a radically new family of optimized magnetic fields for stellarator reactors

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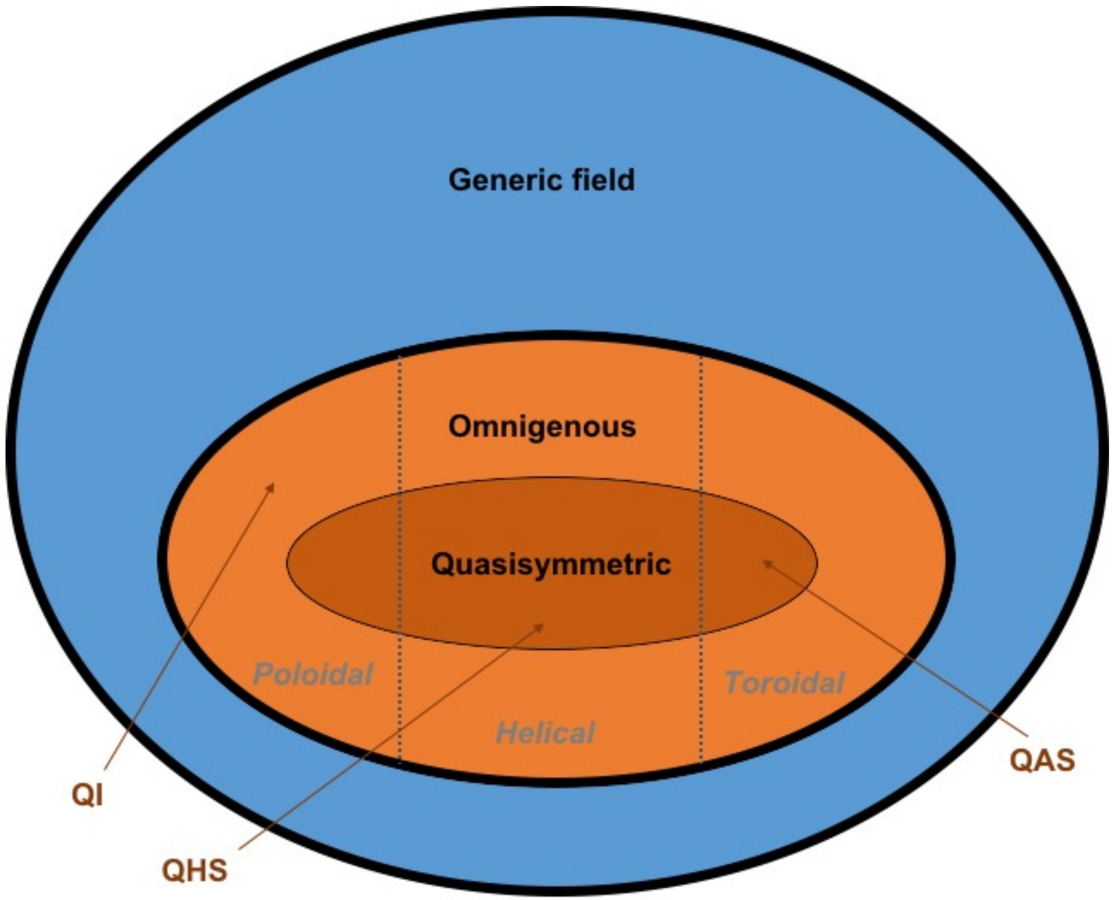
Are there any other $B(\theta, \zeta)$ that confine orbits perfectly?



Piecewise omnigenity generalizes omnigenity:

- Motivation: fulfilling constraints of omnigenity may lead to complicated plasma shapes and coils, and to fields sensitive to design errors.
- Goal: **remove constraint** that all contours of constant B close in the toroidal, poloidal or helical direction while **keeping collisionless confinement of orbits.**

If no global direction of symmetry, **broader region of configuration space.**

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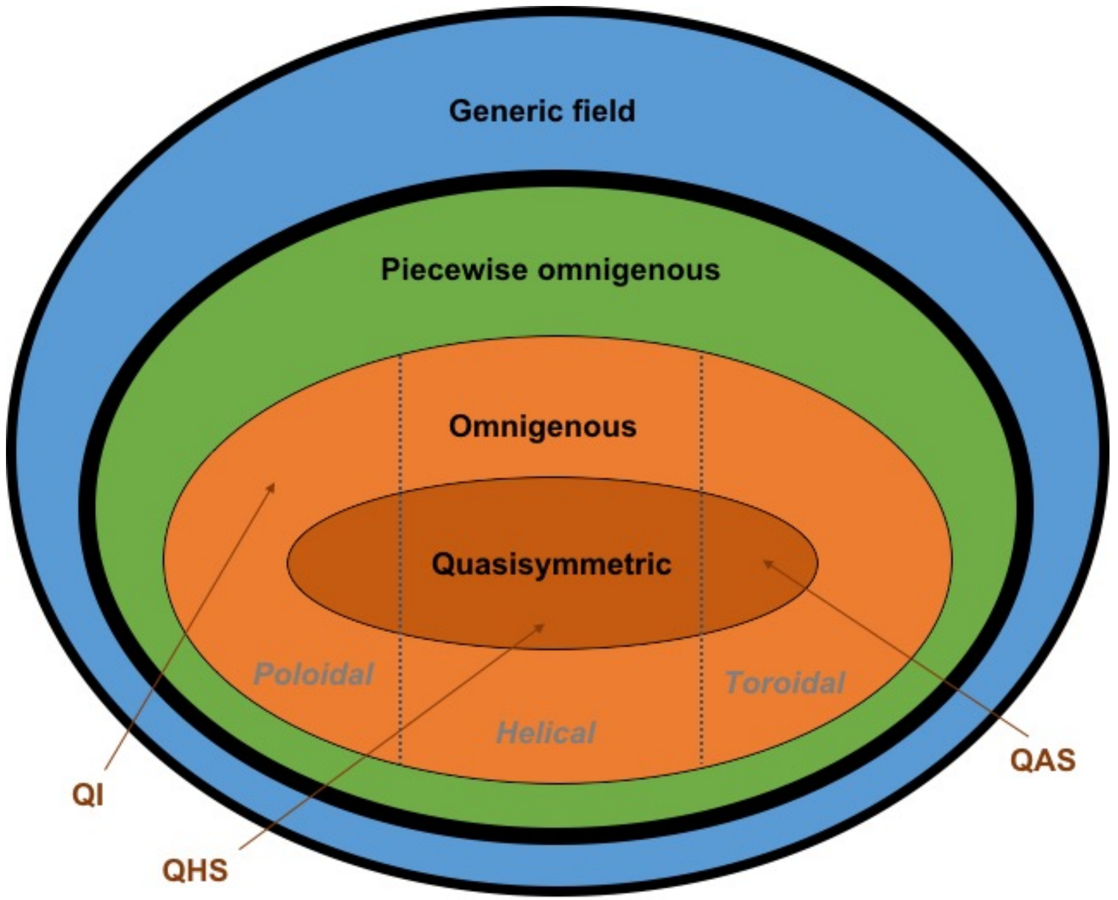

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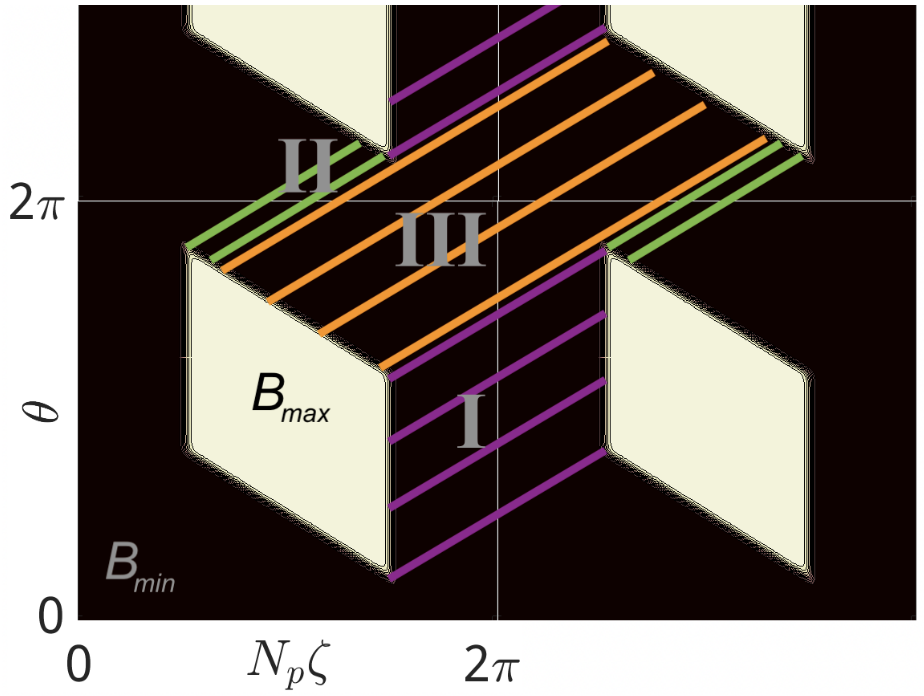
If no global direction of symmetry, **broader region of configuration space.**

Piecewise omnigenity: example of model field [Velasco (2024) PRL]

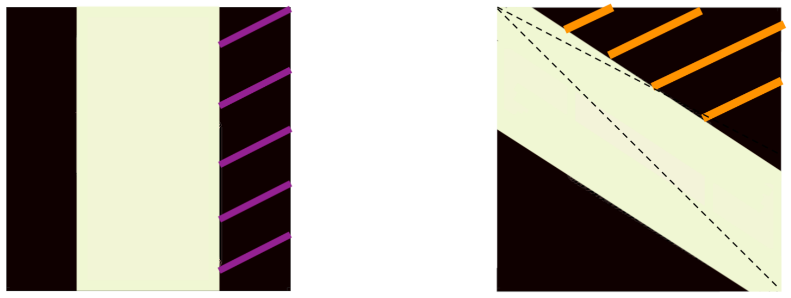
Example of piecewise omnigenous (pwO) field:



- $B(\theta,\zeta) = B_{\max}$ inside a parallelogram.
- $B(\theta,\zeta) = B_{\min}$ outside.
- ι such that corners are connected by field lines.

- **Several regions / classes of orbits:**
  - Purple orbits *locally see* an omnigenous field.
  - Orange (and green) see a *different* omnigenous field.

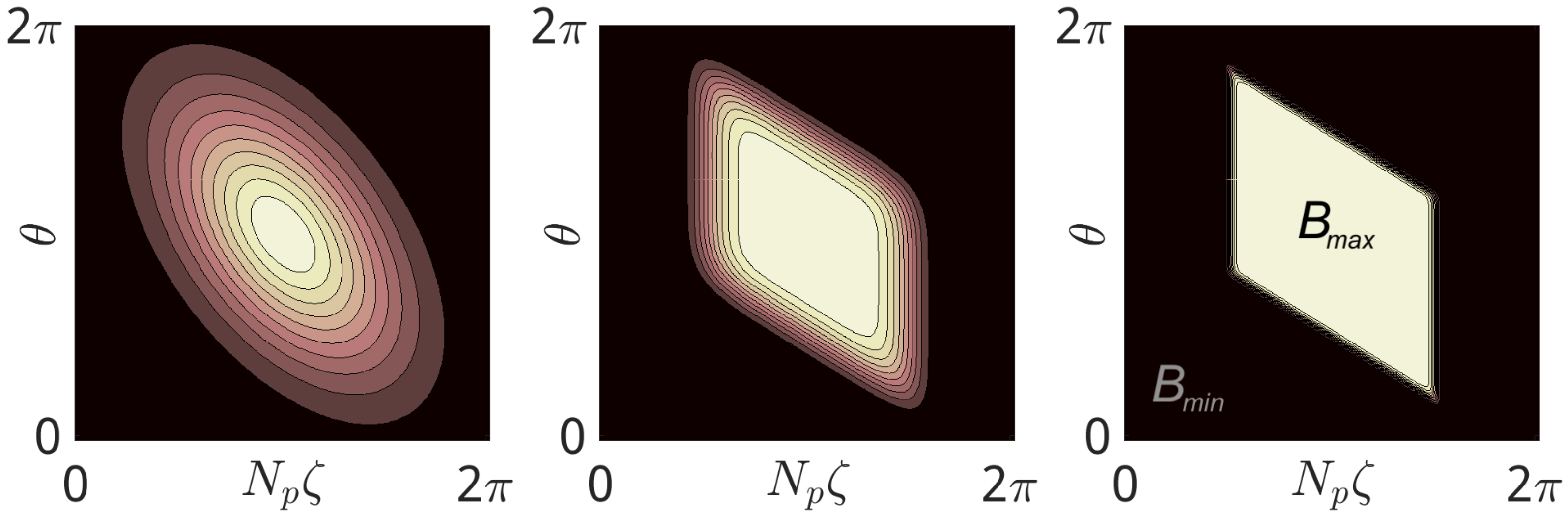
  B-contours not necessarily aligned with $M\theta - N_p N\zeta$.



- $\overline{\mathbf{v_M}\cdot\nabla s} = 0$ **for all classes of orbits.**
- *Junctures* between regions I, II and III do not cause $1/\nu$ transport.

Tokamak-like neoclassical transport ($\varepsilon_{eff} = 0$).

Piecewise omnigenity as a target for a stellarator reactor design

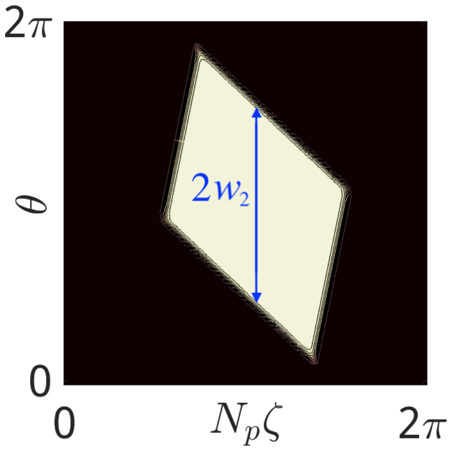


($B(\theta, \zeta)$ from left to right: from less to more piecewise omnigenous model fields).

Neoclassical transport compatible with a reactor requires only *approximate* piecewise omnigenity.

A stellarator reactor requires a divertor:

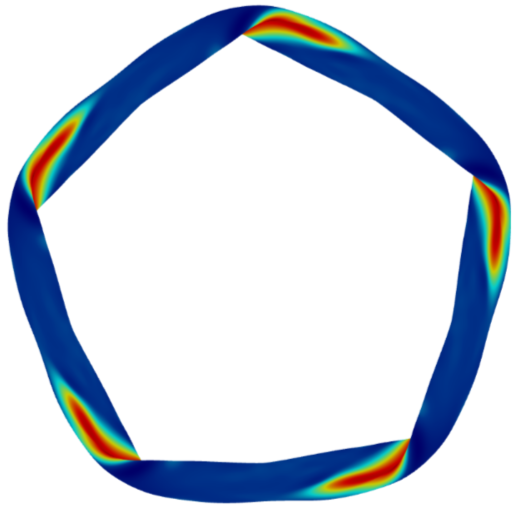
- The most mature concept, the island divertor, was thought to be compatible only with QI fields.
- A subset of pwO fields ($w_2 = \pi$) have reduced bootstrap current, a requirement of an island divertor [Calvo (2025) PRE].

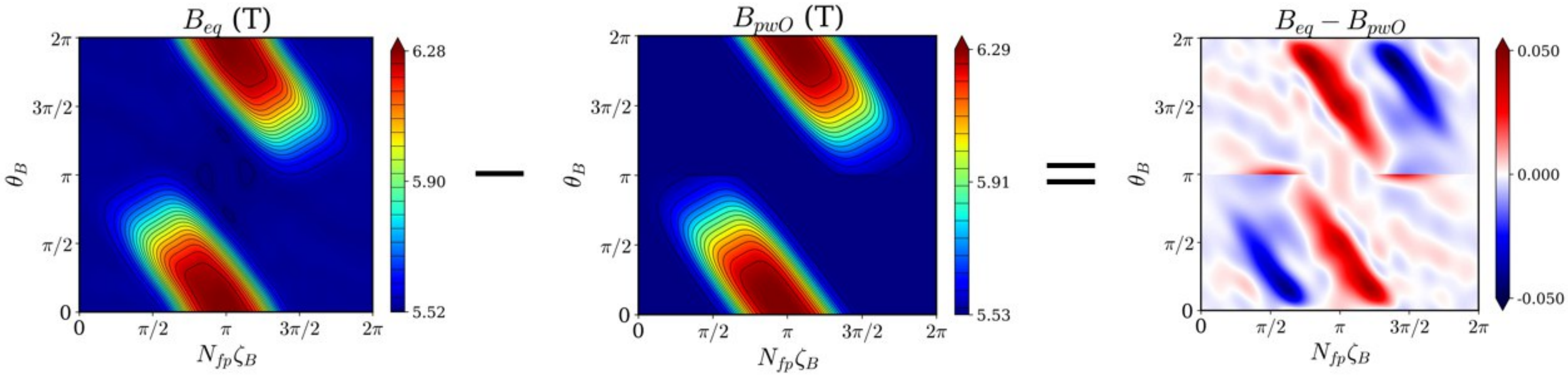


How close to piecewise omnigenity can a magnetic configuration be?

Master Thesis of Víctor Fernández-Pacheco, paper in preparation.

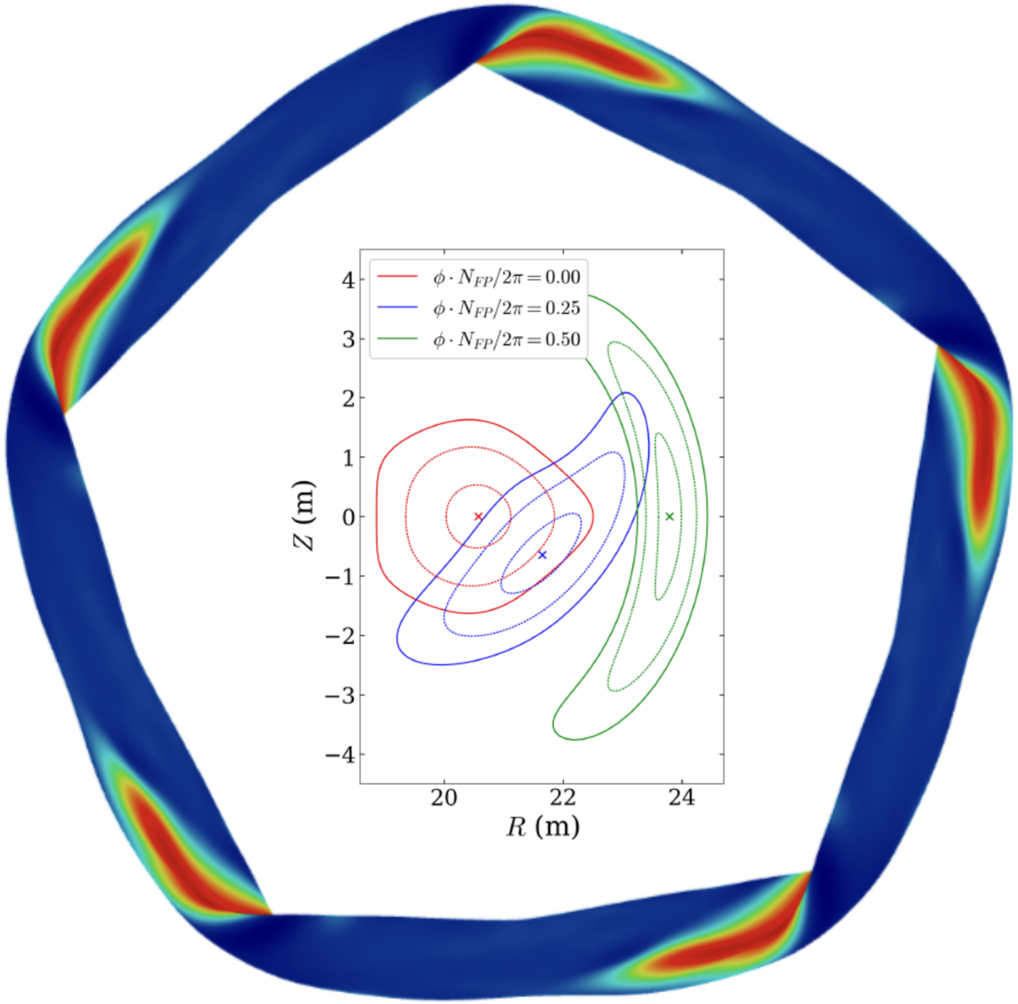
- Optimization w.r.t. piecewise omnigenity implemented in `DESC`.
- Goal of the work: **push the boundaries of the concept** of pwO.
  1. Select family of B_{pwO} as far as possible from omnigenity.
  2. Make B approach B_{pwO} on a flux-surface as much as possible.
     - In a reactor candidate, we can (and we will) relax 1 and 2.
  3. Check that MHD equilibria exist with reasonable shapes and basic properties.

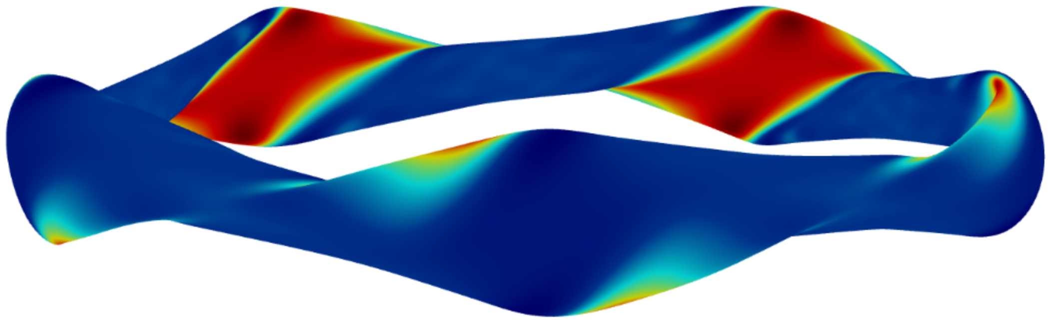




Equilibria with $|B - B_{pwO}|/B_{pwO} < 1\,\%$ (and likely smaller) can be found.

CIEMAT-pw1: reactor-relevant physics properties [Fernández-Pacheco]





- $N_p = 5$, $A = 12.7$, $\kappa = 6.7$
- Mercier MHD stability.
- $4/5 < \iota < 5/5 \Rightarrow$ no low order rationals, compatible with island divertor.
- Reduced neoclassical transport ($\varepsilon_{eff} < 0.005$).
- Reduced bootstrap current ($\Delta\iota_{LCFS} < 1\,\%$).
- Good fast ion confinement at reactor β ($> 95\,\%$ alpha-heating efficiency, most losses for $t > 0.01\,$s).

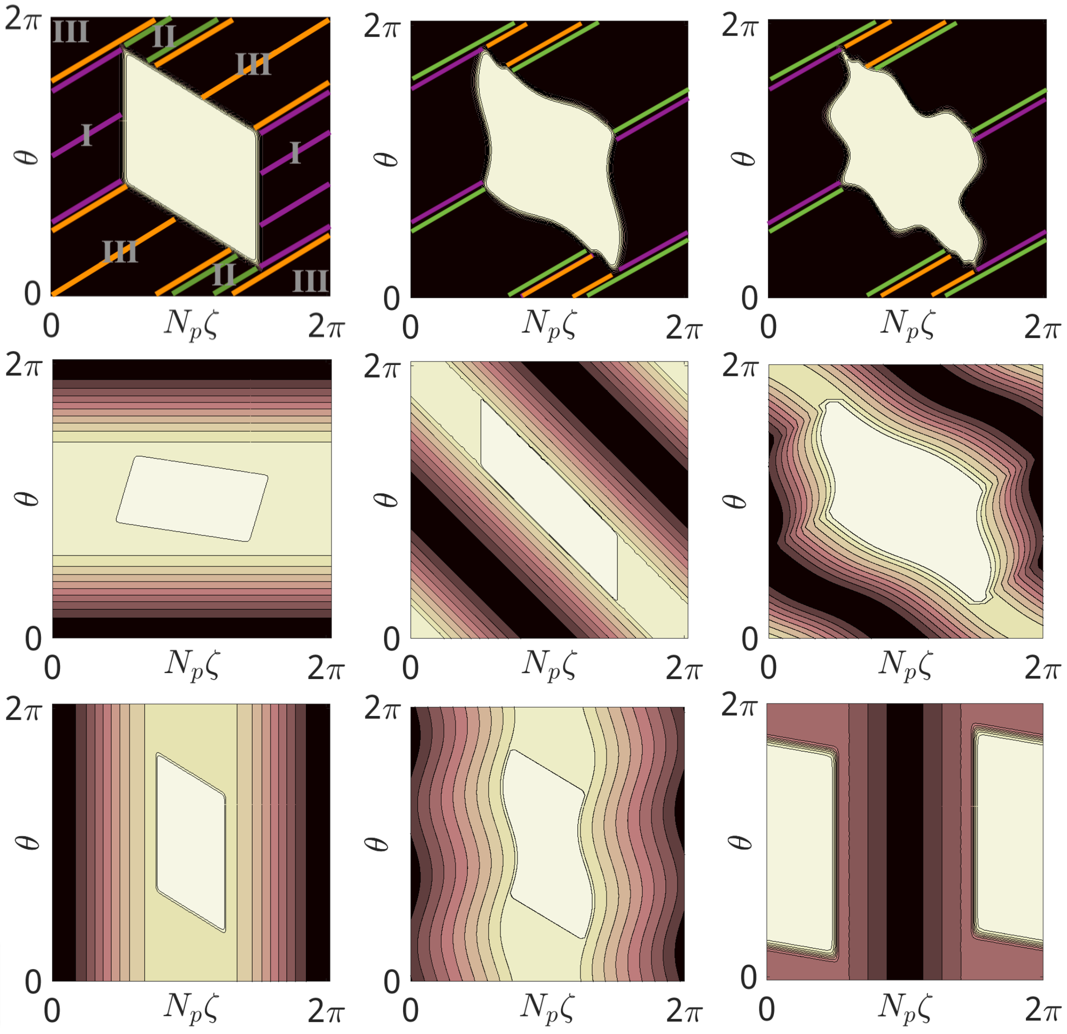
Physics properties compatible with a reactor.

Piecewise omnigenity: vast region of stellarator conf. space [Velasco (2025) NF]

Different types of model fields proposed:

- Fields **completely away from omnigenity** (as in [Velasco (2024) PRL]).
  - Reactor-relevant configurations exist.
- Fields that combine omnigenity with piecewise omnigenity.
  - Deeply and barely trapped particles would behave differently.
- Special relevance: fields with (small) **deviations from QI** that cause negligible neoclassical radial and parallel transport [Calvo (2025) PRE, Velasco, in preparation].
  - $B \approx B_{min}$-contours: closed poloidally
  - $B \approx B_{max}$-contours: parallelogram shape.

The best reactor candidate could lie somewhere in between.

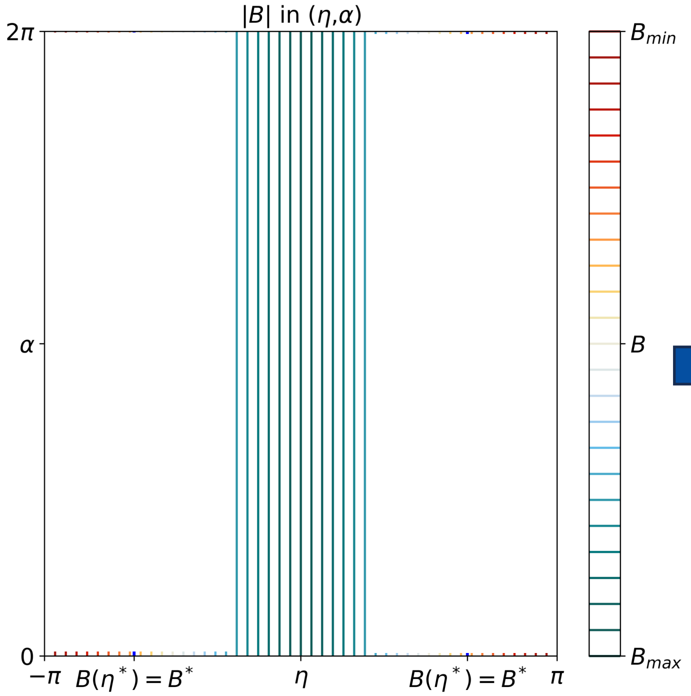


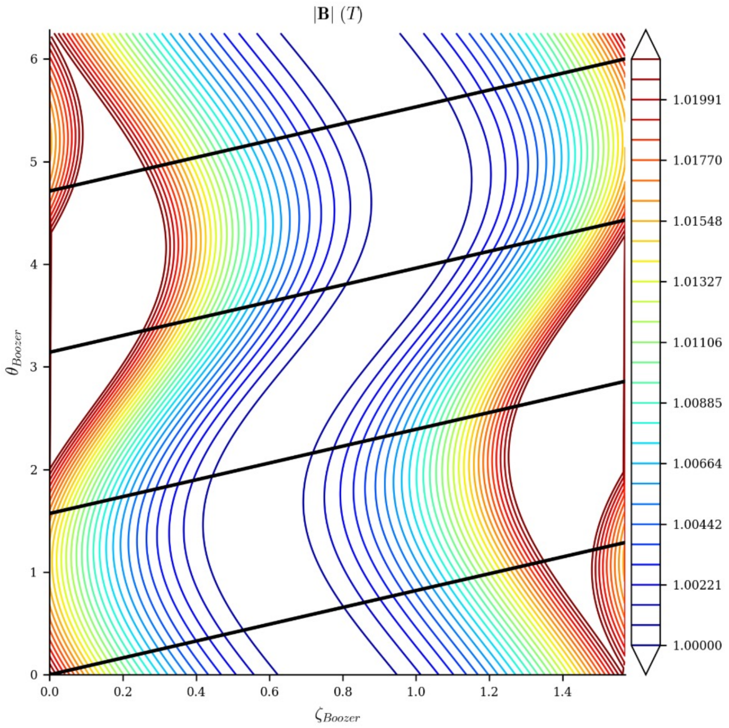
Configurations Combining Omnigenity and Piecewise Omnigenity

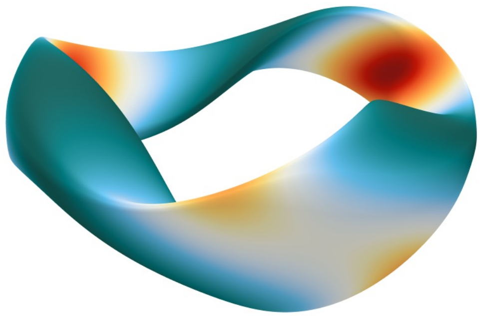
Optimizing pwO using OOPS

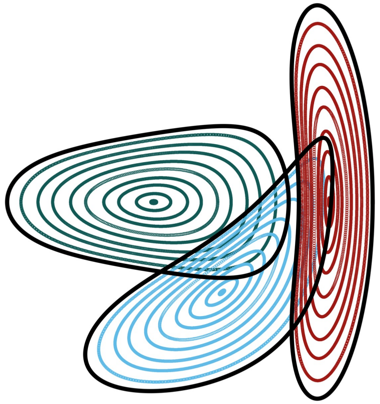
pwO configurations have locally closed contours, however:

- OOPS can still work if we **bound $\boldsymbol{\eta}$** and focus out of the Bmax region.
- B contours have to be **strongly shaped** such that $B_{\max}$ regions can close and form “parallelograms”.



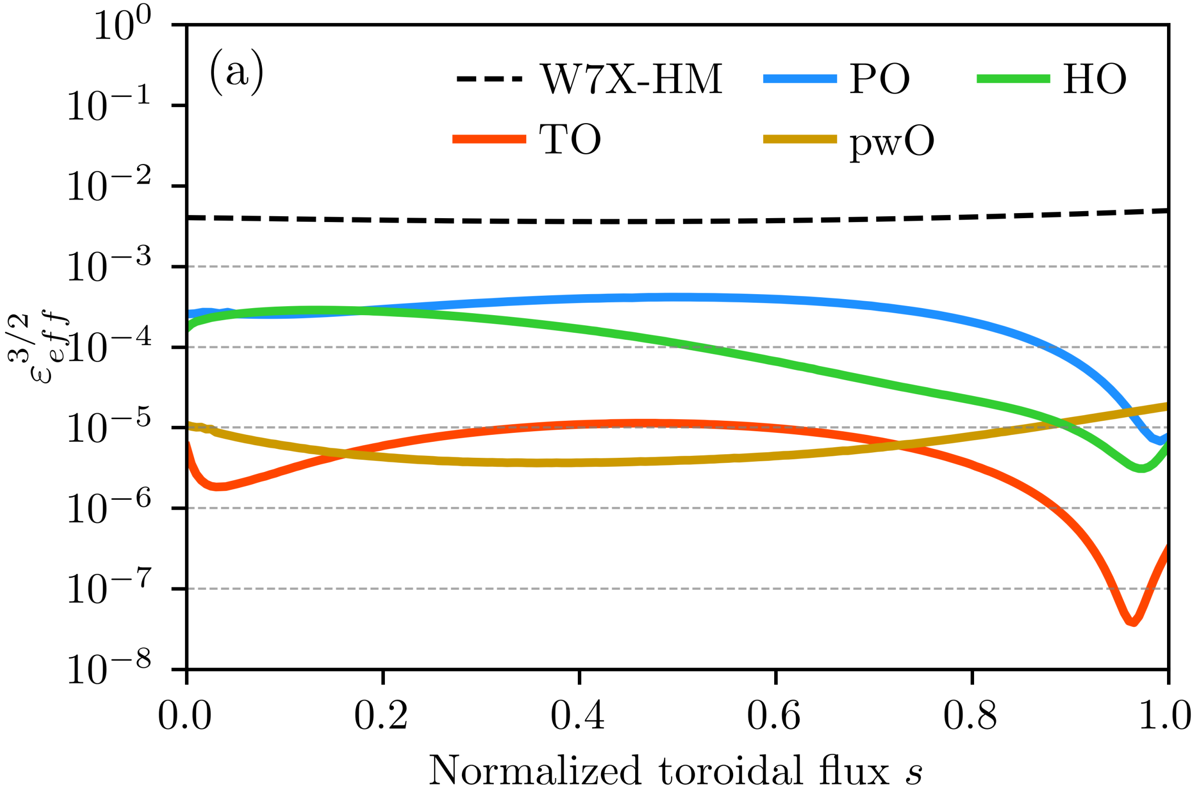




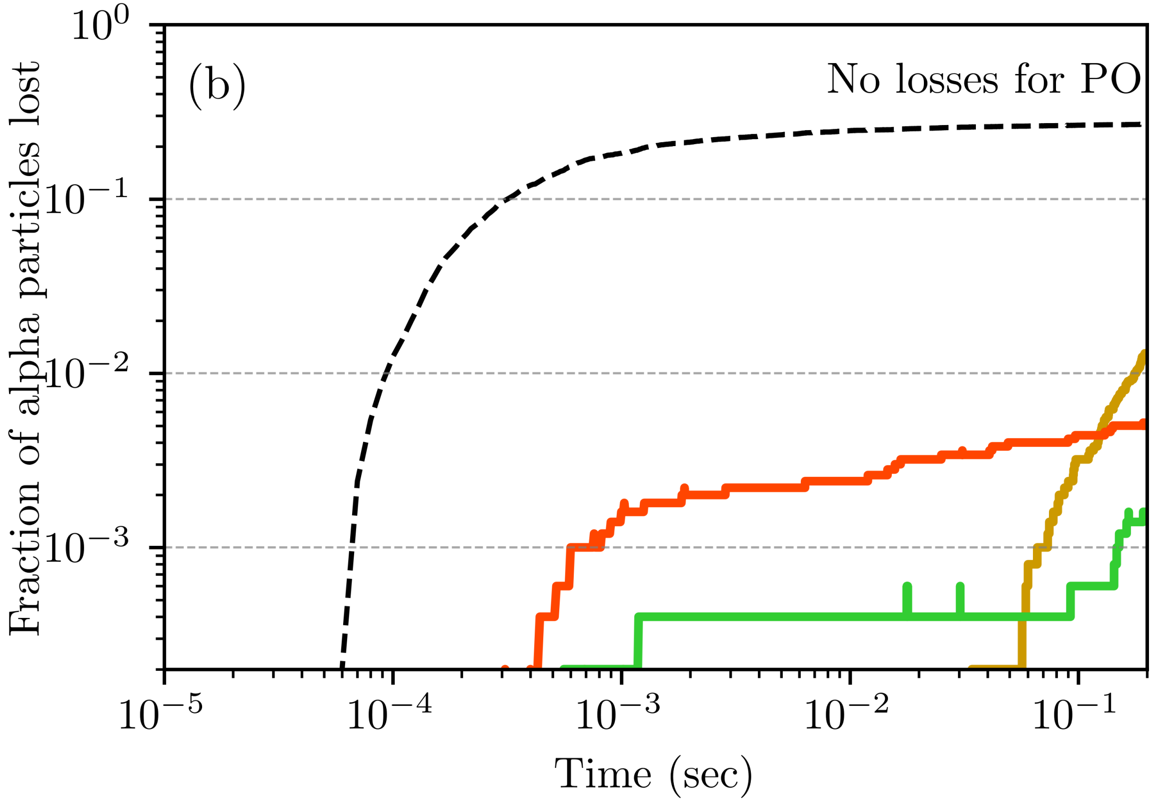


Good confinement properties have been achieved

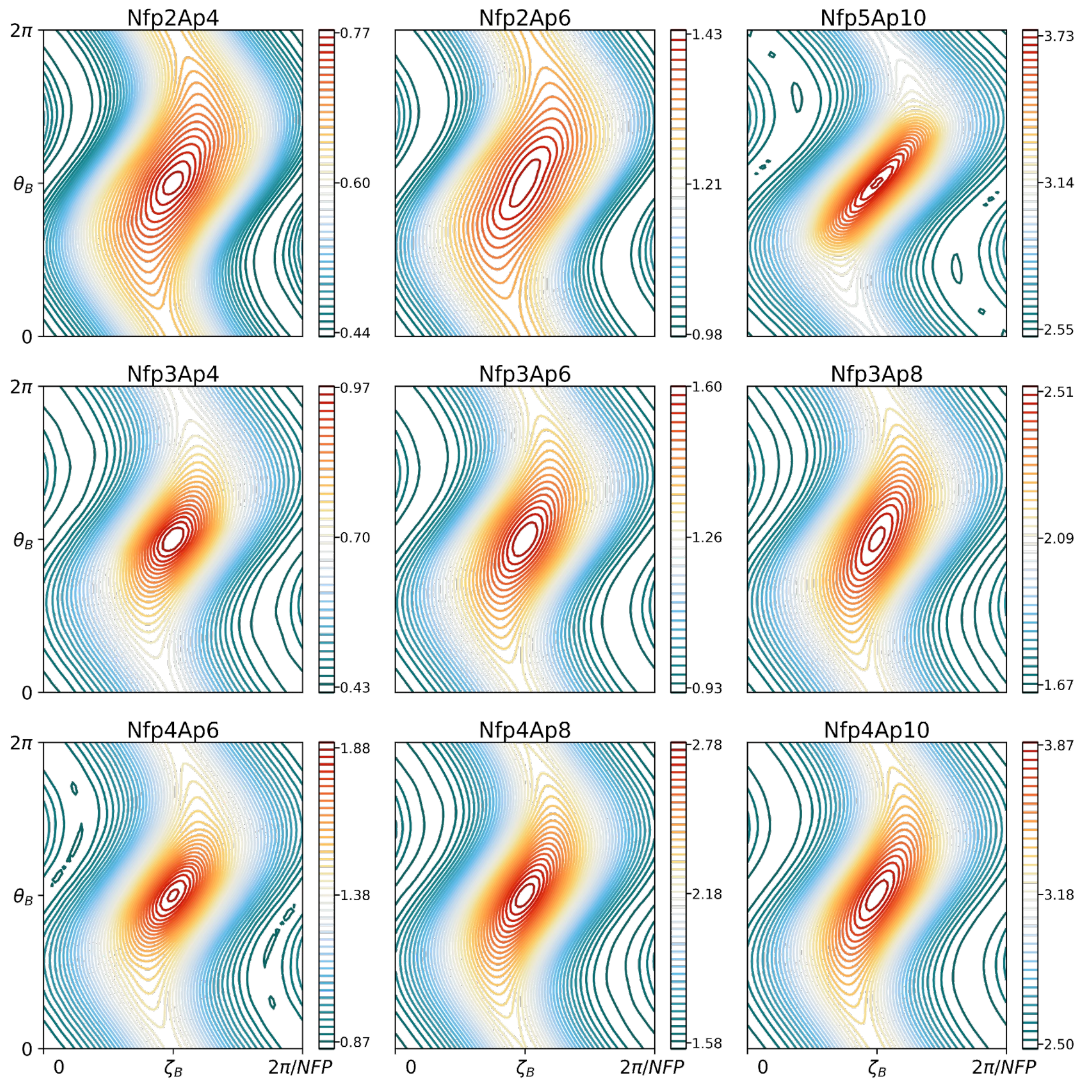
Effiective Ripple $\epsilon_{eff}^{3/2}$

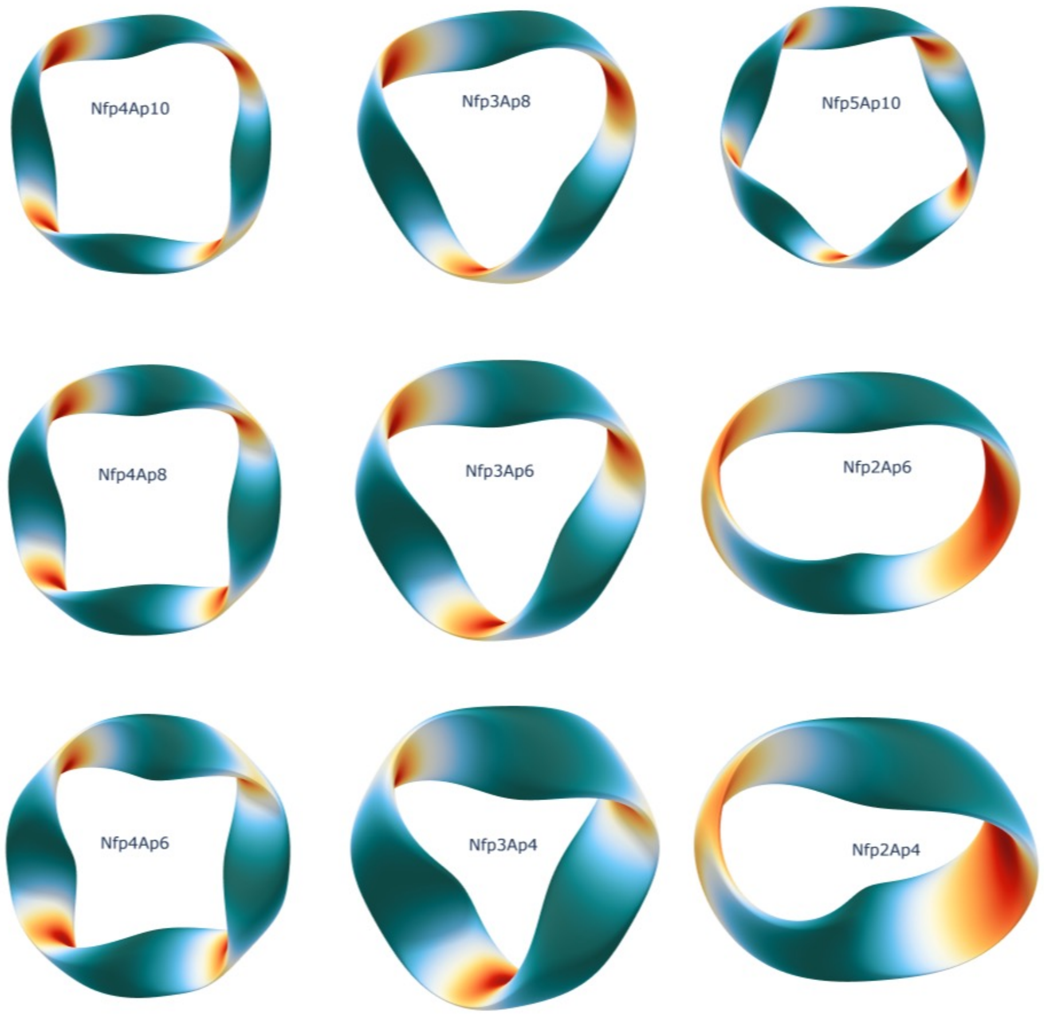


Loss Fraction of Collisionless 3.5 MeV α-Particles



More QI+pwO configurations are available





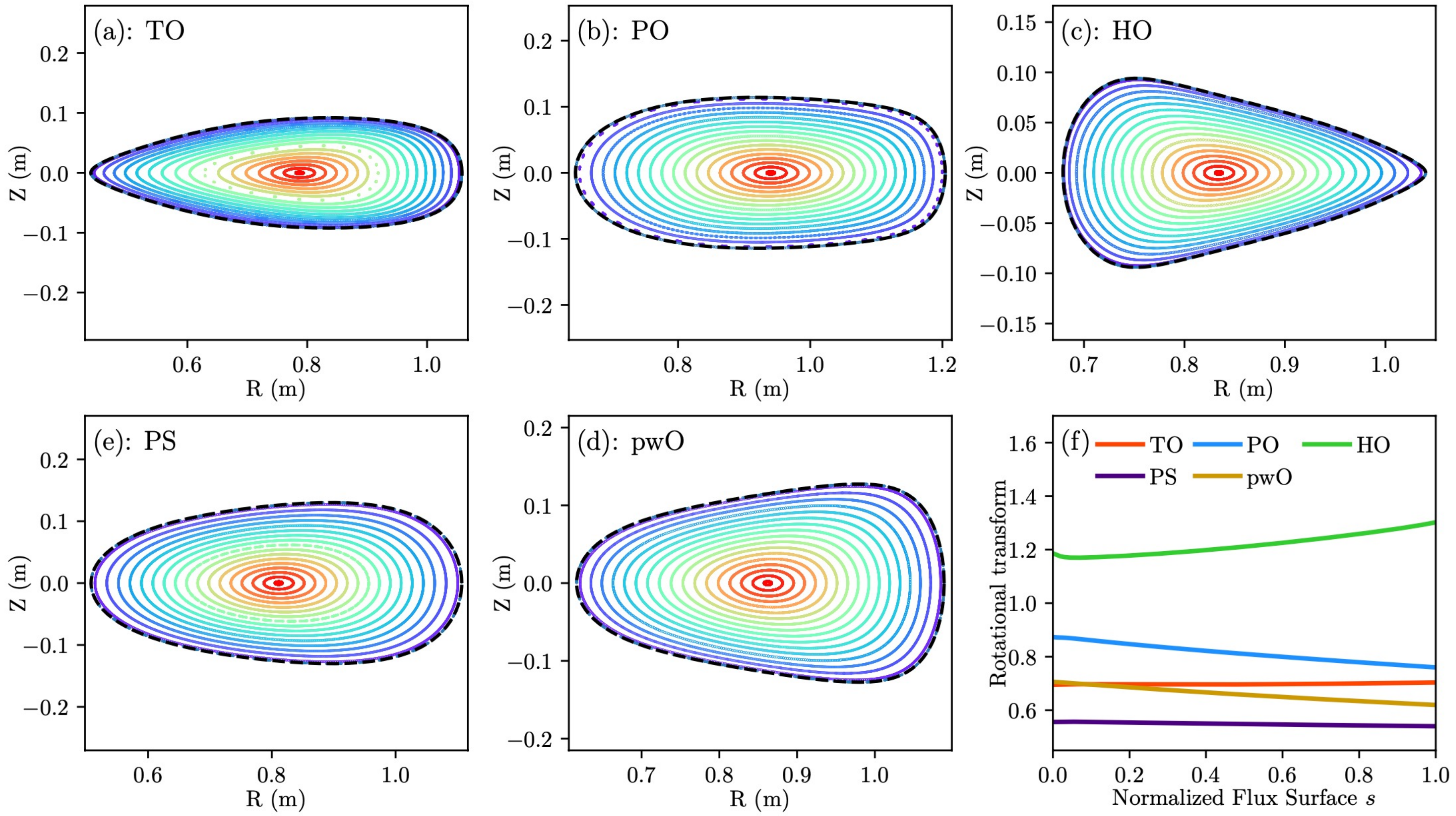
Summary

- A new method, OOPS, has been proposed to optimize omnigenity. Precise, compact omnigenous configurations have been obtained.
- Piecewise omnigenity radically broadens the parameter space of optimized stellarators.
- Various configurations that combine omnigenity and piecewise omnigenity have been obtained using OOPS, with the potential to achieve more compact configurations and/or to use simpler coils while keeping good confinement.

Thanks for your attention! **Both posters available on Saturday morning**

Thank you for your attention

Poincare plots and iota profiles



Omnigenity level

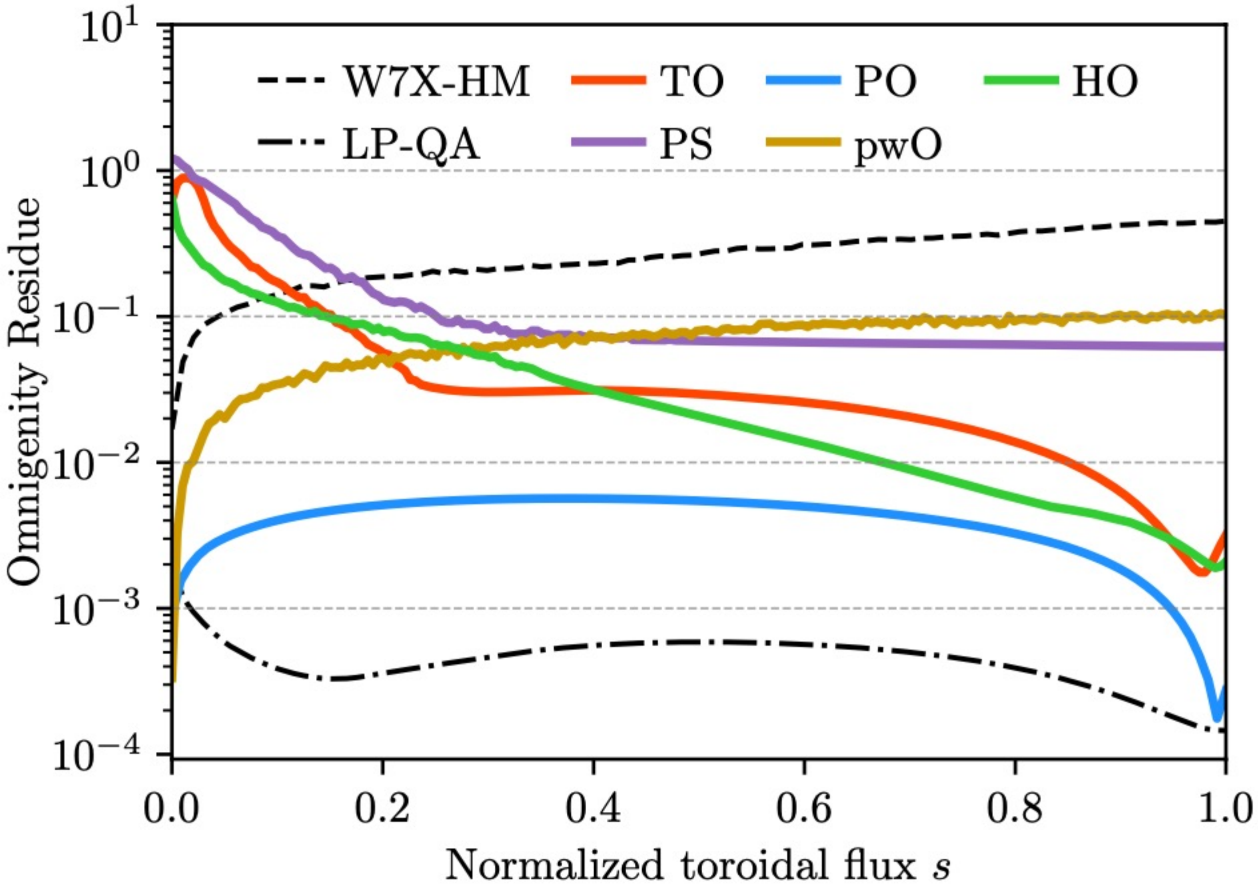
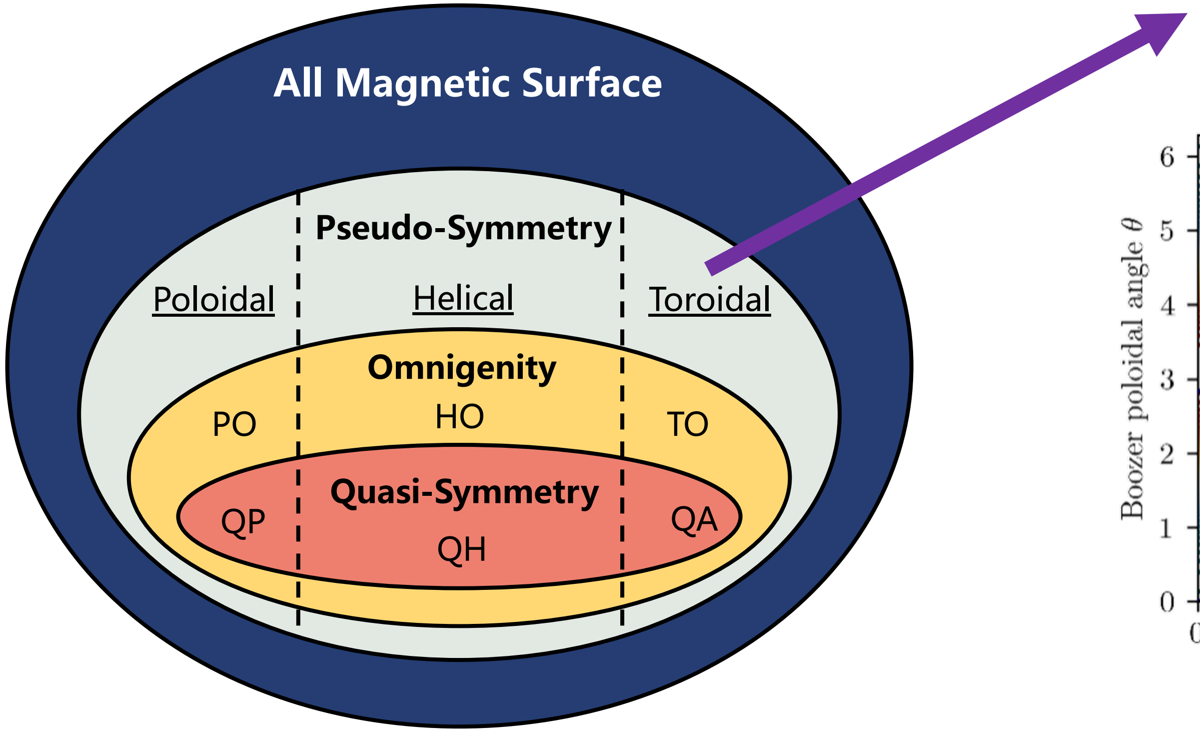


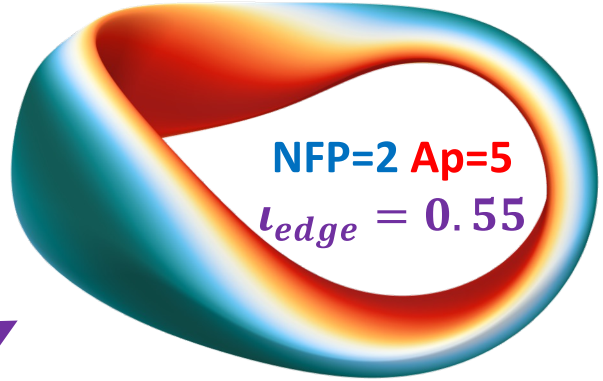
FIG. 3: Radial profiles of the average $\tilde{\mathcal{J}}^{-1}\partial\tilde{\mathcal{J}}/\partial\alpha$ on each flux surface for measuring the omnigenity residue.

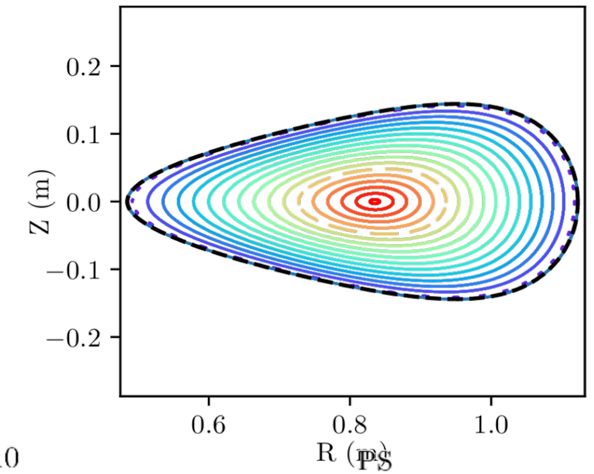
Pseudo-symmetry can also be easily optimized

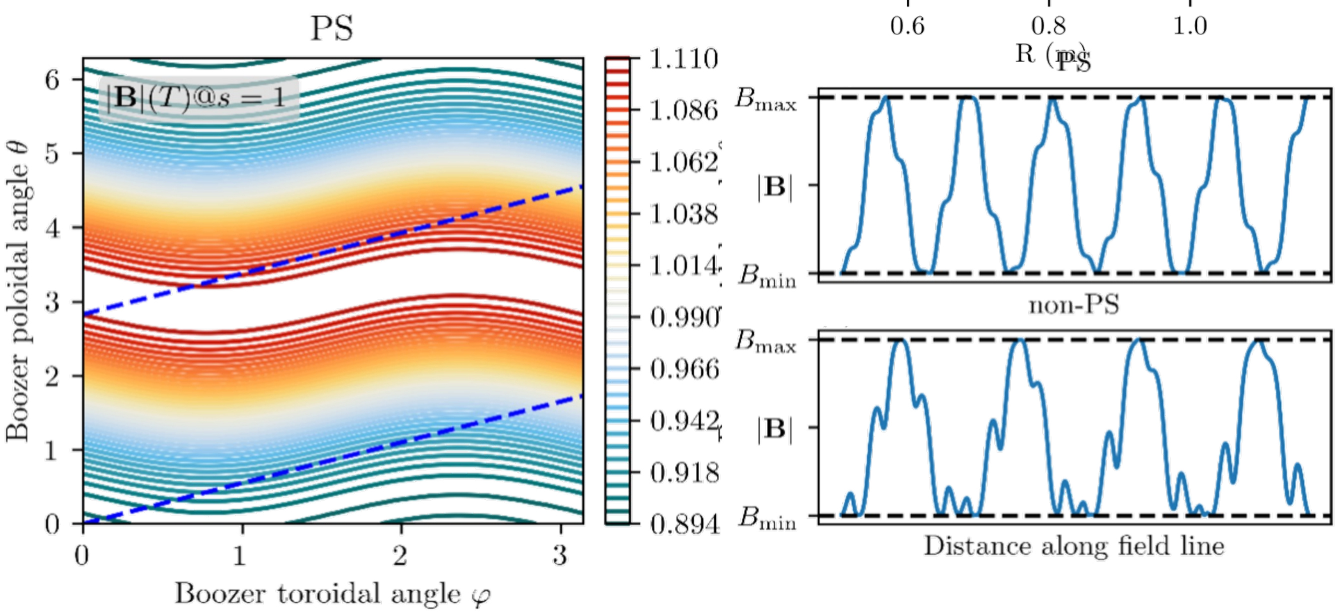
Pseudo-symmetry (PS): Magnetic configurations with no locally trapped particles

- Relax constraint of constant bounce distance
- B has no locally closed contours
- Field lines do not cross B contours twice

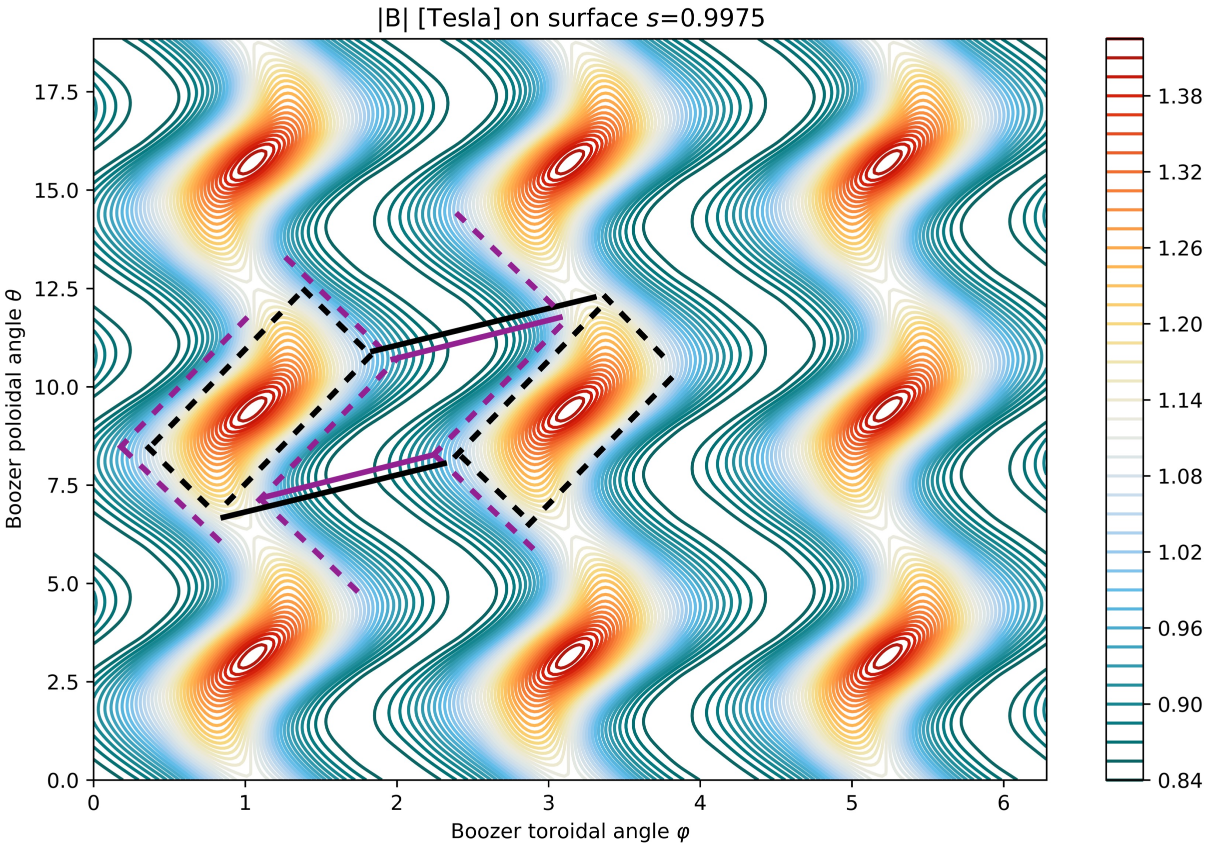






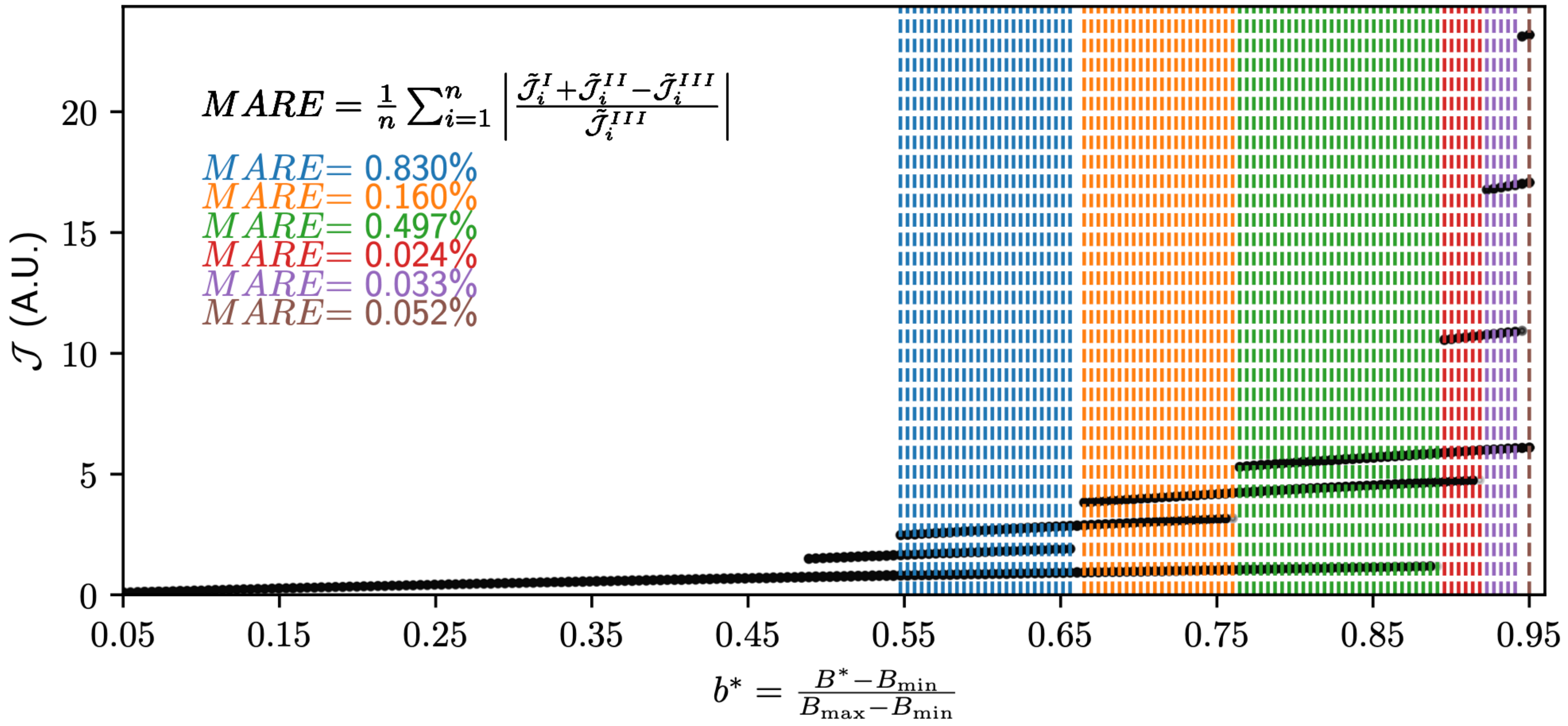


OOPS-pwO3 has the pwO features



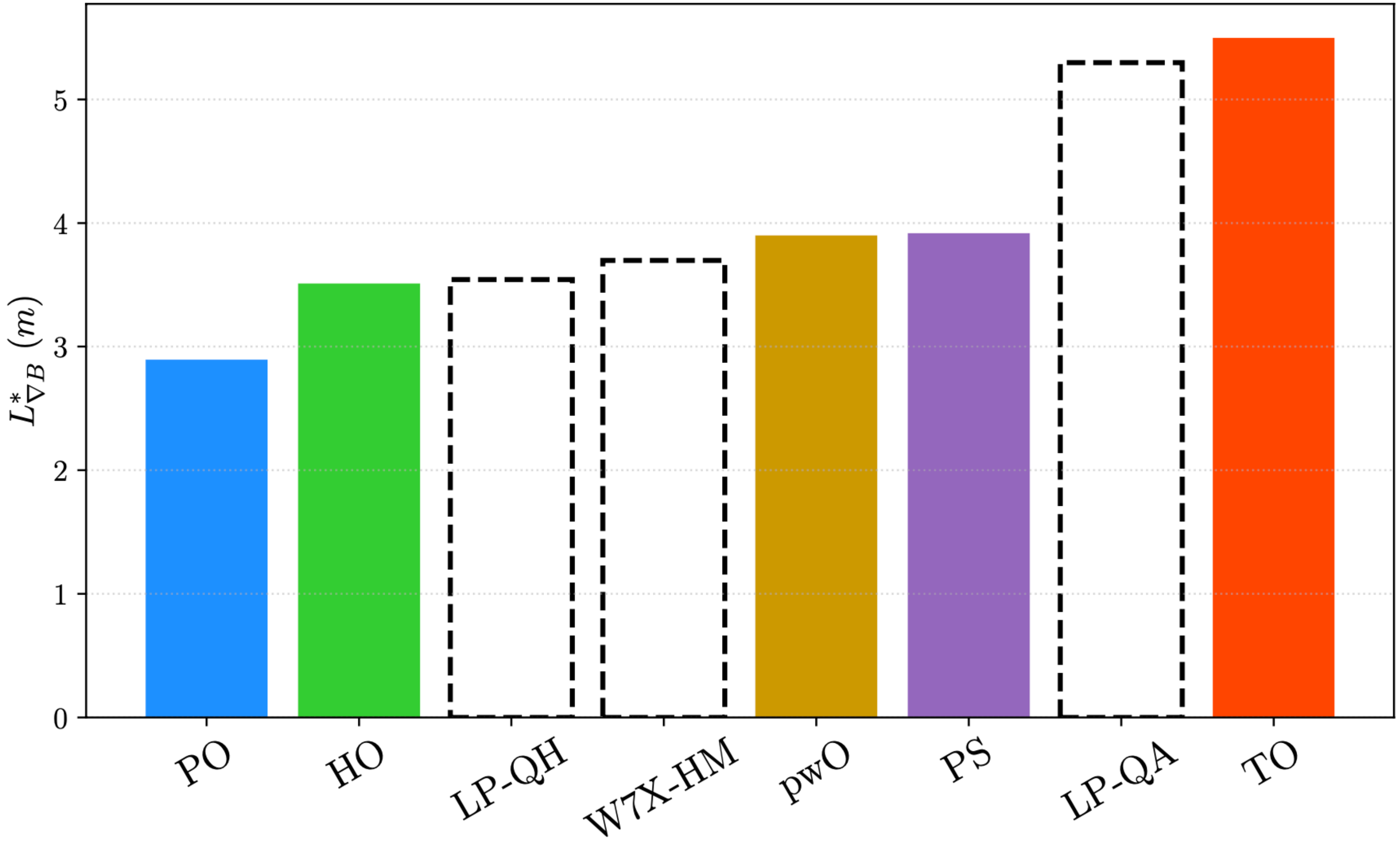
- Omnigenous B-contours (magenta, dashed) approximately composed of two straight segments.
- Magenta B-contour and stellarator-symmetric B-contour rougly enclose a parallelogram (black, dashed).
- Because of omnigenity, inflection points in B-contours are connected by two field lines (magenta, continuous).
- Automatically, field lines of same iota (black, continuous) connect appropriate corners.
- B-contours with larger values of B approximately pwO.

$\mathcal{J}$ distributions confirm the feature of pwO + QI



Relaxed constraints lead to potentially simpler coils

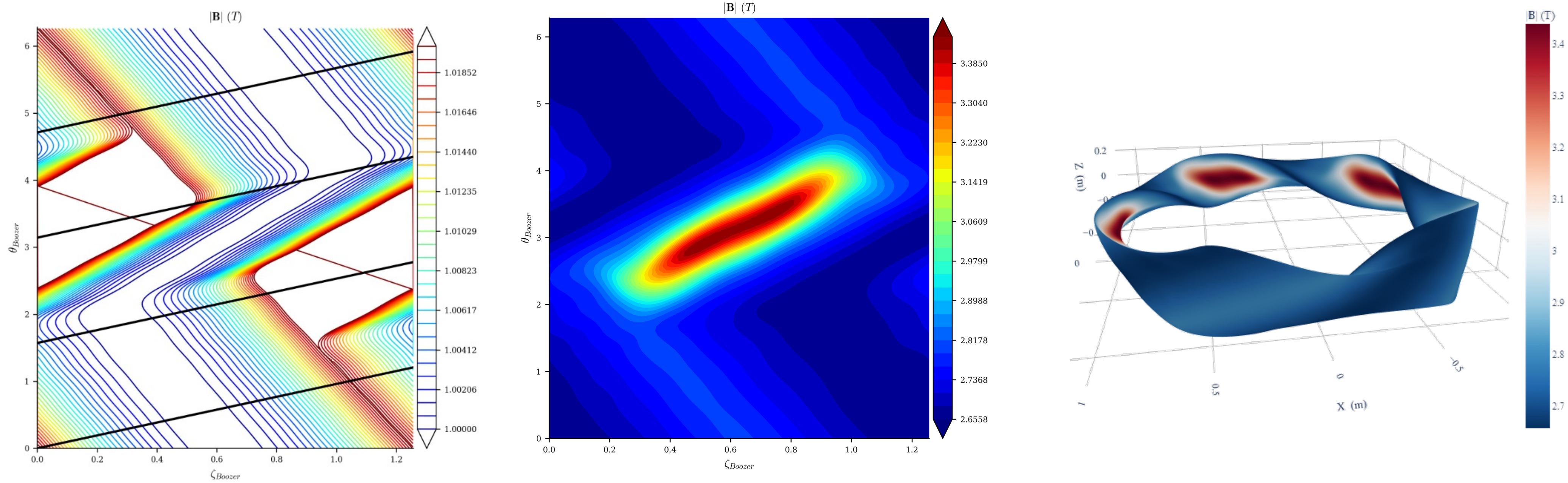
$L^*_{\nabla B}$ [Kappel PPCF 2024]: the minimum magnetic gradient scale length (all scaled to the ARIES-CS size)



Tuning the mapping function to obtain configurations closer to a prototypical pwO

Try to fit the $B_{\max}$ contours in pwO paper [Velasco PRL 2024]

$$B_{pwO} = B_{\min} + (B_{\max} - B_{\min}) \times \lim_{p\to\infty} \exp\left[-\left(\frac{\zeta_B - \zeta_c + t_1(\theta_B - \theta_c)}{w_1}\right)^{2p} - \left(\frac{\theta_B - \theta_c + t_2(\zeta_B - \zeta_c)}{w_2}\right)^{2p}\right]$$



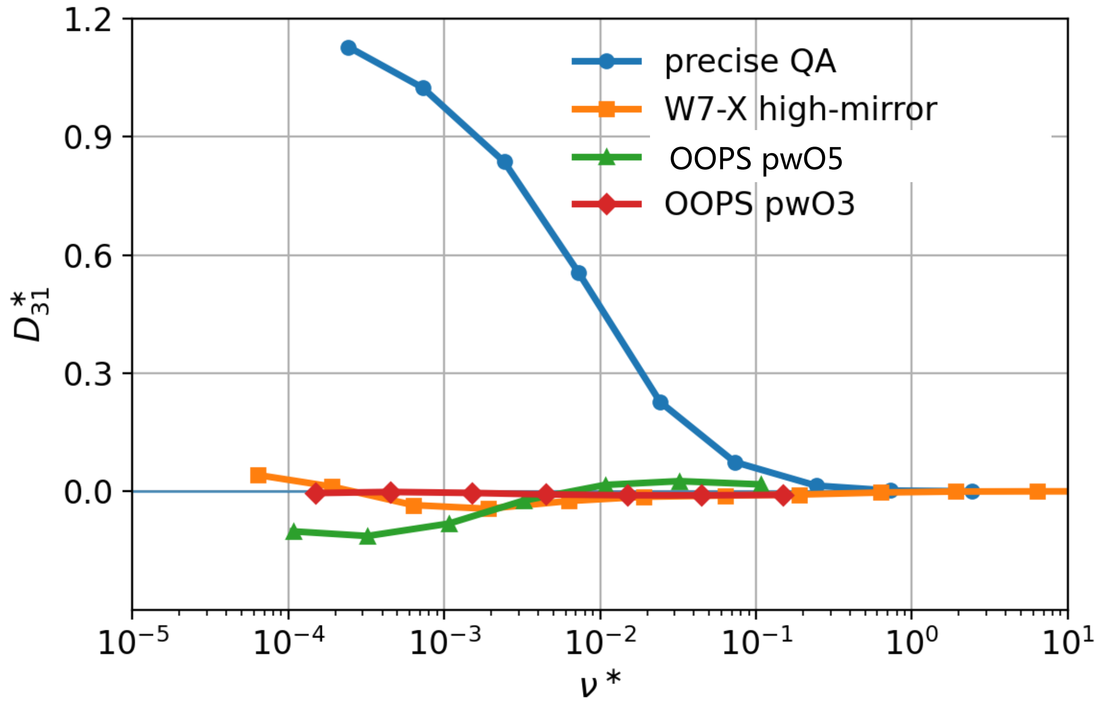
A five-period prototypical pwO (pwO5)

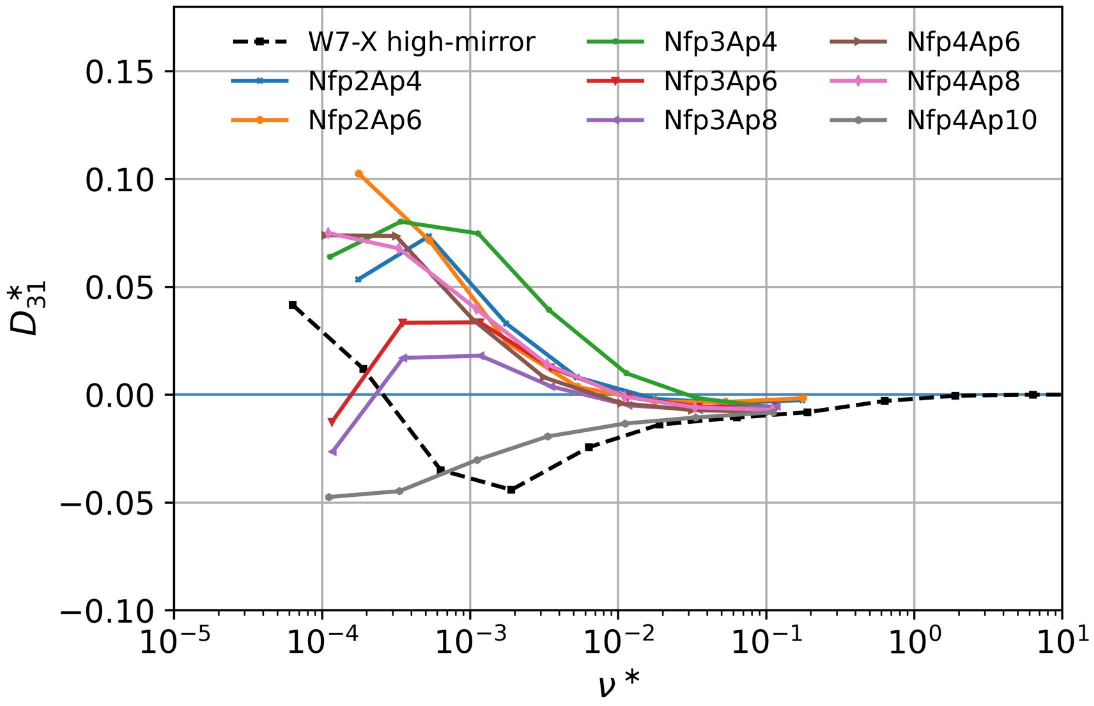
** Prototypical pwOs can also be obtained by PiecewiseOmnigenity objective in DESC [Fernández-Pacheco, in preparation; Velasco EPS 2025]*

D_{31} coefficients show small bootstrap currents

QI: zero bootstrap currents

pwO: zero bootstrap currents if the B=Bmax region has the appropriate shape [Calvo PRE 2025]





D_{31}^* coefficients calculated by MONKES [Escoto NF 2024]