



New Understanding of Resonant Layer Response via Extended MHD

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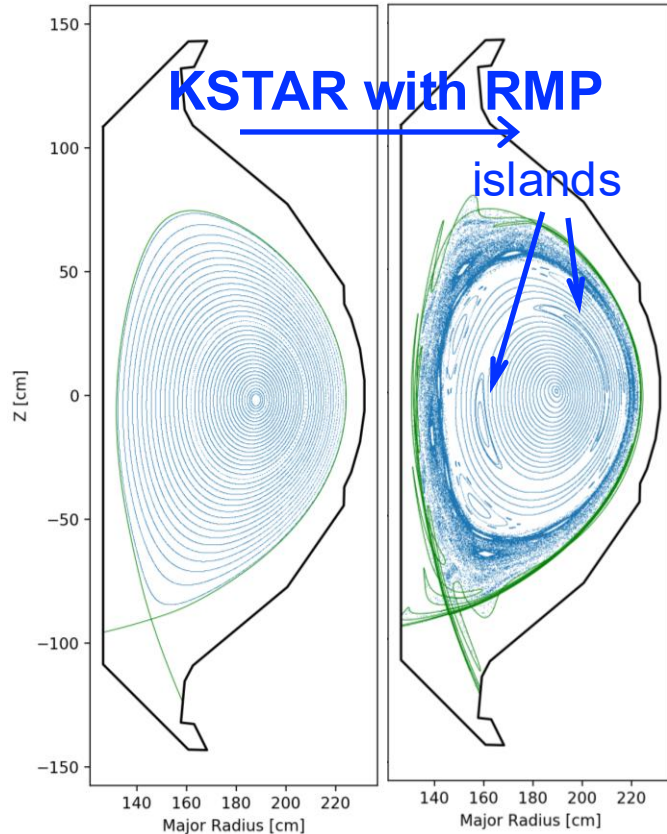
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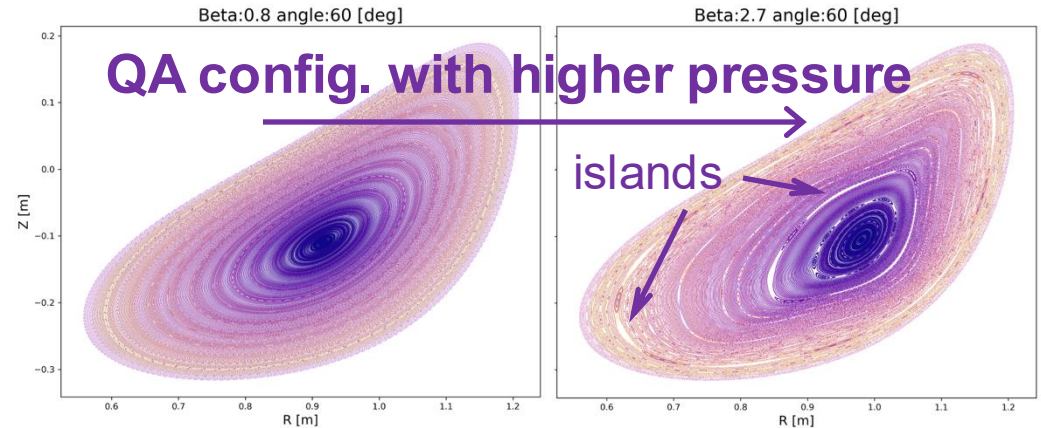
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Mechanism of magnetic island opening in fusion plasmas



- Magnetic fusion needs good magnetic surfaces
- Unwanted magnetic islands must be prevented
- **Mechanism opening islands** must be understood
- Leading to our key question on field penetration



Overview

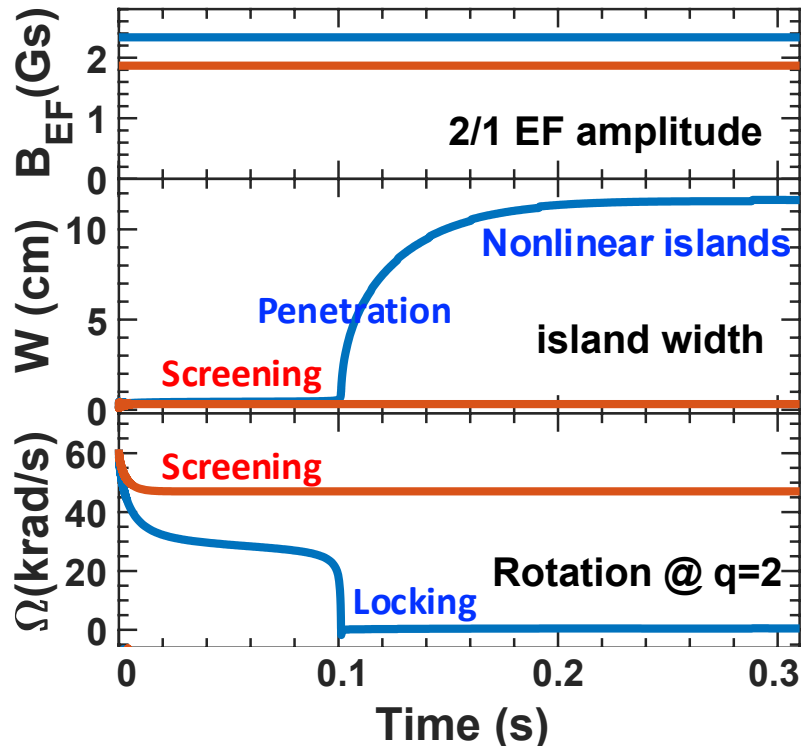
- Introduction to field penetration
 - As a key process in error field, tearing mode, 3D-edge control
- Modeling background
 - Two-fluid drift MHD framework
- New theory and modeling with extended MHD physics
 - **Strong density correlation with electron viscosity**
 - **Shielding by ion parallel flow near natural frequency**
- Implication to parameteric scaling and prediction
- Conclusion remarks

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Field penetration is a bifurcation to large magnetic islands

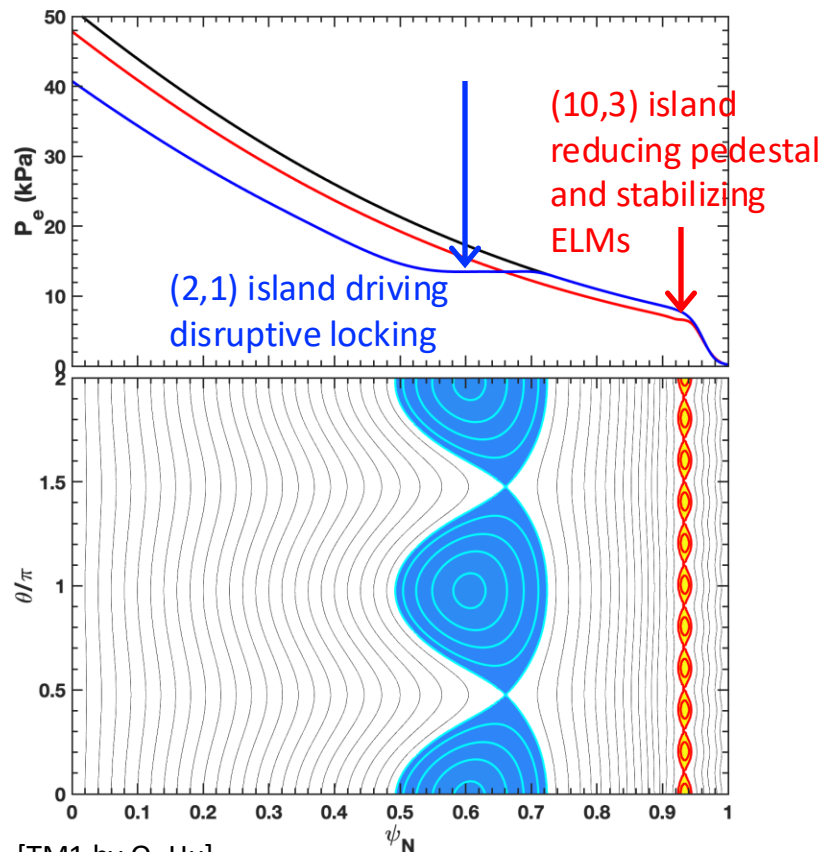
[Hu et al., Nucl. Fusion (2020)]



Local resonant ($m=2, n=1$) field
at just *slightly different* amplitudes,
leading to a very different consequence

- A resonant magnetic perturbation $\delta \vec{B}$ is screened by parallel currents $\delta \vec{j}_{\parallel}$ at the rational surface when its strength is insufficient
- Its electromagnetic torque $\delta \vec{j}_{\parallel} \times \delta \vec{B}$ is balanced by viscous torque
- However, $\delta \vec{B}$ can penetrate to the resonant surface when its strength reaches a threshold – known as **field penetration bifurcation**
- Creating large, nonlinear magnetic islands

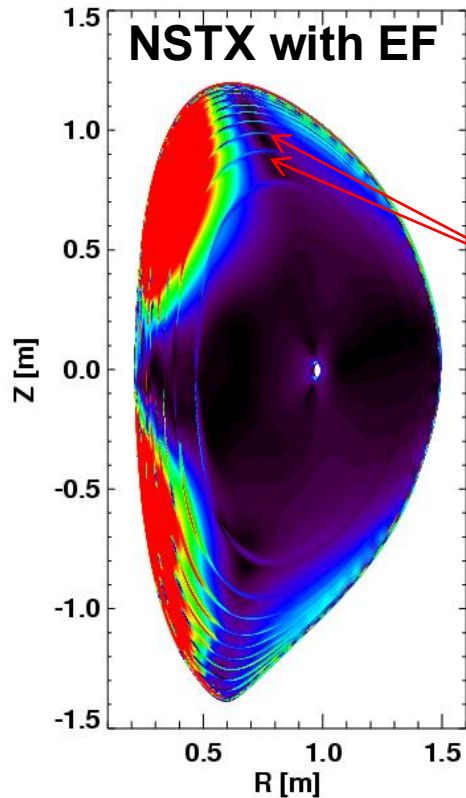
Field penetration is the leading hypothesis for error-field driven disruption and RMP ELM suppression



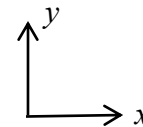
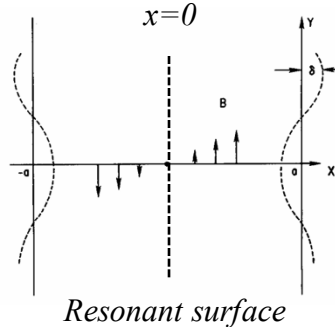
- Disruptive locked modes by intrinsic error field (EF) are created by field penetration in the core, by low (m,n) resonant field
- RMP ELM suppression is believed to come with field penetration in the edge, by mostly high (m,n) resonant fields
- These two represent core vs. edge, or bad vs. good side of non-axisymmetry, but share the key process in common – **Field penetration**

[TM1 by Q. Hu]

Boundary layer theory offers a unique way to gain fundamental understanding of complex MHD interplays in field penetration



- Field penetration is a boundary layer phenomenon
 - Outer layer : Ideal MHD is dominant
 - Inner layer : Ideal MHD breaks down and all subsidiary physics effects can come into a play
- Layer is narrow, with radial scale $\delta \sim S^{-1/3} < 10^{-3}$ in ITER
- Analytic theory is possible in slab representation, targeting physics understanding and verification for full simulation in the future



$$\begin{aligned}
 & - \hat{x} \propto \vec{\nabla} \psi \\
 & - \hat{y} \propto \vec{B} \times \vec{\nabla} \psi \\
 & - \hat{z} \propto \vec{B}
 \end{aligned}$$

As used in theory for islands since FKR (1963)

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Two-fluid drift-MHD adapted to explore layer response

- Following two-fluid drift MHD formulation and including [Fitzpatrick et al., Phys. Plasmas (2005)]
 - First-order drifts and ion gyro-viscous tensor pressure ($\vec{\nabla} \cdot \vec{\Pi}_g$)
 - Phenomenological conductivity or (perpendicular diffusivity), electron/ion viscosity

$$m_i n_0 \left(\frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) \vec{V} = \vec{j} \times \vec{B} - \vec{\nabla} p - m_i n_0 \frac{\tau}{1 + \tau} \vec{V}_* \cdot \vec{\nabla} \vec{V}_\perp + \mu_i \nabla^2 \vec{V}_i + \mu_e \nabla^2 \vec{V}_e$$

$$\vec{E} + \vec{V} \times \vec{B} = \eta \vec{j} - \frac{1}{en_0} \left(\vec{\nabla} p - \vec{j} \times \vec{B} - \frac{\tau}{1 + \tau} (\hat{b} \cdot \vec{\nabla} p) \hat{b} - \mu_e \nabla^2 V_e \right)$$

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) p = -\gamma p (\vec{\nabla} \cdot \vec{V}) + \kappa \nabla^2 p$$

- Drift velocities: $\vec{V} = \vec{V}_E + V_\parallel \hat{b} = \vec{V}_i - \frac{\tau}{1 + \tau} \vec{V}_* \quad \left(= \vec{V}_e - \frac{\tau}{1 + \tau} \vec{V}_* + \frac{\vec{j}}{en_0} \right) \quad \vec{V}_* = \frac{\hat{b} \times \vec{\nabla} p}{en_0 B}$

4-(Scalar)-field model along with reduced MHD

- Reduced MHD: Take $(\hat{b} \cdot \vec{\nabla} \times)$ to remove compressional Alfvén wave
- Force balance indicates $\mathbf{p} = p_0 - B_0 \mathbf{b}_z$
- $\vec{B} = \vec{\nabla} \psi \times \hat{z} + (B_0 + \mathbf{b}_z) \hat{z}$, $\vec{V} = \vec{\nabla} \phi \times \hat{z} + V_z \hat{z}$ in a slab lead to **4-field** (ψ, Z, ϕ, V_z) equations

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= [\phi, \psi] - [Z, \psi] + \eta J - \frac{\mu_e d_\beta (1 + \tau)}{c_\beta} \nabla^2 \left(V_z + \frac{d_\beta}{c_\beta} J \right) \\ \frac{\partial Z}{\partial t} &= [\phi, Z] + d_\beta^2 [J, \psi] + (c_\beta^2 \eta + (1 - c_\beta^2) \kappa) \nabla^2 Z + d_\beta c_\beta [V_z, \psi] + \mu_e d_\beta^2 \nabla^2 (U - \nabla^2 Z) \\ \frac{\partial U}{\partial t} &= [\phi, U] - \frac{\tau}{2} (\nabla^2 [\phi, Z] + [U, Z] + [\nabla^2 Z, \phi]) + [J, \psi] + \mu_i \nabla^2 (U + \tau \nabla^2 Z) - \mu_e \nabla^2 (U - \nabla^2 Z) \\ \frac{\partial V_z}{\partial t} &= [\phi, V_z] + \frac{c_\beta}{d_\beta} [Z, \psi] + \mu_i \nabla^2 V_z + \mu_e \nabla^2 \left(V_z + \frac{d_\beta}{c_\beta} J \right) \end{aligned}$$

$$*Z \equiv \mathbf{b}_z / c_\beta \sqrt{1 + \tau}$$

$$J = \nabla^2 \psi, U = \nabla^2 \phi, \quad c_\beta = \sqrt{\beta / (1 + \beta)}, \quad d_\beta = c_\beta d_i / \sqrt{1 + \tau} \quad \text{Ion skin depth } d_i = \sqrt{m_i / n_0 e^2 \mu_0 / a}$$

$$* \text{Poisson Bracket } [A, B] \equiv \hat{z} \cdot (\vec{\nabla} A \times \vec{\nabla} B) \quad * (t, \nabla, B, \eta, \mu_{i,e}, \kappa) \text{ are properly normalized with } a, V_A$$

Resonant field response is characterized by a single quantity “ Δ_{in} ” in boundary layer theory

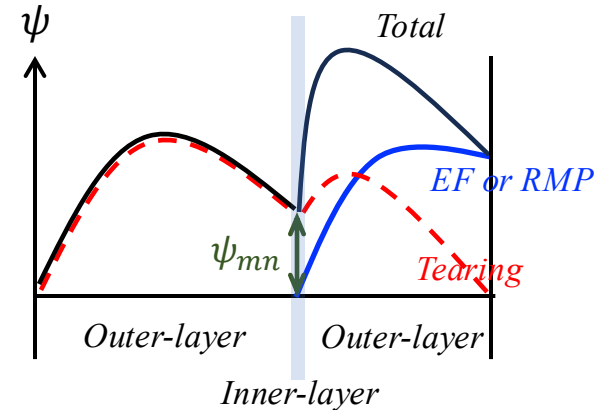
- Boundary layer theory with a small parameter $\epsilon \sim S^{-1/3}$ shows that resonant layer response $(\tilde{\psi}, \tilde{Z}, \tilde{\phi}, \tilde{V}_Z) \propto e^{i(m\theta - n\phi)}$ is characterized by a single quantity:

$$\Delta_{in}(\omega, Q, Q_i, Q_e, c_\beta, C, D, P_i, P_e) \quad \begin{array}{l} * \text{Normalized } \vec{E} \times \vec{B} \text{ rotation } Q, \text{ ion (electron) diamagnetic rotation } Q_{i(e)}, \text{ beta } c_\beta, \\ \text{conductivity (diffusivity) } C, \text{ ion gyroradius } D, \text{ ion (electron) viscosity } P_{i(e)} \end{array}$$

- Asymptotic matching to global outer-layer response (mostly ideal 3D MHD)

$$\lim_{X \rightarrow \infty} \Delta_{in} \rightarrow \left[\frac{\partial}{\partial x} \ln \psi \right]_{-}^{+} \leftarrow \lim_{x \rightarrow 0} \Delta_{out} \left(= \underbrace{\frac{\Delta_{ext}}{\psi_{mn}}}_{\text{External drive } \propto \delta B_{mn}} + \underbrace{\Delta'}_{\text{Tearing mode index}} \right)$$

- Giving growth ω for tearing modes, or seed island size ψ_{mn} driven by external EF or RMP
 - As a function of $Q, Q_i, Q_e, c_\beta, C, D, P_i, P_e$



Field penetration occurs when torque balance breaks down

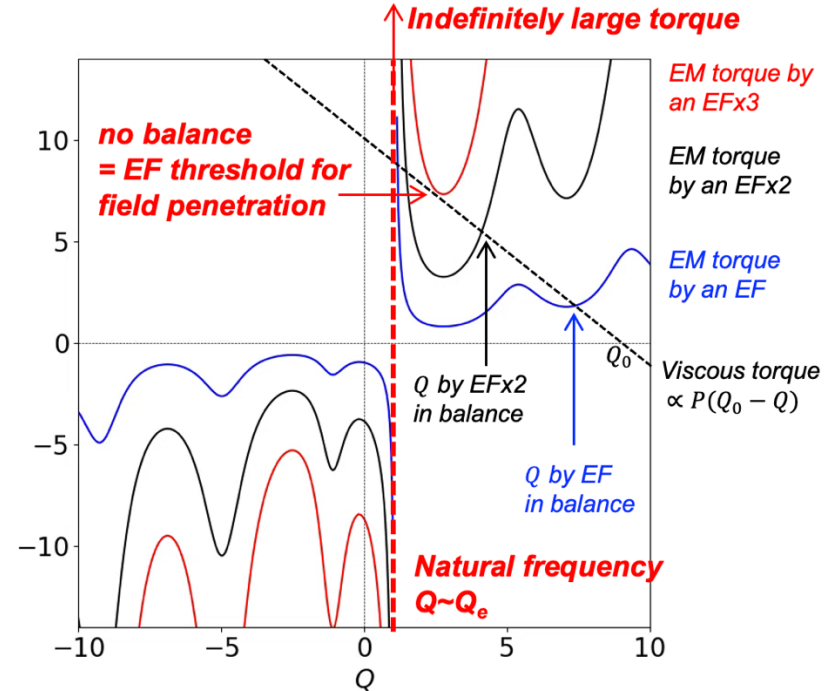
- In the presence of external 3D fields (EF or RMP), electromagnetic $\delta \vec{j}_{\parallel} \times \delta \vec{B}$ torque is induced

$$\tau_{\phi} = -\frac{k}{2} \text{Im}(\Delta_{in}) |\psi_{mn}|^2 \propto \text{Im} \left(\frac{1}{\Delta_{in}} \right)$$

- Field penetration:** When viscous torque cannot make a balance against $\delta \vec{j}_{\parallel} \times \delta \vec{B}$ torque, (ψ_{mn}, τ_{ϕ}) becomes indefinitely large near **natural frequency** $Q \sim Q_e$, breaking linear framework with nonlinear islands

- Field penetration threshold** can be estimated when this torque balance is no longer possible

$$\delta_c = \left[\frac{\delta B_{mn}}{B_{\phi}} \right]_{crit}^2 \approx \max \left[\frac{2P(Q_0 - Q)}{S \text{Im}(1/\hat{\Delta})} \right] * \hat{\Delta} = \epsilon \Delta_{in}$$



Earlier theory already identified 10 distinct regimes

- Earlier theory explored layer response analytically (Cole) and numerically (Park) by ignoring **electron viscosity** and **ion parallel flow** [Cole et al., Phys. Plasmas (2006)] [Park, Phys. Plasmas (2022)]

Fourier transform from $\tilde{F} = (\tilde{\psi}, \tilde{\phi}, \tilde{Z}, \tilde{V}_z)$ to $\bar{F} = (\bar{\psi}, \bar{\phi}, \bar{Z}, \bar{V}_z)$ by $\bar{F}(p) = \int_c \tilde{F}(X) e^{ipX} dX$

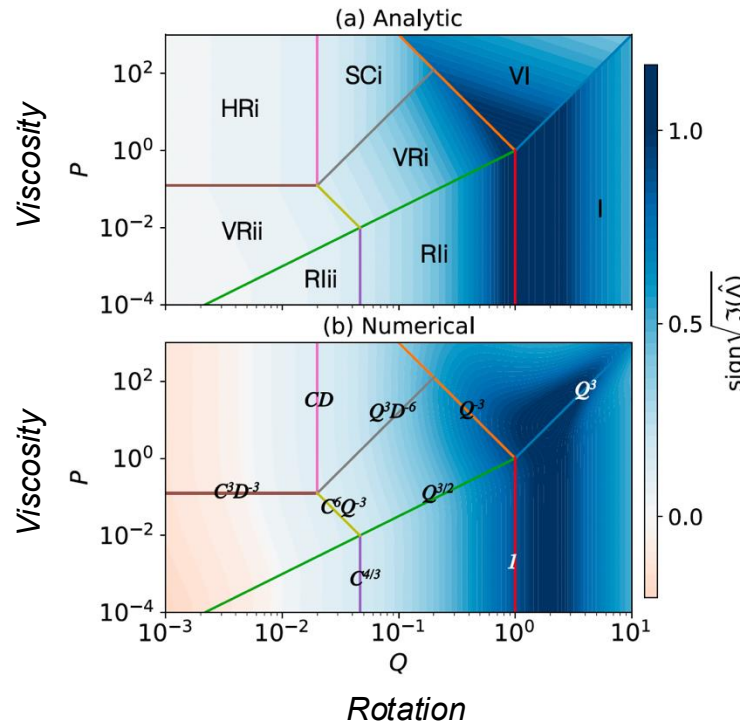
$$i(Q - Q_e)\bar{\psi} = \frac{d(\bar{\phi} - \bar{Z})}{dp} - p^2\bar{\psi} - p^4 P_e \frac{D^2}{c_\beta^2} (1 + \tau)\bar{\psi},$$

$$i(Q - Q_i)p^2\bar{\phi} = \frac{d(p^2\bar{\psi})}{dp} - Pp^4(\bar{\phi} + \tau\bar{Z}) - P_e p^4(\bar{\phi} - \bar{Z}).$$

$$iQ\bar{Z} - iQ_e\bar{\phi} = -D^2 \frac{d^2(p^2\bar{\psi})}{dp^2} - c_\beta^2 p^2 \bar{Z} + c_\beta^2 \frac{d\bar{V}_z}{dp} + P_e D^2 p^4 (\bar{\phi} - \bar{Z}),$$

$$\cancel{iQ\bar{V}_z + iQ_e\bar{\psi}} = \cancel{\frac{d\bar{Z}}{dp} - (P + P_e)p^2\bar{V}_z + P_e \frac{D^2}{c_\beta^2} p^4\bar{\psi}}.$$

HR: Hall-Resistive, SC: Semi-Collisional,
RI: Resistive-Inertial, VR: Visco-Resistive,
VI: Visco-Inertial, I: Inertial, (i: First, ii: Second)



Earlier theories reveal complexity but still without reproducing key observations of field penetration

$$\delta_c \propto n_e^{\alpha_n} B_\phi^{\alpha_B} R_0^{\alpha_R} \omega^{\alpha_\omega}$$

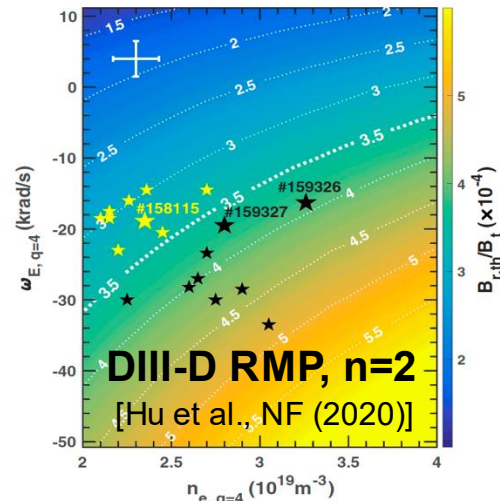
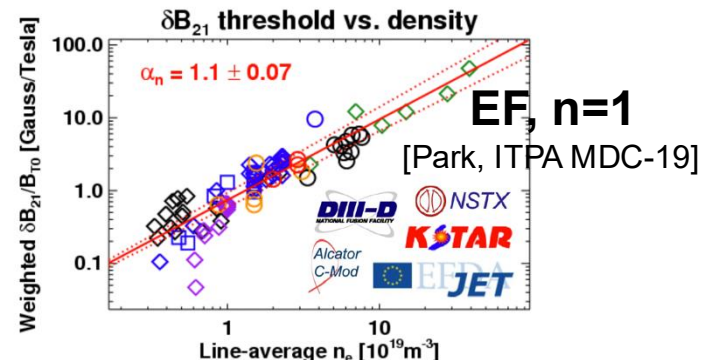
- Most well-known observation is strong density scaling (as well as inverse B_ϕ scaling and rotation scaling) of field penetration threshold

$$\alpha_{n,EF \text{ or } RMP} = 0.55 \sim 1.1$$

- In theory, most relevant regimes for operating or future tokamaks are Semi-collisional (SCi) or Hall-Resistive (HRI), but its density scaling is too weak

$$\alpha_{n,SCi \text{ or } HRI} = 0.25$$

- Nonlinear or toroidal effects? Still, there are missing physics in linear regimes



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Theory has been successfully extended with electron viscosity

- Earlier methods through a single 2nd ODE extended with **electron viscosity**

$$\frac{d}{dp} \left[\underbrace{\frac{p^2}{D^2(\tau+1)P_e \frac{c_\beta^2}{p^4 + p^2 + i(Q-Q_e)}}}_{\text{From parallel Ohm's law}} \frac{dY}{dp} \right] - p^2 G(p) Y = 0, \quad \begin{array}{l} \text{[J. Waybright and J.-K. Park, Phys. Plasmas (2024)]} \\ \text{[Y. Lee et al., Phys. Plasmas (2025)]} \end{array}$$

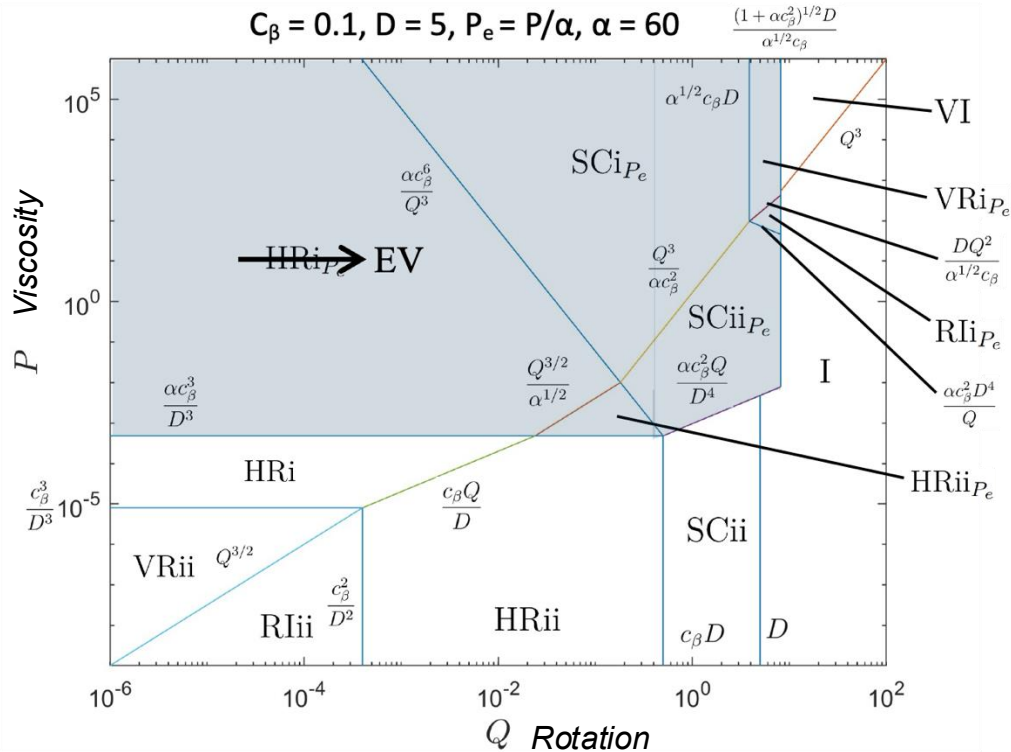
$$G(p) = \frac{\overbrace{PP_e D^2(\tau+1)p^6 + (iP_e D^2(Q-Q_i) + P c_\beta^2)p^4 + i(c_\beta^2 + P)(Q-Q_i)p^2 - Q(Q-Q_i)}^{\text{From the curl of Ohm's law}}}{PD^2(\tau+1)p^4 + (i(Q-Q_i)D^2 + c_\beta^2)p^2 + i(Q-Q_e)}$$

- Identifying two new regimes and reproducing **strong density scaling**, indicating key mechanism may be viscosity rather than (or in addition to) nonlinearity or toroidicity

$$\hat{\Delta}_{SCiPe} = \frac{3\Gamma(1/4)i^{7/4}\pi c_\beta^{1/2}(Q-Q_e)(Q-Q_i)^{3/4}}{8\Gamma(7/4)D^2(\tau+1)P_e^{1/4}} \longrightarrow \delta_{c,SCiPe} \sim n_e^{0.625} B_\phi^{-1.626} R_0^{-0.75}$$

$$\hat{\Delta}_{EV} = \frac{3i\pi\Gamma(5/8)c_\beta^{5/4}(Q-Q_e)}{8^{3/4}\Gamma(11/8)D^{5/4}(\tau+1)^{5/8}P_e^{1/4}} \longrightarrow \delta_{c,EV} \sim n_e^{0.69} B_\phi^{-0.75} R_0^{0.125}$$

Small electron viscosity can change resonant response strongly through delicate balance in generalized Ohm's law



- Electron viscosity can change response strongly even with classical assumption $P_e = P_i \sqrt{m_e/m_i}$
- It is because electron dynamics is in delicate balance, as represented by generalized Ohm's law
 - Electron viscosity should be compared with resistivity, flow, Hall term, in Ohm's law, rather than ion viscosity
- Can play even more important role when electron viscosity is anomalous

New analytic prediction has also been numerically verified

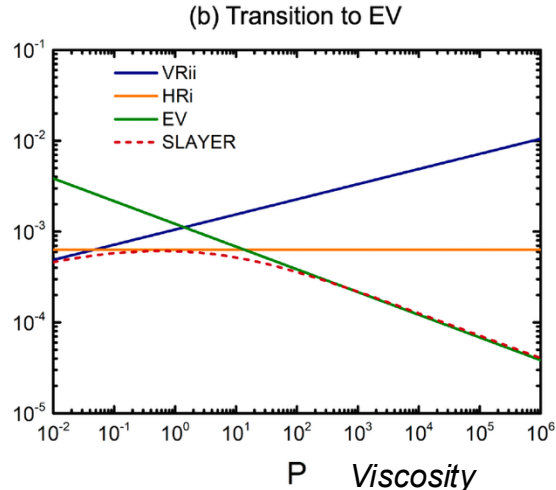
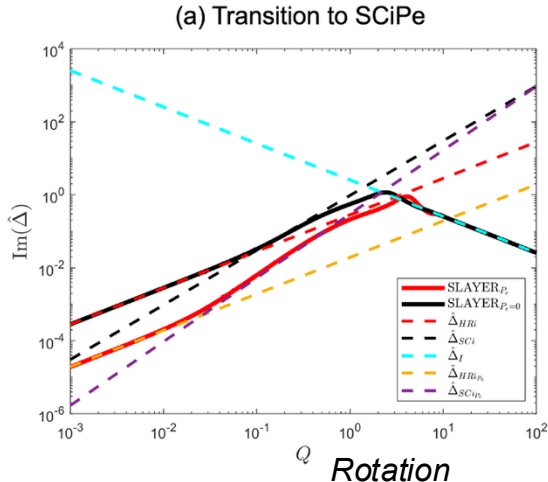
- 2nd order linear ODE with exponential solution behavior → 1st order nonlinear ODE with algebraic solution behavior by Riccati $W = (p/Y)(dY/dp)$

$$\frac{dW}{dp} = W \left(\frac{\frac{4D^2 p^3 (\tau+1) P_e}{c_\beta^2} + 2p}{\frac{D^2 p^4 (\tau+1) P_e}{c_\beta^2} + i(Q - Q_e) + p^2} - \frac{1}{p} \right) - \frac{W^2}{p} + pG(p) \left(\frac{D^2 p^4 (\tau+1) P_e}{c_\beta^2} + i(Q - Q_e) + p^2 \right)$$

[J. Waybright et al., Phys. Plasmas (2024)]

[Y.S. Lee et al., Phys. Plasmas (2025)]

- As implemented in SLAYER code and used for verification
- Each asymptotic regimes are precisely reproduced by SLAYER computation



Ion parallel flow effects require full numerical integration

- Last piece is **ion parallel flow perturbation** – requiring full integration in either

In configuration space

or

In Fourier space

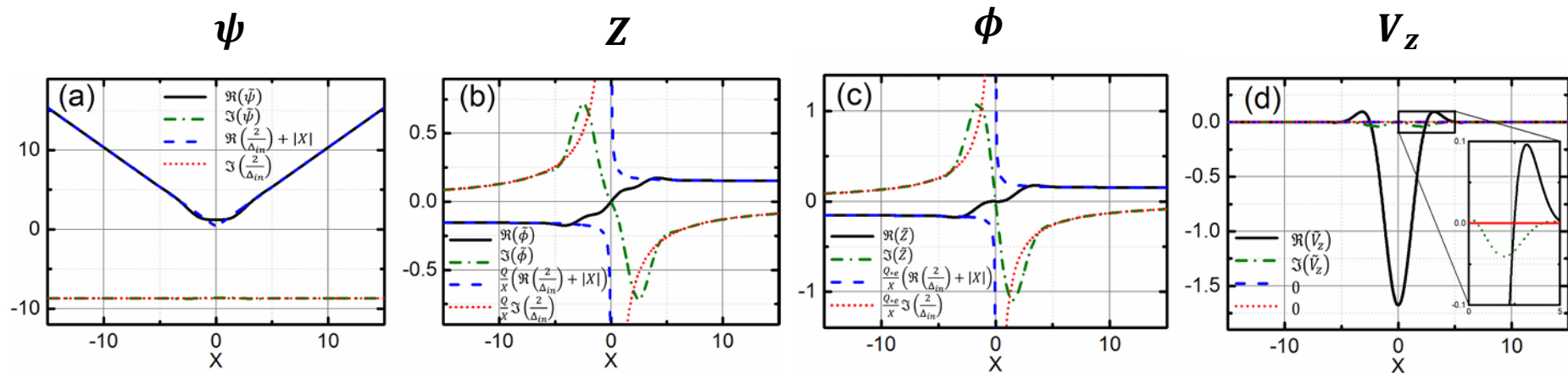
$$\begin{aligned}
 i(Q - Q_e)\tilde{\psi} &= iX(\tilde{\phi} - \tilde{Z}) + \frac{d^2\tilde{\psi}}{dX^2} - (1 + \tau)P_e \left(\frac{d^2\tilde{V}_z}{dX^2} + \frac{D^2}{c_\beta^2} \frac{d^4\tilde{\psi}}{dX^4} \right), \\
 iQ\tilde{Z} &= iQ_e\tilde{\phi} + iD^2X \frac{d^2\tilde{\psi}}{dX^2} + \cancel{i c_\beta^2 X \tilde{V}_z} + (c_\beta^2 + (1 - c_\beta^2)K) \frac{d^2\tilde{Z}}{dX^2} + P_e D^2 \left(\frac{d^4\tilde{\phi}}{dX^4} - \frac{d^4\tilde{Z}}{dX^4} \right), \\
 i(Q - Q_i) \frac{d^2\tilde{\phi}}{dX^2} &= iX \frac{d^2\tilde{\psi}}{dX^2} + P \left(\frac{d^4\tilde{\phi}}{dX^4} + \tau \frac{d^4\tilde{Z}}{dX^4} \right) + P_e \left(\frac{d^4\tilde{\phi}}{dX^4} - \frac{d^4\tilde{Z}}{dX^4} \right), \\
 \boxed{iQ\tilde{V}_z} &= -iQ_e\tilde{\psi} + iX\tilde{Z} + (P + P_e) \frac{d^2\tilde{V}_z}{dX^2} + P_e \frac{D^2}{c_\beta^2} \frac{d^4\tilde{\psi}}{dX^4}.
 \end{aligned}$$

$$\begin{aligned}
 i(Q - Q_e)\bar{\psi} &= \frac{d(\bar{\phi} - \bar{Z})}{dp} - p^2\bar{\psi} - p^4P_e \frac{D^2}{c_\beta^2} (1 + \tau)\bar{\psi}, \\
 i(Q - Q_i)p^2\bar{\phi} &= \frac{d(p^2\bar{\psi})}{dp} - Pp^4(\bar{\phi} + \tau\bar{Z}) - P_e p^4(\bar{\phi} - \bar{Z}), \\
 iQ\bar{Z} - iQ_e\bar{\phi} &= -D^2 \frac{d^2(p^2\bar{\psi})}{dp^2} - c_\beta^2 p^2\bar{Z} + \cancel{c_\beta^2 \frac{d\bar{V}_z}{dp}} + P_e D^2 p^4(\bar{\phi} - \bar{Z}), \\
 \boxed{iQ\bar{V}_z + iQ_e\bar{\psi}} &= \frac{d\bar{Z}}{dp} - (P + P_e)p^2\bar{V}_z + P_e \frac{D^2}{c_\beta^2} p^4\bar{\psi}.
 \end{aligned}$$

- A full solver for these high-order ODE system has also been successfully developed
 - In configuration space first by Y. S. Lee [Y. Lee et al., Nucl. Fusion (2024)]
 - Recently in Fourier space by R. Fitzpatrick's group [Private communication]

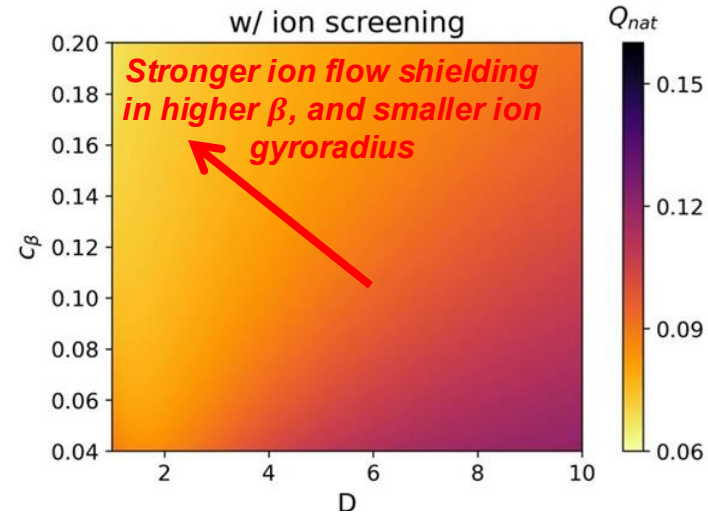
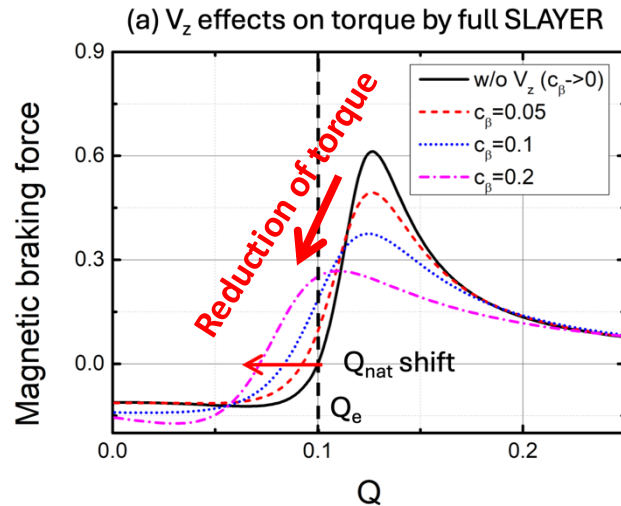
Fully reconstructed solution via Riccati matrix transformation clearly illustrates boundary phenomena across layer

- Solution steepness in $X \in (0, \infty)$ has been controlled by Riccati matrix transformation [Y. Lee et al., Nucl. Fusion (2024)]
- This enables the full solution reconstruction in configuration space, elucidating boundary layer phenomena



Ion parallel flow can shield field penetration strongly in high β

- Linear regime in low Q has $\Delta_{in} \propto (Q - Q_e)$, resulting $\tau_\phi \propto \text{Im}(1/\Delta_{in}) \rightarrow \infty$ as $Q \rightarrow Q_e$
- Singularity point is called **natural frequency** $Q_{nat} = Q_e$, leading to the concept of field penetration to nonlinearly growing islands
- Turns out that ion parallel flow removes singularity by strong screening, especially in high- β reactor conditions



New analytic theory with multiple-layer breakdown verifies ion parallel flow screening and natural frequency shift

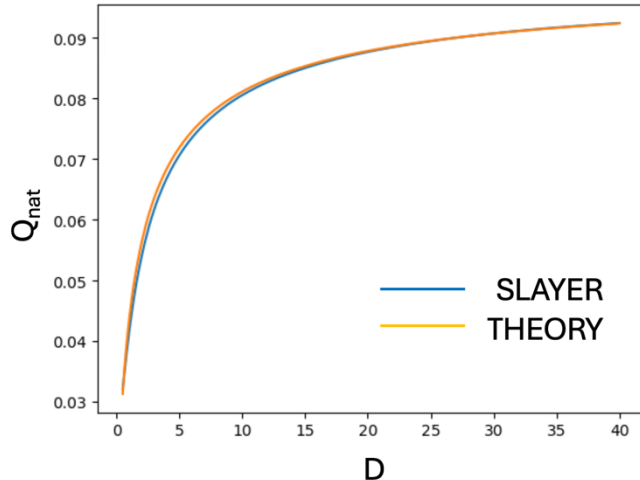
- Extended asymptotic matching with multiple layers has been developed
- For example, in HRi regime:

High-order correction by ion parallel flow

$$\hat{\Delta}_{HRiVz} = i\pi(Q - Q_e) \left(2 \frac{\Gamma(3/4)}{\Gamma(1/4)} \left(\frac{c_\beta^2}{(1+\tau)D^2} \right)^{1/4} + (2^{7/4}) \frac{4}{5} \frac{\Gamma(13/8)}{\Gamma(3/8)} (1+\tau)P \frac{Q}{Q - Q_e} \left(\frac{c_\beta^2}{(1+\tau)D^2 P^2} \right)^{5/8} \right)$$

[On the courtesy of J. Waybright (PU)]

(b) Comparison for natural frequency Q_{nat}



- Incredible agreement verifies strong ion flow screening
- Also verifying surprising implication – **There may be no field penetration at all in high- β reactor conditions** - to be validated in experiments

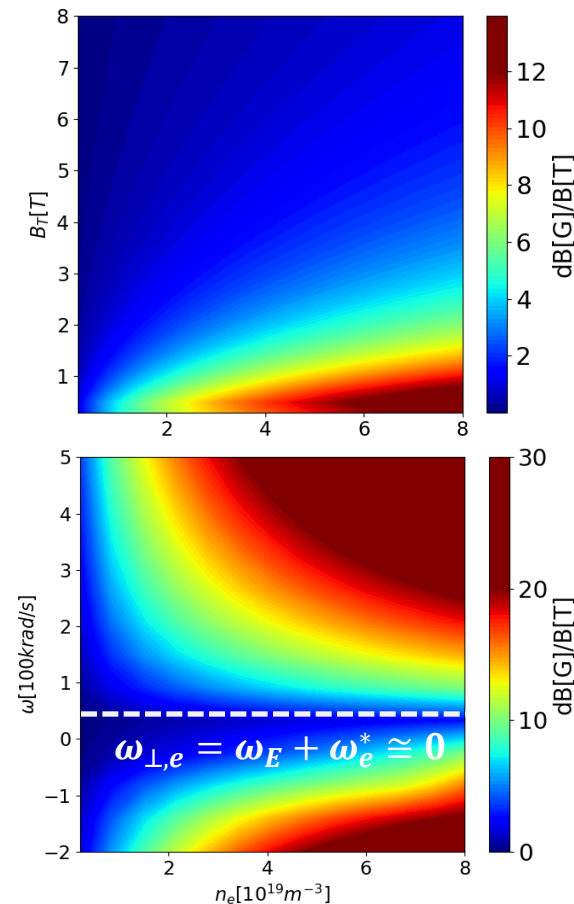
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Extended MHD elevates overall density and B_ϕ scaling

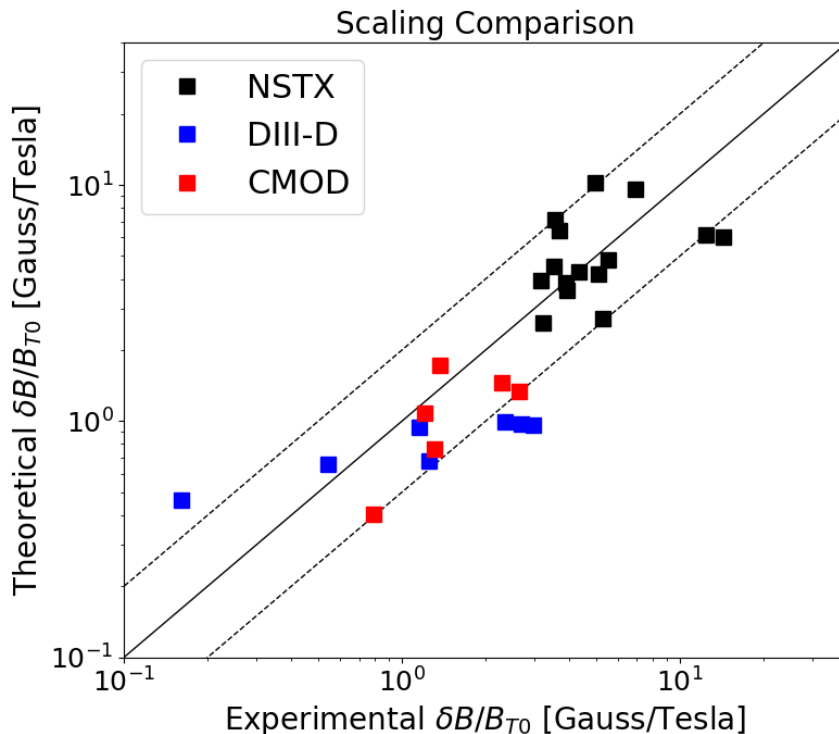
- Layer conditions can be under transition regimes and regimes are not always clearly separable in reality
- Full SLAYER computation for parametric scaling in wide experimental conditions
 - $3 \times 10^{18} m^{-3} < n_e < 8 \times 10^{19} m^{-3}$, $0.3T < B_\phi < 8T$,
 - $200 \text{krad/s} < \omega_\phi < 500 \text{krad/s}$
 - Separating rotation and viscosity (=constant) as an independent variable
- Scaling becomes favorable to experiments, although inter-parameter relation must be carefully accounted

$$\delta_c \sim n_e^{0.82} B_\phi^{-1.07} \omega_{\perp,e}^{0.79}$$



Applications to ITPA EF threshold database show feasibility

- A set of EF locked mode threshold data having TS diagnostics in Ohmically heated plasmas is tested with SLAYER
 - In order to minimize uncertainties in externally driven torque
 - With assumption of viscosity $P = 3.0$, $\omega_E = 0, n_i = n_e, T_i = T_e$
- Initial testing shows a possibility of good field penetration threshold prediction
- SLAYER with electron viscosity and ion parallel flow will also be tested for data with profiles and transport coefficients



Concluding remarks

- Two-fluid drift-MHD analytic theory extended with electron viscosity and parallel ion flow offers new insights for field penetration to magnetic islands
- Key findings:
 - Electron viscosity can enhance positive density correlation for field penetration threshold, better aligned with empirical scaling
 - Parallel ion flow perturbation becomes more important in high- β reactor conditions and can substantially shield resonant field along with shifted natural frequency
- These new findings are verified numerically, which is being tested against empirical scaling and EF database for validation
- Future work will also include the effects of NTV effects, directly incorporating anisotropic tensor, as its effects can be important in low-collisionality conditions