NUCLEAR ENGINEERING
서울대학교 공과대학 원자핵공학과

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3DST

Modelling of mildly relativistic runaway electrons

Development of reduced-kinetic model and validation in KSTAR ohmic startup

Y. Lee[1], P. C. de Vries[2], P. Aleynikov[3], H.-T. Kim[4], J. Lee[5] , J. Gwak[1] , K. Park[1], G. Nam[1], J. Jang[5], J.-K. Park[1] and Y.-S. Na[1*]

[1]*Department of Nuclear Engineering, Seoul National University, Seoul, South Korea*
[2]*ITER Organization, France*
[3]*Max-Planck Institute fur Plasmaphysik, Greifswald, Germany*
[4]*United Kingdom Atomic Energy Authority, United Kingdom of Great Britain and Northern Ireland*
[5]*Korean Institute of Fusion Energy, South Korea*

leeys1996@snu.ac.kr
ysna@snu.ac.kr *

Main references

[1] H.-T. Kim et. al., Development of full electron magnetic plasma burn-through model and validation in MAST, *Nucl. Fusion*. 62, 126012 (2022)
[2] Y. Lee et. al., Kinetic modelling of start-up runaway electrons in KSTAR and ITER, *Nucl. Fusion*. 63, 106011 (2023)
[3] Y. Lee et. al., Binary Nature of Collisions Facilitates Runaway Electron Generation in Weakly Ionized Plasmas, *Phys. Rev. Lett*. 133, 175102 (2024)

30th IAEA Fusion Energy Conference (IAEA FEC 2025)
Oct 16th, 2025, Cheongdu

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- Introduction
- Model development
  - Reduced kinetic model of mildly relativistic runaway electrons
  - Coupling with DYON
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- Conclusions

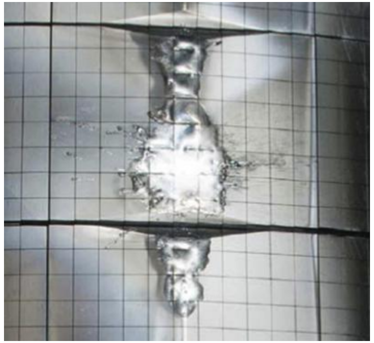
Lay physical and engineering foundation of designing a RE-free scenario in future fusion reactors

Introduction

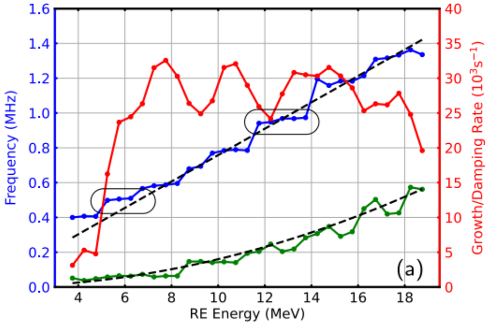
Runaway Electrons

- Runaway Electrons (REs)
  - Runaway electrons are the energetic electrons that will be continuously accelerated by the electrical field due to the decrease in the Coulomb collision frequency with increasing energy [Wilson1925].

- Hazards of REs
  - Damage to devices [Matthews et al., 2016]
  - Trigger instabilities [Liu et al., 2023]



[JET in-vessel image]



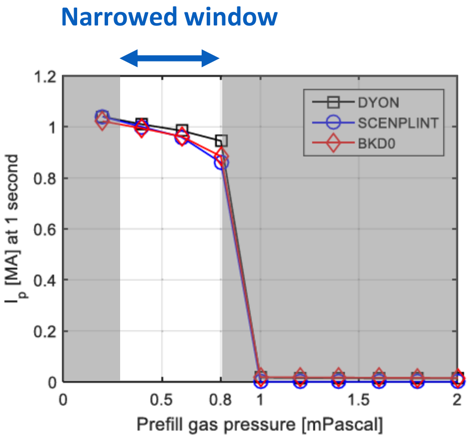
[CAW excited by REs]

- Disruption REs
  - Main interest of RE research community [Breizman et al., 2019].

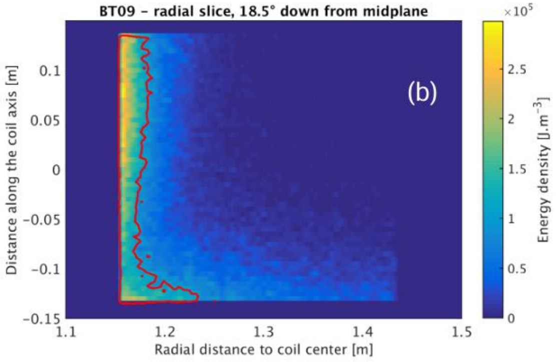
Startup runaway electrons – Obstacles of ITER startup

- Strict conditions for burn-through success
  - Plasma-driven failure at high prefill pressure (>= 0.8 mPa) Kim2020NF
  - Runaway-driven failure at low prefill pressure (<= 0.3 mPa) Gribove2018EPS Hoppe2022JPP
    - ➢ The conservative prediction suggested the failure at 0.6 mPa. Lee2024PRL

- Risk of catastrophic threat
  - Toroidal field coil quench
    - ➢ Reported in WEST



Kim2020NF

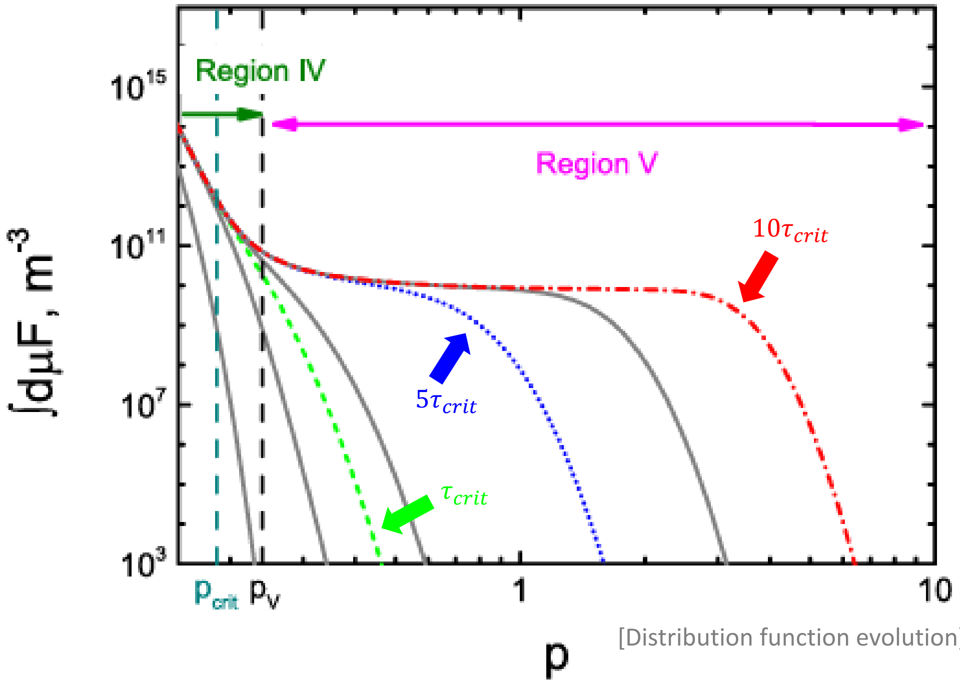


Ruex2021FEC

Safe plasma startup in ITER requires understanding of startup REs.

Improved understanding of startup runaway physics

- Runaway electrons are born at the region-based critical momentum p_V.
  - Not the force-free momentum p_{crit} often referred to as the critical momentum.

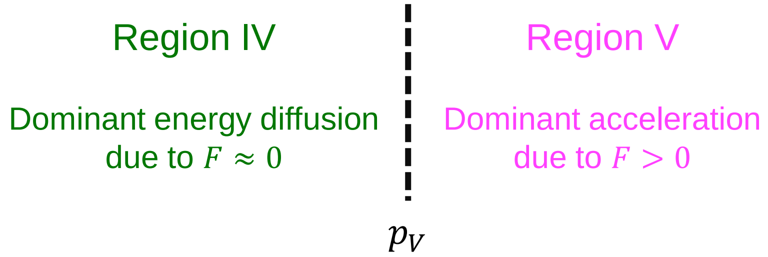


[Distribution function evolution]

Force-free critical momentum p_{crit}

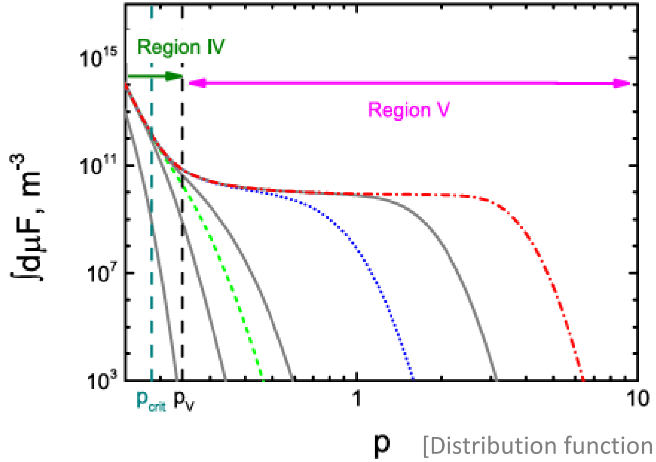
$$F = F_{electric} - F_{fric} = 0$$

Region-based critical boundary p_V

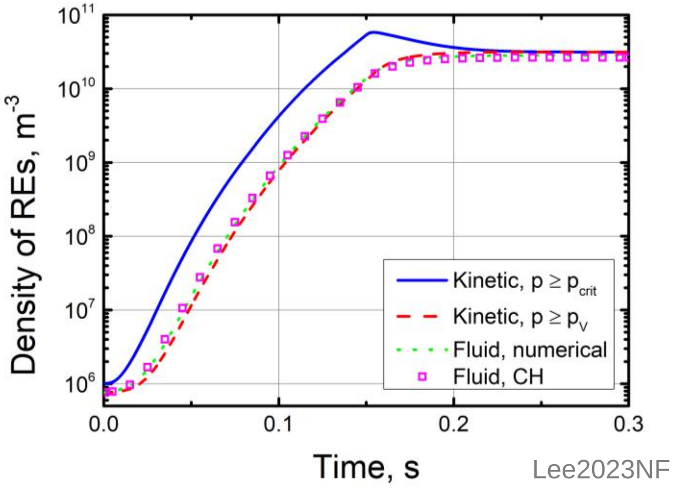


Improved understanding of startup runaway physics

- The initial startup runaway momentum is the region-based momentum p_V.
  - Not the force-free momentum p_{crit} often referred to as the critical momentum.



[Distribution function evolution]



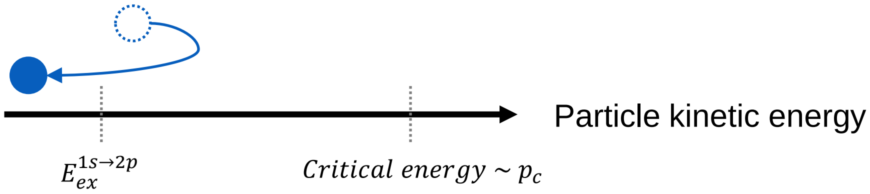
Lee2023NF

  - Main Conclusions
    - ➢ Fluid description of runaway density evolution is good.
    - ➢ Primary runaway particles are born with initial momentum p_V.

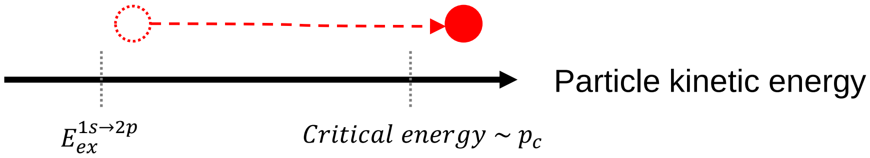
Improved understanding of startup runaway physics

- In early startup, runaway generation mechanism in non-diffusive.

Inelastic collision prohibits electron acceleration

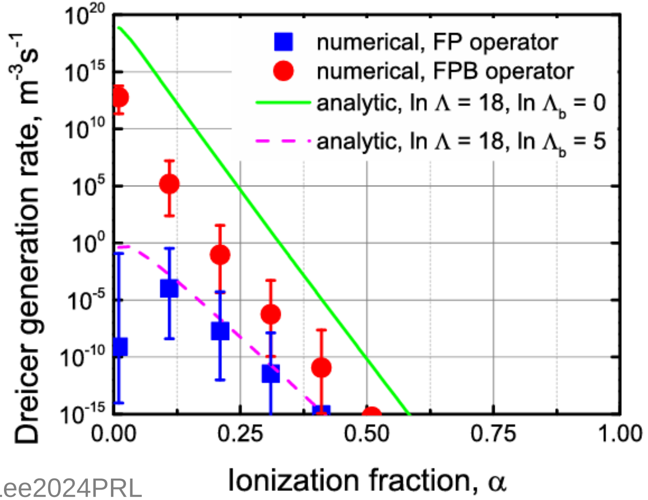


Collisionless acceleration allows electron energy gain above p_c

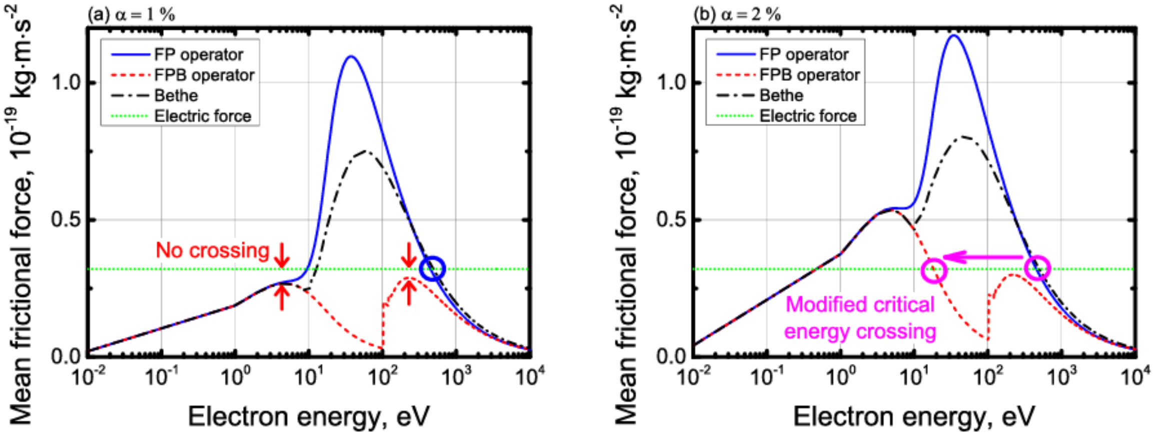


Improved understanding of startup runaway physics

- In early startup, runaway generation mechanism in non-diffusive.
  - The Fokker-Planck description severely underestimates the generation rate.
    - ➢ The effective critical momentum can be significantly reduced or even disappear.



Lee2024PRL



Lee2025arXiv

This implies $v_{RE} \ll c$ and thereby necessitates a mildly relativistic correction

Model development

Reduced kinetic model of mildly relativistic runaway electrons
Coupling with DYON

Model development

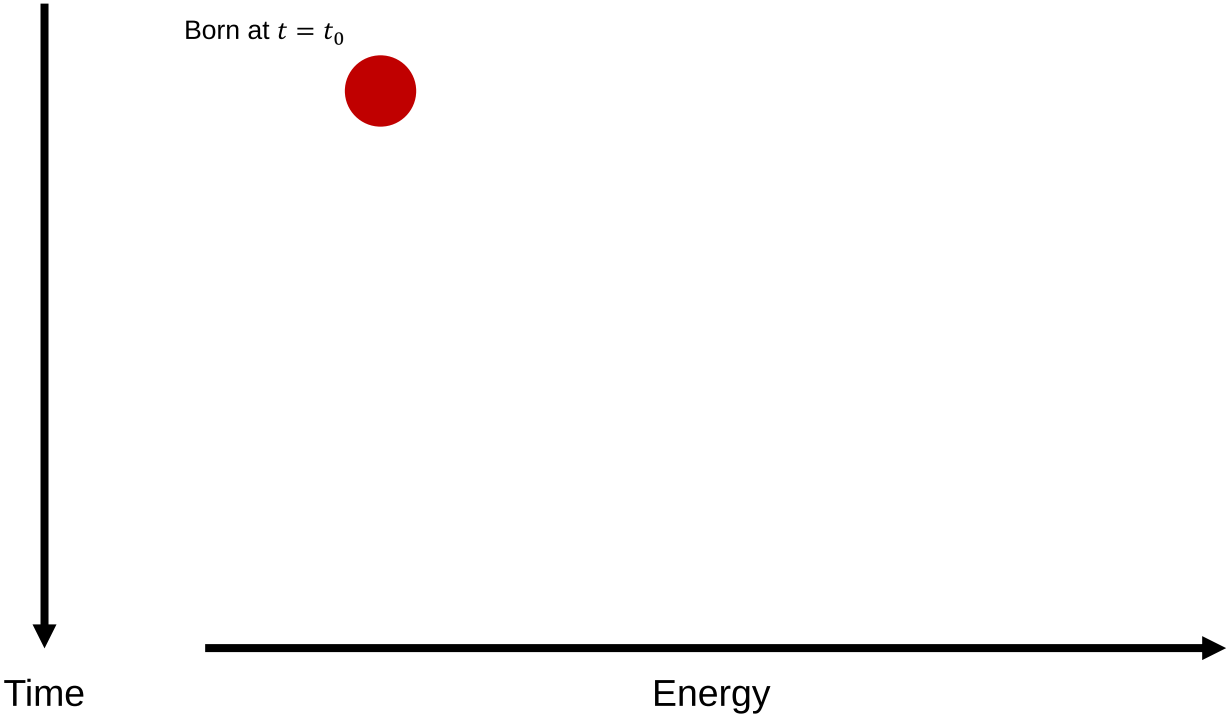
Reduced kinetic model of mildly relativistic runaway electrons

Coupling with DYON

Multi fluid approximation of mildly relativistic runaway current

- Multi-fluid description of runaway electrons

$$t_0 = t_0$$

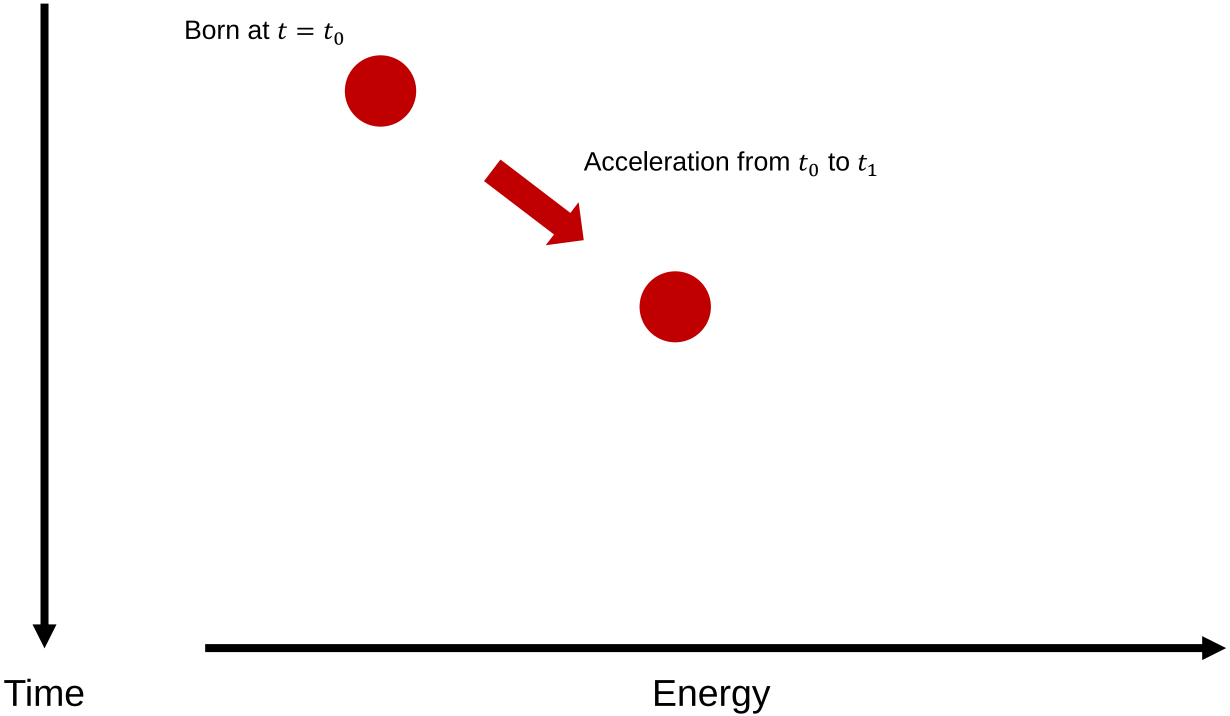


Multi fluid approximation of mildly relativistic runaway current

- Multi-fluid description of runaway electrons

$$t_0 = t_0$$

$$t_1 = t_0 + dt$$



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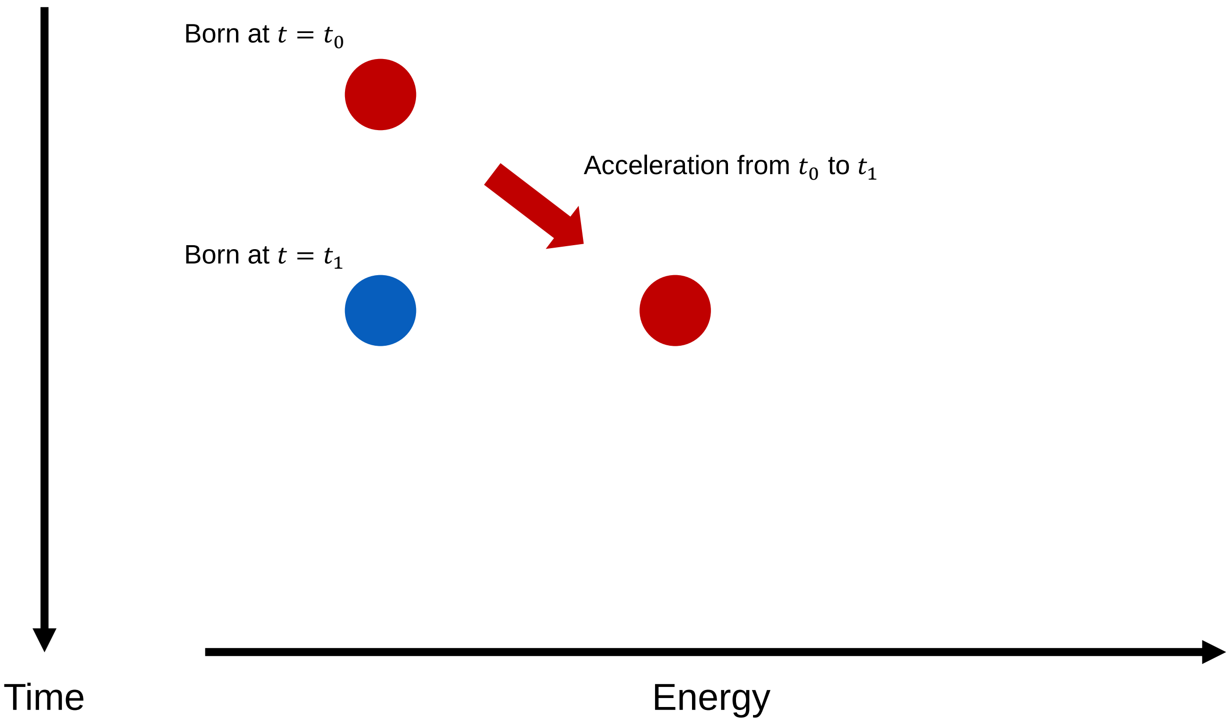

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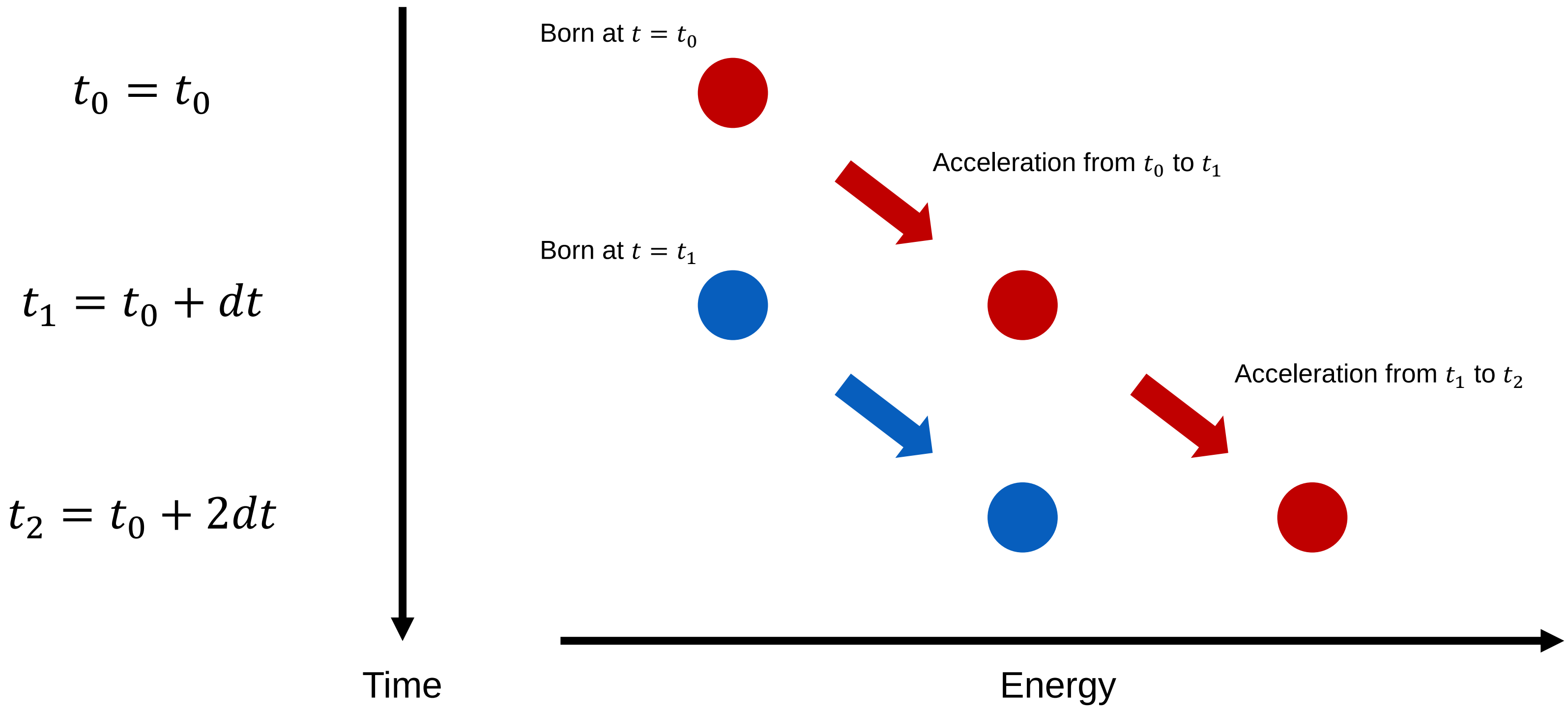


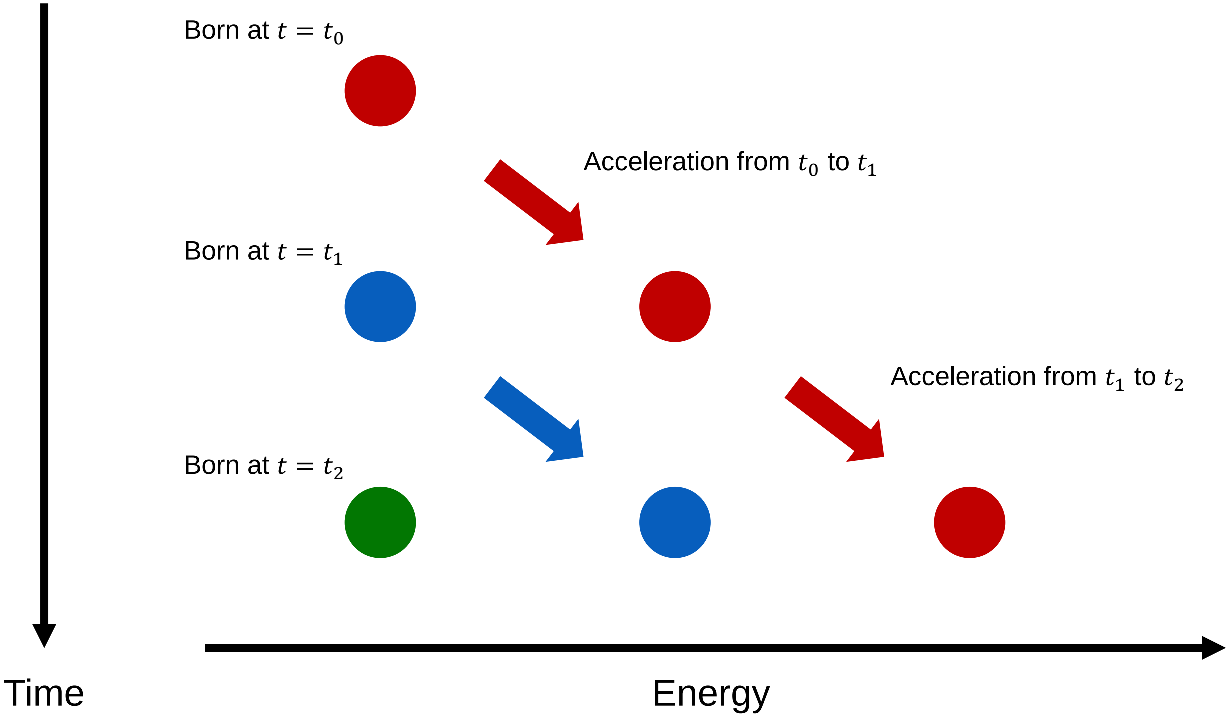
Multi fluid approximation of mildly relativistic runaway current

- Multi-fluid description of runaway electrons

$$t_0 = t_0$$

$$t_1 = t_0 + dt$$

$$t_2 = t_0 + 2dt$$



Multi fluid approximation of mildly relativistic runaway current

- The multi-fluid runaway model is consistent with the kinetic model.
  - The total runaway current density is "sum" of every runaway fluid currents

$$j_{RE,i+1} = ec \sum_{i' \leq i} dn_{RE,i'+\frac{1}{2}} \beta_{i'+\frac{1}{2}}(t_{i+1}),$$

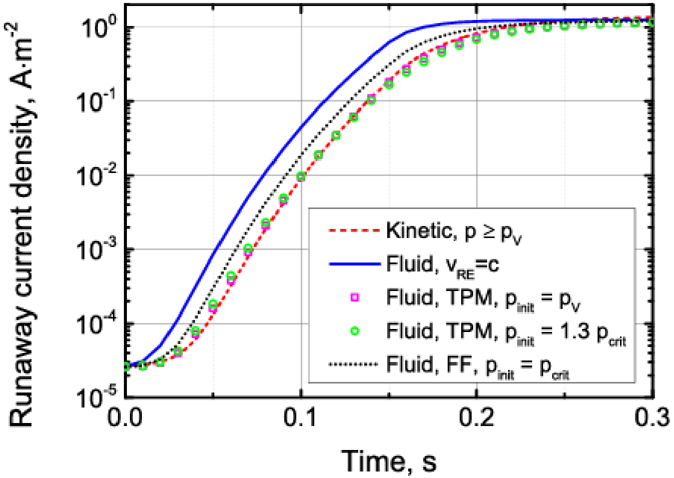
  - Fluid runaway velocity

$$\beta_{i'+\frac{1}{2}}(t_{i+1}) = \begin{cases} \beta_V \approx p_V & \text{if } i = i' \\ \beta_{i'+\frac{1}{2}}(t_i) + \left(\frac{d\beta}{dt}\right)_{i'+\frac{1}{2}}(t_{i+1} - t_i) & \text{if } i > i' \end{cases}.$$

Seed velocity

Accelerated velocity

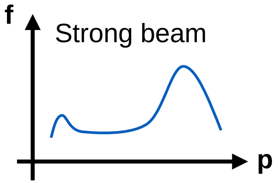
Test Particle Method (TPM)

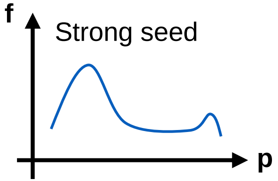


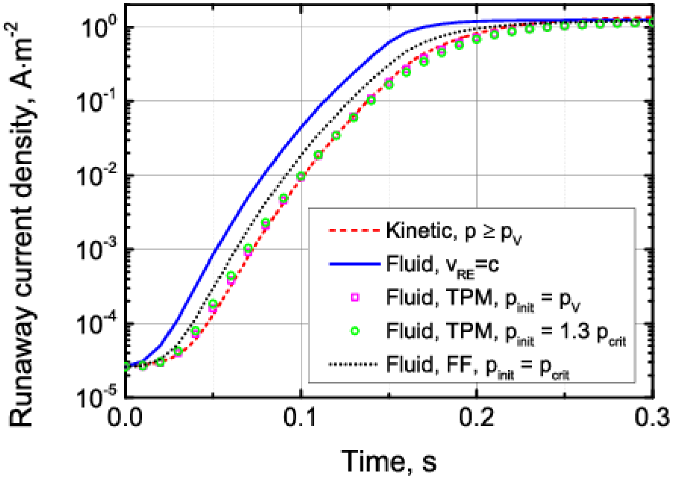
Successful verification !

Single fluid approximation of mildly relativistic runaway current

- Simple idea is to introduce the mean runaway velocity $\beta_{RE,i} \equiv \frac{j_{RE,i}}{ecn_{RE,i}}$
  - Runaway current density : $\frac{dj_{RE}}{dt} = ec\beta_V \frac{dn_{RE}^{seed}}{dt} + ec\frac{d\beta_{RE}}{dt} n_{RE}^{beam} + \ ...$
    - ➢ Single fluid model has comparable accuracy to multi-fluid free fall model.
  - Two critical asymptotes
    - ➢ When runaway seeding is weak
      - ▪ β_{RE} goes to runaway "beam" velocity.
    - ➢ When runaway seeding is strong
      - ▪ β_{RE} goes to runaway "seed" velocity.







Model development

Reduced kinetic model of mildly relativistic runaway electrons

Coupling with DYON

Transport features of runaway electrons during startup

- Runaway transport mechanism relies on magnetic configuration.

Closed field configuration

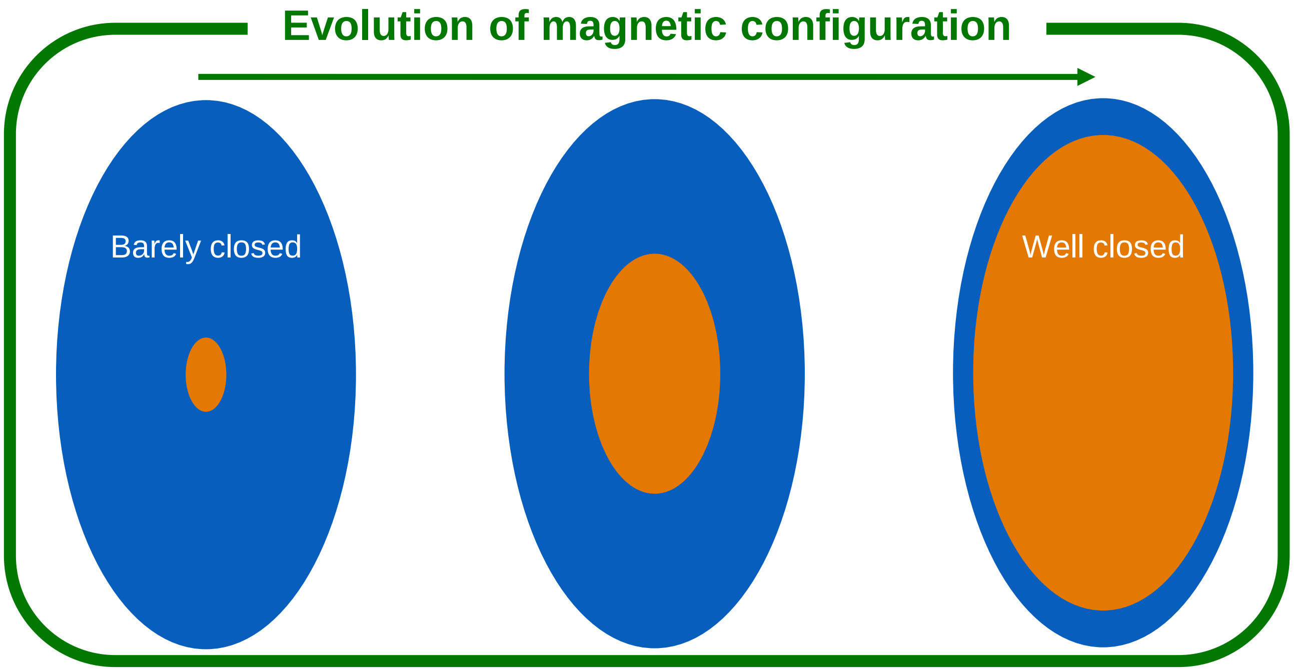
Radial stochastic transport

$$\tau_{RE} \propto \frac{a^2}{v_{RE}}$$

Open field configuration

Parallel streaming loss

$$\tau_{RE} \approx \frac{L}{v_{RE}}$$



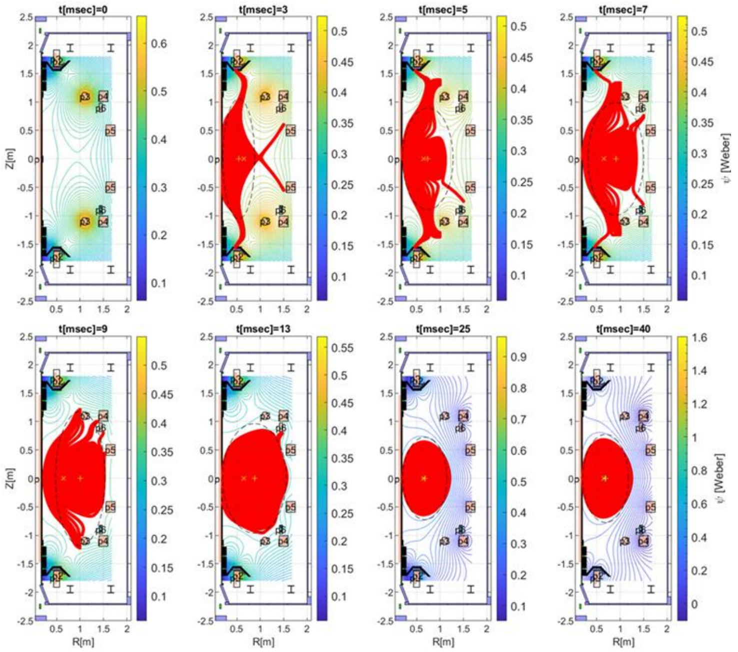
Need to be coupled with mildly relativistic runaway velocity $v_{RE} \neq c$!

Reliable startup design by multi-machine validated code, DYON

- Self-consistent coupling with DYON

- Key code features
  - Multi-machine validated code.
    - ➢ MAST-U, DIII-D, EAST, KSTAR, VEST
  - Unique approach for *model-based* description of magnetic configuration.
  - Minimize free parameters and reduce solution non-uniqueness.
    - ➢ Critical lesson from startup RE model validation in JET [deVries2025NF].

Regular oral talk on model validation
by H.-T. Kim et al
17th Oct, 11AM



Kim2022NF

Two macroscopic populations on dual magnetic configuration

- Describe species in closed (cl) and open (op) magnetic configurations
  - The particle-conserving evolution of runaway particle density

$$\frac{dn_{RE}^{cl}}{dt} = S_p^{cl} + (\gamma_{ava}^{cl,i} - \frac{1}{\tau_{RE,\perp}^{cl}})n_{RE}^{cl} - n_{RE}^{cl}\frac{d}{dt}\log V_p^{cl},$$

Particle-conserving correction

$$\frac{dn_{RE}^{op}}{dt} = S_p^{op} + (\gamma_{ava}^{op,i} - \frac{1}{\tau_{RE}^{op}})n_{RE}^{op} + \frac{V_p^{cl}}{V_p^{op}}\frac{n_{RE}^{cl}}{\tau_{RE,\perp}^{cl}} - n_{RE}^{op}\frac{d}{dt}\log V_p^{op}.$$

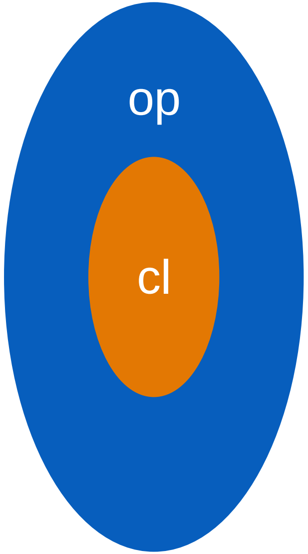
  - The momentum-conserving evolution of runaway current density

$$\frac{dj_{RE}^{cl}}{dt} = ecS_p^{cl}\beta_V^{cl} + (\gamma_{ava}^{cl} - \frac{1}{\tau_{RE,\perp}^{cl}})j_{RE}^{cl} + j_{RE}^{cl}\frac{d}{dt}\log\beta_{RE}^{cl} - j_{RE}^{cl}\frac{d}{dt}\log V_p^{cl},$$

Momentum-conserving correction

$$\frac{dj_{RE}^{op}}{dt} = ecS_p^{op}\beta_V^{op} + (\gamma_{ava}^{op} - \frac{1}{\tau_{RE}^{op}})j_{RE}^{op} + j_{RE}^{op}\frac{d}{dt}\log\beta_{RE}^{op} + \frac{V_p^{cl}}{V_p^{op}}\frac{j_{RE}^{cl}}{\tau_{RE,\perp}^{cl}} - j_{RE}^{op}\frac{d}{dt}\log V_p^{op}.$$

Transport from closed to open field region



S_p : Primary generation rate

γ_{ava} : Runaway avalanche growth rate

τ_{RE} : Runaway confinement time

V_p : Plasma volume

Self-consistent coupling with DYON

- Self-consistent *runaway current* evolution

Amended circuit equations

$$\frac{dI_p}{dt} = \frac{1}{L_p}\Big(V_{loop} - R_p(I_p - I_{RE}^{op} - I_{RE}^{cl}) - \frac{dL_p}{dt} I_p\Big),$$

$$\frac{dI_{RE}^{cl}}{dt} = ecA^{cl} S_p^{cl} \beta_V^{cl} + I_{RE}^{cl} \frac{d}{dt}\log \beta_{RE}^{cl} + (\gamma_{ava}^{cl,j} - \frac{1}{\tau_{RE,\perp}^{cl}}) I_{RE}^{cl},$$

$$\frac{dI_{RE}^{op}}{dt} = ecA^{op} S_p^{op} \beta_V^{op} + (\gamma_{ava}^{op} - \frac{1}{\tau_{RE}^{op}}) I_{RE}^{op} + I_{RE}^{op} \frac{d}{dt}\log \beta_{RE}^{op} + \frac{R^{cl}}{R^{op}} \frac{I_{RE}^{cl}}{\tau_{RE,\perp}^{cl}}.$$

Dominant balancing

$$n_{RE}^{op} \approx S_p^{op} \tau_{RE,\parallel}^{op},$$

$$I_{RE}^{op} \approx ecA^{op} S_p^{op} \beta_V^{op} \tau_{RE,\parallel}^{op},$$

- Self-consistent *runaway confinement* on dual magnetic configurations

$$\tau_{RE,\parallel}^{op} \approx \frac{< L_{op} >}{v_{RE}^{op}}$$

$$\tau_{RE,\perp}^{cl} \approx c \frac{a_{cl}^2 \tilde{b}_r^2}{2\pi R_0 v_{RE}^{cl} B_\phi^2}$$

DYON model

Runaway model

$$\tau_{RE}^{cl} \approx \tau_{RE,\perp}^{cl},$$

$$(\tau_{RE}^{op})^{-1} \approx (\tau_{RE,\perp}^{op})^{-1} + (\tau_{RE,\parallel}^{op})^{-1}$$

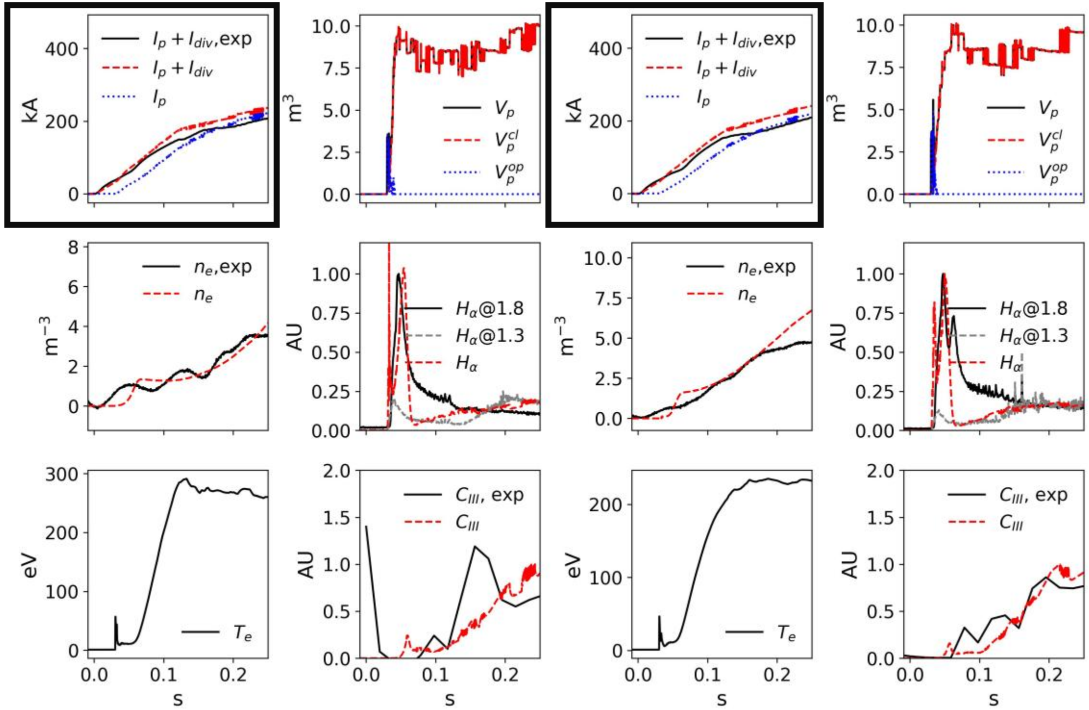
Model validation

Validation in KSTAR ohmic startup

Validation of DYON-RE in KSTAR Ohmic startup

- Reliable prediction of plasma parameters by coupled DYON-RE

26031 (RE rich) 27340 (RE scarce)



1. **Measured rogoski coil current agrees with synthetic signal ($I_p + I_{div}$)**

2. Measured line-averaged electron density agrees with synthetic averaged electron density (n_e)

3. Measured line missions agree with synthetic line emission intensity (H_α, C_{III})
 * Indirect Te validation

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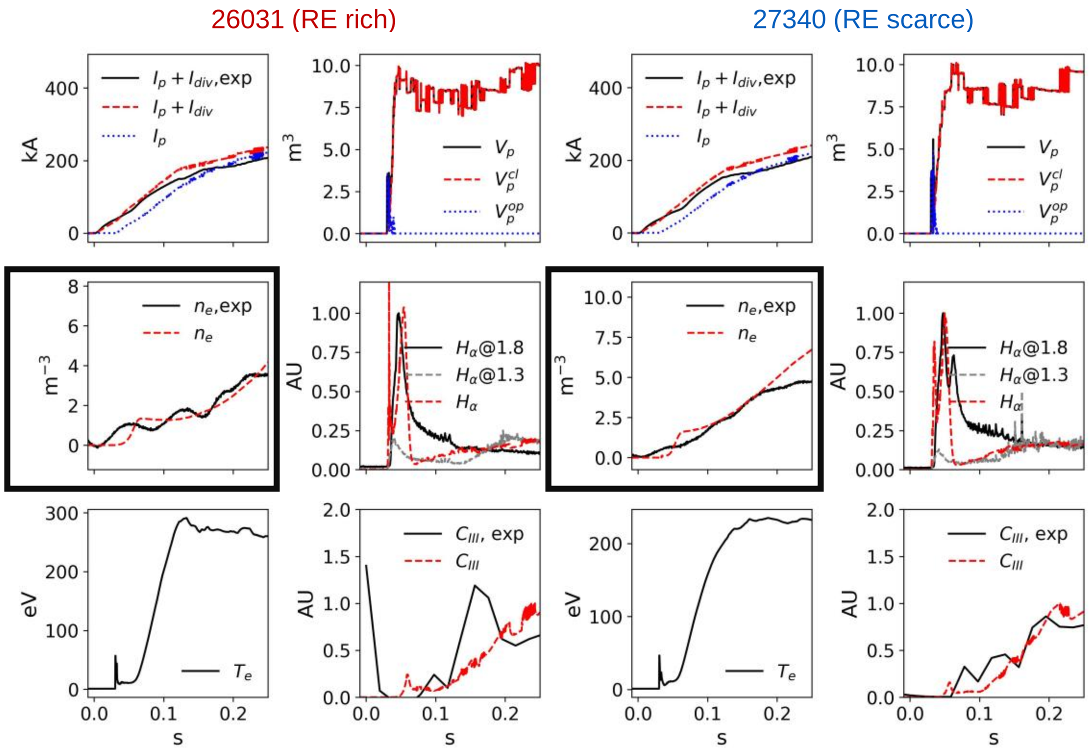

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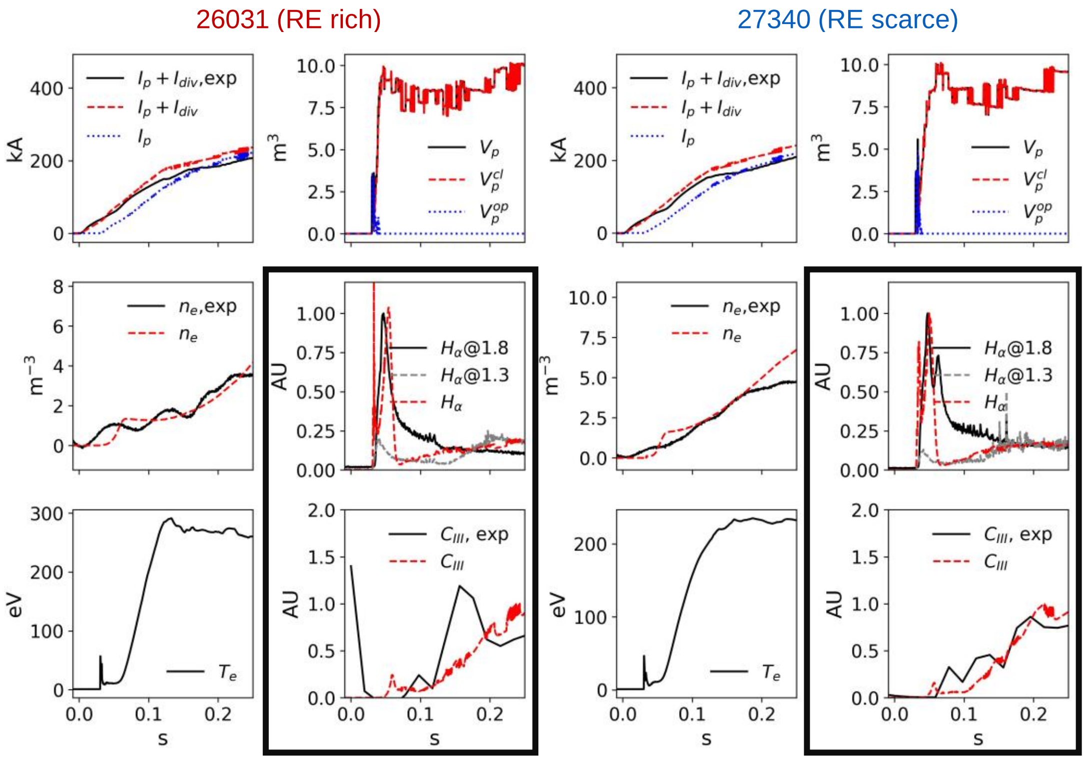

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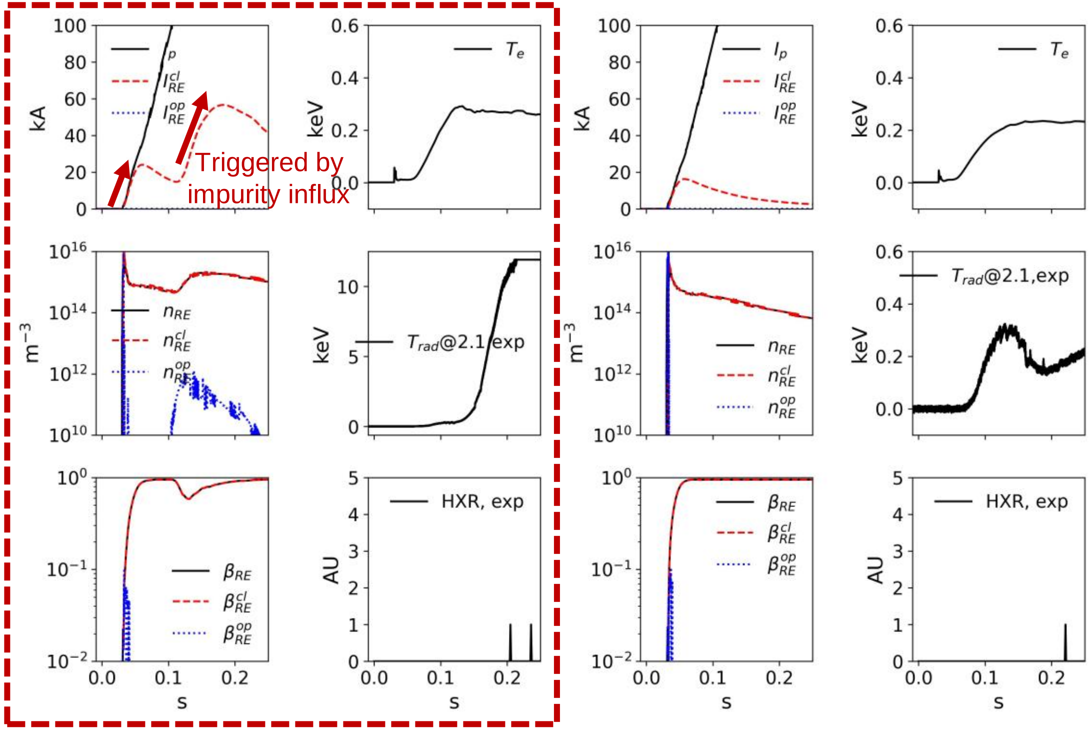
Validation of DYON-RE in KSTAR Ohmic startup

- Qualitative validation of runaway signatures

26031 (RE rich) 27340 (RE scarce)



In RE rich discharge,

- Two-stage runaway generations are observed. 2nd stage trigger is impurity influx.
- Strong runaway seeding (~ t=0.1 s) triggers anomalous increase in nonthermal ECE radiation (> 12 keV [exp] $\gg 0.25$ keV [DYON]).
- During strong runaway production, mildly relativistic correction renders $\beta < 1$.
- Note that highly relativistic runaway electrons do not satisfy the resonance condition so can't explain the anomalous increase.

Validation of DYON-RE in KSTAR Ohmic startup

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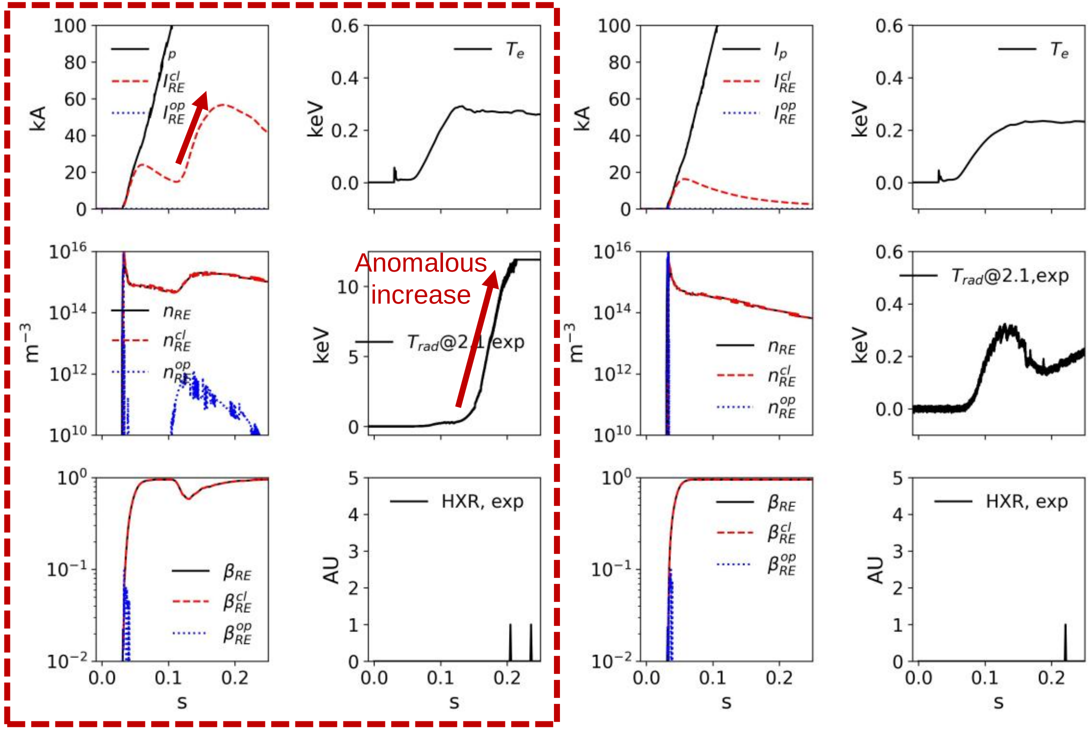

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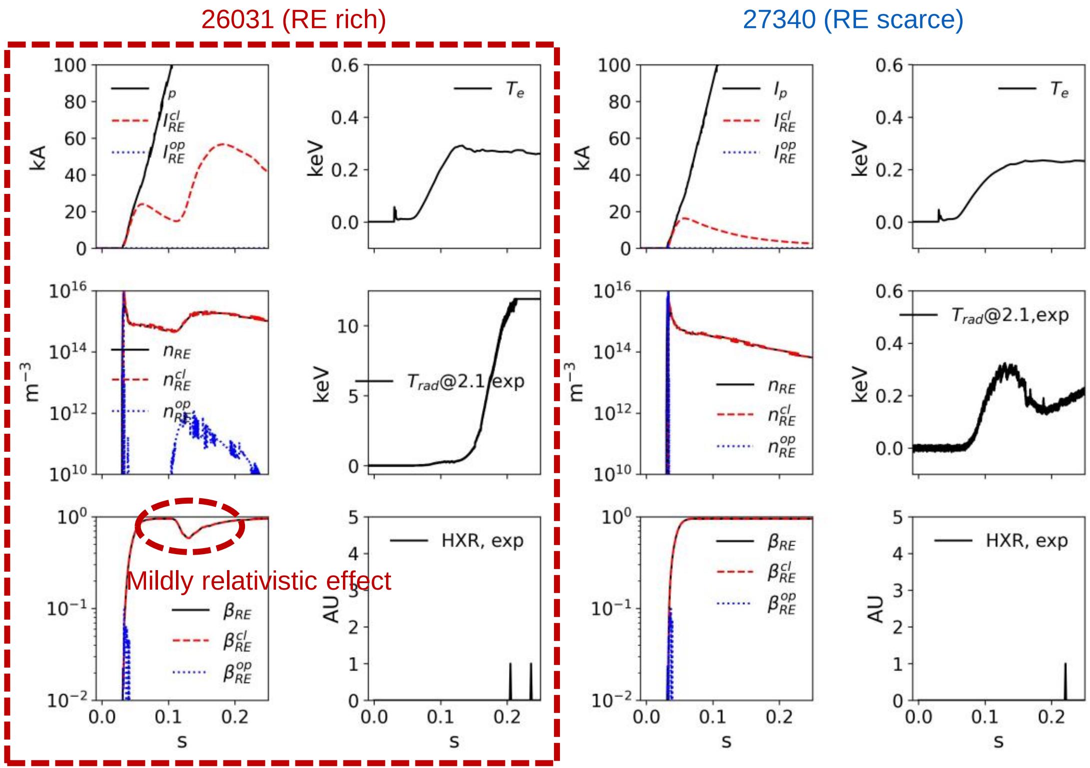

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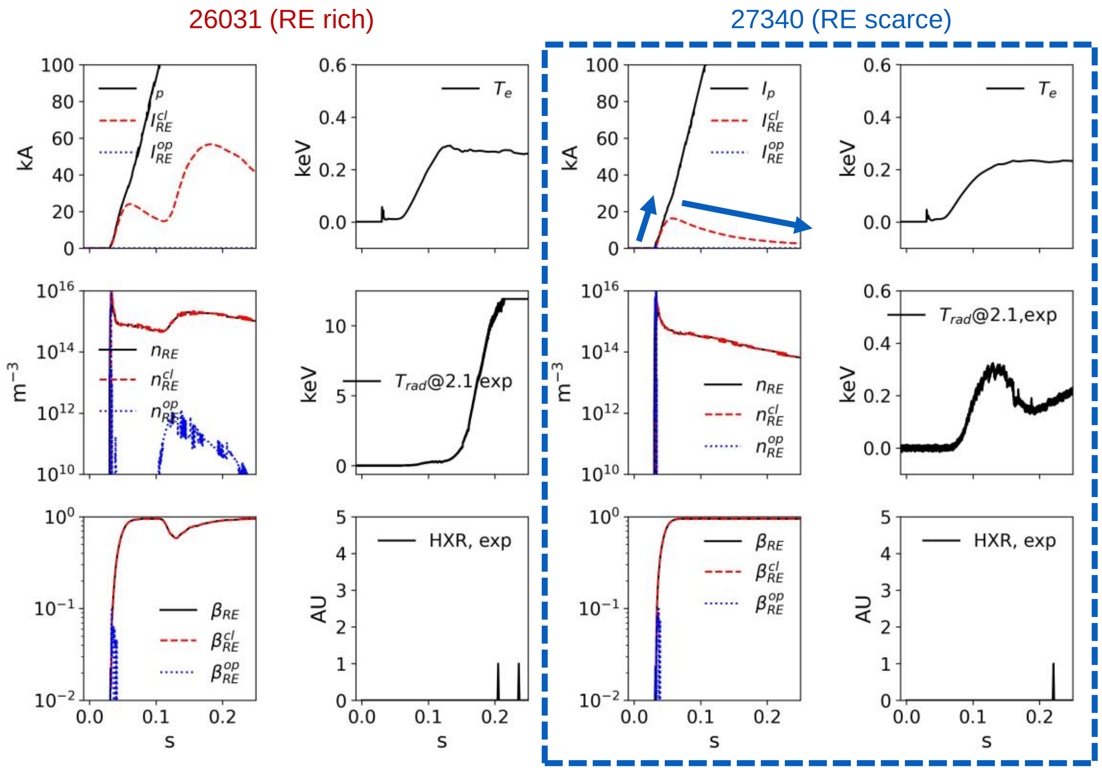
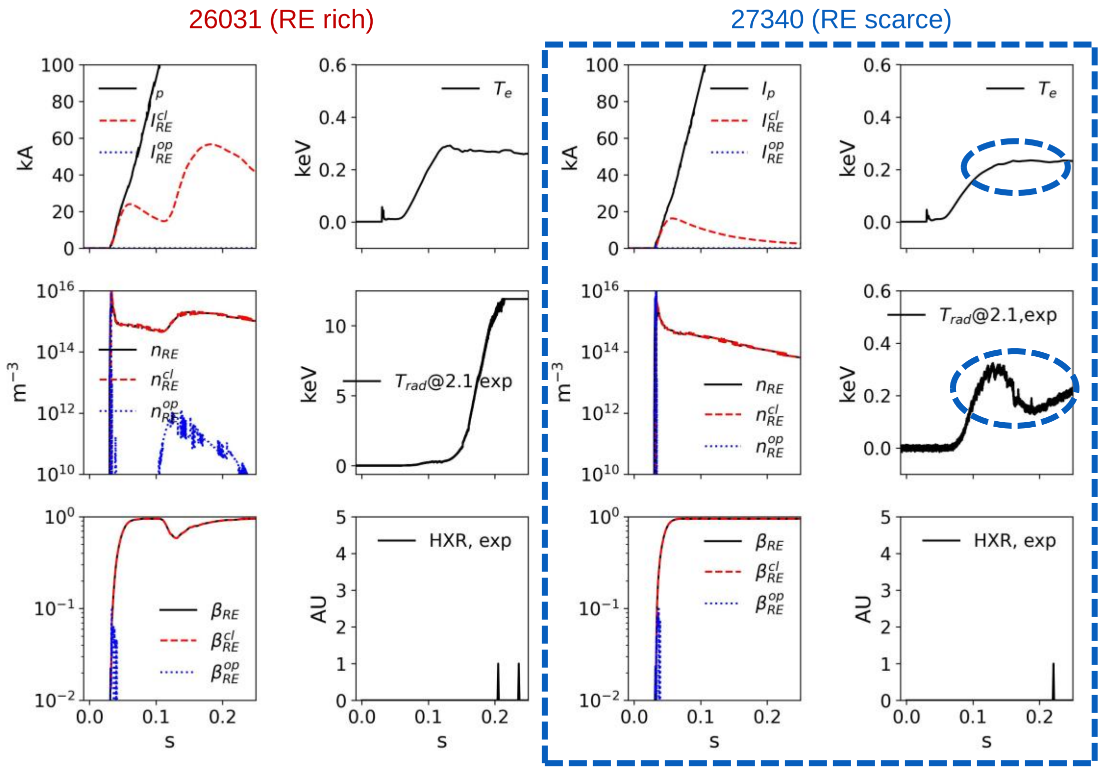


In RE scarce discharge,

- Strong startup runaway seed forms (~ 20 kA) and decays.
- Electron temperature (~ 0.23 keV) has a comparable order to measured ECE intensity (0.2 ~ 0.3 keV).
- DYON prediction suggested kinetic instability is triggered by runaway beam but stabilized due to their deconfinement.

Validation of DYON-RE in KSTAR Ohmic startup

- Qualitative validation of runaway signatures



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- Qualitative validation of runaway signatures

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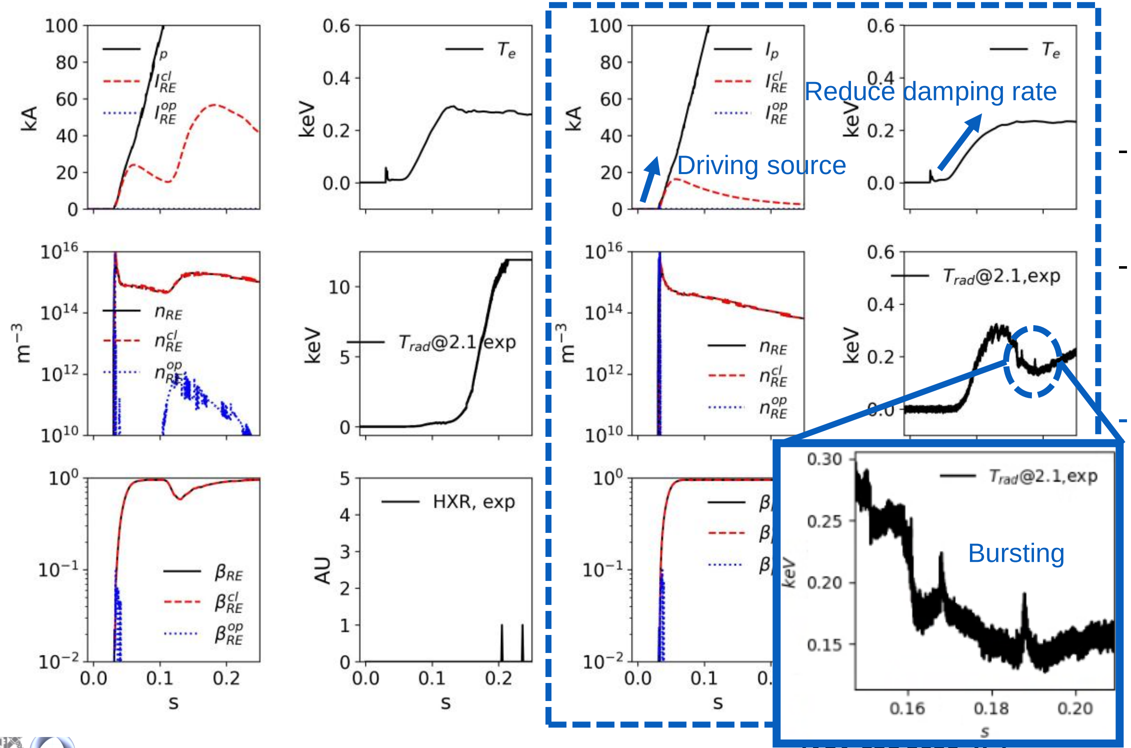

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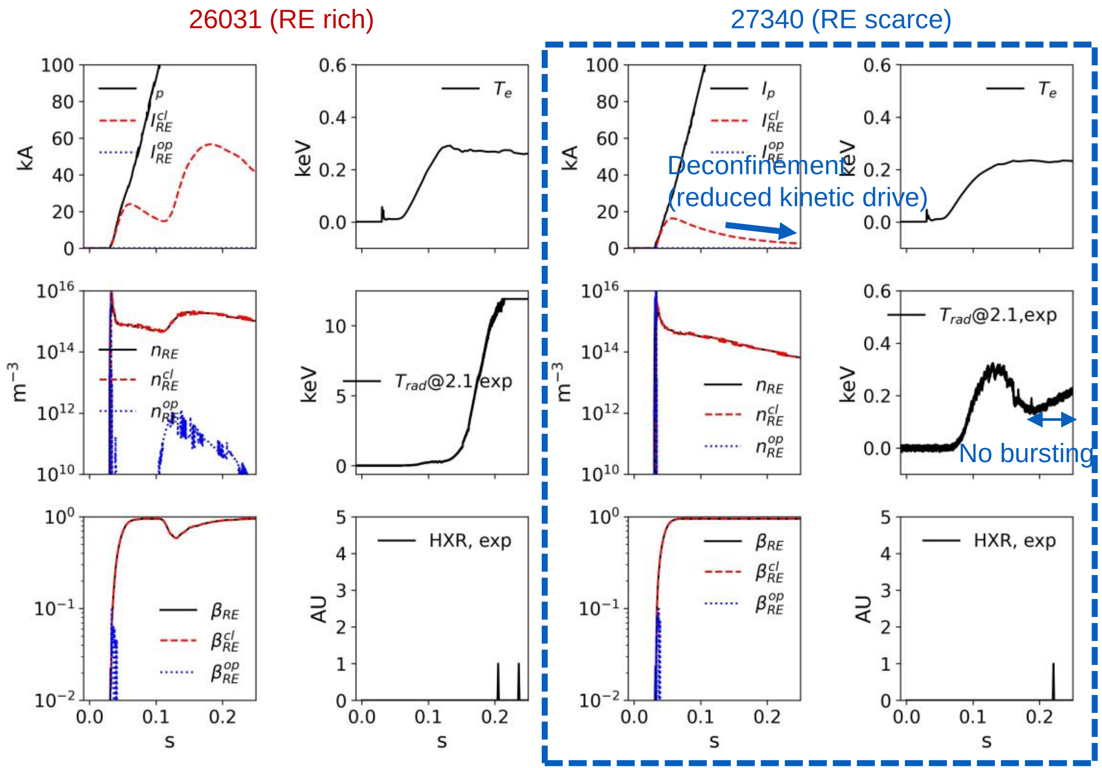

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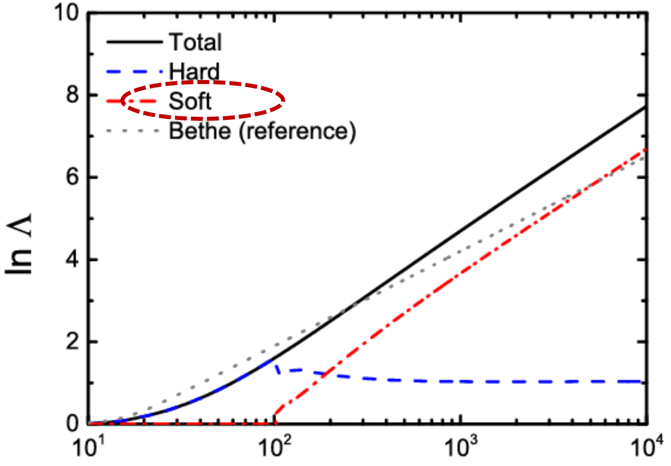
Conclusions

- Lay physical and engineering foundation of designing a RE-free scenario in future fusion reactors
  - Model development of reduced startup runaway model
    - ➢ **Single fluid model** with non-light-speed velocity $v_{RE} \neq c$
      - No overestimation in **runaway current density**
      - No underestimation in **runaway confinement time**.
    - ➢ **Self-consistent coupling** with reliable startup code DYON.
  - Model validation in KSTAR Ohmic startup
    - ➢ In RE rich shot,
      - Timing of strong runaway seed formation coincides with that of anomalous increase in non-thermal ECE intensity.
    - ➢ In RE scarce shot,
      - Measured ECE intensity is comparable to DYON's electron temperature.
      - Bursting characteristics in ECE intensity is consistent with formation and loss of runaway beam.

Effective critical electron field

- Ad-hoc treatment to include inelastic collisions in the critical electric field.
  - We only consider “soft” collisions to account for non-diffusive mechanism.

$$E_c \equiv \frac{e^3 n_e \ln\Lambda_{free}}{4\pi\varepsilon_0^2 mc^2} \rightarrow E_c^{eff} \equiv \frac{e^3(n_e \ln\Lambda_{free} + n_H \ln\Lambda_{bound}^{H,soft} + \sum_I n_I \ln\Lambda_{bound}^I)}{4\pi\varepsilon_0^2 mc^2}$$



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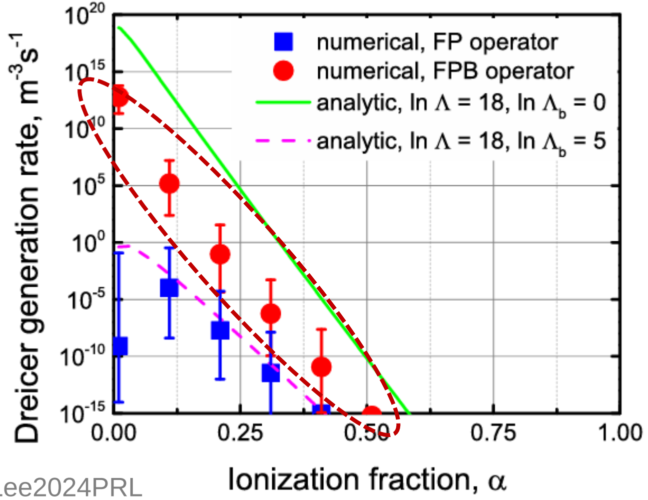

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