Electric Field Effects During Disruptions

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Reference: *Electric field effects during disruptions*, A. H. Boozer, https://arxiv.org/pdf/2404.09744

As magnetic surfaces break, an individual magnetic field line covers much of the plasma volume but requires many toroidal transits to do so. Called magnetic field line chaos.

Electrons move rapidly along field lines compared to ions and the electric potential $\Phi(\ell) \sim T_e/e$ must depend on the distance ℓ along each line to enforce quasi-neutrality.

Neighboring chaotic field lines exponentiate apart and have different $\Phi(\ell)$, which implies large $\vec{E} \times \vec{B}$ drifts.

Faraday's Law Is Of Advection-Diffusion Form

Mathematics implies an arbitrary electric field can always be represented as

$$\vec{E} = -\vec{u}_\perp \times \vec{B} - \vec{\nabla}\Phi + \frac{V_\ell}{2\pi}\vec{\nabla}\varphi, \text{ so}$$

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times (\vec{u}_\perp \times \vec{B}) - \frac{\vec{\nabla} V_\ell}{2\pi} \times \vec{\nabla}\varphi$$

V_ℓ is the loop voltage, is constant along the magnetic field line, given by $\vec{E}\cdot\vec{B}$, and must be non-zero for reconnection. When $V_\ell = 0$, $\vec{u}_\perp$ is the magnetic field line velocity.

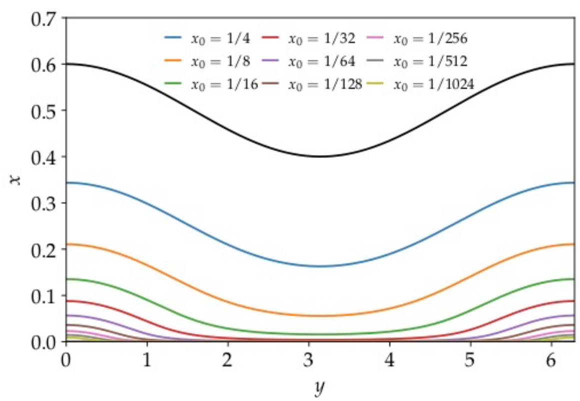
Faraday's law is of the advection-diffusion form. When $\vec{u}_\perp$ is chaotic, reconnection only depends on η logarithmically; has a timescale about ten times the ideal timescale.

Note $\vec{E} + \vec{v} \times \vec{B} = \eta\vec{j}$ implies $\partial\vec{B}/\partial t + \vec{\nabla} \times (\vec{v} \times \vec{B}) = (\eta/\mu_0)\nabla^2\vec{B}$.

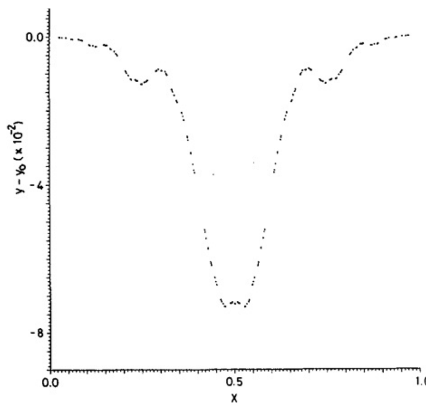
2D differs fundamentally from 3D reconnection.

Magnetic Surfaces Can Rapidly Break

Ideal non-axisymmetric perturbations create a large variation in separation between neighboring surfaces near rational magnetic surfaces. An arbitrarily small resistivity η/μ_0 can diffuse lines at the locations of closest approach. *The irrational surfaces are the last to break and become Cantori, which have gaps called turnstiles.*



Ideal perturbation near a rational magnetic surface
Huang et al, PoP **20**,032513 (2022).

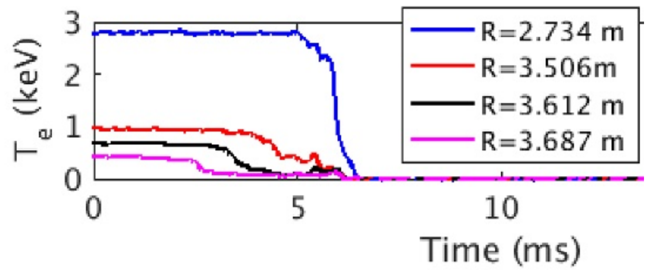


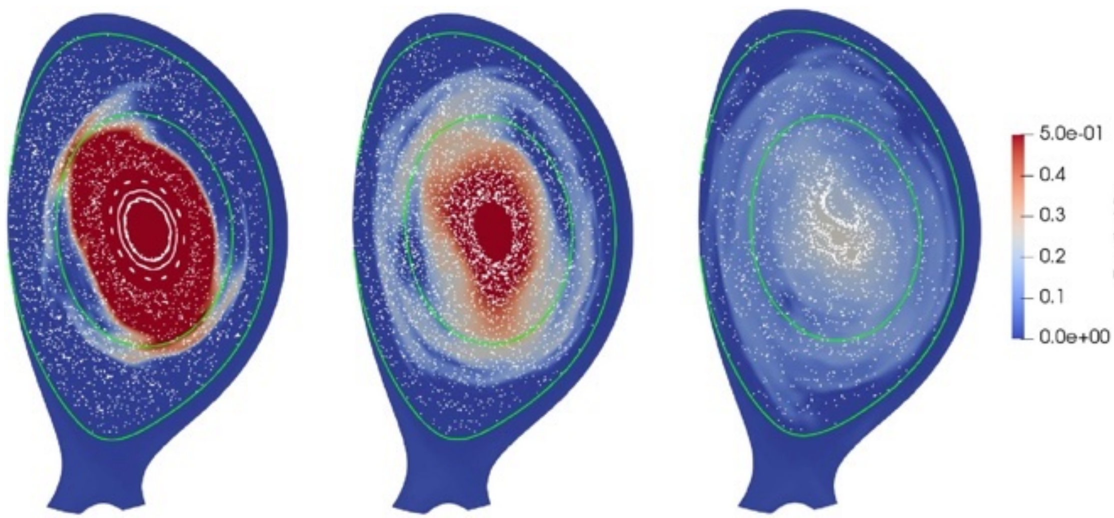
Cantorus in standard map
MacKay et al, Physica **13D**, 55 (1984).

MacKay et al, Physica **13D**, 55 (1984): "**Most of the gaps in a cantorus are very small, since their total length is finite. Even when there are large gaps, orbits can take a long time to get through.**"

Temperature Gradients Remain When $\vec{B}$ Is Chaotic

Nardon et al, Plasma Phys. Control. Fusion 63,115006 (2021), analyzed an intentional disruption in JET triggered by a massive gas injection of argon and found large temperature gradients remain as surfaces break.





4.62 ms 4.91 ms 5.09 ms

Nardon et al, Plasma Phys. Control. Fusion **63,** 115006 (2021)

Quasi-Neutrality Electric Field

The ideal response of ions and electrons is related to the electric field by the equations

$$\vec{E} = -\vec{v}_h \times \vec{B} + \frac{1}{e}\left(m_h \frac{d\vec{v}_i}{dt} - \frac{\vec{\nabla}\cdot\overleftrightarrow{p}_i}{n}\right) \qquad \text{for ions}$$

$$= -\vec{v}_e \times \vec{B} + \frac{1}{e}\frac{\vec{\nabla}p_e}{n}, \qquad \text{for electrons}$$

$$m_i \frac{d\vec{v}_i}{dt} = \vec{j} \times \vec{B} - \vec{\nabla}\cdot\overleftrightarrow{p} \qquad \text{eliminating } \vec{E}$$

$$\frac{\partial \Phi}{\partial \ell} = -\frac{1}{en}\frac{\partial p_e}{\partial \ell} \qquad E_{||} \text{ equation}$$

$$= -\frac{1}{e}\left(\frac{\partial \ln n}{\partial \ell}T_e + \frac{\partial T_e}{\partial \ell}\right) \qquad \text{with } p_e = nT_e.$$

Where $\partial T_e/\partial \ell$ is large $\Phi = -(c_0 + T_e)/e$.

When $\partial T_e/\partial \ell = 0$, then $\Phi = -(T_e/e)\ln(n/n_0)$.

Quasi-neutrality potential $\Phi \approx -\langle T_e \rangle/e$ on each line.

Transport due to Quasi-Neutrality Electric Field

Consider a small spherical region within a volume in which $\vec{B}$ is chaotic. Each B-line that passes through the sphere eventually comes arbitrarily close to every point in the entire chaotic volume by a long trajectory $\vec{x}(\ell)$ divergent from that of neighboring lines.

$\Phi(\ell) = \langle T_e \rangle / e$ given by an integration of $\partial\Phi/\partial\ell = -(\partial p_e/\partial\ell)/en$ from each plasma point. Implies two types of $\vec{V}_q \equiv (\vec{B} \times \vec{\nabla}\Phi)/B^2$ transport effects.

1. A correlated flow on the large scale of the T_e variation, a_T comparable to the plasma radius, $V_{ls} \sim \frac{T_e}{eBa_T}$.

2. A diffusive transport $D_q \approx \Delta^2/\tau_{cor}$ due to the short scale, Δ over which Φ is correlated across $\vec{B}$. Correlation time $\tau_{cor} \approx \Delta/|\vec{V}_q|$ and $|\Delta\Phi| \approx \langle T_e \rangle / e$ the variation in Φ across the field lies. **Expected diffusion approximately Bohm-like:**

$$D_q \approx \frac{\langle T_e \rangle}{eB} = 10^3 \frac{T_{\text{kev}}}{B_{\text{Tesla}}} \frac{\text{m}^2}{\text{s}}.$$

Simulations of $B = 3$ T JET argon-induced disruptions, Nardon et al, NF **63**, 056011 (2023) found a $D \approx 10^3$m^2/s and a large scale flow ≈ 5000m/s.

Summary

- When magnetic surfaces break in a disruption, the magnetic field lines become chaotic.
- Each line in a chaotic volume comes arbitrarily close it every point in that volume and exponentiates away from a neighboring line as $\exp(\sigma(\ell))$.
- Electrons move faster than ions along $\vec{B}$. Quasi-neutrality requires an electric potential $\partial\Phi/\partial\ell = (\partial p_e/\partial\ell)/en$ to balance electron pressure with $|\Phi| \sim \langle T_e \rangle/e$.
- When $\sigma|\partial p_e/\partial\ell|$ is large, the potential Φ has a short correlation across $\vec{B}$, which gives diffusion $D_q \approx \langle T_e \rangle/eB$ and large scale flow $V_{ls} \approx D_q/a_T$, where T_e varies on scale a_T.
- Diffusion and flow spread impurities, which $\vec{E} \times \vec{B}$ drift.