NEW UNDERSTANDING OF RESONANT LAYER RESPONSE VIA EXTENDED DRIFT MHD

Implications of high order physics on application of resonant magnetic perturbations

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Notable progress has been achieved in understanding and predicting resonant layer response under non-axisymmetric magnetic perturbations in tokamaks, by incorporating high order physics into the two-fluid drift-MHD layer model: (1) Electron viscosity is shown to play an important role in high temperature and low torque plasmas [Fig. 1(a)] since then the electric field is induced through the delicate balance of generalized Ohm's law [1]. It leads to a strong density scaling of field penetration threshold, consistent with locked mode experiments. (2) Parallel ion flow is shown to be critical in high β by maintaining screening effects in low rotation, shifting the natural rotating frequency [Fig. 1(b)]. As a result, electromagnetic torque remains finite even when electron flow becomes stationary without field penetration – a surprising prediction for reactor-relevant regimes. (3) Another high order physics under investigation is the ion neoclassical toroidal viscosity, which may require the proper modification of non-resonant layer response. These new elements for layer model are being integrated to the general perturbed equilibrium framework [3-4] to develop a reliable predictive model for field penetration thresholds – a central subject of error field correction against locked modes [Fig. 1(c)] and resonant magnetic perturbation (RMP) ELM suppression.

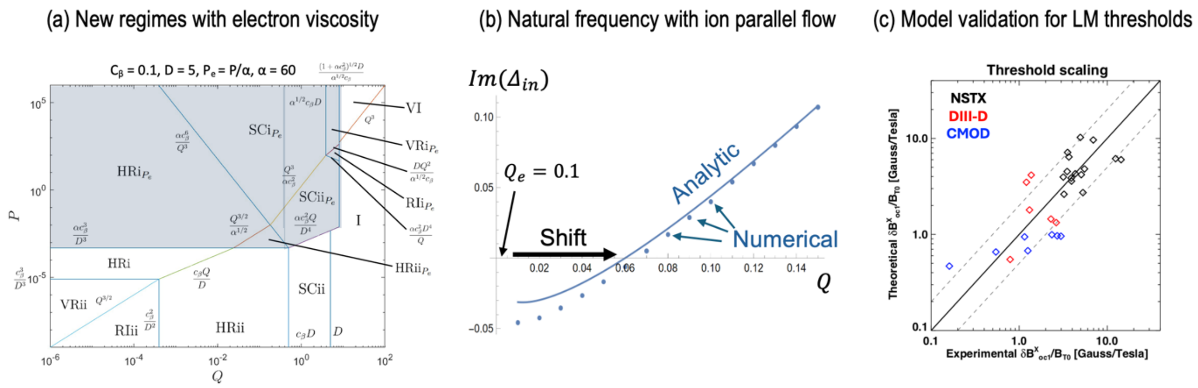

Fig. 1 (a) Newly identified regimes due to electron viscosity as a function of normalized viscosity (P) and rotation (Q) as colored in blue. (b) Shift of natural frequency Q for $\Delta_{in} = 0$ from Q_e due to ion parallel flow in high β $(c_\beta \to 1, D = 0, P = 0)$, as verified by high-order matching theory and numerical modeling. (c) Comparison of locked mode thresholds between GPEC SLAYER modeling and experimental data.

Field penetration is a bifurcation process in which magnetic islands form near resonant surfaces in response to magnetic perturbations. Predicting its onset requires resolving complex boundary layer phenomena. An efficient approach to include the extended MHD is the asymptotic matching along with the simplified geometry for the narrow layers [5,6]. In the linear phase, layer response is fully characterized by a scalar quantity called the inner layer Δ_{in}, representing the screening currents within the layers. Δ_{in} must be consistent with the outer-layer response and serves as the matching parameter in asymptotic boundary layer theory. Earlier theories identified 10 distinct linear drift-MHD regimes, highlighting the complexity of layer response. However, none of these regimes fully align with experimental parametric scaling, sparking extensive discussions on nonlinearity. Nonetheless, it remains possible that these discrepancies stem from missing non-ideal MHD physics.

One of such is the electron viscosity in the generalized Ohm's law $\vec{E} + \vec{V} \times \vec{B} + (1/en)\big(\vec{\nabla}p - (\tau/1 + \tau)(\hat{b} \cdot \vec{\nabla}p) - \vec{j} \times \vec{B} - \mu_e \nabla^2 \vec{V}_e\big) = \eta \vec{j}$ [5], where delicate parallel balance must be maintained in high temperature and low rotation. A new analytic theory incorporating electron viscosity $\mu_e \nabla^2 \vec{V}_e$ has uncovered additional response regimes, despite μ_e being smaller than ion viscosity μ_i, by a factor of the square root of the ion-to-electron mass ratio. This is illustrated in Fig. 1(a) with the new branches of the Hall-Resistive, Semi-Collisional, Visco-Resistive, Resistive-Inertial (HR, SC, VR, RI, respectively) regimes due to electron viscosity. Here the magnetic Prandtl number is $P = \tau_R/\tau_V$ with the resistive and viscous time respectively, Q is the $E \times B$ rotation normalized by $S^{1/3}\tau_H$ with Lunquist number S and hydromagnetic time τ_H, D is the normalized ion Lamour radius, and $c_\beta = \sqrt{\beta/(1+\beta)}$. A significant implication is the modification of the parametric scaling of the error field penetration threshold, particularly in the two most relevant tokamak regimes: $b_r/B_0 \sim n_e R_0^{-1/2} B_\phi^{-2}$ for HR and $b_r/B_0 \sim n_e^{5/8} T_e^{1/16} R_0^{-3/4} B_\phi^{-13/8}$ for SC [1]. For the first time in linear theory, these predictions align with experimental scaling laws.

On the other hand, a common feature in linear response is $\Delta_{in} \to 0$ as $Q \to Q_e$, implying the electromagnetic torque becomes indefinitely large when the electron flow becomes stationary. This defines the bifurcation to the nonlinear phase, characterized by the formation of magnetic islands, also known as field penetration. However, this singularity can be resolved by including the ion parallel flow, for instance, with V_z for $\vec{V} = \vec{\nabla}\phi \times \hat{z} + V_z \hat{z}$ in a slab model. The ion parallel flow becomes important in high β, but the effects of ion parallel flow remained unclear due to the analytical intractability of the model. To address this, a numerical approach has been developed extending Riccati transformation to solve the high-order ODE system in asymptotic matching. The key result is that a non-zero minimum Δ_{in} occurs at an offset rotation from electron stationary point, implying that the torque $\tau \sim Im(\Delta_{\mathrm{in}}^{-1})$ and the size of magnetic islands also remain finite due to additional ion screening effects. This numerical prediction has also been reproduced analytically by higher-order asymptotic matching as illustrated in Fig. 1(b). This leads to a surprising conclusion to be tested - no field penetration under sufficiently high-β conditions, a scenario that appears achievable locally in ITER advanced regimes [2].

Another effect that can be potentially important in reactor-relevant regimes is ion neoclassical toroidal viscosity (NTV), represented by $\vec{\nabla} \cdot \left(\delta p_\perp I + (\delta p_\parallel - \delta p_\perp)\hat{b}\hat{b}\right)$. Unlike previous studies that considered only second-order NTV effects [7], our formulation incorporates the anisotropic pressure tensor directly into the first-order ion momentum balance in the layer. Inner-layer expansion suggests that NTV corrections may also be required in the outer layer matching, which has already been implemented in the General Perturbed Equilibrium Code (GPEC) [3]. This NTV-driven term is expected to be particularly relevant at very low collisionality, a condition anticipated in the core region of ITER.

These new elements above are being integrated into the GPEC package as an inner-layer solver called SLAYER [4]. SLAYER calculates the key quantities such as Δ_{in}, helical flux, island size, electromagnetic torque, enabling predictions of rotation evolution and field penetration thresholds. Presently only a portion of the layer model has been implemented but the initial tests demonstrate strong feasibility. Figure 1(c) compares SLAYER's predictions for field penetration thresholds with experimental data from Ohmic locked mode (LM) experiments in NSTX, DIII-D, and C-Mod, showing agreement within a factor of 2. This initial comparison assumes $T_i = T_e$ and $\omega_{E\times B} = 0$, both reasonable approximations for Ohmic plasmas. The large uncertainty lies in the Prandtl number which is set to $P = 3$, but can vary as $P = 1 \sim 10$. This variation does not alter qualitative nature of the prediction but can shift the prediction quantitatively up to a factor of 2-3. The validation efforts are being extended to the ITPA database for error field and RMP threshold for ELM suppression, where the high-order physics presented in this paper is being progressively incorporated to enhance predictive accuracy.

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