Information theory applied to numerical simulations of resistive drift-wave turbulent in tokamak plasmas

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ABSTRACT

- Turbulence dominates the radial transport at the edge region of tokamak plasmas, reducing magnetic confinement in fusion experiments, and its control remains a challenge in physics and engineering. Information theory can provide useful tools to quantify the degree of order/disorder of turbulent fluids and plasmas.
- In this work we analyze numerical simulations of a simplified nonlinear model of turbulence induced by drift waves in tokamak plasmas. By varying a control parameter, we construct a bifurcation diagram of a transition from a turbulent regime to a regime dominated by zonal flows, in which turbulence is mostly suppressed. This transition is then characterized by computing the normalized spectral entropy of the turbulent patterns observed in the numerical simulations.
- Our results show that the turbulent regime displays a higher degree of entropy, the regime dominated by zonal flows is characterized by lower values of entropy, and the transition from the low-to-high confinement occurs abruptly. This work demonstrates that information theory can improve our understanding of the turbulent fluctuations that arise in the edge region of tokamak plasmas.

METHODS

HASEGAWA-WAKATANI MODEL

The Hasegawa-Wakatani equations describe a simplified model of the drift wave turbulence behavior in the edge region of tokamak. Thus, the model is described by

$$\frac{\partial}{\partial t}\zeta + [\varphi, \zeta] = \alpha(\varphi - n) - D\nabla^4\zeta \qquad (1)$$

$$\frac{\partial}{\partial t}n + [\varphi, \zeta] = \alpha(\varphi - n) - \kappa\frac{\partial \varphi}{\partial y} - \nabla^4 n, \qquad (2)$$

where ζ is the vorticity, parameter κ corresponds to the background density gradient and α is the adiabacity operator associated with the resistive coupling force between density, n, and the plasma electrostatic potential φ. D is the dissipation coefficient; the square brackets indicate Poisson bracket notation and represent nonlinear terms.

In this paper we set $D = 10^{-4}$ and $\kappa = 10^{-1}$ and choose α as a control parameter. Equations (1)-(2) are solved using the finite-differences method for the spatial derivatives, and the fourth-order Runge-Kutta method for the time integration. A spatial grid of 256x256 in 2D allows for accurately solving the equations while maintaining a reasonable computing time.

SPECTRAL ENTROPY

The spectral entropy employs the information of amplitudes of Fourier modes to measure the degree of spatial disorder (Rempel et al., 2007). It can be written as

$$H(t) = -\sum_{k=1}^{N} p_{k,t} \ln p_{k,t}, \qquad (3)$$

where $p_{k,t}$ is the relative Fourier weight of mode k, which is represented by

$$p = \frac{|\widehat{U}(k,t)|^2}{\sum_k |\widehat{U}(k_1 t)|^2} \qquad (4)$$

being $\widehat{U}(k,t)$ is a real function obtained from the Fourier transform.

Normalizing this equation, we infer that $p_{k,t} \epsilon [0,1]$ and $\sum_{k=1}^{N} p_{k,t} = 1$.

Thus, we can conclude that entropy will be maximum $p(k,t) = 1/N$, that is, when the distribution is uniform.

RESULTS AND DISCUSSION

The electrostatic potential φ is shown in figure 1 for two different values of α. For $\alpha = 0.01018$ (left side panel), φ displays turbulent patterns with the presence of vortices that can be recognized as regions of local minima and maxima of φ. For $\alpha = 0.01104$ (right side panel), a large-scale pattern can be easily recognized, due to the presence of zonal flows.

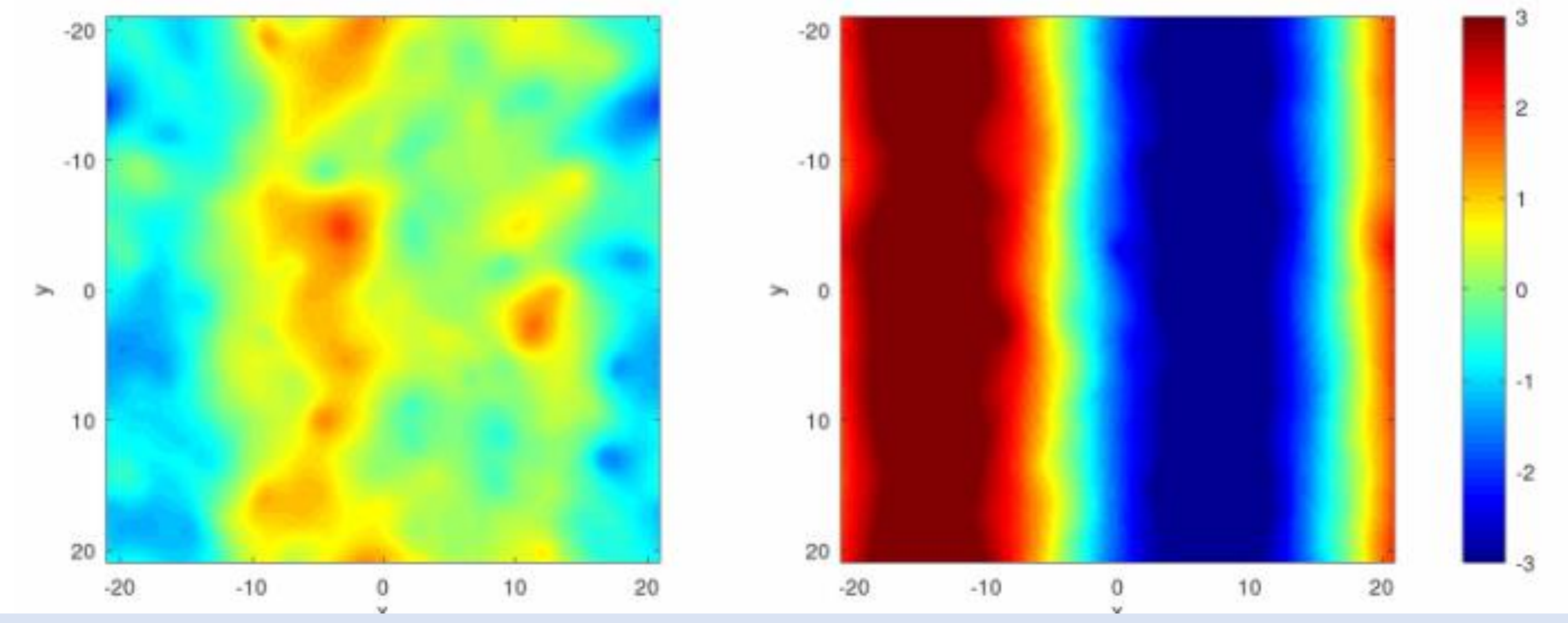

Figure 1 Plasma electrostatic potential obtained from the numerical simulations. On the left is the turbulent regime, on the right is the zonal flow.

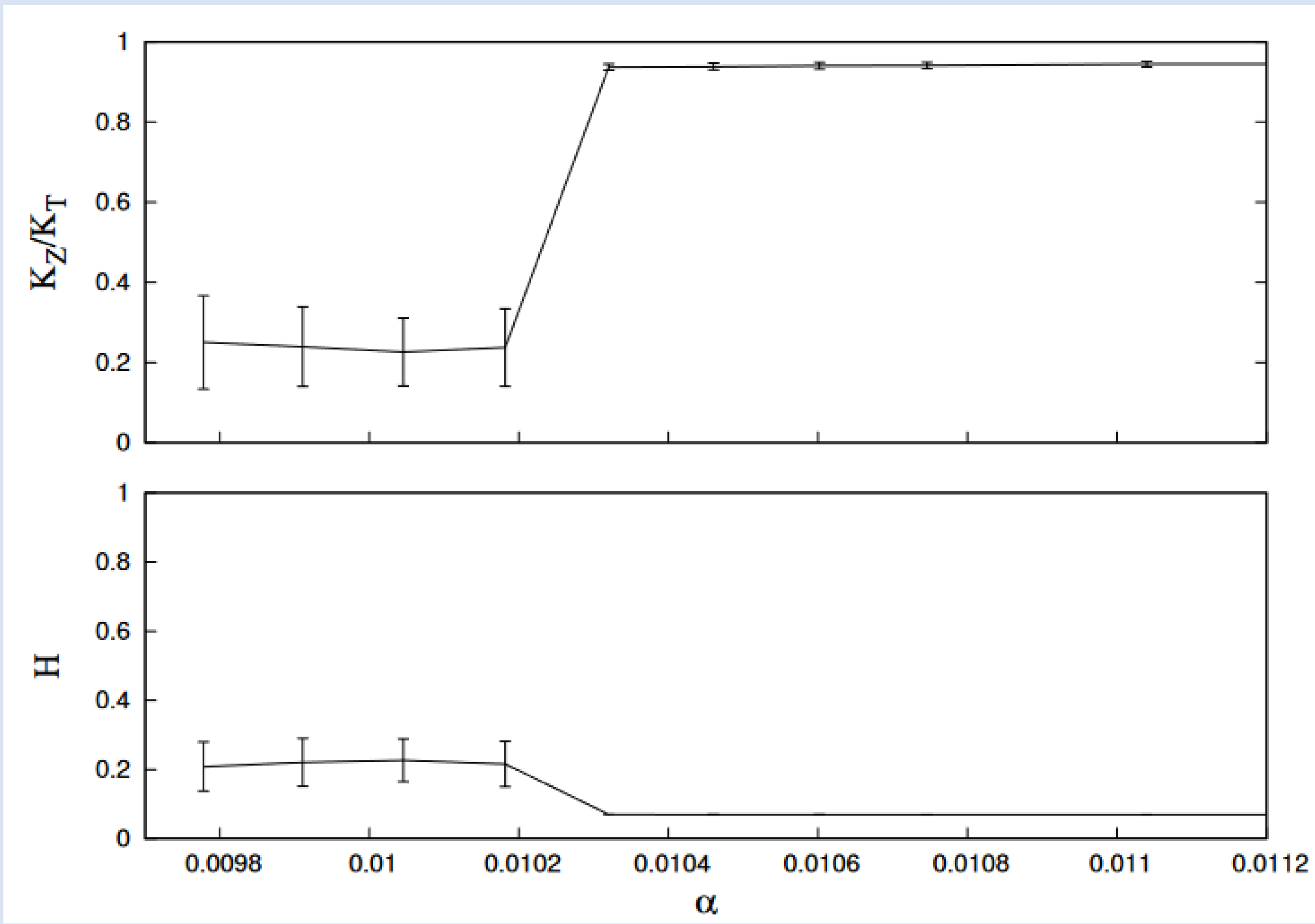

Figure 2 Bifurcation diagram of the ratio between the kinetic energy of zonal flows and the total kinetic energy, and the spectral entropy per α.

A comparison of the averaged spectral entropy before and after the L-H transition (α~0.0103) shows that the turbulent regime is characterized by values of the entropy higher than the values of the zonal-flow regime. This demonstrates that the emergence of zonal flows for α > 0.0103 results in a decrease of the degree of disorder of the electrostatic potential.

CONCLUSION

- The turbulent regime displays higher values of entropy compared to the zonal flow regime. This agrees with the spatial patterns of the electrostatic potential φ, in which the turbulent regime displays disordered patches of φ, whereas the zonal-flow regime is characterized by a large-scale coherent structure that suppresses turbulence and acts as barriers of horizontal flux .

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