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Reconstruction of plasma equilibrium using physics-informed neural network on EAST

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Plasma Equilibrium Reconstruction

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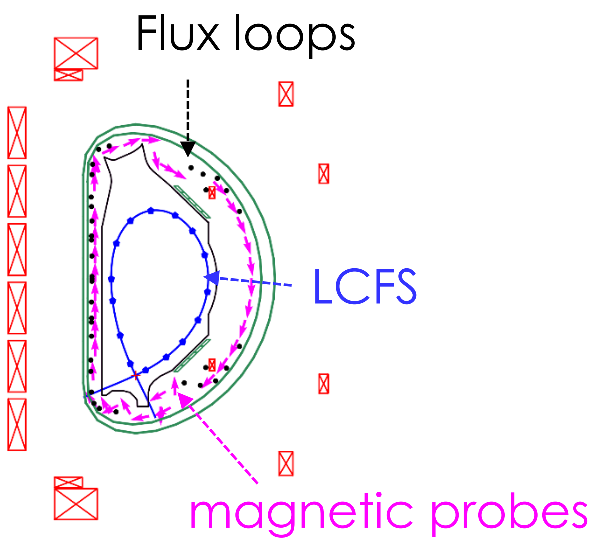
■ Equilibrium Reconstruction – Solving Grad-Shafranov equation



G-S equation: $$R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial\psi}{\partial R}+\frac{\partial^2\psi}{\partial Z^2}=-\mu_0R^2\frac{d}{d\psi}p-F\frac{d}{d\psi}F$$

□ **Known:**

External diagnostics, e.g., plasma current Ip, magnetic probes, flux loops, etc.

Internal diagnostics, e.g., **MSE、POINT;**

□ **Solving:**

$\psi(R,Z)$ that satisfies the G-S equation on the domain Ω_c.

EFIT:

□ **Distributed current model:**

$$p'(\psi)=\sum_n \alpha_n\psi_N^n$$

$$F(\psi)F'(\psi)=\sum_n \gamma_n\psi_N^n$$

(H-mode) → **Difficult for complex form**

□ **Picard iterations:**

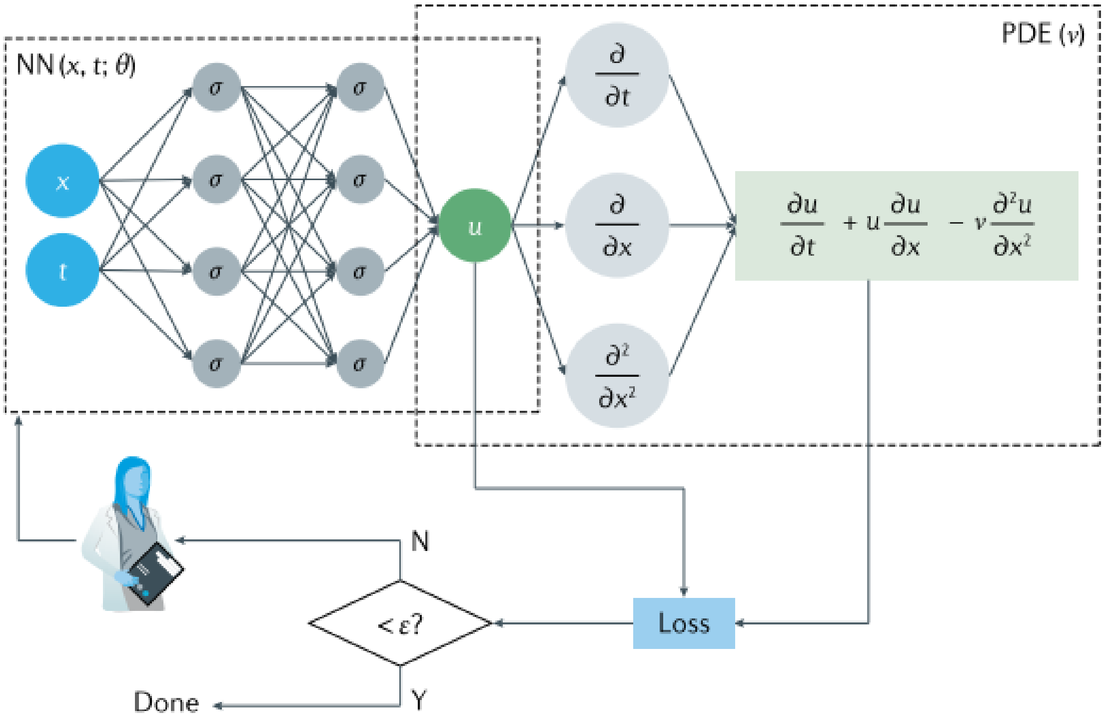
$$C_j^{(m+1)}(r_j,z_j)=\sum_{n=1}^{n_{PF}}G_{cD_j}(r_j,z_j,r_n,z_n)I_n^{(m+1)}+\int_{\Omega^{(m)}}dRdZG_{pD_j}(r_j,z_j,R',Z')J_\varphi\left(R',Z',\psi^{(m)},\vec{\alpha}^{(m+1)},\vec{\gamma}^{(m+1)}\right)$$

□ **Least squares fitting:**

$$\chi^2=\sum_{n=1}^{n_M}\left(\frac{M_j-Cj}{\sigma_j}\right)^2$$

→ **Difficult to add new diagnostics**

Physics-Informed Neural Network (PINN)



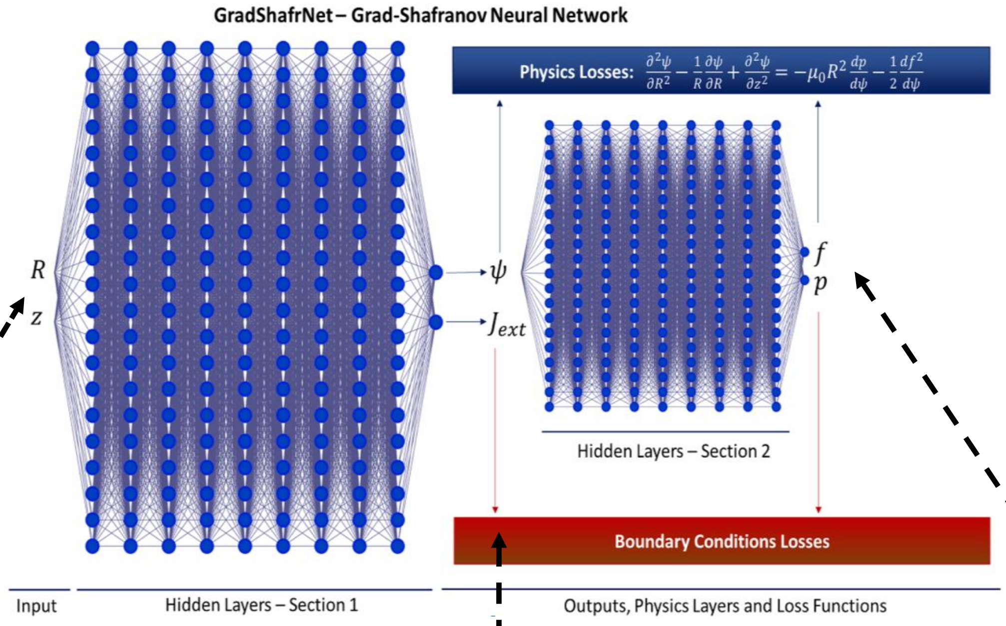
The basic structure of PINN

	Traditional numerical methods	PINN
Model building	$p'(\psi) = \sum_n \alpha_n \psi_N^n$ $F(\psi)F'(\psi) = \sum_n \gamma_n \psi_N^n$	$p'(\psi) = NN_1(\psi)$ $F(\psi)F'(\psi) = NN_2(\psi)$
New diagnostics	New response matrix, new model assumptions, etc.	New boundary conditions
Applicability to complex problems	Difficult	Easy

- **PINN can be used for plasma equilibrium reconstruction.**

Karniadakis G E, et al, Nat. Rev. Phys, 3(2021) 422–440

Reconstruction by PINN on JET



9 layers, 20 neurons each

3 layers, 20 neurons each

input:
5×10^5 **collocation points** (R, Z) **(Sobol sequence generated)**

middle:
Flux value ψ **of collocation points and external current distribution** J_{ext}

output:
Poloidal current flux function f **and pressure** p

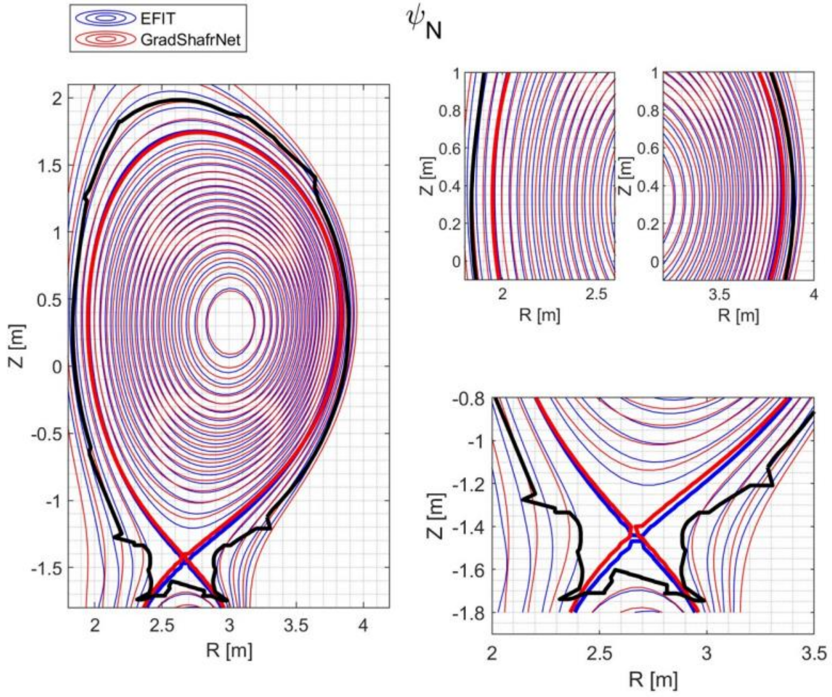
- **Optimizer: Adam**

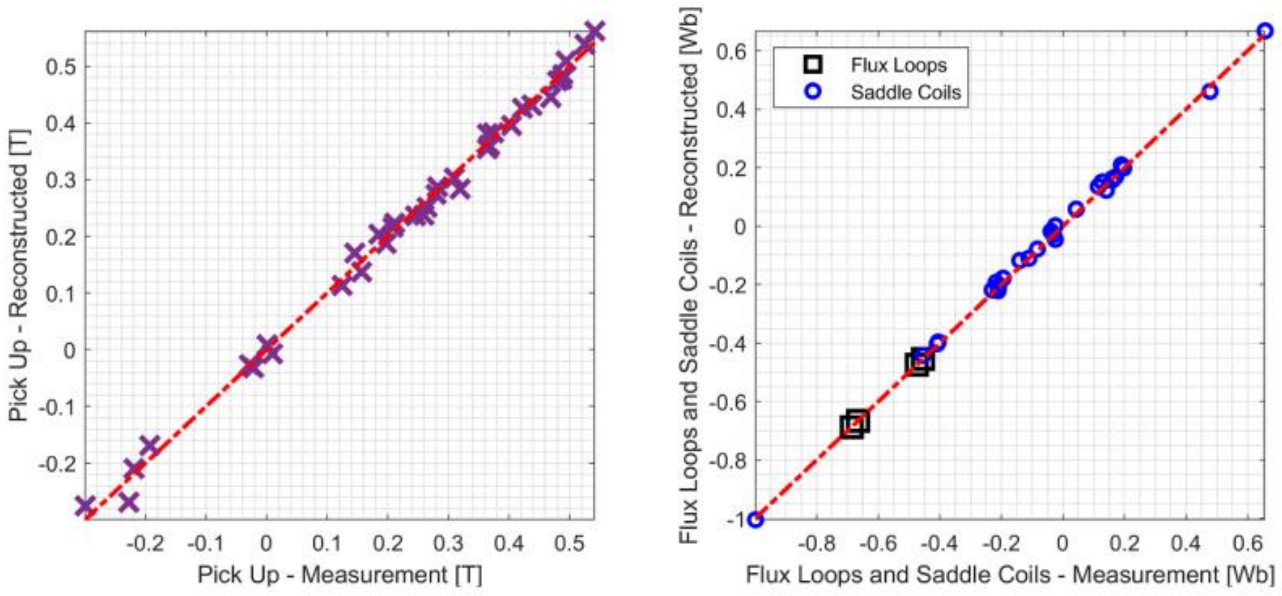
  - **Boundary Conditions Losses:**
  - **Pick-up coils**
  - **Flux loops**
  - **Saddle coils**
  - **Pressure on the wall**

Rossi R, et al, *Nucl. Fusion*, 63 (2023) 126059

Reconstruction by PINN on JET

Shot 94217 t=9s

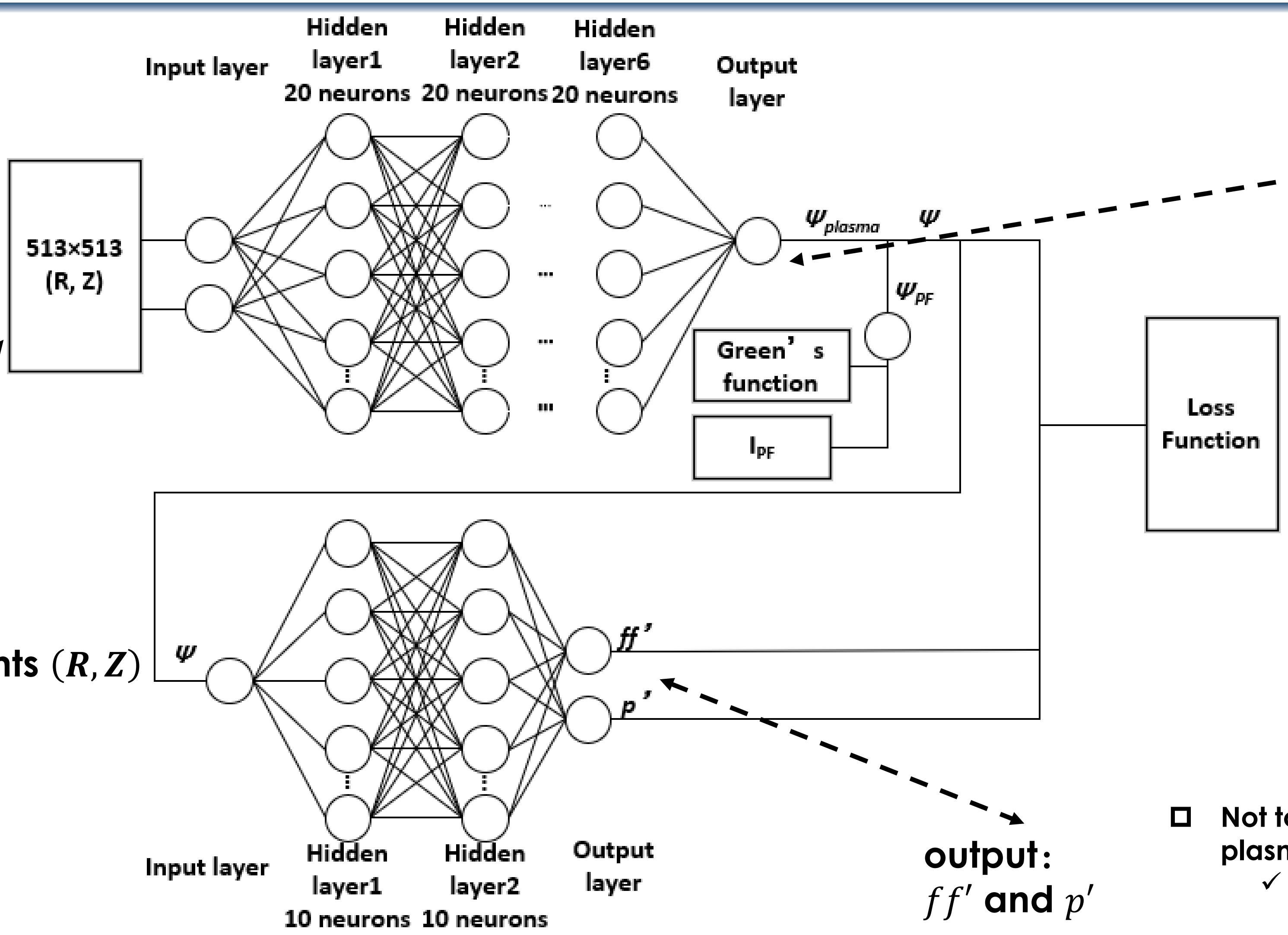




- ✓ **Relative error of $\psi_N < 5\%$**
- ✓ **Measurements and predictions in good agreement**
- **Unreasonable assumptions of external current distribution**
- **Conflation of plasma boundary and limiter**
- **Great error**

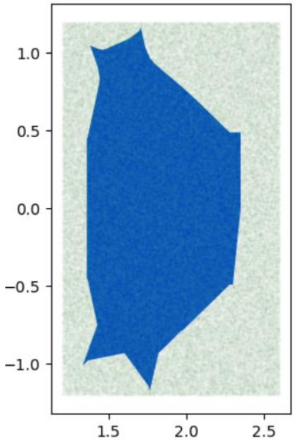
Rossi R, et al, *Nucl. Fusion*, 63 (2023) 126059

PINN Structure for reconstruction on EAST (I_{PF} known)



middle：
Flux value ψ_{plasma} of grid points

input：
513×513 grid points (R, Z)



output：
ff' and p'

- Not total flux but flux contributed by plasma is reconstructed.
  - ✓ To avoid conflicts between different current sources
  - ✓ To separate the external current

Loss Functions (I_{PF} known)

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$$\mathcal{L} = w_{p,inside}\mathcal{L}_{p,inside} + w_{p,outside}\mathcal{L}_{p,outside} + w_{b,MP}\mathcal{L}_{b,MP} + w_{b,FL}\mathcal{L}_{b,FL} + w_{p',p,LCFS}\mathcal{L}_{p',p,LCFS} + w_{ff',p,LCFS}\mathcal{L}_{ff',p,LCFS} + w_{ip}\mathcal{L}_{ip} + w_{b,MPG}\mathcal{L}_{b,MPG} + w_{b,FLG}\mathcal{L}_{b,FLG} + w_{p,max}(\mathcal{L}_{p,insidemax} + \mathcal{L}_{p,outsidemax})$$

Inside the LCFS:

$$\mathcal{L}_{p,inside} = \frac{1}{M_p}\sum_{i_p=1}^{M_p}\left[\left(\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial Z^2} + \mu_0 R^2\frac{dp}{d\psi} + \frac{1}{2}\frac{df^2}{d\psi}\right)^2\right]_{(R_{i_p},Z_{i_p})_{inside}}$$

Outside the LCFS:

$$\mathcal{L}_{p,outside} = \frac{1}{M_p}\sum_{i_p=1}^{M_p}\left[\left(\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial Z^2}\right)^2\right]_{(R_{i_p},Z_{i_p})_{outside}}$$

p' on the LCFS:

$$\mathcal{L}_{p',p,LCFS} = \frac{1}{M_{p,LCFS}}\sum_{i_{p,LCFS}=1}^{M_{p,LCFS}}\left(p'\left(R_{i_{p,LCFS}}, Z_{i_{p,LCFS}}\right)\right)^2$$

ff' on the LCFS:

$$\mathcal{L}_{ff',p,LCFS} = \frac{1}{M_{p,LCFS}}\sum_{i_{p,LCFS}=1}^{M_{p,LCFS}}\left(ff'\left(R_{i_{p,LCFS}}, Z_{i_{p,LCFS}}\right)\right)^2$$

$\mathcal{L}_{p,insidemax}$ → **top max 5%** $\mathcal{L}_{p,inside}$

$\mathcal{L}_{p,outsidemax}$ → **top max 5%** $\mathcal{L}_{p,outside}$

- **Strong Constraints for points that are difficult to converge**

■ **Physics loss**

Magnetic probes:

$$\mathcal{L}_{b,MP} = \frac{1}{M_{b,MP}}\sum_{i_{b,MP}=1}^{M_{b,MP}}\frac{\left(B_{R_{plasma}}\left(R_{i_{b,MP}}, Z_{i_{b,MP}}\right)\cos\left(\theta_{i_{b,MP}}\right) + B_{Z_{plasma}}\left(R_{i_{b,MP}}, Z_{i_{b,MP}}\right)\sin\left(\theta_{i_{b,MP}}\right) - B_{plasma}{}^{b}_{i_{b,MP}}\right)^2}{\left(\sigma^b_{i_{b,MP}}\right)^2}$$

$$B_{plasma}{}^{b}_{i_{b,MP}} = B_{total}{}^{b}_{i_{b,MP}} - G_{bc}J_{ext}$$

$$\mathcal{L}_{b,MPG} = \frac{1}{M_{b,MP}}\sum_{i_{b,MP}=1}^{M_{b,MP}}\frac{\left(G_{MP}Ip_{grid} - B_{plasma}{}^{b}_{i_{b,MP}}\right)^2}{\left(\sigma^b_{i_{b,MP}}\right)^2}$$

Flux loops:

$$\mathcal{L}_{b,FL} = \frac{1}{M_{b,FL}}\sum_{i_{b,FL}=1}^{M_{b,FL}}\frac{\left(\psi_{plasma}\left(R_{i_{b,FL}}, Z_{i_{b,FL}}\right) - \psi_{plasma}{}^{b}_{i_{b,FL}}\right)^2}{\left(\sigma^b_{i_{b,FL}}\right)^2}$$

$$\mathcal{L}_{b,FLG} = \frac{1}{M_{b,FL}}\sum_{i_{b,FL,}=1}^{M_{b,FL}}\frac{\left(G_{FL}Ip_{grid} - \psi_{plasma}{}^{b}_{i_{b,FL}}\right)^2}{\left(\sigma^b_{i_{b,FL}}\right)^2}$$

$$\psi_{plasma}{}^{b}_{i_{b,FL}} = \psi_{total}{}^{b}_{i_{b,FL}} - G_{fc}J_{ext}$$

Ip:

$$\mathcal{L}_{ip} = \left(\sum_{i_p=1}^{M_p} Ip_{grid}\,{}_{(R_{i_p},Z_{i_p})_{inside}} - ip_{measure}\right)^2$$

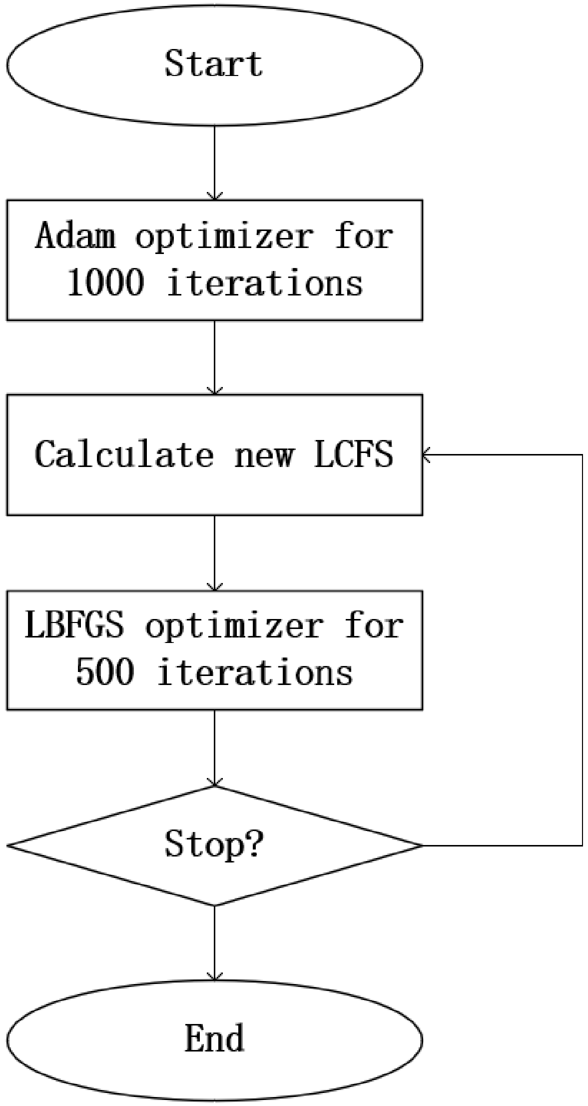
$$Ip_{grid} = \left(Rp' + \frac{ff'}{\mu_0 R}\right)S_{grid}$$

■ **Boundary loss**

Algorithm and weights

$$\mathcal{L} = w_{p,inside}\mathcal{L}_{p,inside} + w_{p,outside}\mathcal{L}_{p,outside} + w_{b,MP}\mathcal{L}_{b,MP} + w_{b,FL}\mathcal{L}_{b,FL} + w_{p',p,\boldsymbol{LCFS}}\mathcal{L}_{p',p,\boldsymbol{LCFS}} + w_{ff',p,\boldsymbol{LCFS}}\mathcal{L}_{ff',p,\boldsymbol{LCFS}} + w_{ip}\mathcal{L}_{ip} + w_{b,MPG}\mathcal{L}_{b,MPG} + w_{b,FLG}\mathcal{L}_{b,FLG} + w_{p,max}(\mathcal{L}_{p,insidemax} + \mathcal{L}_{p,outsidemax})$$

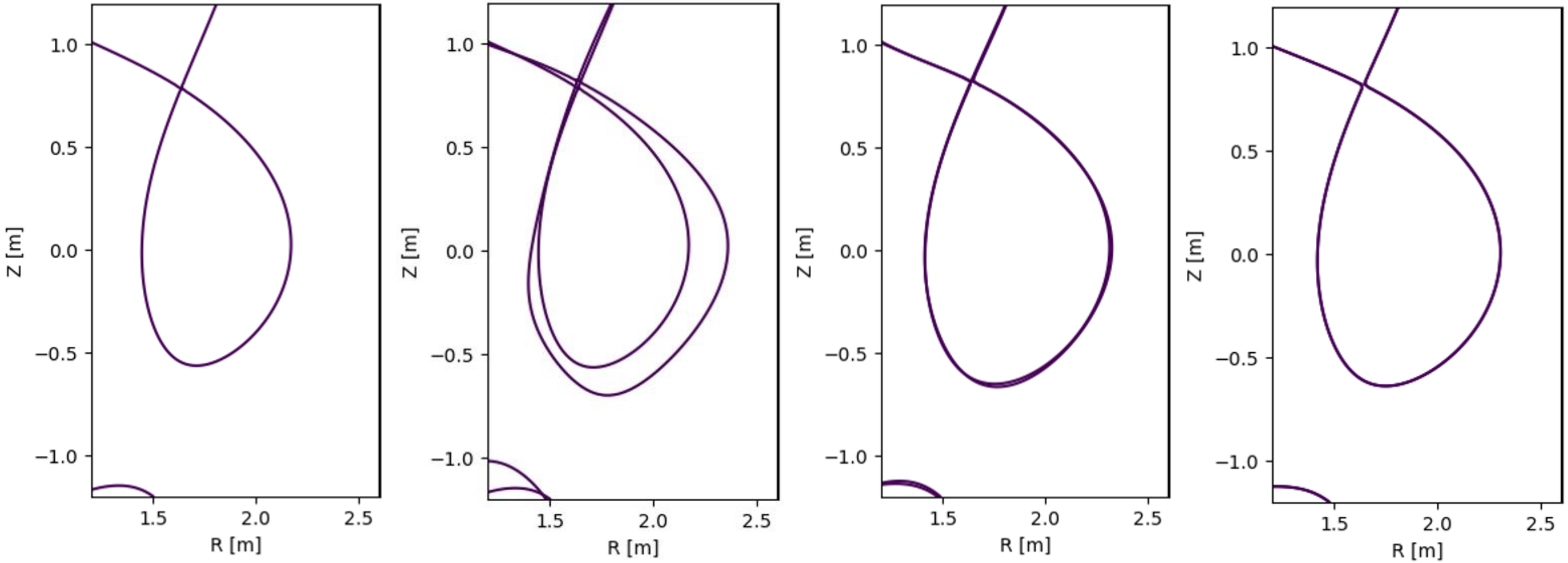
- **Optimizer: Adam, LBFGS**



	Adam	LBFGS(below 3500 iterations)	LBFGS(above 3500 iterations)
$w_{p,inside}$	1	10	10
$w_{p,outside}$	1	10	10
$w_{b,MP}$	1	1	1
$w_{b,FL}$	1	1	1
$w_{b,MPG}$	1	10	10
$w_{b,FLG}$	1	10	10
$w_{p',p,\boldsymbol{LCFS}}$	1	10	10
$w_{ff',p,\boldsymbol{LCFS}}$	0.5	5	5
w_{ip}	0.5	1	1
$w_{p,max}$	0	1	10

- **Stop conditions:**
  - **LCFS unchanged between two LBFGS optimization phase**
  - **More than 50,000 iterations**

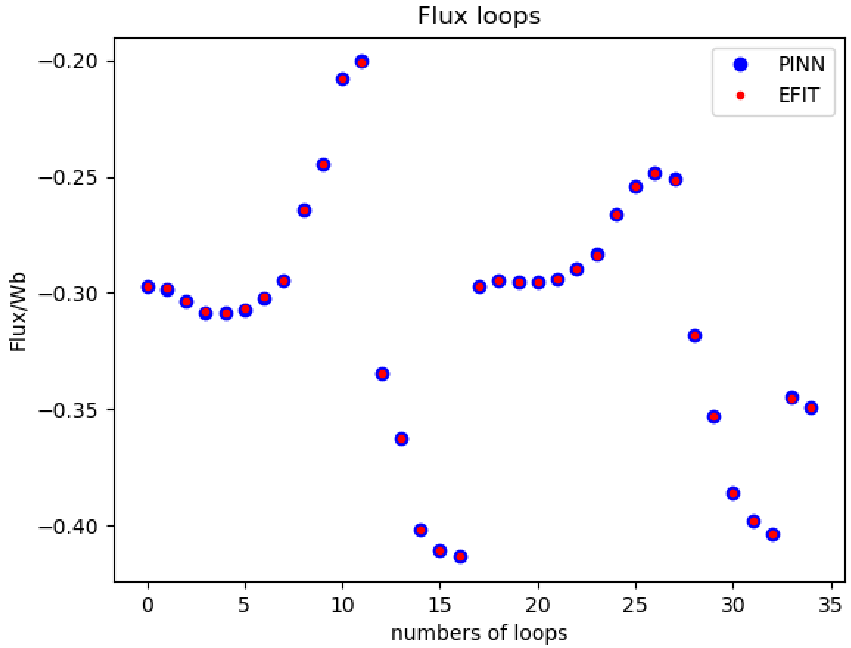
LCFS Convergence Process

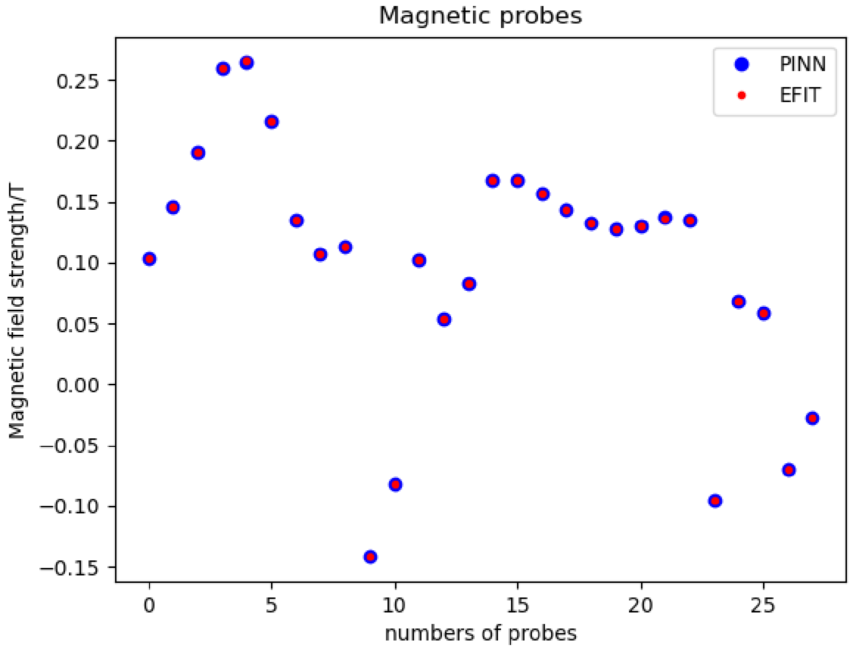


After Adam 1000 iterations | After LBFGS 500 iterations | After LBFGS 2000 iterations | After LBFGS 4000 iterations

LCFS gradually converges during the optimization process

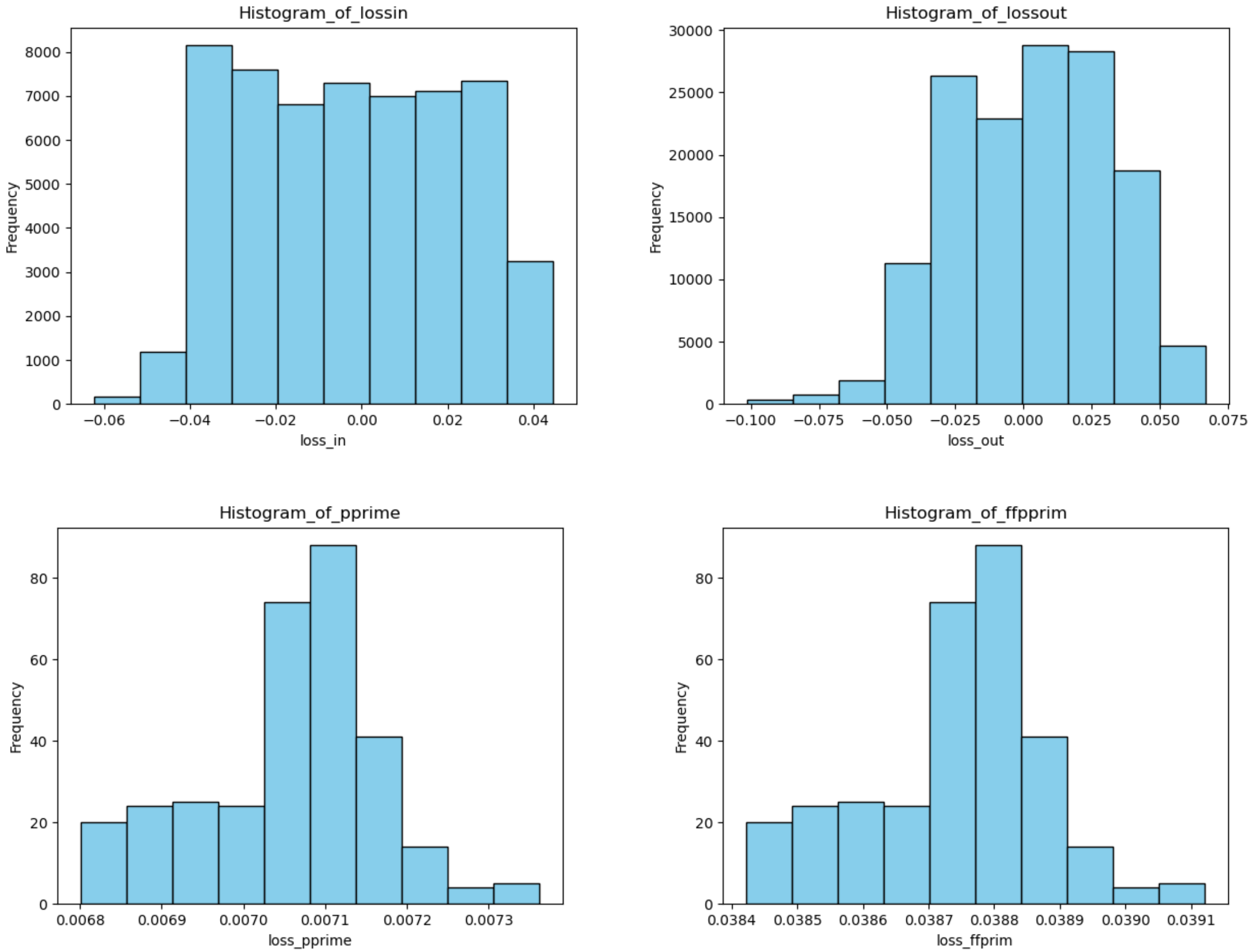
Convergence of diagnostics





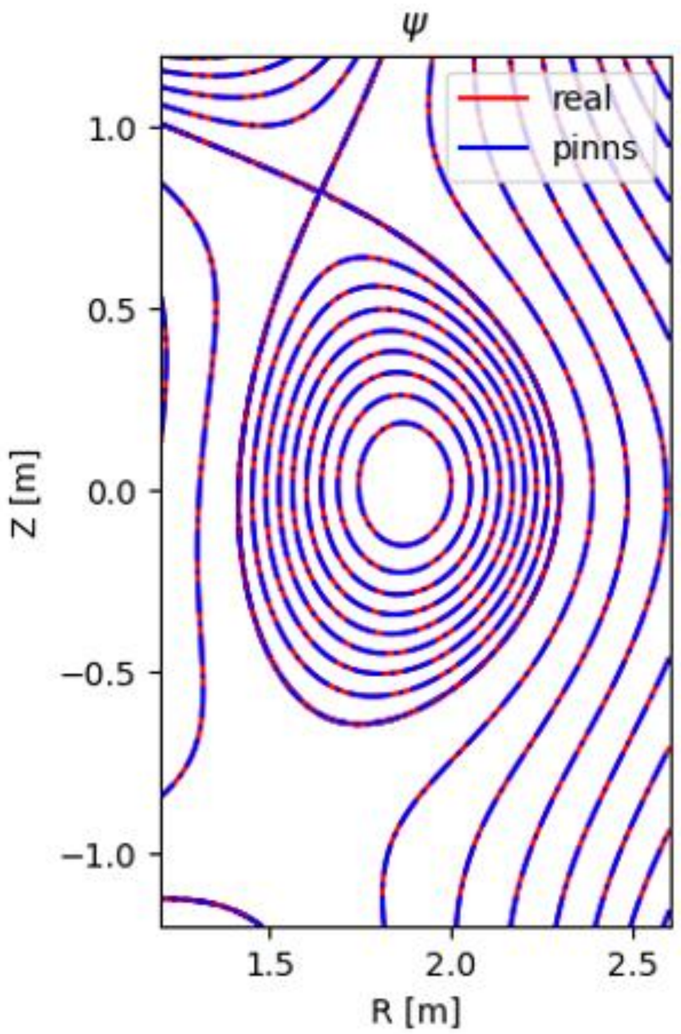
- ✓ **Maximum relative error 0.22% on flux loops**
- ✓ **Maximum relative error 0.26% on magnetic probes**

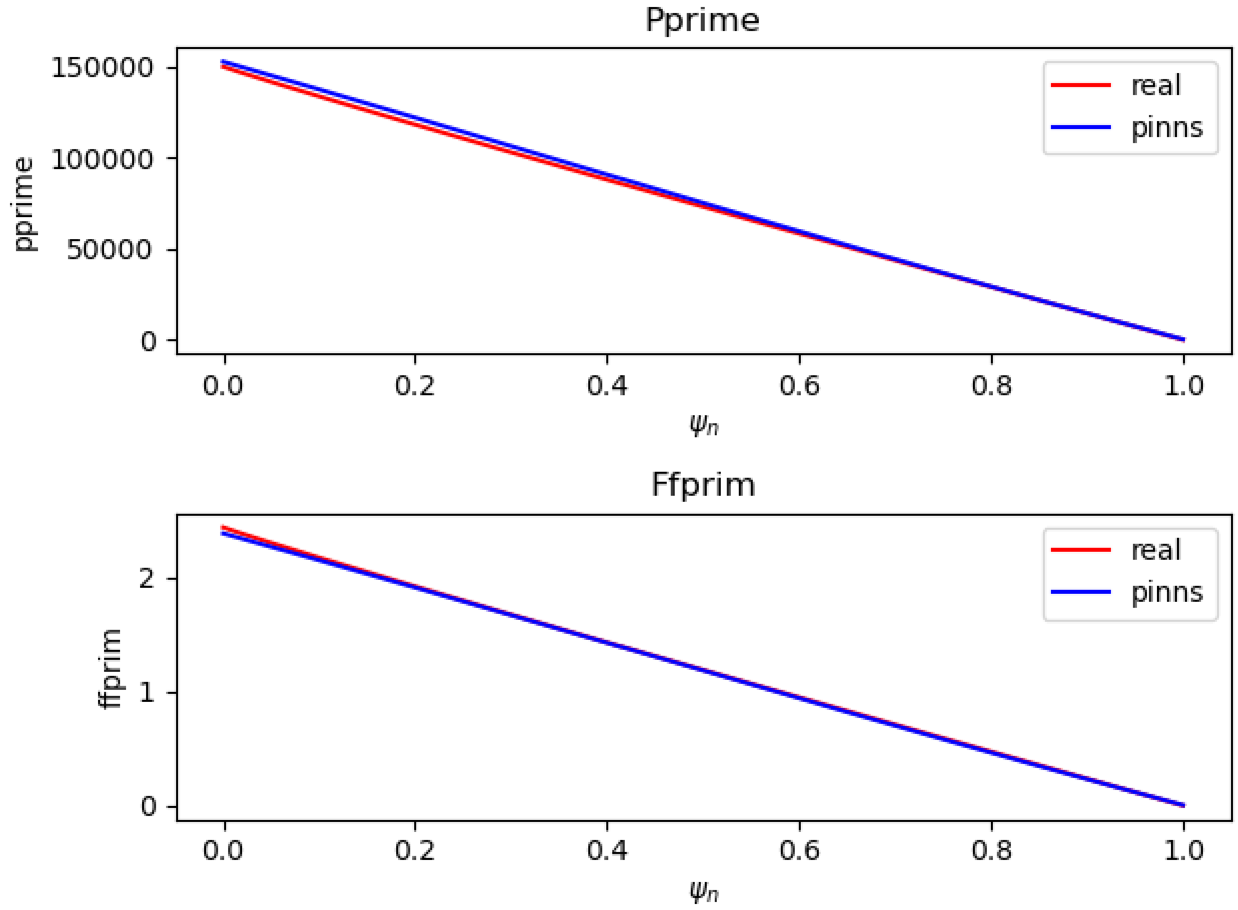
Convergence of physics loss



- ✓ **an absolute error below 0.06 in LCFS and below 0.1 outside LCFS**
- ☐ **convergence is still not enough for equilibrium reconstruction**

Reconstruction Results

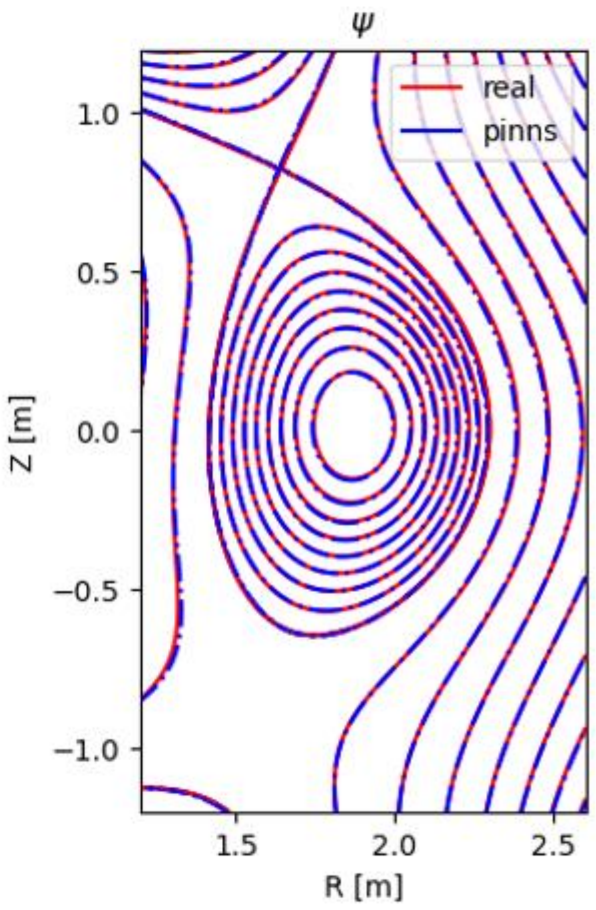


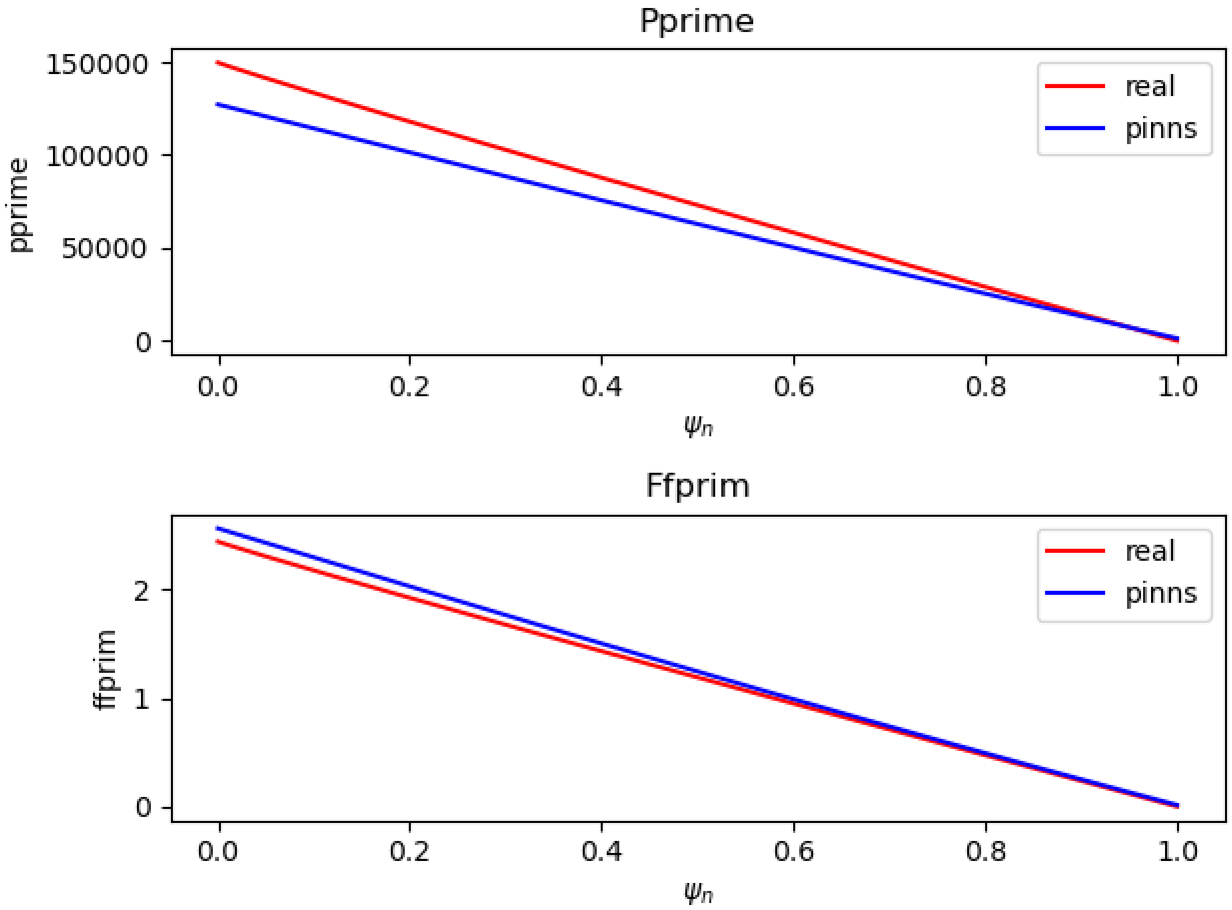


simulated equilibrium reconstruction results

- ✓ **a relative flux error below 0.1%**

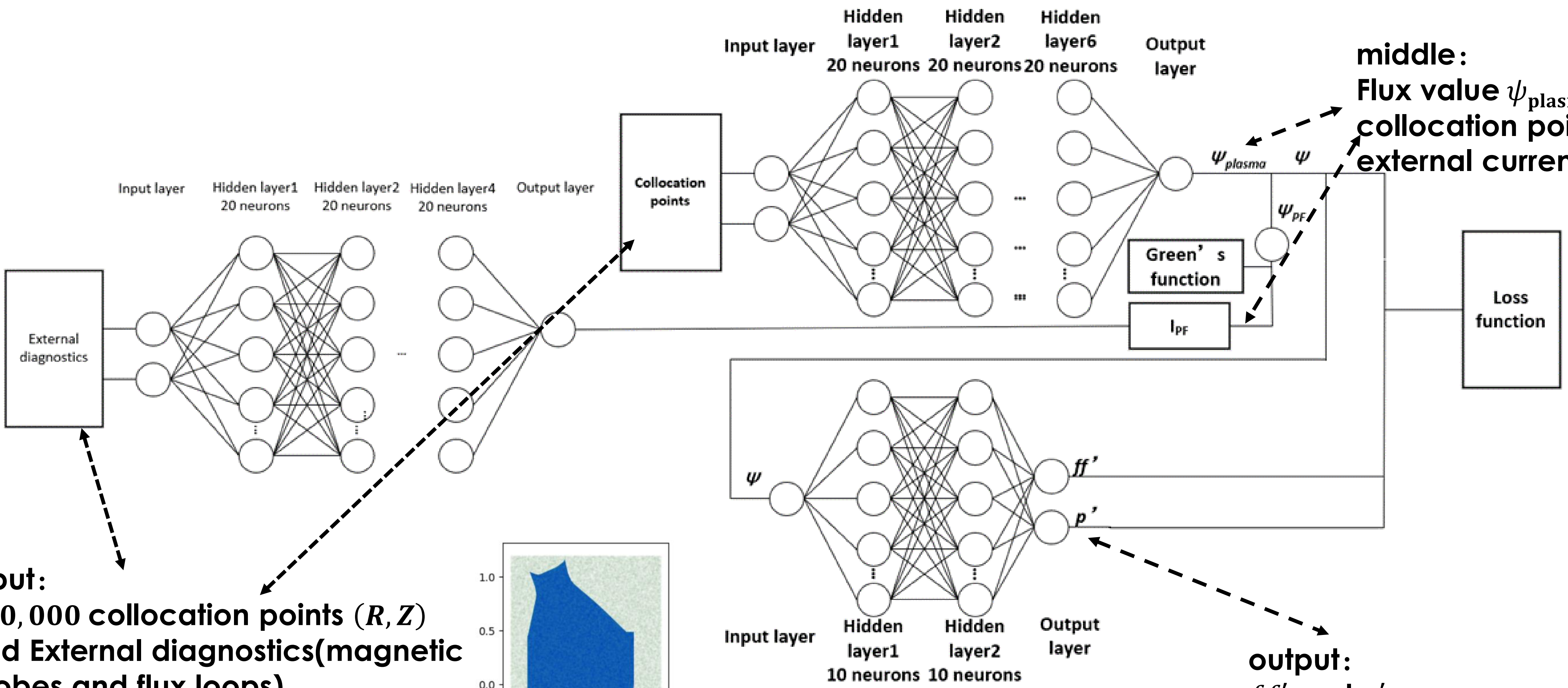
Reconstruction Results(3% relative diagnostics error)





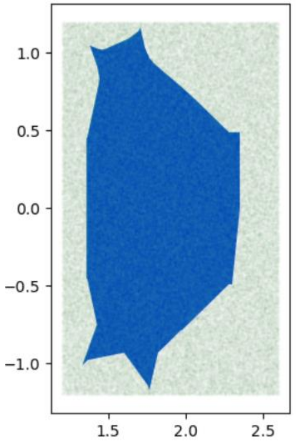
simulated equilibrium reconstruction results (with diagnostics error)

PINN Structure for reconstruction on EAST (I_{PF} unknown)



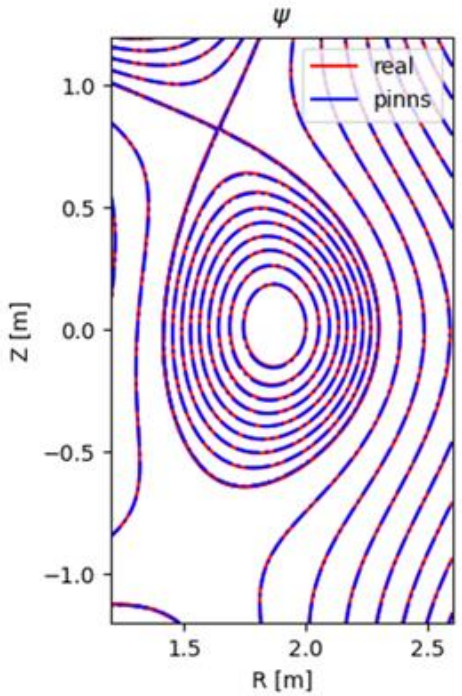
middle：
Flux value $\psi_{\mathbf{plasma}}$ of collocation points and external current I_{PF}

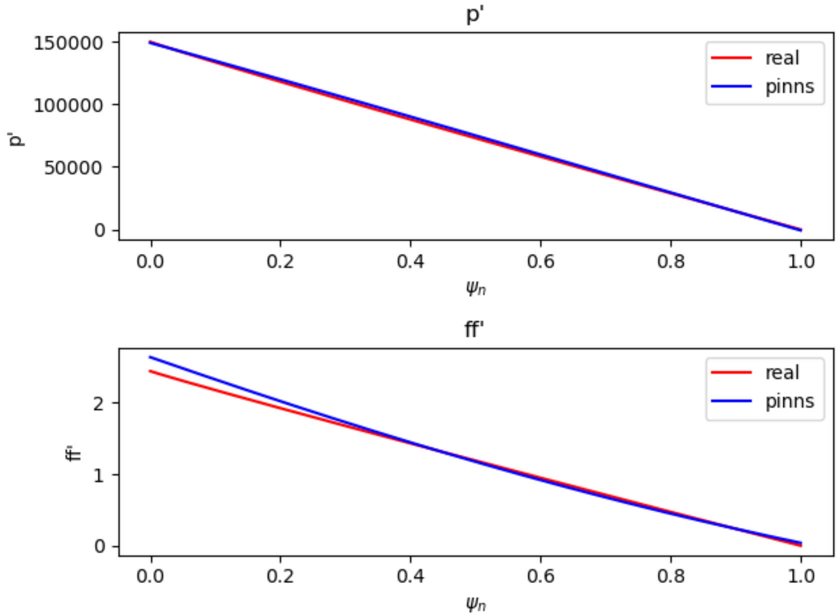
input：
$200,000$ collocation points (R, Z) and External diagnostics(magnetic probes and flux loops)

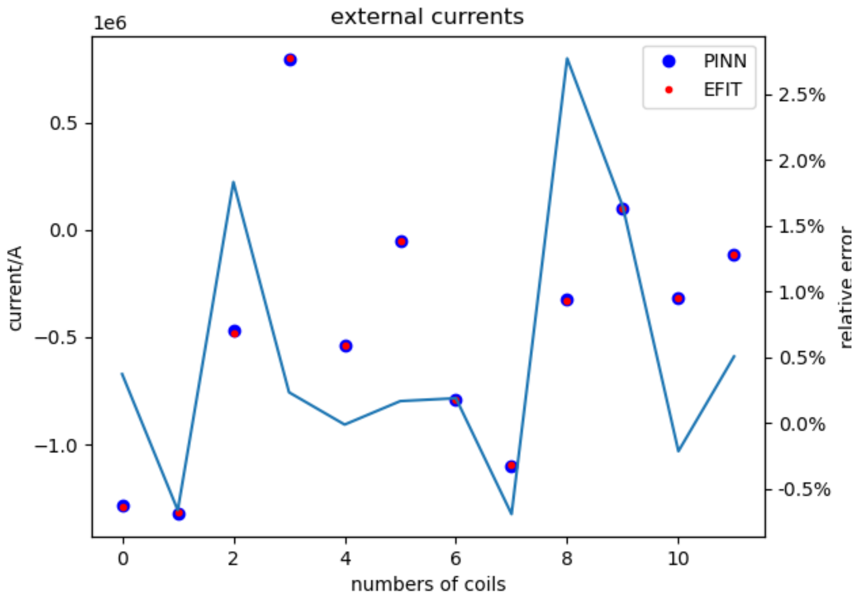


output：
ff' and p'

Reconstruction Results







simulated equilibrium reconstruction results

- ✓ **a relative error below 0.5% of flux**
- ✓ **a relative error below 3% of external currents**

- ☐ **much harder to converge due to the complexity of the loss function**

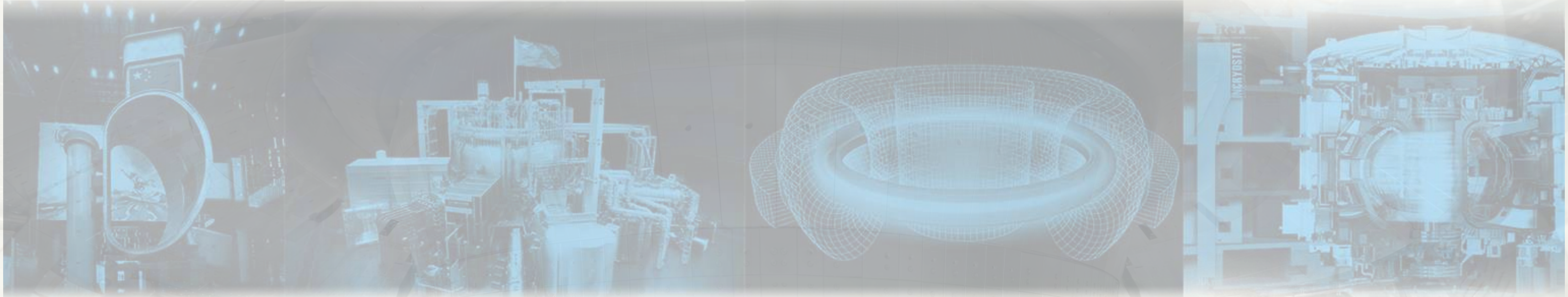
Conclusion & Future research plans

- ☐ Develop a PINN structure for equilibrium reconstruction with external current known/unknown

- ☐ Design an algorithm, including optimizers and weights adjustment for loss functions

- ✓ The results are all consistent with EFIT, proving its high reliability

- Reconstruction of equilibrium including external currents with relative error added on diagnostics
- Reconstruction of different plasma shapes and complex current distribution form
- Fast equilibrium reconstruction



Thank you !