Divertor-safe Nonlinear Burn Control in Long-Pulse Reactor Operation: Integrating Edge Constraints by Reference Governor

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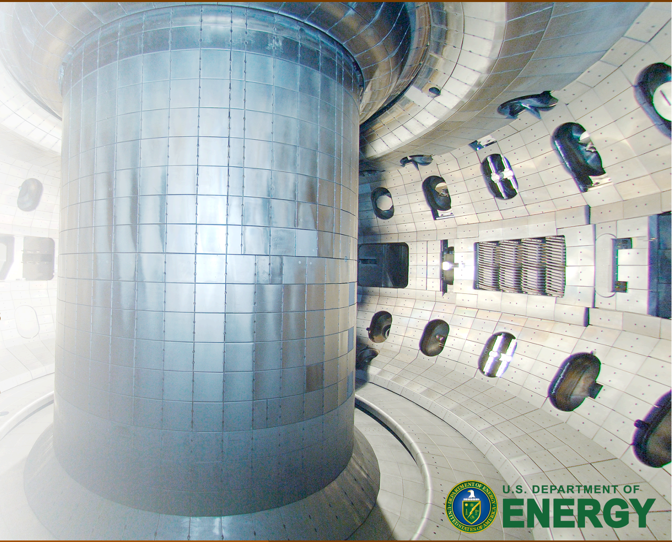

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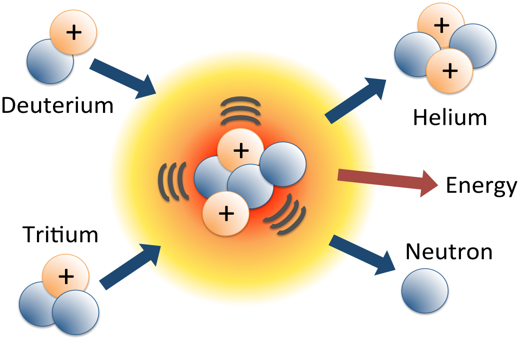
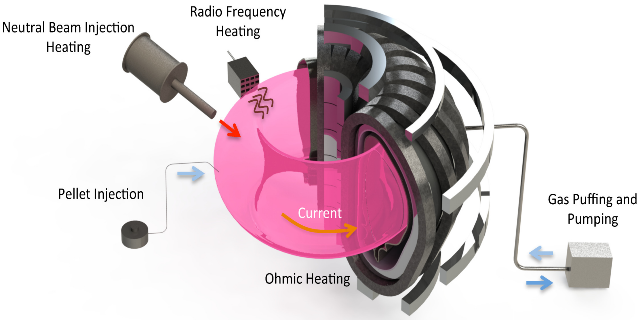
Burn Control ≡ Density and Temperature Control of All Species

$$ {}_1^2D + {}_1^3T \rightarrow \underbrace{{}_2^4He}_{3.5MeV} + \underbrace{{}_0^1n}_{14.1MeV}, \qquad Q_{DT} = \Delta mc^2 = 17.6MeV $$

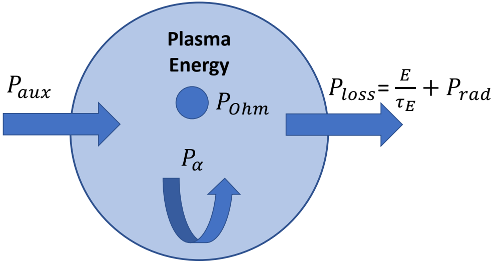




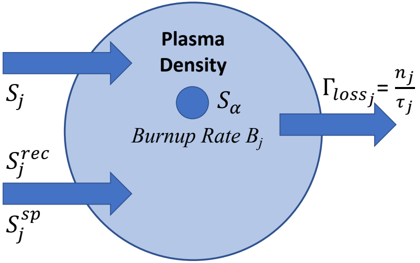
- **Regulation of fusion power in DT plasma ≡ Control of density/temperature of all species**

Burning Plasma Model (Controls): 0D Balance Equations

- Control goal is 0D ⇒ 0D response model is what is needed for control synthesis
  - Energy balance equations for ions and electrons ($E = E_i + E_e$ with $-\frac{E}{\tau_E} = -\frac{E_i}{\tau_{E_i}}) - \frac{E_e}{\tau_{E_e}})$
- Control synthesis: 0D model → Performance assessment (simulations): 1D model
  - Asses coupling with other space-dependent control goals such as profile control



$$\frac{dE}{dt} = -P_{loss} + P_\alpha + P_{Ohm} + P_{aux}$$



$$\frac{dn_j}{dt} = -\Gamma_{loss_j} \pm B_j + S_j + S_j^{rec} + S_j^{sp}$$

Nonlinear Plasma Response Model for Burn Control Synthesis

Ion Energy: $$\frac{dE_i}{dt} = -\frac{E_i}{\tau_{E_i}} + \phi_\alpha \overbrace{Q_\alpha S_\alpha}^{P_\alpha} + P_{ei} + P_{aux_i}^{others} + P_{aux_i} \qquad \left(E_i = \frac{3}{2} n_i T_i\right)$$

Electron Energy: $$\frac{dE_e}{dt} = -\frac{E_e}{\tau_{E_e}} + \bar{\phi}_\alpha Q_\alpha S_\alpha - P_{ei} - P_{rad} + P_{Ohm} + P_{aux,e}^{others} + P_{aux,e} \qquad \left(E_e = \frac{3}{2} n_e T_e\right)$$

Alpha particles: $$\frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + S_\alpha$$

Deuterium: $$\frac{dn_D}{dt} = -\frac{n_D}{\tau_D} - S_\alpha + S_D^{rec} + S_D^{others} + S_D$$

Tritium: $$\frac{dn_T}{dt} = -\frac{n_T}{\tau_T} - S_\alpha + S_T^{rec} + S_T$$

Impurities: $$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I^{sp} + S_I$$

(actuators/disturbances in red/blue)

Quasi-neutrality: $$n_e = n_D + n_T + 2n_\alpha + Z_I n_I$$

Density: $$n = \overbrace{n_\alpha + n_D + n_T + n_I}^{n_i} + n_e = 2n_D + 2n_T + 3n_\alpha + (Z_I + 1)\, n_I$$

Nonlinear Plasma Response Model for Burn Control Synthesis

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(actuators/disturbances in red/blue)

- $P_{aux_{i,e}}^{others}$, S_D^{others} represent effect of actuators (NBI, RF H&CD) under other competing controllers.
- P_{aux}, S_D, S_T, S_I are actuators available for burn control.

Nonlinear Plasma Response Model for Burn Control Synthesis

Ion Energy: $$\frac{dE_i}{dt} = -\frac{E_i}{\tau_{E_i}} + \phi_\alpha \overbrace{Q_\alpha S_\alpha}^{P_\alpha} + P_{ei} + P_{aux_i}^{others} + P_{aux_i} \qquad (E_i = \frac{3}{2} n_i T_i)$$

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(actuators/disturbances in red/blue)

- Reaction rate: $S_\alpha = n_D n_T \langle \sigma v \rangle \rightarrow S_\alpha = \gamma(1-\gamma) n_{DT}^2 \langle \sigma v \rangle$. Tritium fraction: $\gamma \triangleq n_T / n_{DT}$, $n_{DT} = n_T + n_D$.
- The DT reactivity $\langle \sigma v \rangle$ is a highly nonlinear function of the plasma temperature. $Q_\alpha = 3.52 MeV$.
- Fraction ϕ_α of P_α going to ions is highly nonlinear function of plasma state ($\bar{\phi}_\alpha = 1 - \phi_\alpha$).

Nonlinear Plasma Response Model for Burn Control Synthesis

Ion Energy: $\frac{dE_i}{dt} = -\frac{E_i}{\tau_{E_i}} + \phi_\alpha \overbrace{Q_\alpha S_\alpha}^{P_\alpha} + P_{ei} + P_{aux_i}^{others} + P_{aux_i}$ $\quad (E_i = \frac{3}{2} n_i T_i)$

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Impurities: $\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I^{sp} + S_I$ (actuators/disturbances in red/blue)

- Confinement scaling (IPB98(y,2)): $\tau_E = 0.0562 H_H (I_{coils}^{non-axi}) I_p^{0.93} B_T^{0.15} P^{-0.69} n_{e19}^{0.41} M^{0.19} R^{1.97} \epsilon^{0.58} \kappa_{95}^{0.78}$.
- Particle confinement assumed proportional to τ_E, i.e. $\tau_\alpha = k_\alpha \tau_E$, $\tau_D = k_D \tau_E$, $\tau_T = k_T \tau_E$, $\tau_I = k_I \tau_E$.
- Ion/Electron energy confinement assumed proportional to τ_E, i.e. $\tau_{E_i} = \zeta_i \tau_E$, $\tau_{E_e} = \zeta_e \tau_E$.

Nonlinear Plasma Response Model for Burn Control Synthesis

Ion Energy: $$\frac{dE_i}{dt} = -\frac{E_i}{\tau_{E_i}} + \phi_\alpha \overbrace{Q_\alpha S_\alpha}^{P_\alpha} + P_{ei} + P_{aux_i}^{others} + P_{aux_i} \qquad \left(E_i = \frac{3}{2} n_i T_i\right)$$

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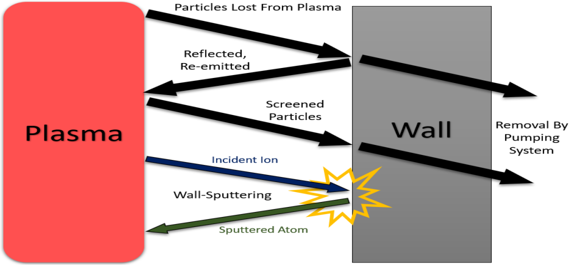
(actuators/disturbances in red/blue)

- Impurity sputtering source: $S_I^{sp} = f_I^{sp}\left(\frac{n}{\tau_I} + \dot{n}\right) \quad (0 \le f_I^{sp} << 1) \Rightarrow n_I^{sp} = f_I^{sp} n \quad (n_I = n_I^{inj} + n_I^{sp})$.
- Fuel recycling is included in the model through nonlinear functions S_D^{rec} and S_T^{rec} of the states.
- P_{rad}, P_{Ohm}, P_{ei} are also highly nonlinear functions of the states (and magnetic properities like I).

Nonlinear Plasma Response Model for Burn Control Synthesis

$$S_D^R = \frac{f_{eff}}{1-f_{ref}(1-f_{eff})}\left\{f_{ref}\frac{n_D}{\tau_D} + \left(1-\gamma^{PFC}\right)\times\left[\frac{(1-f_{ref}(1-f_{eff}))R^{eff}}{1-R^{eff}(1-f_{eff})} - f_{ref}\right]\left(\frac{n_D}{\tau_D}+\frac{n_T}{\tau_T}\right)\right\}$$

$$S_T^R = \frac{f_{eff}}{1-f_{ref}(1-f_{eff})}\left\{f_{ref}\frac{n_T}{\tau_T} + \gamma^{PFC}\times\left[\frac{(1-f_{ref}(1-f_{eff}))R^{eff}}{1-R^{eff}(1-f_{eff})} - f_{ref}\right]\left(\frac{n_D}{\tau_D}+\frac{n_T}{\tau_T}\right)\right\}$$



Recycled fluxes S_D^R, S_T^R are functions of†,[1]:

- f_{ref} : Reflection Fraction
- f_{eff} : Recycling Efficiency
- R^{eff} : Recycling Coefficient
- γ^{PFC} : Plasma-Facing-Component Tritium Fraction

$$P_{rad} = \underbrace{5.5\times 10^{-37} n_e^2 \sqrt{T(keV)Z_{eff}}}_{P_{brem}\ \text{(Bremmstrahlung)}} + P_{line} + P_{rec}, \quad Z_{eff} = \frac{n_D + n_T + 4n_\alpha + Z_I^2 n_I}{n_e}, \quad P_{Ohm} = 2.8\times 10^{-9}\frac{Z_{eff} I^2}{a^4 T^{3/2}}$$

† Ehrenberg J. 1996 Physical Processes of the Interaction of Fusion Plasmas with Solids (New York: Academic)

[1] M.D. Boyer and E. Schuster, Nuclear Fusion 55 (2015) 083021 (24pp).

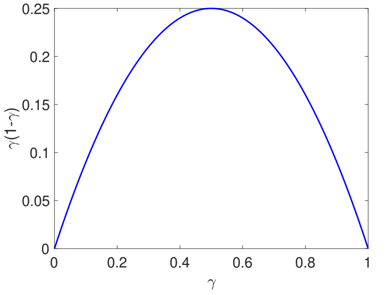
Burn Control Challenges: Nonlinearity Lead to Multiple Equilibria

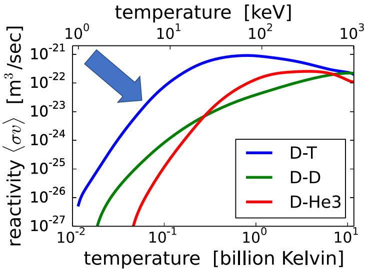
- Fusion power is highly nonlinear function of plasma state

$$P_f = n_D n_T \langle\sigma\nu\rangle Q_{DT} = \gamma\,(1-\gamma) n_{DT}^2 \langle\sigma\nu\rangle Q_{DT}$$

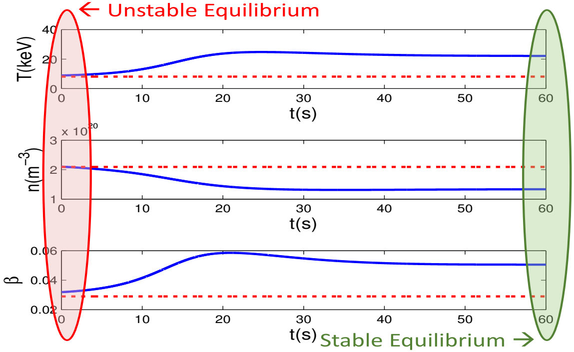
  where $n_{DT} \triangleq n_T + n_D$ and $\gamma \triangleq n_T / n_{DT}$.

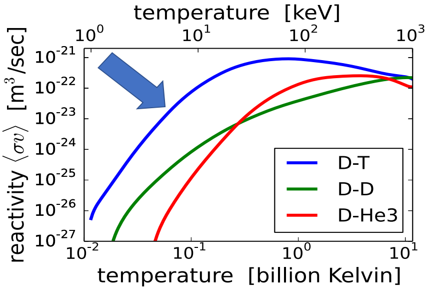
- The function $\gamma\,(1-\gamma)$ achieves its maximum of 0.25 at $\gamma = 0.5$ and decreases steeply for smaller/larger γ's
- The reactivity $\langle\sigma\nu\rangle$ is a highly nonlinear function of ion temperature (steep derivative in low-temperature region)
- **How can we regulate the fusion power P_f?**
  - Fuel density n_{DT} → Fueling
  - Tritium fraction γ → Isotopic fuel tailoring
  - Reactivity $\langle\sigma\nu\rangle$ → Heating (ion)
- **Key observation: Multiple Solutions!**
  - Several operating points (plasma states) with the same P_f
  - Full density/temperature control → Desired operating point





Burn Control Challenges: Stability Is a Property of Equilibrium





$$P_f = n_D n_T \langle \sigma \nu \rangle Q_{DT} = \gamma (1 - \gamma) n_{DT}^2 \langle \sigma \nu \rangle Q_{DT}$$

- The DT reactivity introduces a *positive-feedback mechanism* in the low-temperature region
- Potential for **thermal instability of operating equilibrium** → **excursions and quenching**
  - Left fig.: Thermal excursion from perturbed unstable equilibrium to stable equilibrium (same inputs)
- Active burn control could enable operation at (higher-performance) unstable operating points
  - Scenario development work for burning plasmas should be carried out with active burn controller
- Operation at stable equilibria still needs active control (transient performance, disturbance rejection)

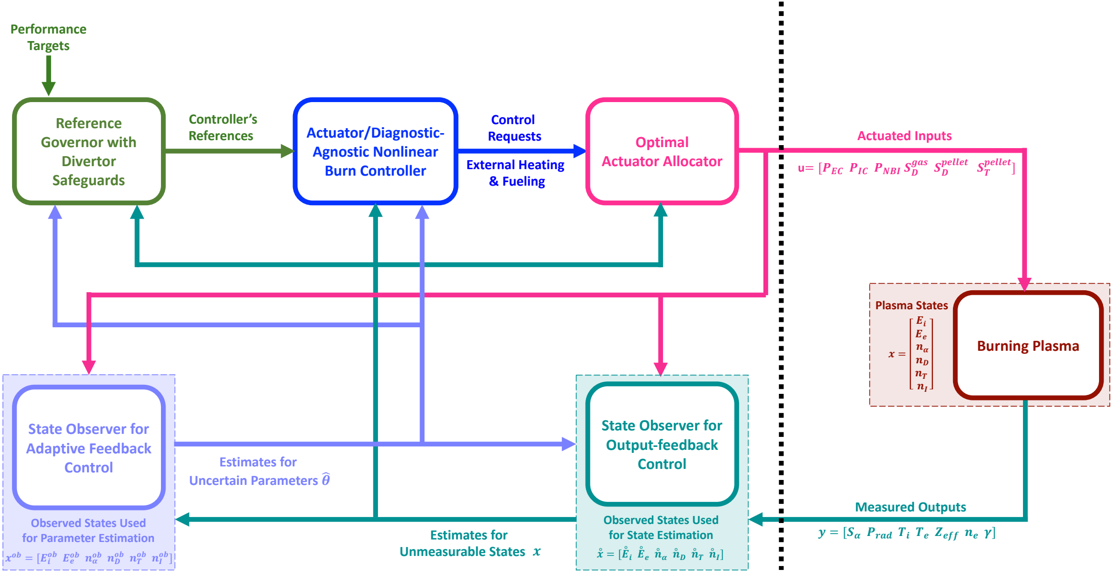
Burn Control Synthesis: Needs, Constraints, Approaches

- **Nonlinearity, dimensionality, actuator sharing, control-goal coupling** $\rightarrow$ **Model-based Control**
- **Modeling Challenges:** Limited eliability & readiness of predictive models for burning plasmas
  - Transport (1D) models (control simulations) are still under development (particle transport is complex)
  - Volume-averaged (0D) balance models (control synthesis) include uncertain parameters
- **Integration Challenges:** Coupling though confinement/actuator-sharing with other control goals
  - Equilibrium (shape, current), ELM control, RWM/NTM stabilization, profile control
- **Controllability Challenges:**
  - $Q \uparrow \Leftrightarrow P_{\alpha} >> P_{aux}$: **control by heating may not be effective**
  - **Wall recycling: control by (isotopic) fueling may not be effective**
- **Observability Challenges:**
  - **Limited and noisy set of diagnostics**
- **Long-pulse Operation Challenges:**
  - Wall heat/particle load tolerance may impose constraints on core burn regulation
  - Burn controller should be able to change operating conditions to assist divertor/wall protection
  - Effort on core/edge-compatible scenario development $\rightarrow$ Effort on integrated core-edge control

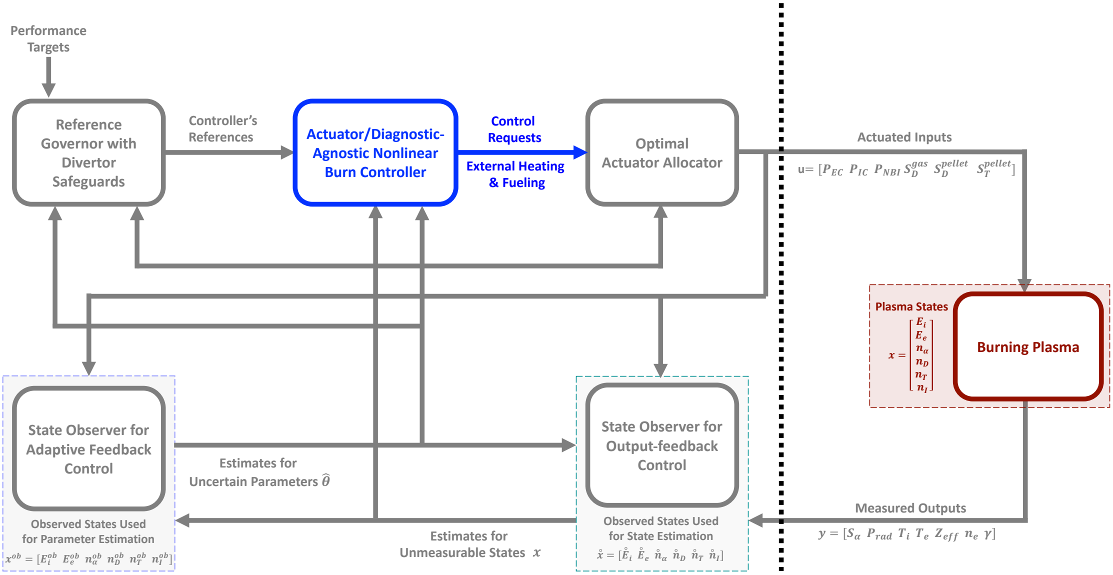
Burn Control Solution: Overview of Proposed Approach

1. **An Actuator/diagnostic-agnostic Nonlinear Burn Controller**
   – Embeds knowledge (model) of coupled nonlinear dynamics and multiple input/output configuration
2. **A State Observer for Output-feedback Control**
   – Estimates non-measurable components of plasma state from a limited set of noisy diagnostics
3. **State Observer for Adaptive Feedback Control**
   – Learns uncertain model (plasma confinement, wall recycling) parameters in real time
4. **Optimal Actuator Allocator**
   – Converts virtual → physical actuation requests incorporating actuator constraints/dynamics
5. **Reference Governor with Divertor Safeguards**
   – Determines the controller's references in order to achieve operator-defined performance metrics
   – Incorporates divertor safeguards as optimization constraints to prevent divertor damage

Burn Control Solution: Overview of Proposed Approach



Burn Control Solution: Controller



Nonlinear Feedback Control: Summary of Synthesis Technique

1. Choose operating **equilibrium point**: $\bar{E}_i, \bar{E}_e, \bar{n}_\alpha, \bar{n}_D, \bar{n}_T, \bar{n}_I \equiv 0$ given by $\bar{P}_{aux_i}, \bar{P}_{aux_e}, \bar{S}_D, \bar{S}_T, \bar{S}_I \equiv 0$
2. Write **dynamics of deviations** of states from desired operating point:

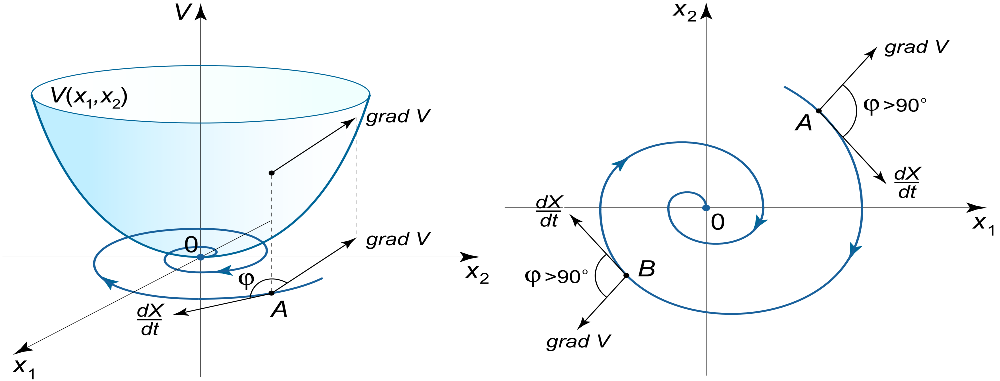
$$\tilde{E}_i \triangleq E_i - \bar{E}_i, \tilde{E}_e \triangleq E_e - \bar{E}_e, \tilde{n}_\alpha \triangleq n_\alpha - \bar{n}_\alpha, \tilde{n}_D \triangleq n_D - \bar{n}_D, \tilde{n}_T \triangleq n_T - \bar{n}_T, \tilde{n}_I \triangleq n_I$$

3. Defining $x \triangleq [\ \tilde{E}_i \quad \tilde{E}_e \quad \tilde{n}_\alpha \quad \tilde{n}_D \quad \tilde{n}_T \quad \tilde{n}_I\ ]$ and $u \triangleq [\ \tilde{P}_{aux_i} \quad \tilde{P}_{aux_e} \quad \tilde{S}_D \quad \tilde{S}_T \quad \tilde{S}_I\ ]$, the burning-plasma model can be written as $\dot{x} = f(x,u)$ with $f(0,0) = 0$
4. **Control Design Challenge:**
   - Choose a **Lyapunov function candidate** $V(x)$, where $V(0) = 0$ and $V(x) > 0$ for all $x \neq 0$
   - Choose a **feedback control law** $\alpha(x)$ with $\alpha(0) = 0$ s.t. $\dot{x} = f(x, \alpha(x)) \triangleq f^*(x)$ with $f^*(0) = 0$ and

$$\dot{V}(x) = \frac{\partial V}{\partial x}\dot{x} = \frac{\partial V}{\partial x} f^*(x) < 0 \text{ for all } x \neq 0$$

   Actuator laws ($u = \alpha(x)$) are chosen to cancel nonlinear and possibly destabilizing terms, and to adjust response time, robustness to uncertainties, and sensitivity to noise.
5. This technique **avoids linearization** around a particular operating point, which satisfies goals:
   - ✓ **Regulation around a desired burning equilibrium point**
   - ✓ **Drive plasma from one operating point to another (Modify Q or P_f)**
   - ✓ **Access to and exit from the burning plasma mode**

Nonlinear Feedback Control: Lyapunov Theory in a Nutshell

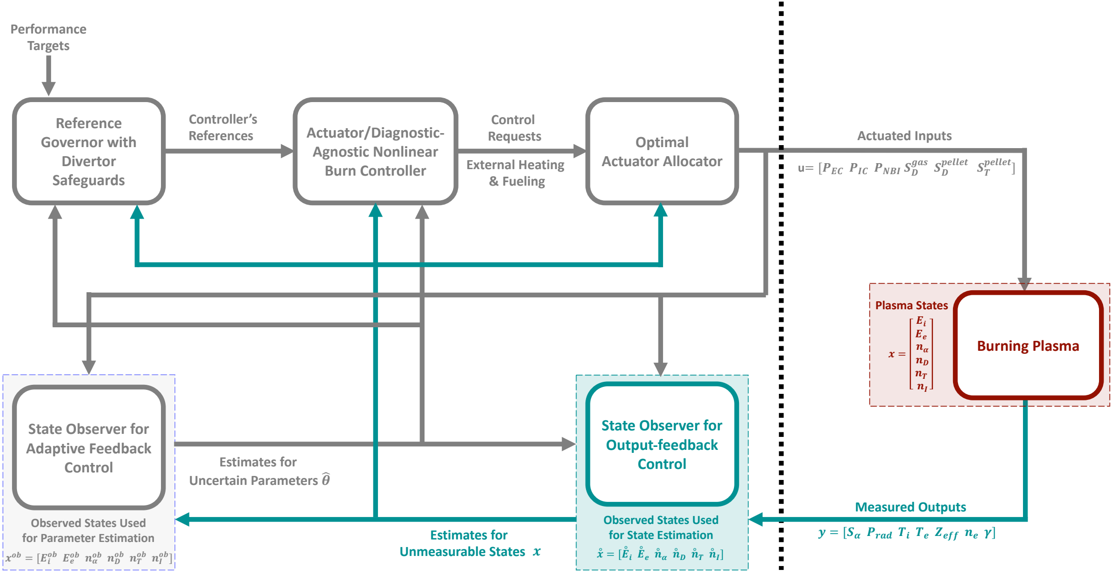


If the derivative $\frac{dV}{dt} = \frac{\partial V}{\partial x} f^*(x) = \frac{\partial V}{\partial x} f(x, \alpha(x))$ along a phase trajectory is everywhere negative, then the trajectory x tends to $V \equiv 0$ and therefore to the origin, i.e. the system is asymptotically stable

$\dot{V} \equiv \frac{dV}{dt}$ is negative as long as angle ϕ between grad $V \equiv \frac{\partial V}{\partial x}$ and $\dot{x} \equiv \frac{dx}{dt} = f^*(x)$ is higher than 90°

[1] M.D. Boyer and E. Schuster, Nuclear Fusion 55 (2015) 083021 (24pp).
[2] A. Pajares, E. Schuster, Nuclear Fusion 59 (2019) 096023 (18pp).
[3] V. Graber and E. Schuster, Nuclear Fusion 62 (2022) 026016 (18pp).

Burn Control Solution: State Observer



Burn Control Solution: State Observer

We define an observer as ($T_i = T_e$ case)

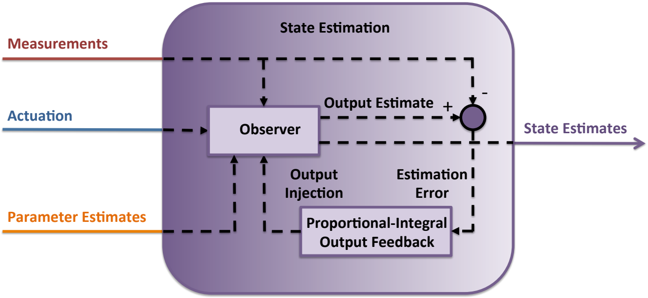
$$\dot{\mathring{E}} = -\hat{\theta}_1 \frac{\mathring{E}}{\tau_E^{sc}} + P_\alpha - P_{rad} + P_{Ohm} + P_{aux} + L_E,$$

$$\dot{\mathring{n}}_\alpha = -\hat{\theta}_2 \frac{\mathring{n}_\alpha}{\tau_E^{sc}} + S_\alpha + L_\alpha,$$

$$\dot{\mathring{n}}_D = -\hat{\theta}_3 \frac{\mathring{n}_D}{\tau_E^{sc}} + \hat{\theta}_4 \frac{\mathring{n}_T}{\tau_E^{sc}} - S_\alpha + S_D + L_D,$$

$$\dot{\mathring{n}}_T = \hat{\theta}_5 \frac{\mathring{n}_D}{\tau_E^{sc}} - \hat{\theta}_6 \frac{\mathring{n}_T}{\tau_E^{sc}} - S_\alpha + S_T + L_T,$$

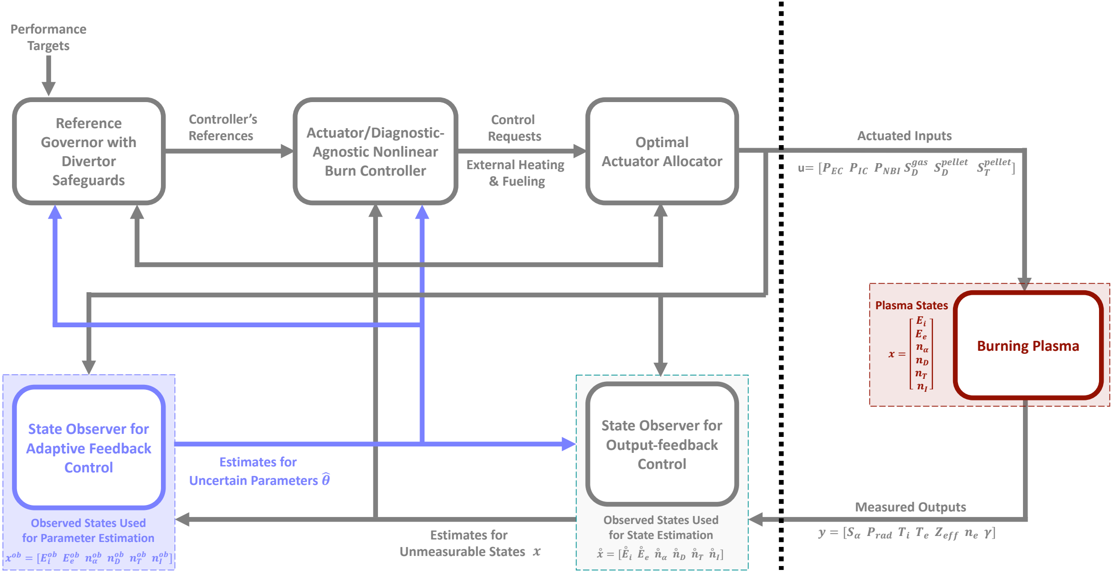
$$\dot{\mathring{n}}_I = -\hat{\theta}_7 \frac{\mathring{n}_I}{\tau_E^{sc}} + S_I^{sp} + S_I + L_I,$$



- We consider a general nonlinear output map $y = h(E, n_\alpha, n_I, n_D, n_T)$
- The system is augmented with an additional state, $\check{z}$, governed by $\dot{\check{z}} = \mathring{y} - y = \check{y}$
- Lyapunov analysis $\rightarrow$ injection terms L_E, L_α, L_D, L_T, L_I adopt a proportional-integral form

[1] M. D. Boyer, E. Schuster, International Federation of Automatic Control World Congress (2014).

Burn Control Solution: Parameter Estimator



Burn Control Solution: Parameter Estimator

We define a system observer as ($T_i = T_e$ case)

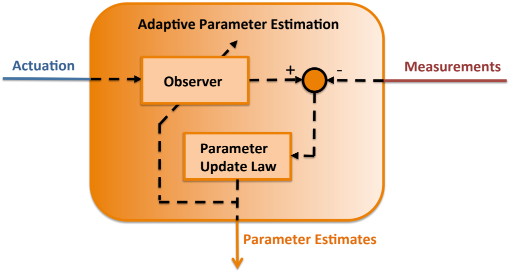
$$\dot{E}^{ob} = -\hat{\theta}_1 \frac{E}{\tau_E} + P_\alpha - P_{rad} + P_{aux} + P_{Ohm} - K_E^{ob}\left(E^{ob} - E\right)$$

$$\dot{n}_\alpha^{ob} = -\hat{\theta}_2 \frac{n_\alpha}{\tau_E} + S_\alpha - K_\alpha^{ob}\left(n_\alpha^{ob} - n_\alpha\right)$$

$$\dot{n}_D^{ob} = -\hat{\theta}_3 \frac{n_D}{\tau_E} + \hat{\theta}_4 \frac{n_T}{\tau_E} - S_\alpha + S_D - K_D^{ob}\left(n_D^{ob} - n_D\right)$$

$$\dot{n}_T^{ob} = \hat{\theta}_5 \frac{n_D}{\tau_E} - \hat{\theta}_6 \frac{n_T}{\tau_E} - S_\alpha + S_T - K_T^{ob}\left(n_T^{ob} - n_T\right)$$

$$\dot{n}_I^{ob} = -\hat{\theta}_7 \frac{n_I}{\tau_E} + S_I + S_I^{sp} - K_I^{ob}\left(n_I^{ob} - n_I\right)$$



The dynamics of the error $\tilde{\theta} = \theta - \hat{\theta}$ can be asymptotically stabilized by taking

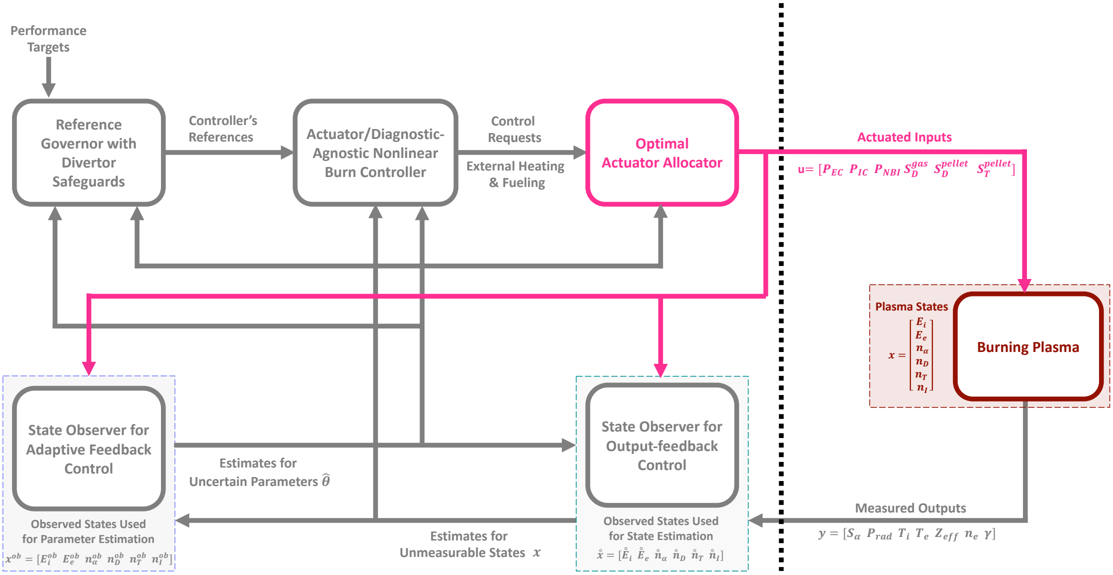
$$\dot{\hat{\theta}} = -\frac{1}{\tau_E}\Gamma\left[\ \tilde{n}_\alpha^{ob} n_\alpha \quad \tilde{E}^{ob} E \quad \tilde{n}_D^{ob} n_D \quad -\tilde{n}_D^{ob} n_T \quad -\tilde{n}_T^{ob} n_D \quad \tilde{n}_T^{ob} n_T \quad \tilde{n}_I^{ob} n_I\ \right]^T, \Gamma > 0$$

where

$$\tilde{n}_\alpha^{ob} = n_\alpha^{ob} - n_\alpha, \tilde{E}^{ob} = E^{ob} - E, \tilde{n}_I^{ob} = n_I^{ob} - n_I, \tilde{n}_D^{ob} = n_D^{ob} - n_D, \tilde{n}_T^{ob} = n_T^{ob} - n_T.$$

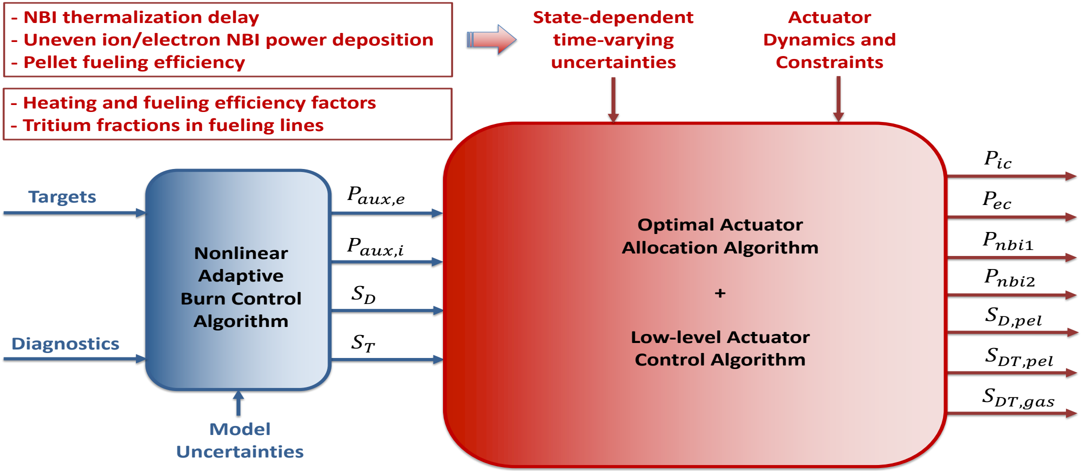
[1] M.D. Boyer and E. Schuster, Plasma Physics and Controlled Fusion 56 104004 (2014).

Burn Control Solution: Actuator Allocator



Burn Control Solution: Actuator Allocator

- Virtual control inputs ↔ Effector System ↔ Physical control inputs
- Optimally handle competition by multiple actuators for available actuation



[1] V. Graber and E. Schuster, Nuclear Fusion 62 (2022) 026016 (18pp).

Burn Control Solution: Actuator Allocator

The Effector System maps the control efforts v to the actuator efforts u:

$$v = [P_{aux,i}\ P_{aux,e}\ S_D\ S_T]^T \quad \longleftrightarrow \quad u = [P_{ic}\ P_{ec}\ P_{nbi_1}\ P_{nbi_2}\ S_{D_{pel}}\ S_{DT_{pel}}\ S_{DT_{gas}}]^T$$

$$P_{aux,i} = \eta_{ic} P_{ic} + \eta_{nbi_1}\phi_{nbi} P_{nbi_1} + \eta_{nbi_2}\phi_{nbi} P_{nbi_2}$$

$$P_{aux,e} = \eta_{ec} P_{ec} + \eta_{nbi_1}\bar{\phi}_{nbi} P_{nbi_1} + \eta_{nbi_2}\bar{\phi}_{nbi} P_{nbi_2} \qquad \text{(where } \bar{\phi}_{nbi} = 1 - \phi_{nbi}\text{)}$$

$$S_D = \eta_{nbi_1}\frac{P_{nbi_1}}{\varepsilon_{nbi_0}} + \eta_{nbi_2}\frac{P_{nbi_2}}{\varepsilon_{nbi_0}} + \eta_{pel_1} S_{D_{pel}} + \eta_{pel_2}(1-\gamma_{pel}) S_{DT_{pel}} + \eta_{gas}(1-\gamma_{gas}) S_{DT_{gas}}$$

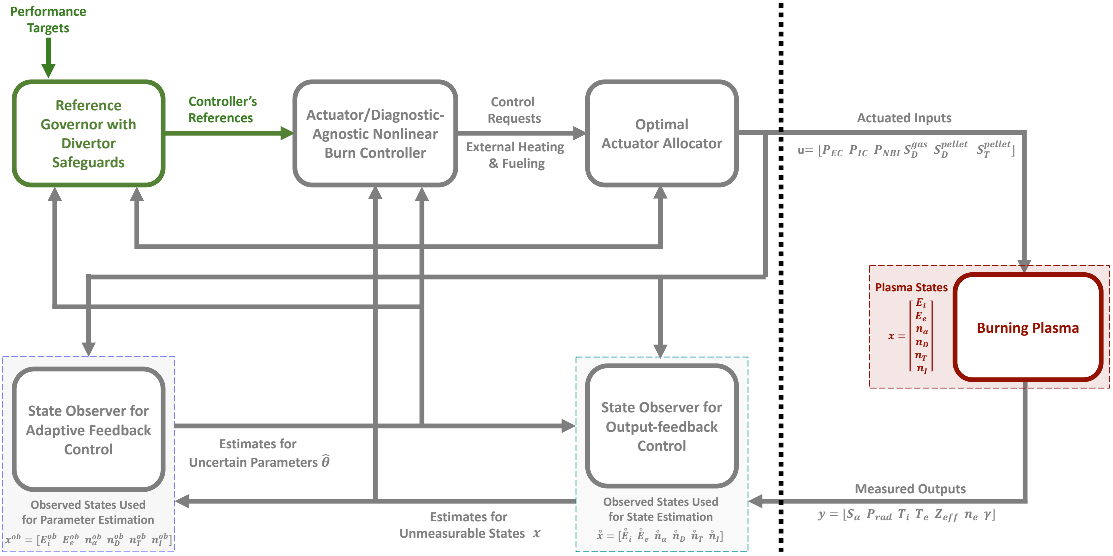
$$S_T = \eta_{pel_2}\gamma_{pel} S_{DT_{pel}} + \eta_{gas}\gamma_{gas} S_{DT_{gas}}$$

Uncertain Parameters

- Ion cyclotron, electron cyclotron & NBI heating: P_{ic}, P_{ec}, P_{nbi_1}, P_{nbi_2}
- DT pellet & gas injection with Tritium fractions γ_{pel} & γ_{gas}: $S_{D_{pel}}$, $S_{DT_{pel}}$, $S_{DT_{gas}}$
- Efficiency factors: η_{ic}, η_{ec}, η_{nbi_1}, η_{nbi_2}, η_{pel_1}, η_{pel_2}, η_{gas}
- Pellet fueling efficiency decreases with increasing plasma energy: $\eta_{pel_i} = \rho_{pel_i}(1 - E/E_0)$, $i \in \{1, 2\}$
- The NBI ion-heating fraction $\phi_{nbi} = \rho_{nbi}\phi^\star_{nbi}$ contains uncertainty (ρ_{nbi}).
- NBI thermalization delay contains uncertainty: ρ_{th}

$$\tau^{lag}_{nbi} = \rho_{th}\tau^\star_{nbi} = -\rho_{th}\frac{2}{3B}\ln\left[\frac{\left(\frac{\varepsilon_{nbi_{th}}}{\varepsilon_{nbi_0}}\right)^{3/2} + \left(\frac{\varepsilon_c}{\varepsilon_{nbi_0}}\right)^{3/2}}{1 + \left(\frac{\varepsilon_c}{\varepsilon_{nbi_0}}\right)^{3/2}}\right] \qquad (\varepsilon_{nbi_{th}} = T_i)$$

Burn Control Solution: Real-time Optimal Reference Governor

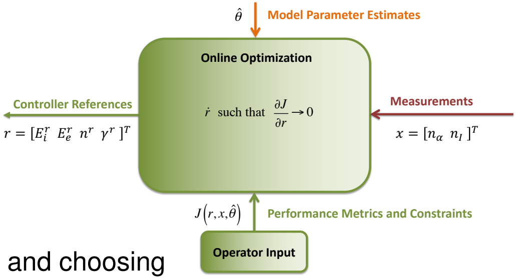


Burn Control Solution: Real-time Optimal Reference Governor

$$J = \underbrace{\frac{w_{T_e}}{2}}^{\text{weight}}\Big(T_e - \underbrace{T_e^{des}}_{\text{target}}\Big)^2 + \underbrace{\frac{w_{n_e}}{2}}^{\text{weight}}\Big(n_e - \underbrace{n_e^{des}}_{\text{target}}\Big)^2 + \underbrace{\frac{w_{P_f}}{2}}^{\text{weight}}\Big(P_f - \underbrace{P_f^{des}}_{\text{target}}\Big)^2 + \underbrace{\frac{w_{\gamma}}{2}}^{\text{weight}}\Big(\gamma - \underbrace{\gamma^{des}}_{\text{target}}\Big)^2 \overbrace{- \frac{1}{\eta_c}\sum_{i=1}^{K}\ln(-g_i)}^{constraints}$$

- A **reference for the controlled states** $r = [E_i^r, E_e^r, n^r, \gamma^r]^T$ determines burn condition
- T_e^{des}, n_e^{des}, P_f^{des}, γ^{des} are desired targets
- w_{T_e}, w_{n_e}, w_{P_f}, w_{γ} are tracking weights
- Constraints by **barrier function**

$$g_i(E, n, \gamma, n_\alpha, n_I) < 0$$



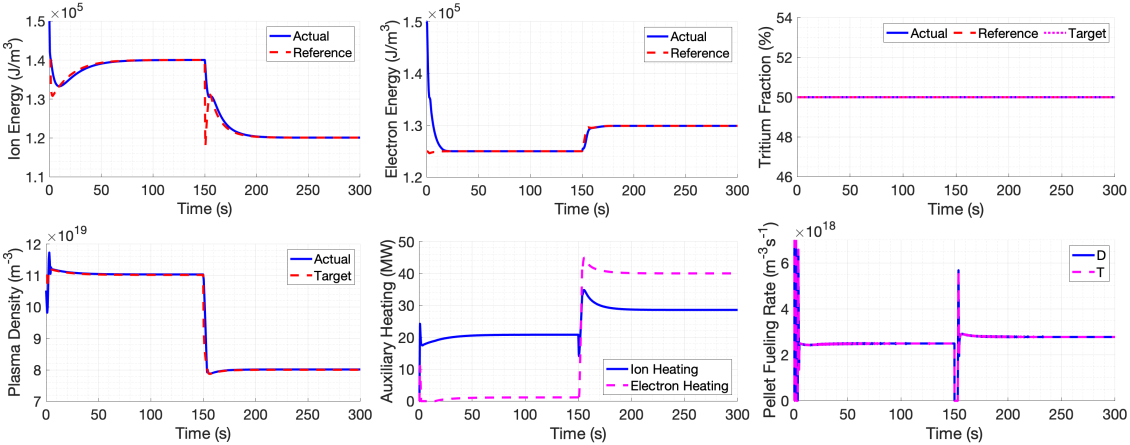
- Optimization is achieved by defining $V_r = \frac{1}{2}\left(\frac{\partial J}{\partial r}\right)^T \frac{\partial J}{\partial r}$ and choosing

$$\dot{r} = -\left(\frac{\partial^2 J}{\partial r^2}\right)^{-1}\left[K_{RTO}\frac{\partial J}{\partial r} + \frac{\partial^2 J}{\partial r \partial x}\dot{x} + \frac{\partial^2 J}{\partial r \partial \hat{\theta}}\dot{\hat{\theta}}\right] \Rightarrow \dot{V}_r \le 0 \Rightarrow \frac{\partial J}{\partial r} \to 0 \Rightarrow r \to r^{des}$$

- The cost function is user-defined! More sophisticated optimization problems are possible!

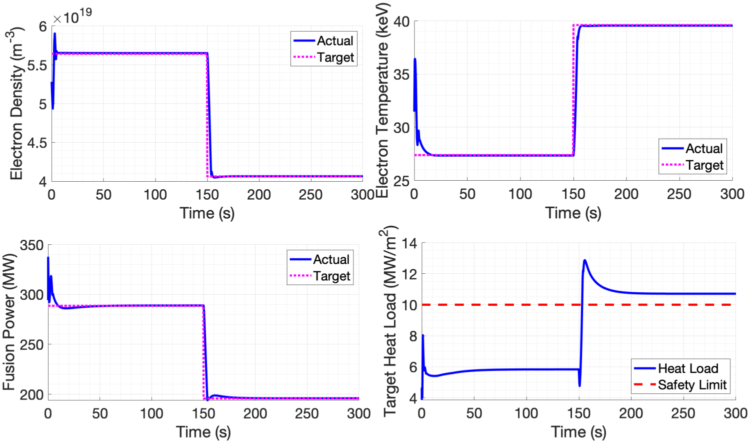
[1] M.D. Boyer and E. Schuster, Plasma Physics and Controlled Fusion 56 104004 (2014).

Real-time Optimal Reference Governor Without Divertor Constraint



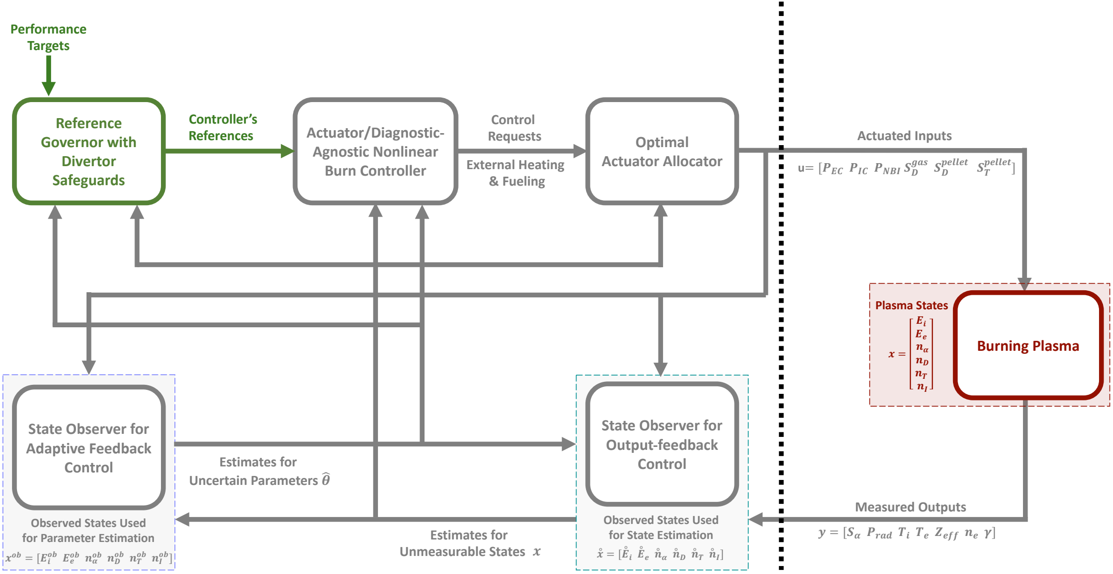
[Scenario 1]: The controller drives the states E_i, E_e, γ, n (blue-solid lines) to their reference values (dashed-red lines), which are updated over time by the reference governor, by commanding the external heating $P_{aux,i}$, $P_{aux,e}$ and fueling rates S_D, S_T to be provided by the actuators.

Real-time Optimal Reference Governor Without Divertor Constraint

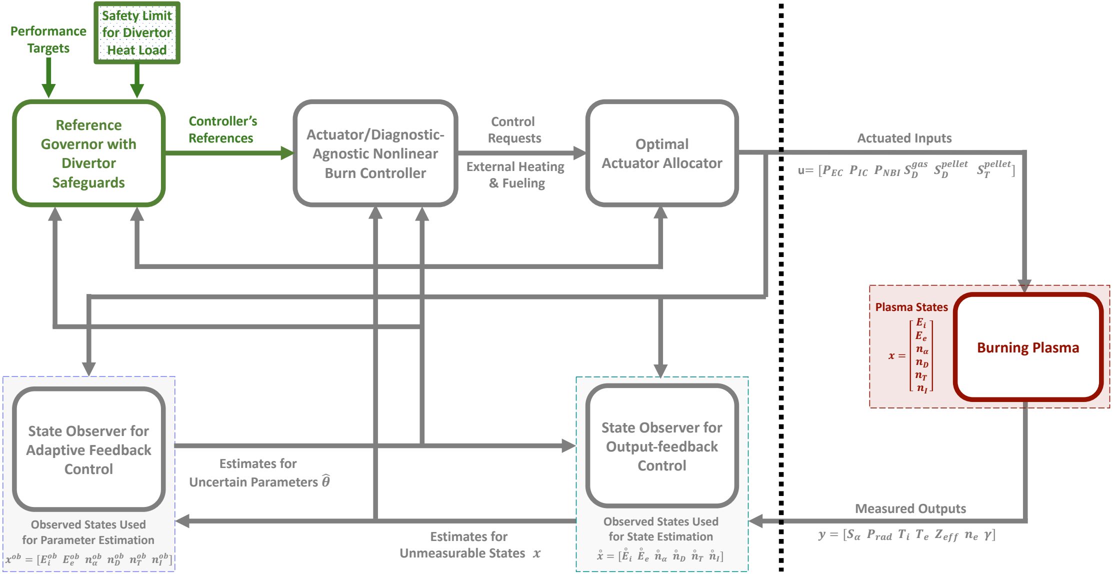


[Scenario 1]: Reference governor successfully drives electron density n_e, electron temperature T_e, and fusion power P_f (blue-solid lines) to target values (magenta-dotted lines) by updating references sent to controller for states E_i, E_e, γ, n (shown in previous figure). Because the reference optimization was unconstrained in this simulation, the peak heat load on the divertor targets exceeds safety limit.

Burn Control Solution: Real-time Optimal Reference Governor



Burn Control Solution: Real-time Optimal Reference Governor

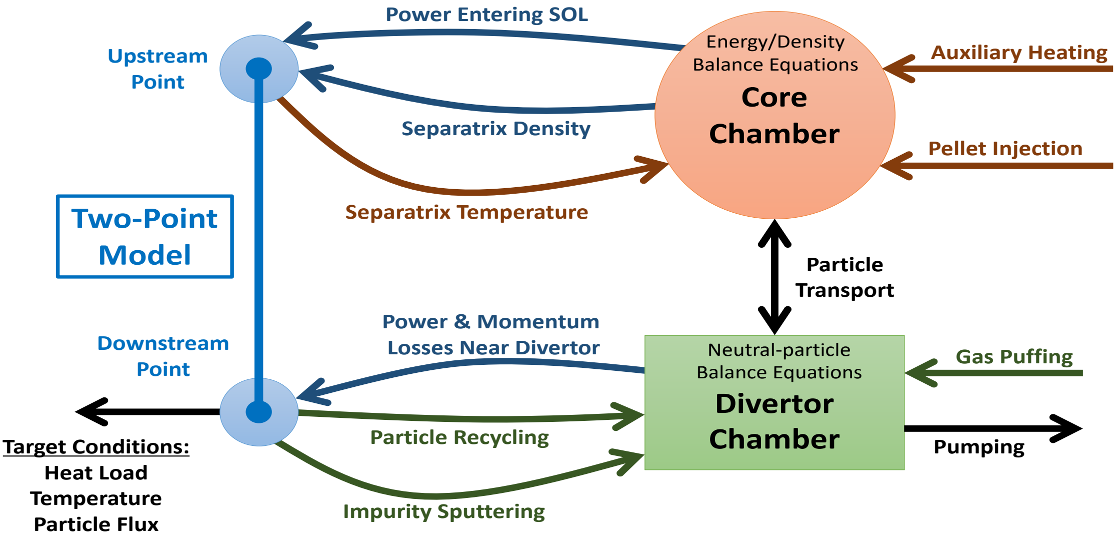


Plasma Modeling for Divertor-Safe Burn Control in ITER

- Core and edge coupling → Integrated burn and divertor control more challenging:
  - Increased fusion power → Increased power flowing across separatrix into SOL and onto divertor
  - High target heat loads ($q_{pk} > 10$ MW/m^2) can cause catastrophic melting
- Divertor-safe burn controller: Control-oriented core-plasma + SOL/divertor models
  - Two considered SOL/Divertor models: Two-point model† and SOLPS-ITER parameterizations‡
  - The optimization by the reference governor is constrained by safety limits for the divertor
- Results from SOLPS-ITER simulations have been parameterized
  - Simulations: High-power DT operation with full-tungsten divertor and impurity seeding (neon/nitrogen)

† P.C. Stangeby, “The Plasma Boundary of Magnetic Fusion Devices,” IoP Publishing, 2000.
‡H.D. Pacher *et al.*, Impurity seeding in ITER DT plasmas in a carbon-free environment, J. Nucl. Mater. 463 (2015).

Two-Point Model: Coupled Core-Edge Burning Plasma Model



[1] V. Graber and E. Schuster, Fusion Engineering and Design 171 (2021) 112516.

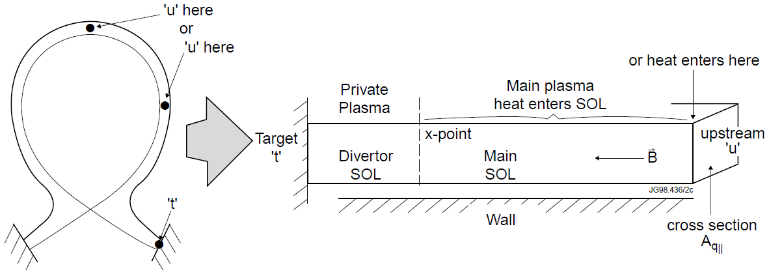
Two-Point Model: Straightening Out the SOL Plasma

- Relates upstream (separatrix) and downstream (target) conditions
- **Upstream**: density n_u, temperature T_u, parallel power flux density $q_{||}$
- **Downstream**: density n_t, temperature T_t

Two Point Model Equations:

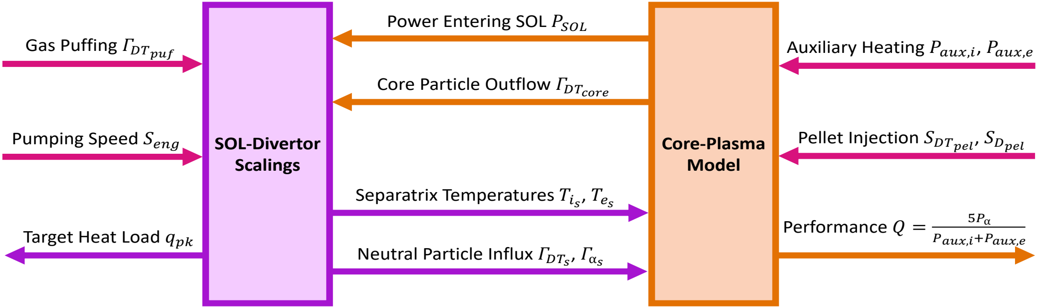
$$2n_tT_t = f_{mom}n_uT_u, \qquad T_u^{\frac{7}{2}} = T_t^{\frac{7}{2}} + \frac{7}{2}\frac{f_{cond}q_{||}L}{\kappa_0}, \qquad (1-f_{pow})q_{||} = \gamma_s n_t T_t c_{st}$$

The Two-Point Model can be solved in terms of the **electron density** and **power entering the SOL** which are controllable with core-plasma actuators (pellet injection and auxiliary power).



SOLPS-ITER Scalings: Coupled Core-Edge Burning Plasma Model

- The SOLPS parameterizations yielded scalings for:
  - the divertor target heat load (q_{pk})
  - the ion and electron separatrix temperatures (T_{i_s} and T_{e_s})
  - the particle influxes into the core-plasma region (Γ_{DT_s} and Γ_{α_s})
- These scalings were coupled with a control-oriented core-plasma model.
- **Actuators:** Auxiliary Heating, Pellet Injection, Gas Puffing, Pumping
- **Divertor Constraint:** Heat Load $q_{pk} < 10$ MW/m^2



[1] V. Graber and E. Schuster, 29th IAEA Fusion Energy Conference, London, UK, October 16-21 2023.

SOLPS-ITER Scalings: Peak Power Load on Target

- The SOLPS-ITER scalings depend on the following:
  - The power flowing into the SOL: $\bar{P}_{SOL} = P_{SOL}$ [MW]$/100$
  - The DT flux into the SOL [Pa m^3/s]: $\Gamma_{DT_{SOL}} = \Gamma_{DT_{core}} + \Gamma_{DT_{puf}}$
    - The controlled gas puffing rate: $\Gamma_{DT_{puf}}$
    - DT core outflow $\Gamma_{DT_{core}} = (\frac{n_D}{\tau_D} + \frac{n_T}{\tau_T}) \times V$
  - The engineering pumping speed: $\bar{S}_{eng} = S_{eng}$ [m^3/s]$/57$
    - 65 m^3/s $\leq S_{eng} \leq$ 107 m^3/s | slow response time: 0 to 100% in 10 s†
  - Separatrix impurity concentration: $\bar{c}_{Z_{sep}} = c_{Z_{sep}}/0.004$ for $Z \in \{\text{Ne}, \text{N}\}$
- Parameter μ is the neutral pressure normalized to one at full detachment:

$$\mu = \left(\frac{\Gamma_{DT_{SOL}}}{250\bar{S}_{eng}}\right)^{0.83} \bar{P}_{SOL}^{-0.52} \qquad (\mu \leq 1 \rightarrow \text{detached})$$

- Peak power load on the target [MW/m^2]:

$$\begin{bmatrix} q_{pk}|^{\text{Ne}} \\ q_{pk}|^{\text{N}} \end{bmatrix} = \max\left(\begin{bmatrix} 4.01 \\ 3.45 \end{bmatrix} \bar{P}_{SOL}^{1.44}\, \mu^{-0.83}, \quad 5.819\bar{P}_{SOL}^{1.12}\, \mu^{-0.32} \begin{bmatrix} \bar{c}_{\text{Ne}_{sep}}^{-0.29} \\ \bar{c}_{\text{N}_{sep}}^{-0.19} \end{bmatrix} \right)$$

† J.A. Snipes *et al.*, Actuator and diagnostic requirements of the ITER Plasma Control System, Fusion Engineering and Design 87 (2012).

SOLPS-ITER Scalings: Separatrix Temperatures and Fluxes

- The electron and ion temperatures at the separatrix [eV]:

$$\begin{bmatrix} T_{e_s}|^{\text{Ne}} \\ T_{e_s}|^{\text{N}} \end{bmatrix} = \bar{P}_{SOL}^{0.31} \left(\frac{\bar{P}_{SOL,e}}{\bar{P}_{SOL,i}} \right)^{0.05} \max \left(140\mu^{-0.093} \begin{bmatrix} \bar{c}_{\text{Ne}_{sep}}^{0.046} \\ \bar{c}_{\text{N}_{sep}}^{0.037} \end{bmatrix}, \quad 150 \begin{bmatrix} \bar{c}_{\text{Ne}_{sep}}^{0.092} \\ \bar{c}_{\text{N}_{sep}}^{0.063} \end{bmatrix} \right)$$

$$\begin{bmatrix} T_{i_s}|^{\text{Ne}} \\ T_{i_s}|^{\text{N}} \end{bmatrix} = \bar{P}_{SOL}^{0.27} \left(\frac{\bar{P}_{SOL,e}}{\bar{P}_{SOL,i}} \right)^{-0.13} \left(1 + 0.08 \left(1 - \frac{\Gamma_{DT_{puf}}}{\Gamma_{DT_{SOL}}} \right) \right)^{-1} \max \left(200\mu^{-0.19} \begin{bmatrix} \bar{c}_{\text{Ne}_{sep}}^{0.23} \\ \bar{c}_{\text{N}_{sep}}^{0.12} \end{bmatrix}, \quad 230\bar{c}_{Z_{sep}}^{0.105} \right)$$

  - where $P_{SOL,e} \equiv (1-\phi_\alpha)P_\alpha + P_{ohm} - P_{rad} - P_{ei} + P_{aux,e}$ and $P_{SOL,i} \equiv \phi_\alpha P_\alpha + P_{ei} + P_{aux,i}$

- The DT neutral flux across separatrix [Pa m^3/s]:

$$\begin{bmatrix} \Gamma_{DT_s}|^{\text{Ne}} \\ \Gamma_{DT_s}|^{\text{N}} \end{bmatrix} = \begin{bmatrix} \bar{c}_{\text{Ne}_{sep}}^{0.86} \\ \bar{c}_{\text{N}_{sep}}^{0.58} \end{bmatrix} 0.0053\bar{P}_{SOL}^{-1.6} \mu^{-0.65}\bar{S}_{eng}^{-0.38} \Gamma_{DT_{SOL}} \left(1 + 0.25 \left(1 - \frac{\Gamma_{DT_{puf}}}{\Gamma_{DT_{SOL}}} \right) \right)$$

- The He neutral flux across the separatrix [Pa m^3/s]:

$$\Gamma_{\alpha_s} = 2\bar{P}_{SOL}^{-1} \mu^{-0.33}\bar{c}_{Z_{sep}}^{0.35}\bar{S}_{eng}^{-0.93} f_{He}$$
$$\times \max \left(0.0016\mu^{-1.9}\bar{c}_{Z_{sep}}^{-0.35}, \quad \min \left(\begin{bmatrix} 0.008 \\ 0.024 \end{bmatrix} \mu^{-0.46}\bar{c}_{Z_{sep}}^{-0.57}, \quad \begin{bmatrix} 0.0055 \\ 0.014 \end{bmatrix} \bar{P}_{SOL}^{1.18} \mu^{-1.42} \right) \right)$$

  - where $f_{He} = (1.05 \times P_\alpha)/P_{SOL}$

Plasma Modeling for Divertor-Safe Burn Control in ITER

Peaked Electron and Ion Temperature Profiles

$$T_e(t,\psi) = (T_{e,0} - T_u)(1 - \psi/\psi_0)^2 + T_u$$

$$T_i(t,\psi) = (T_{i,0} - T_u)(1 - \psi/\psi_0)^2 + T_u$$

Central Ion Temperature: $T_{i,0}$

Central Electron Temperature: $T_{e,0}$

Upstream Separatrix Temperature: T_u

Flat Density Profiles

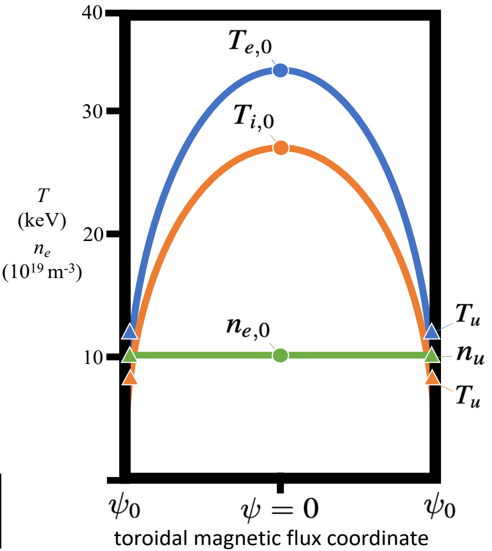
$$n_e(t,\psi) = n_{e,0} = n_u$$

Central Electron Density: $n_{e,0}$

Upstream Separatrix Density: n_u

Radial profiles couple core conditions (T_0 / n_0) with conditions at the separatrix (T_u / n_u)

NOTE: This notation is consistent with TPM

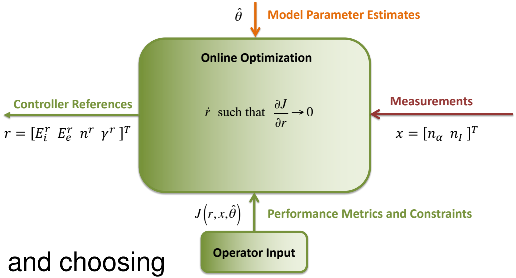


Burn Control Solution: Real-time Optimal Reference Governor

$$J = \frac{\overbrace{w_{T_e}}^{\text{weight}}}{2}\Big(T_e - \underbrace{T_e^{des}}_{\text{target}}\Big)^2 + \frac{\overbrace{w_{n_e}}^{\text{weight}}}{2}\Big(n_e - \underbrace{n_e^{des}}_{\text{target}}\Big)^2 + \frac{\overbrace{w_{P_f}}^{\text{weight}}}{2}\Big(P_f - \underbrace{P_f^{des}}_{\text{target}}\Big)^2 + \frac{\overbrace{w_\gamma}^{\text{weight}}}{2}\Big(\gamma - \underbrace{\gamma^{des}}_{\text{target}}\Big)^2 \overbrace{- \frac{1}{\eta_c}\sum_{i=1}^{K}\ln(-g_i)}^{\text{constraints}}$$

- A **reference for the controlled states** $r = [E_i^r, E_e^r, n^r, \gamma^r]^T$ determines burn condition
- T_e^{des}, n_e^{des}, P_f^{des}, γ^{des} are desired targets
- w_{T_e}, w_{n_e}, w_{P_f}, w_γ are tracking weights
- Constraints by **barrier function**

$$g_i(E, n, \gamma, n_\alpha, n_I) < 0$$



- Optimization is achieved by defining $V_r = \frac{1}{2}\left(\frac{\partial J}{\partial r}\right)^T \frac{\partial J}{\partial r}$ and choosing

$$\dot{r} = -\left(\frac{\partial^2 J}{\partial r^2}\right)^{-1}\left[K_{RTO}\frac{\partial J}{\partial r} + \frac{\partial^2 J}{\partial r \partial x}\dot{x} + \frac{\partial^2 J}{\partial r \partial \hat{\theta}}\dot{\hat{\theta}}\right] \Rightarrow \dot{V}_r \le 0 \Rightarrow \frac{\partial J}{\partial r} \to 0 \Rightarrow r \to r^{des}$$

- The cost function is user-defined! More sophisticated optimization problems are possible!

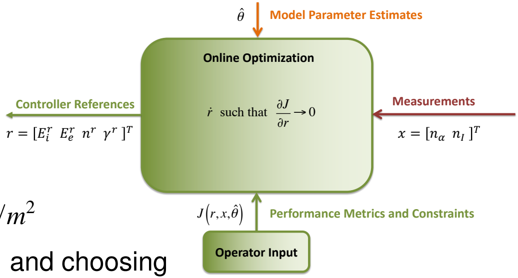
[1] M.D. Boyer and E. Schuster, Plasma Physics and Controlled Fusion 56 104004 (2014).

Burn Control Solution: Real-time Optimal Reference Governor

$$J = \overbrace{\frac{w_{T_e}}{2}}^{\text{weight}} \left(T_e - \underbrace{T_e^{des}}_{\text{target}} \right)^2 + \overbrace{\frac{w_{n_e}}{2}}^{\text{weight}} \left(n_e - \underbrace{n_e^{des}}_{\text{target}} \right)^2 + \overbrace{\frac{w_{P_f}}{2}}^{\text{weight}} \left(P_f - \underbrace{P_f^{des}}_{\text{target}} \right)^2 + \overbrace{\frac{w_{\gamma}}{2}}^{\text{weight}} \left(\gamma - \underbrace{\gamma^{des}}_{\text{target}} \right)^2 \overbrace{- \frac{1}{\eta_c} \sum_{i=1}^{K} \ln(-g_i)}^{\text{constraints}}$$

- A **reference for the controlled states** $r = [E_i^r, E_e^r, n^r, \gamma^r]^T$ determines burn condition
- T_e^{des}, n_e^{des}, P_f^{des}, γ^{des} are desired targets
- w_{T_e}, w_{n_e}, w_{P_f}, w_{γ} are tracking weights
- Constraints by **barrier function**

$$g_i(E, n, \gamma, n_\alpha, n_I) < 0 \Rightarrow g_0 = q_{pk} - q_{pk}^{max}, q_{pk}^{max} = 10MW/m^2$$



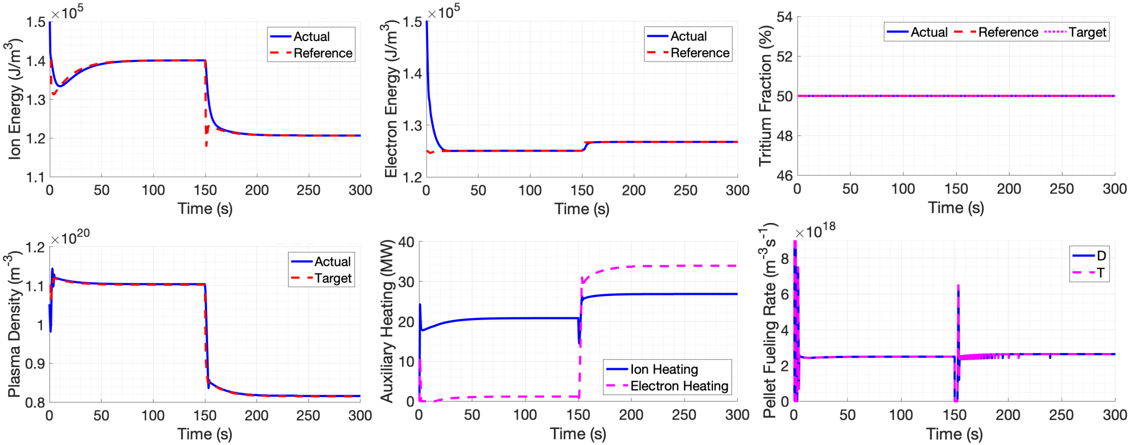
- Optimization is achieved by defining $V_r = \frac{1}{2} \left(\frac{\partial J}{\partial r} \right)^T \frac{\partial J}{\partial r}$ and choosing

$$\dot{r} = - \left(\frac{\partial^2 J}{\partial r^2} \right)^{-1} \left[K_{RTO} \frac{\partial J}{\partial r} + \frac{\partial^2 J}{\partial r \partial x} \dot{x} + \frac{\partial^2 J}{\partial r \partial \hat{\theta}} \dot{\hat{\theta}} \right] \Rightarrow \dot{V}_r \leq 0 \Rightarrow \frac{\partial J}{\partial r} \rightarrow 0 \Rightarrow r \rightarrow r^{des}$$

- The cost function is user-defined! More sophisticated optimization problems are possible!

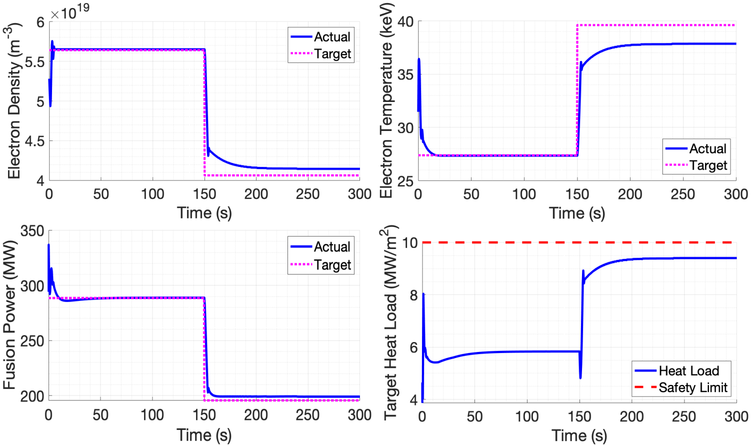
[1] V. Graber and E. Schuster, Nuclear Fusion 64 (2024) 086007 (15pp).

Real-time Optimal Reference Governor With Divertor Constraint



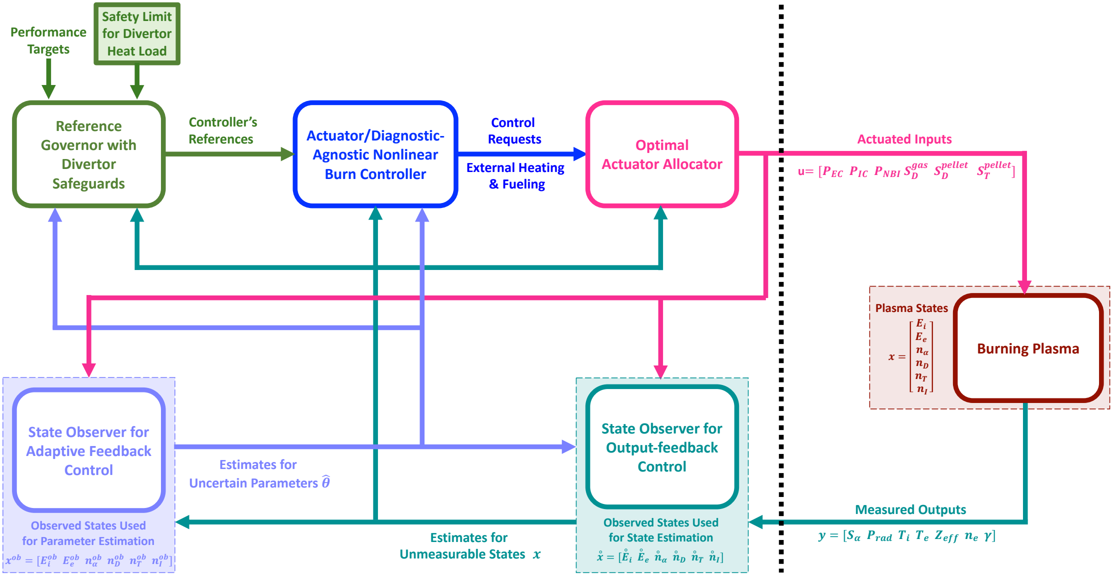
[Scenario 2]: The controller drives the states E_i, E_e, γ, n (blue-solid lines) to their reference values (dashed-red lines), which are updated over time by the reference governor, by commanding the external heating $P_{aux,i}$, $P_{aux,e}$ and fueling rates S_D, S_T to be provided by the actuators.

Real-time Optimal Reference Governor With Divertor Constraint



[Scenario 2]: Reference governor attempts to drive electron density n_e, electron temperature T_e, and fusion power P_f (blue-solid lines) to target values (magenta-dotted lines) by updating references sent to controller for states E_i, E_e, γ, n (shown in previous figure). Because the reference optimization was constrained in this simulation, the peak heat load remains below safety limit at the expense of not achieving targets for n_e, T_e, P_f.

Burn Control Solution: Overview of Proposed Approach



Core-Edge Control Integration is Critical for Safe Reactor Operation

- **Burn-control:** feedback controller, state observer, parameter estimator, actuator allocator
  - *Techniques handling nonlinearities and uncertainties → Performance ↑ + Robustness/Adaptiveness ↑*
  - *Techniques decoupling controller and observer/allocator → Fault Tolerance ↑ + Integration ↑*
- **Reference Gorvernor:** Solves trade-off between core performance and divertor protection
  - Integration of core control with additional machine safety limits → Complement local edge control
- **Integration of burn controller with other competing/coupled controllers**
  - ELM's, profiles, heating maximization (plasma/actuator coupling)
- **Testing of proposed density → burn-control algorithms in 1D simulations is needed [1]**
  - Further work on actuator/diagnostic/transport modeling and core-edge integration is needed
  - Assessment: 1- simultaneous burn and profile control for advanced scenarios; 2- actuator dynamics
  - *Need for multi-zone response model for control synthesis could be determined from simulation results*
- **Testing of proposed density→ burn-control algorithms in present devices is needed**
  - Emulation of α heating, and even particle recycling, is possible through different mechanisms
  - Emulation of ITER's actuators and diagnostics is also possible
- **Combination of data-based and model-based control approaches**
  - Physics-based model → AI/ML controller [2] vs AI/ML model → Model-based controller

[1] V. Graber and E. Schuster, 33rd Symposium on Fusion Technology (SOFT), Dublin, Ireland, September 22-27, 2024.
[2] I. Ward and E. Schuster, 66th APS-DPP Meeting, Atlanta, GA, USA, October 7-11, 2024.