Universidade de Brasília

Faculdade UnB Gama

ROLE OF LAGRANGIAN VORTICES IN NUMERICAL SIMULATIONS OF RESISTIVE DRIFT-WAVE TURBULENCE IN PLASMAS

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Outline

1)Introduction.

2)Lagrangian vortices in neutral fluids.

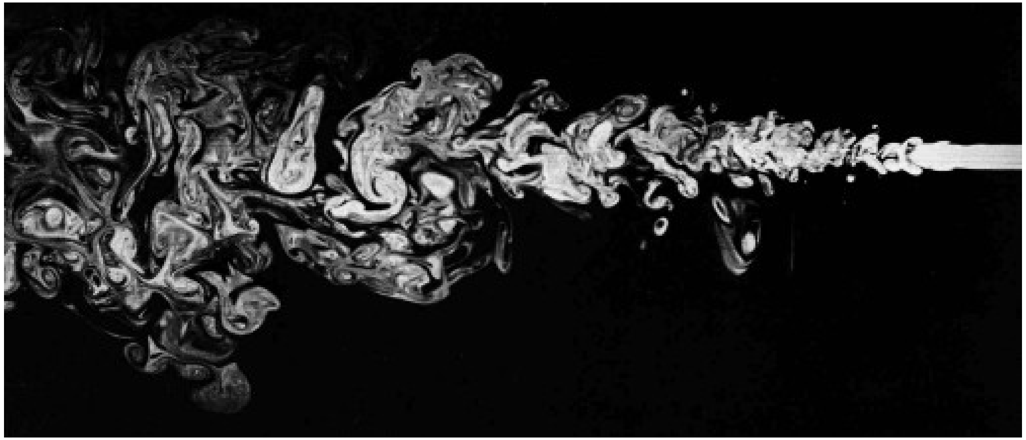
3)Lagrangian vortices in drift-wave plasma turbulence.

a) Turbulent regime.

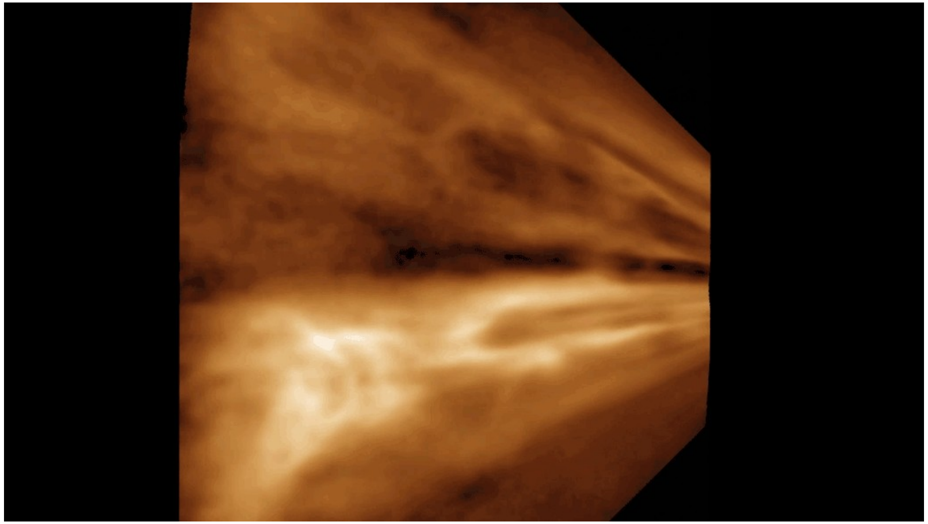
b) Zonal flow regime.

4)Conclusions.

1: Introduction



Turbulent water jet. From Van Dyke (1982)



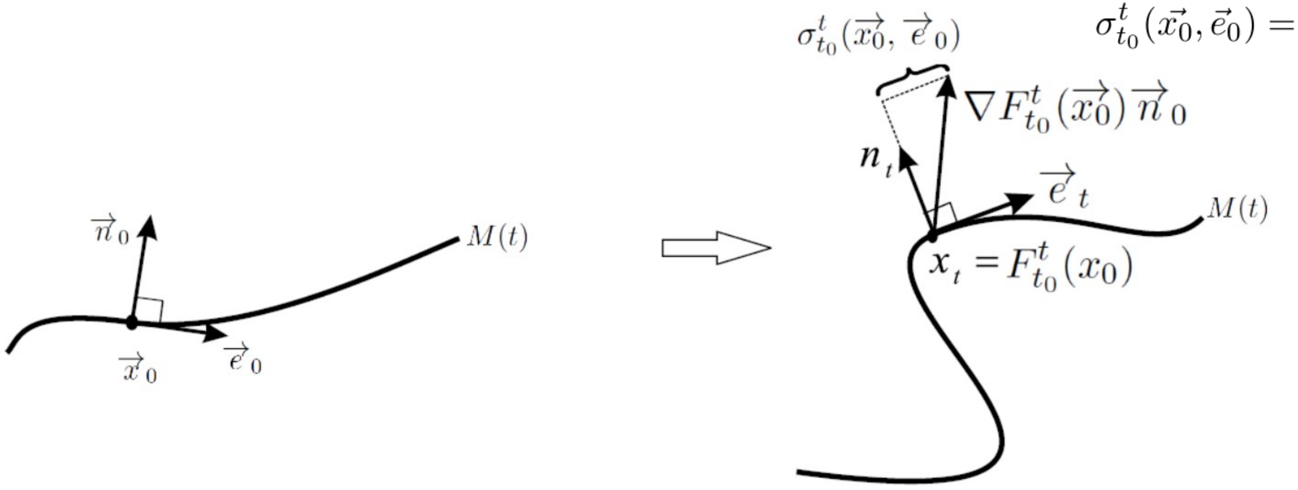
Processed STEREO data of the solar wind.
Data credit: Craig DeForest, SwRI https://svs.gsfc.nasa.gov/12329

2: Lagrangian coherent structures in fluids

Mount Edna blowing volcanic vortex smoke rings
https://youtu.be/Q2cUGICVEhI

2: Lagrangian coherent structures in fluids



$$\sigma_{t_0}^t(\vec{x_0}, \vec{e_0}) = \langle \vec{e}_t, \nabla F_{t_0}^t(\vec{x_0})\vec{n_0} \rangle$$

Defining the Cauchy-Green strain tensor as $C_{t_0}^t(\vec{x_0}) = \left(\nabla F_{t_0}^t(\vec{x_0})\right)^T \left(\nabla F_{t_0}^t(\vec{x_0})\right)$ computing its eigenvalues and eigenvectors, and seeking extrema of the Lagrangian shear one can obtain

$$\vec{\eta_\pm} = \sqrt{\frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1}+\sqrt{\lambda_2}}}\vec{\xi_1} \pm \sqrt{\frac{\sqrt{\lambda_1}}{\sqrt{\lambda_1}+\sqrt{\lambda_2}}}\vec{\xi_2}.$$

Haller, Physica D 2011; Haller & Beron-Vera, Physica D 2012

2: Lagrangian coherent structures in fluids

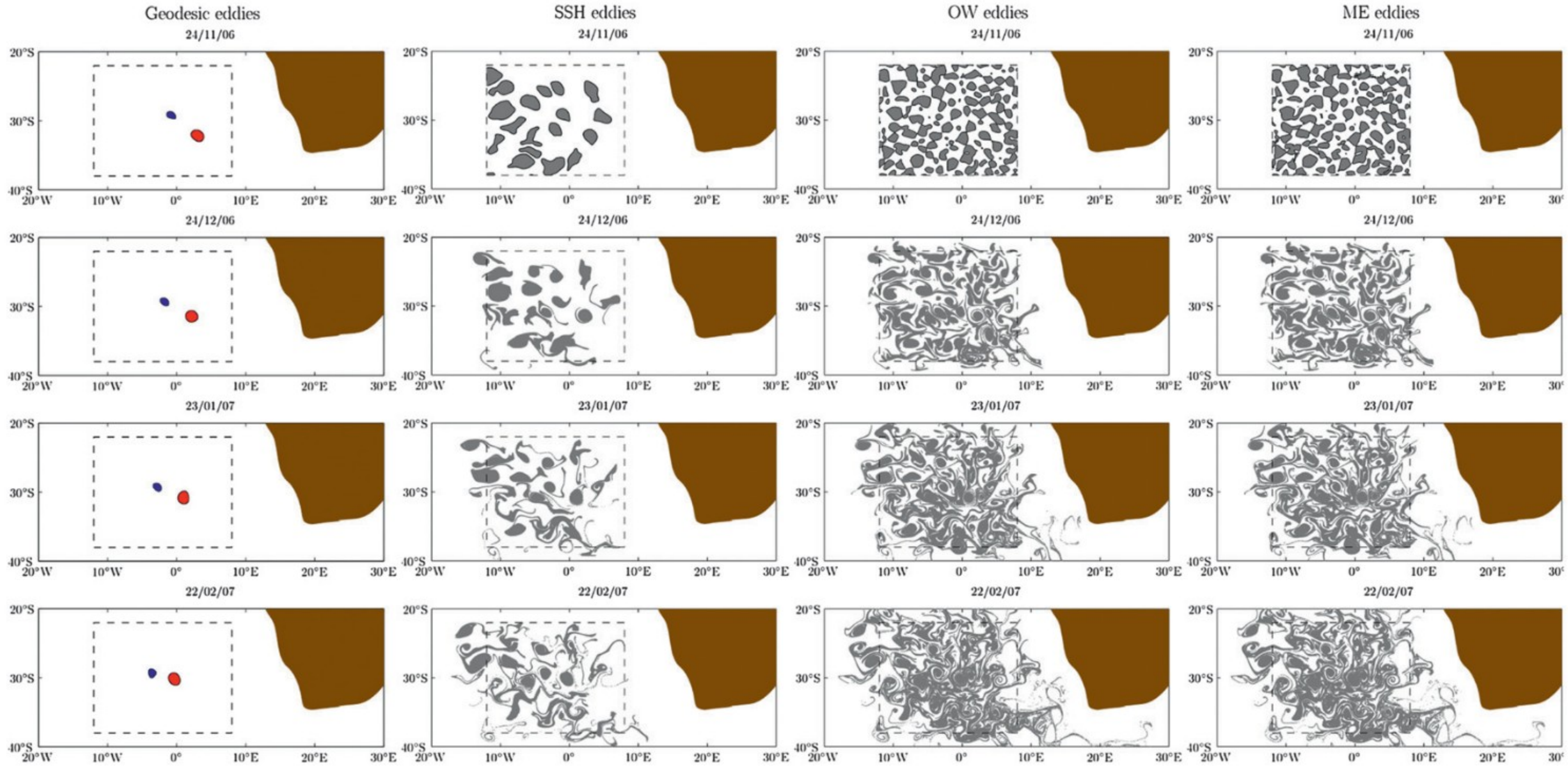


FIG. 7. (left) Selected snapshots of the 90-day evolution of fluid inside eddies identified by geodesic eddy detection; (middle left) the method of Chelton et al. (2011a) with $U/c > 1$ over at least 90 days; (middle right) the Okubo–Weiss (OW) criterion; and (right) the criterion of Mézic et al. (2010).

Beron-Vera et al., J. Phys. Oceanogr. (2013)
https://doi.org/10.1175/JPO-D-12-0171.1

2: Lagrangian coherent structures in fluids

The Navier-Stokes equations are given by

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} + \mathbf{f},$$
$$\nabla \cdot \mathbf{u} = 0$$

where $\mathbf{u}$ represents the fluid velocity, p is the pressure, $\mathbf{f}$ represents an external force, and Re is the Reynolds number.

We solve the 2D Navier-Stokes equations using a spectral code available at

https://gitlab.com/rmiracer/jade

2: Lagrangian coherent structures in fluids

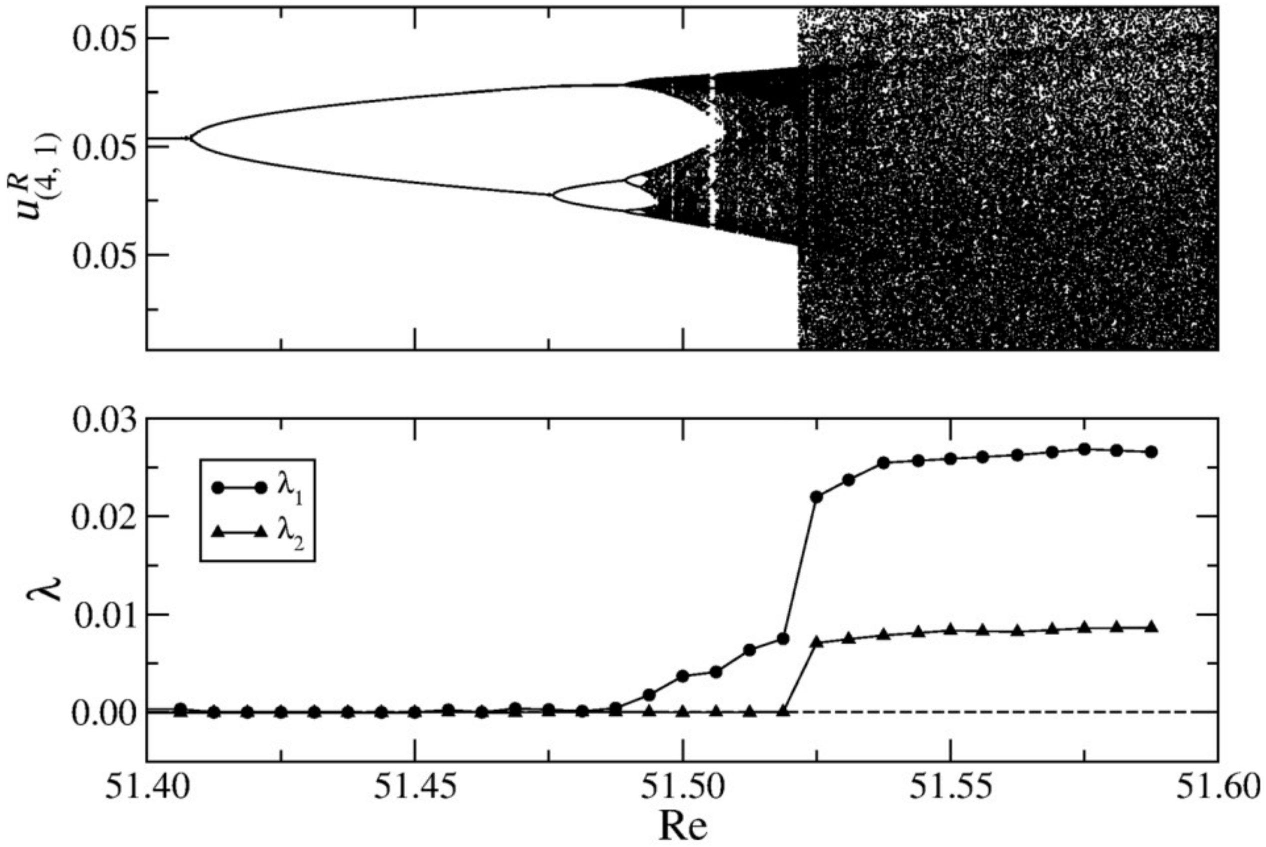
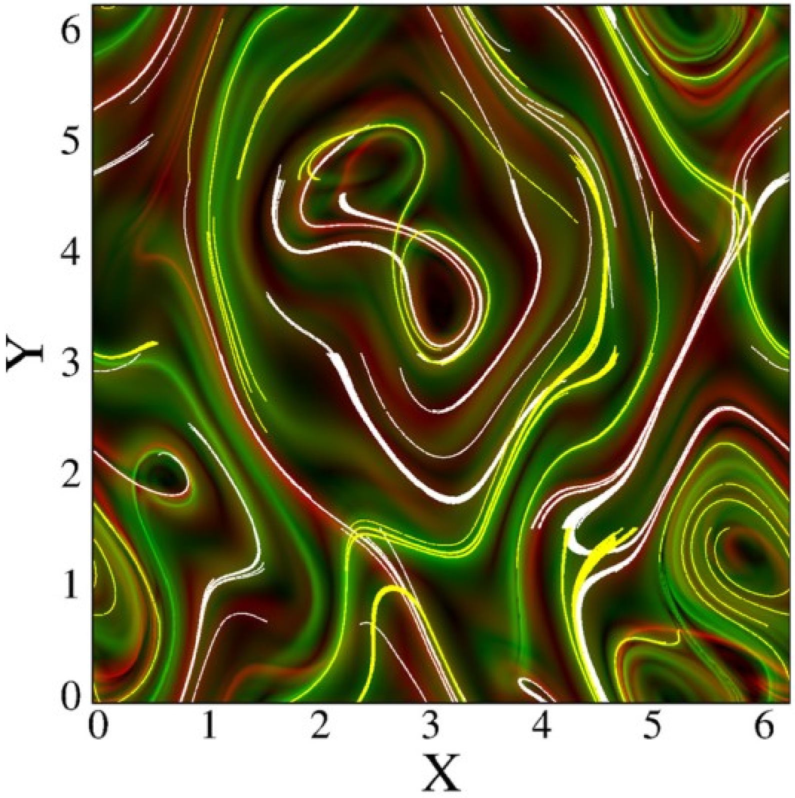


FIG. 2. Upper panel: bifurcation diagram of $u^R_{(4,1)}$ as a function of the Reynolds number Re for A_1. Lower panel: the two largest Lyapunov exponents as a function of Re.

Miranda et al., CHAOS 2013
https://doi.org/10.1063/1.4811297

2: Lagrangian coherent structures in fluids



Spatiotemporal patterns in the 2D Navier-Stokes equations for Re = 51.52 (chaotic regime)

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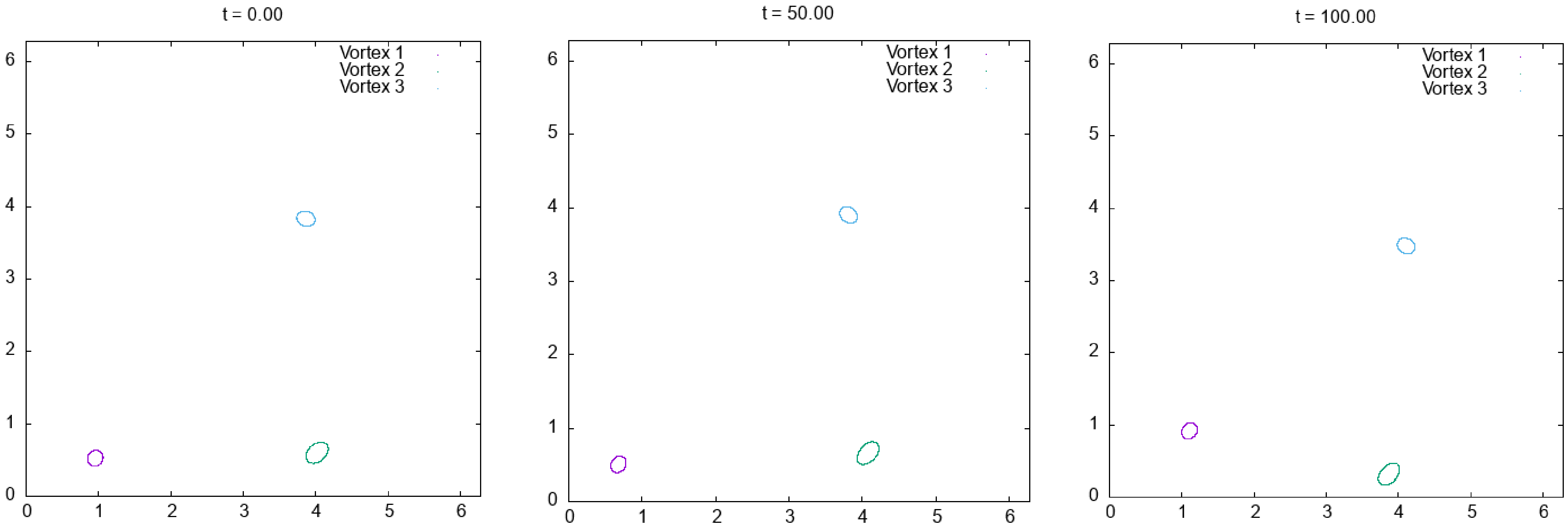
Miranda et al., CHAOS 2013
https://doi.org/10.1063/1.4811297

2: Lagrangian coherent structures in fluids



Spatiotemporal dynamics of Lagrangian vortices in a chaotic regime of the Navier-Stokes equations.

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Miranda et al., under preparation

3: Lagrangian coherent structures in plasmas

Let us define the physical setting of the model in a constant magnetic field equilibrium $B = B_0\nabla z$, and a nonuniform density $n_0 = n_0(x)$ in the edge region. The modified Hasegawa-Wakatani equations are given by

$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha\left(\widetilde{\varphi} - \widetilde{n}\right) - D\nabla^4\zeta, \qquad (2.11)$$

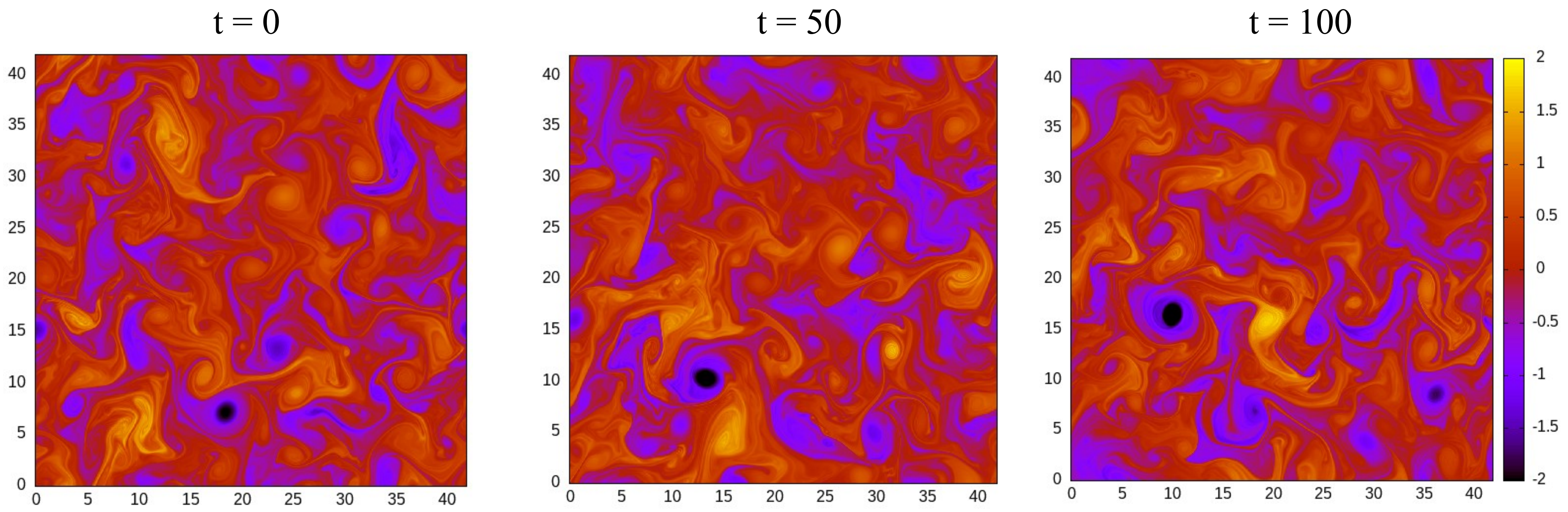
$$\frac{\partial}{\partial t}n + \{\varphi, \zeta\} = \alpha\left(\widetilde{\varphi} - \widetilde{n}\right) - \kappa\frac{\partial\varphi}{\partial y} - \nabla^4 n, \qquad (2.12)$$

where the zonal component of a variable a is given by

$$\widetilde{a} = a - \langle a \rangle\,, \qquad \langle a \rangle = \frac{1}{L}\int a dy \qquad (2.13)$$

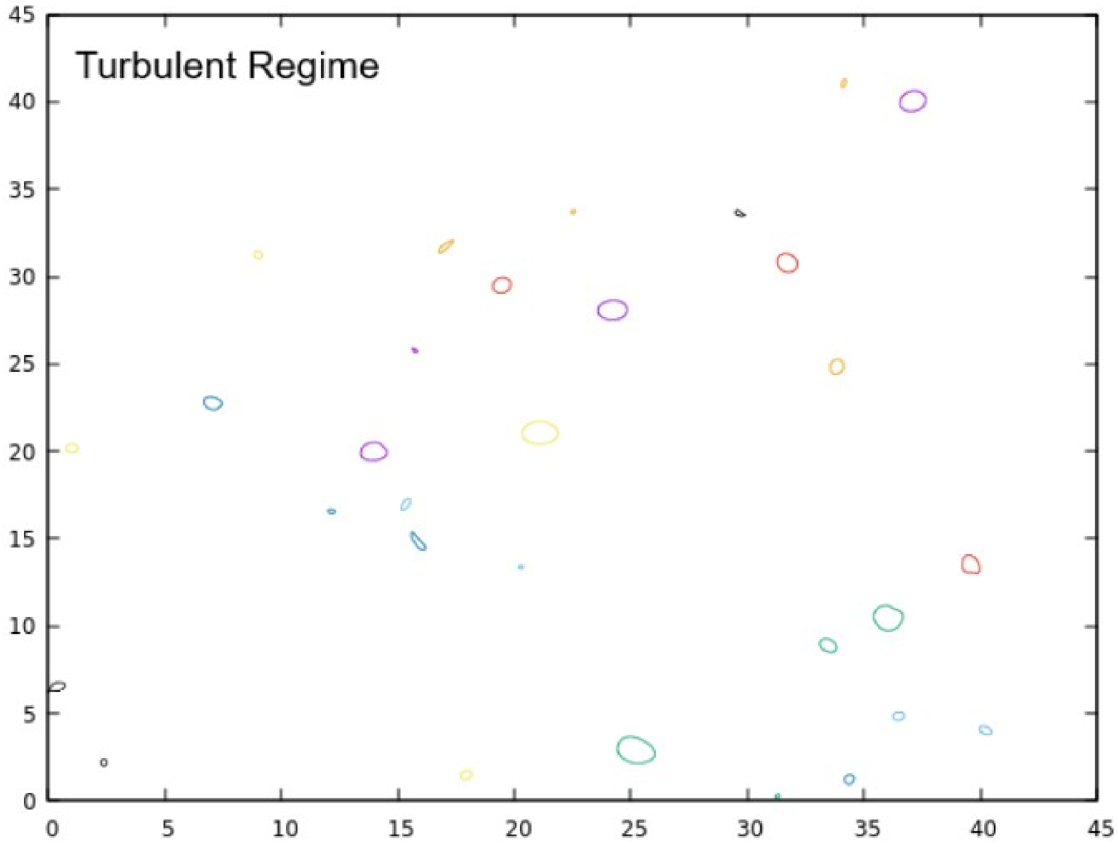
n is the density fluctuation, φ is the electrostatic potential, and the ion vorticity $\zeta \equiv \nabla^2\varphi$. Symbols $\{a, b\}$ denote the Poisson bracket, . The background density $\kappa \equiv (\partial/\ \partial x)\ln n_0$ has a constant exponential profile, and D represents the dissipation coefficient. We set the adiabaticity operator $\alpha = 0.01$ (turbulent regime) and $\alpha = 0.018$ (zonal flow regime).

3: Lagrangian coherent structures in drift-wave turbulence



Spatiotemporal dynamics of the perturbed plasma density, in the turbulent regime.

3: Lagrangian coherent structures in drift-wave turbulence



Lagrangian vortices detected using geodesic theory

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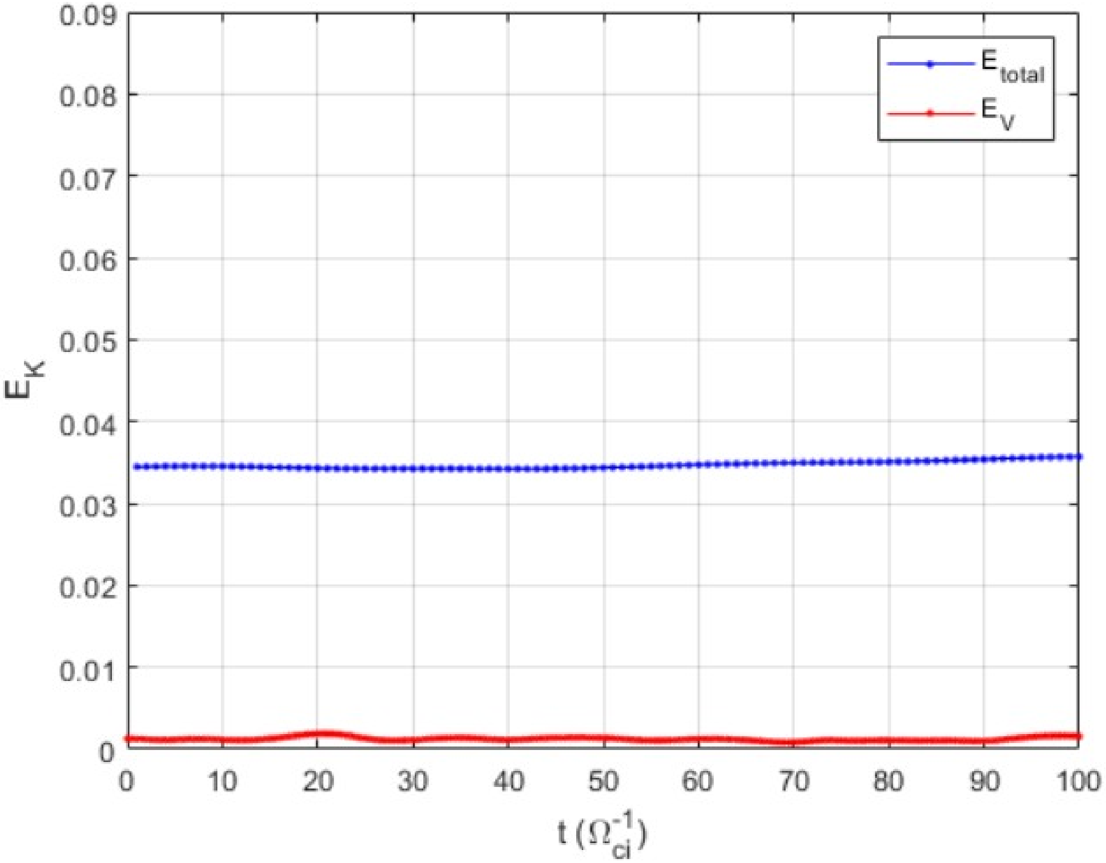
Miranda et al., under preparation

3: Lagrangian coherent structures in drift-wave turbulence



The total kinetic energy E_{total} and the kinetic energy contained in one selected vortex E_V , from t = 0 to t = 100.

Miranda et al., under preparation

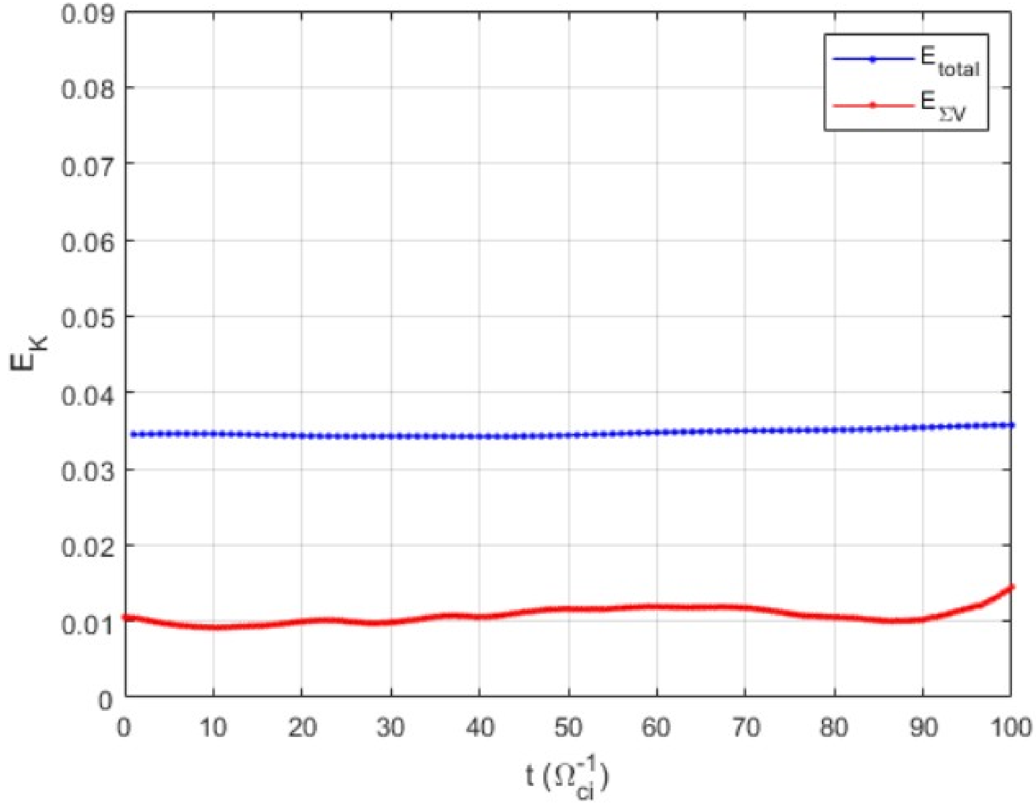
3: Lagrangian coherent structures in drift-wave turbulence

Time (Ω_{ci}^{-1})	Turbulent Regime		
	E_{total}	E_V	$\frac{E_V}{E_{total}}$ (%)
$t = 0$ to $t = 100$	0.0347	0.0012	3.46

Numerical values of the E_{total} and E_V

Miranda et al., under preparation

3: Lagrangian coherent structures in drift-wave turbulence



The total kinetic energy E_{total} and the total kinetic energy contained in all vortices $E_{\sum V}$, from t = 0 to t = 100.

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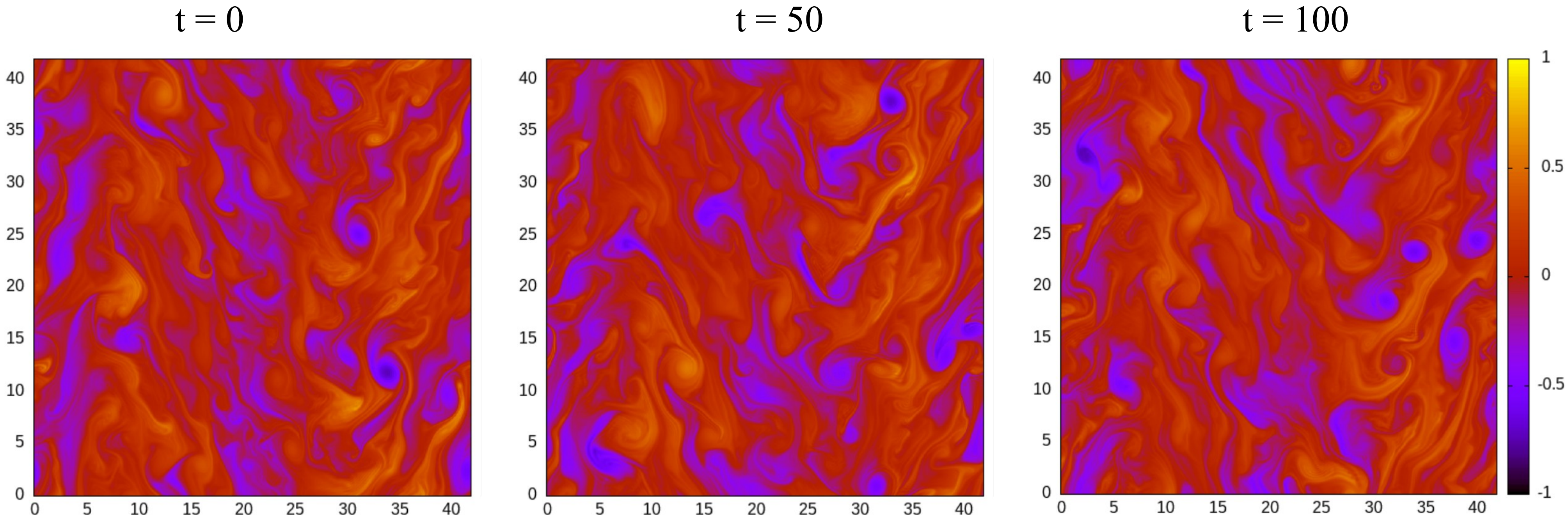
3: Lagrangian coherent structures in drift-wave turbulence

Time (Ω_{ci}^{-1})	**Turbulent Regime**		
	E_{total}	$E_{\sum V}$	$\frac{E_{\sum V}}{E_{total}}$ (%)
$t = 0$ to $t = 100$	0.0347	0.0107	30.84
$t = 100$ to $t = 200$	0.0361	0.0117	32.41
$t = 200$ to $t = 300$	0.0372	0.0191	51.34
$t = 300$ to $t = 400$	0.0350	0.0140	40.00
$t = 400$ to $t = 500$	0.0330	0.0039	11.81

Miranda et al., under preparation

3: Lagrangian coherent structures in zonal flows



Spatiotemporal dynamics of the perturbed plasma density, in the zonal flow regime.

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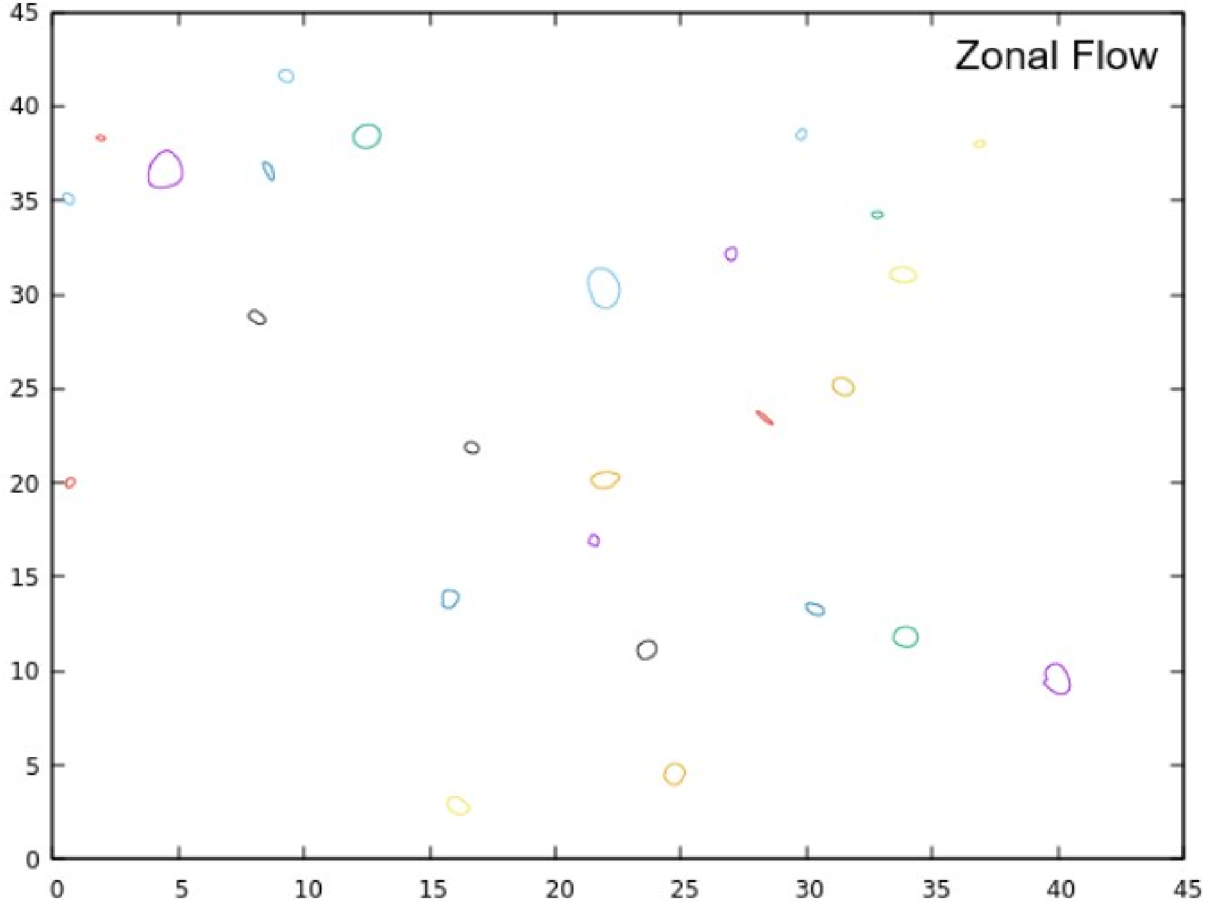
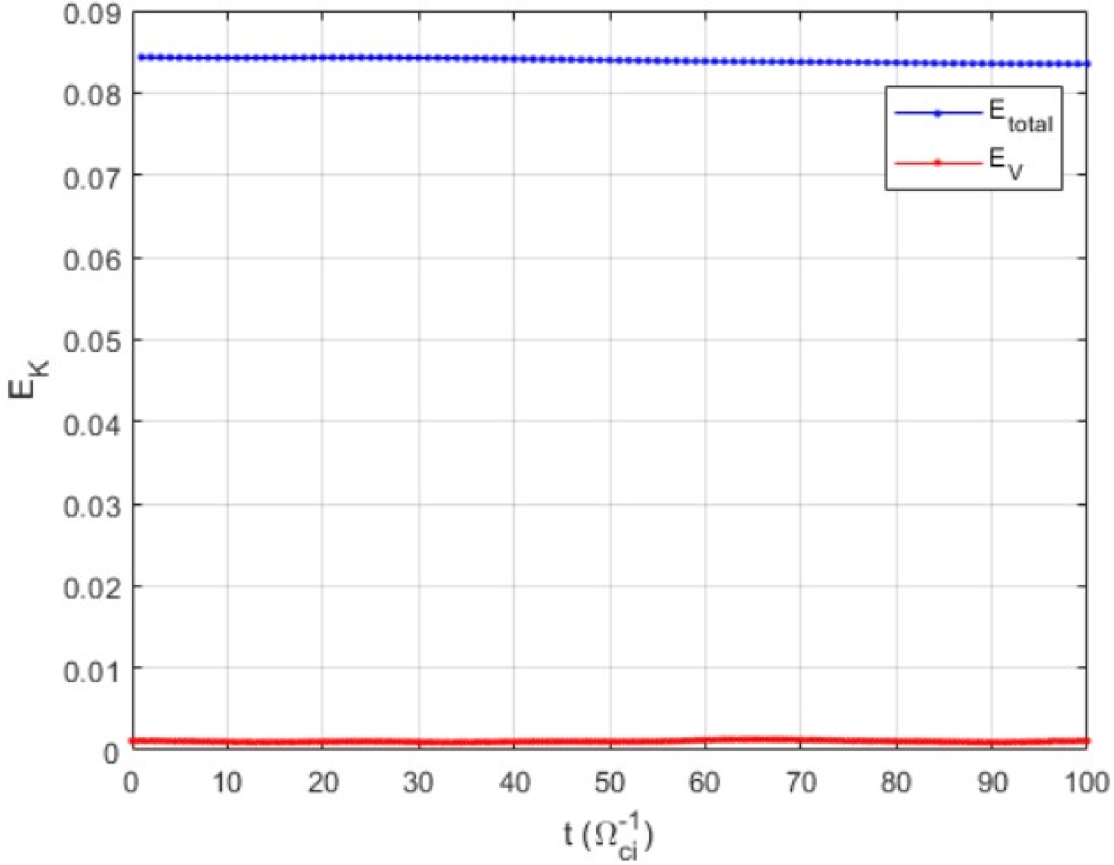
3: Lagrangian coherent structures in zonal flows



Lagrangian vortices detected using geodesic theory

Miranda et al., under preparation

3: Lagrangian coherent structures in zonal flows



The total kinetic energy E_{total} and the kinetic energy contained in one selected vortex E_V , from t = 0 to t = 100.

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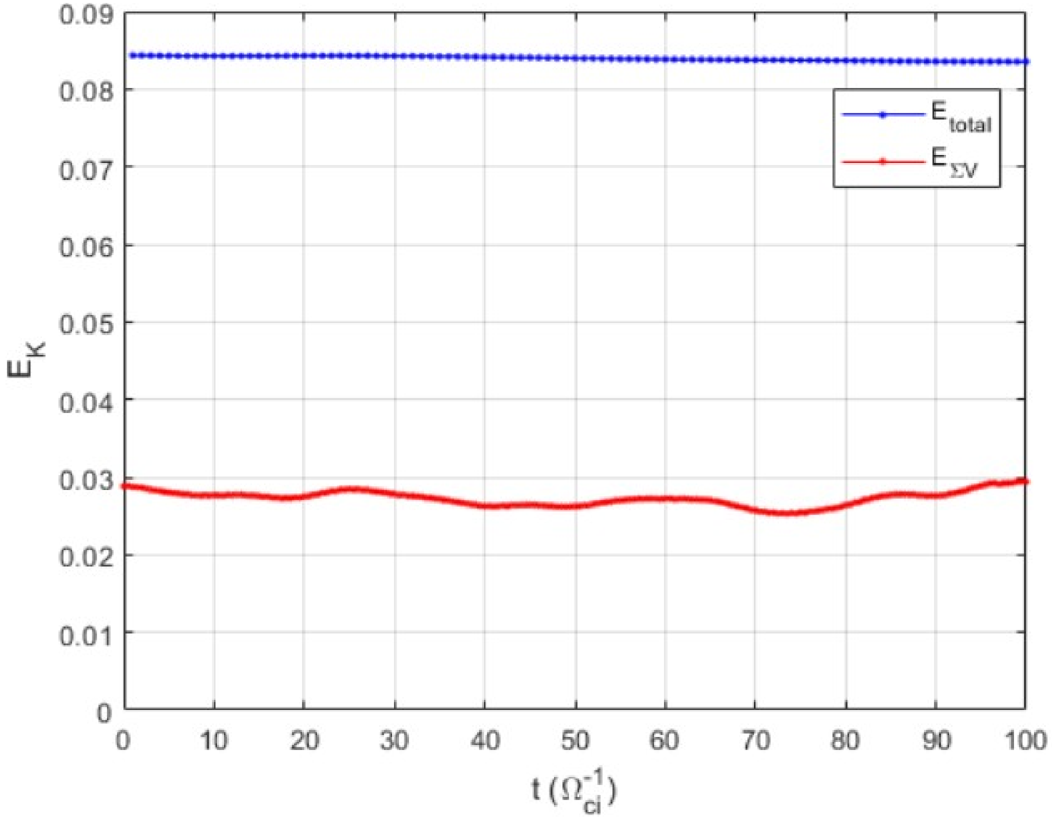
3: Lagrangian coherent structures in zonal flows

Time (Ω_{ci}^{-1})	Turbulent Regime			Zonal Flow		
	E_{total}	E_V	$\frac{E_V}{E_{total}}$ (%)	E_{total}	E_V	$\frac{E_V}{E_{total}}$ (%)
$t = 0$ to $t = 100$	0.0347	0.0012	3.46	0.0839	0.0011	1.31

Numerical values of the E_{total} and E_V

Miranda et al., under preparation

3: Lagrangian coherent structures in zonal flows



The total kinetic energy E_{total} and the total kinetic energy contained in all vortices $E_{\sum V}$, from t = 0 to t = 100.

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3: Lagrangian coherent structures in zonal flows

Time (Ω_{ci}^{-1})	Turbulent Regime			Zonal Flow		
	E_{total}	$E_{\sum V}$	$\frac{E_{\sum V}}{E_{total}}$ (%)	E_{total}	$E_{\sum V}$	$\frac{E_{\sum V}}{E_{total}}$ (%)
$t = 0$ to $t = 100$	0.0347	0.0107	30.84	0.0839	0.0273	32.54
$t = 100$ to $t = 200$	0.0361	0.0117	32.41	0.0835	0.0200	23.95
$t = 200$ to $t = 300$	0.0372	0.0191	51.34	0.0831	0.0191	22.98
$t = 300$ to $t = 400$	0.0350	0.0140	40.00	0.0828	0.0170	20.53
$t = 400$ to $t = 500$	0.0330	0.0039	11.81	0.0822	0.0264	32.12

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4: Conclusions

- Robust vortices using geodesic theory were detected in numerical simulations of electrostatic drift-wave turbulence.
- Lagrangian vortices contain ~ 50% or less of the total kinetic energy in drift-wave turbulence in plasmas.

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Numerical codes available at
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