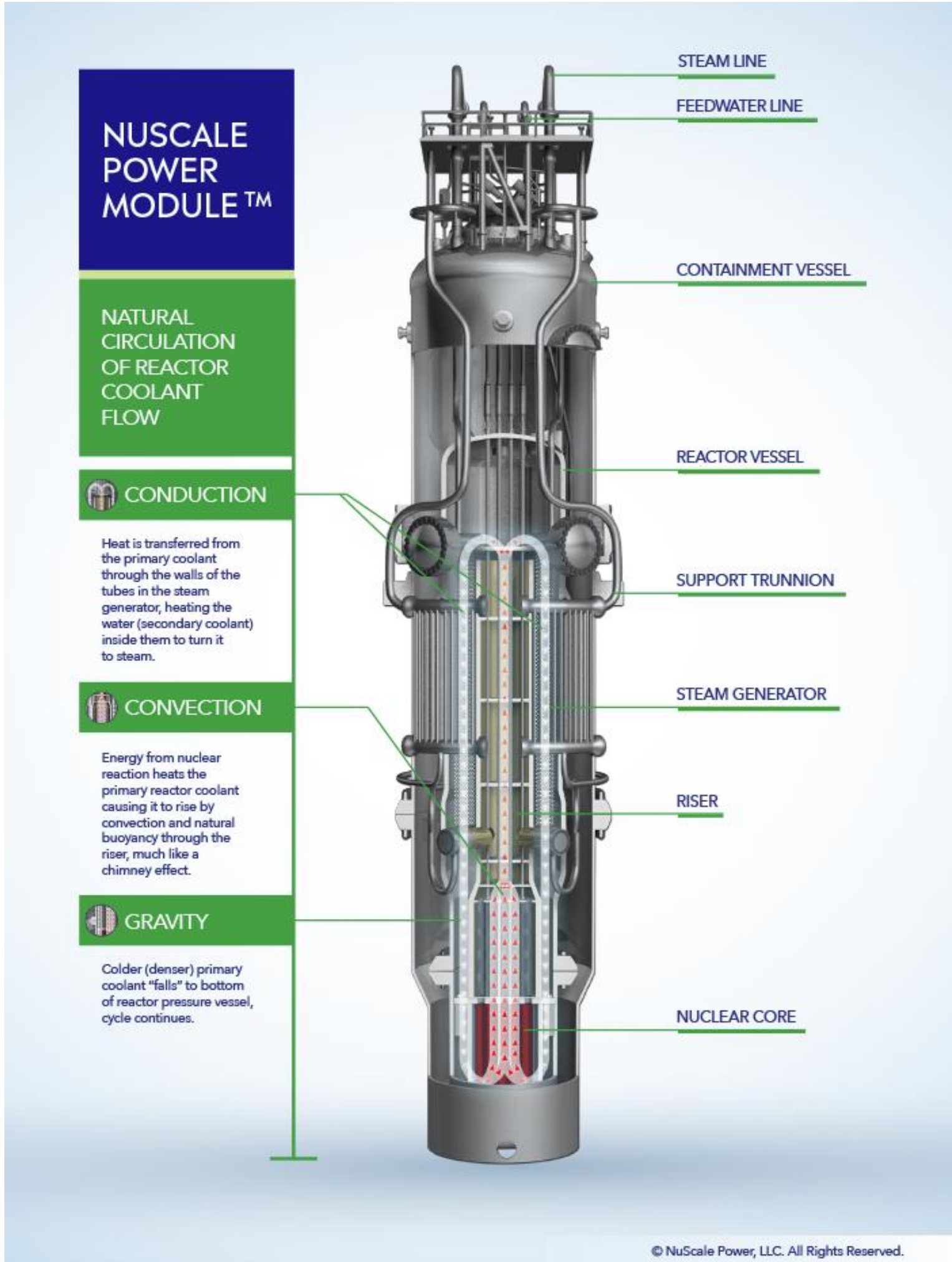
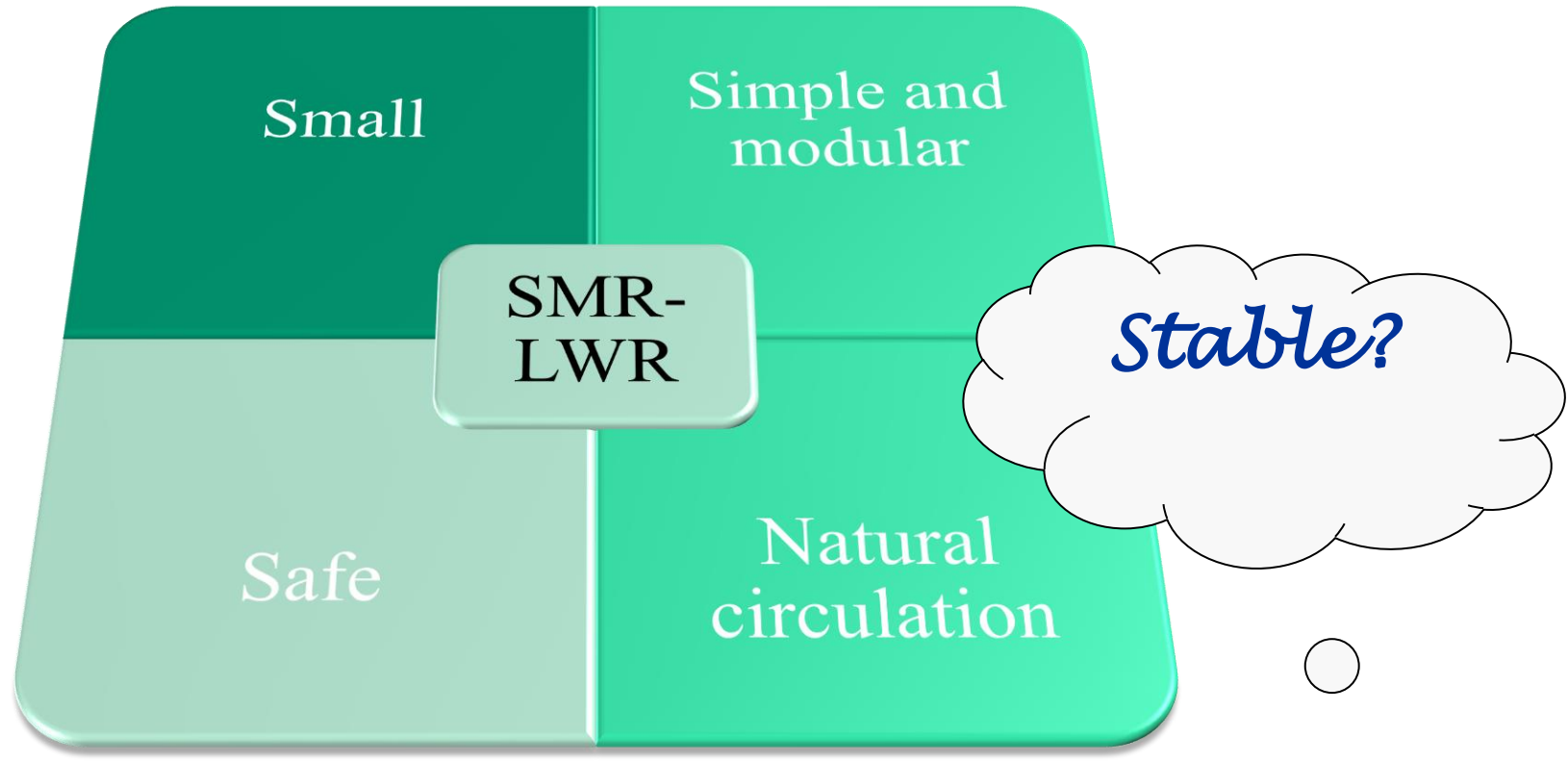


Analysis of the Stability of a SMR with Lyapunov Methods

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Introduction

- Light water SMRs offer numerous advantages such as transportability, availability, and operability compared to large Generation II reactors, and some designs additionally feature improved safety by natural convection cooling.
- This work addresses the study of the **neutronic-thermohydraulic stability** of these reactors using Lyapunov methods, as an alternative to conventional time-domain and frequency-domain methods.



An example of a stable SMR with natural circulation.
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Point Kinetics

$$\frac{dP}{dt} = \frac{\rho - \beta}{\Lambda} P + \sum_1^6 \lambda_i C_i$$
$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} P - \lambda_i C_i \quad i = 1, \dots, 6$$
$$\rho = \rho_{ext} + \alpha_f (T_f - T_{f0}) + \alpha_w (T_w - T_{w0})$$

Heat transfer

$$\frac{dT_f}{dt} = [P - UA_f(T_f - T_c)] / (m_f c p_f)$$
$$\frac{dT_c}{dt} = [UA_f(T_f - T_c) - \mu(T_c - T_w)^{1.33}] / (m_c c p_c)$$
$$\frac{dT_w}{dt} = [\mu(T_c - T_w)^{1.33} - 2w_w c p_w(T_w - T_{win})] / (m_w c p_w)$$



Flow momentum

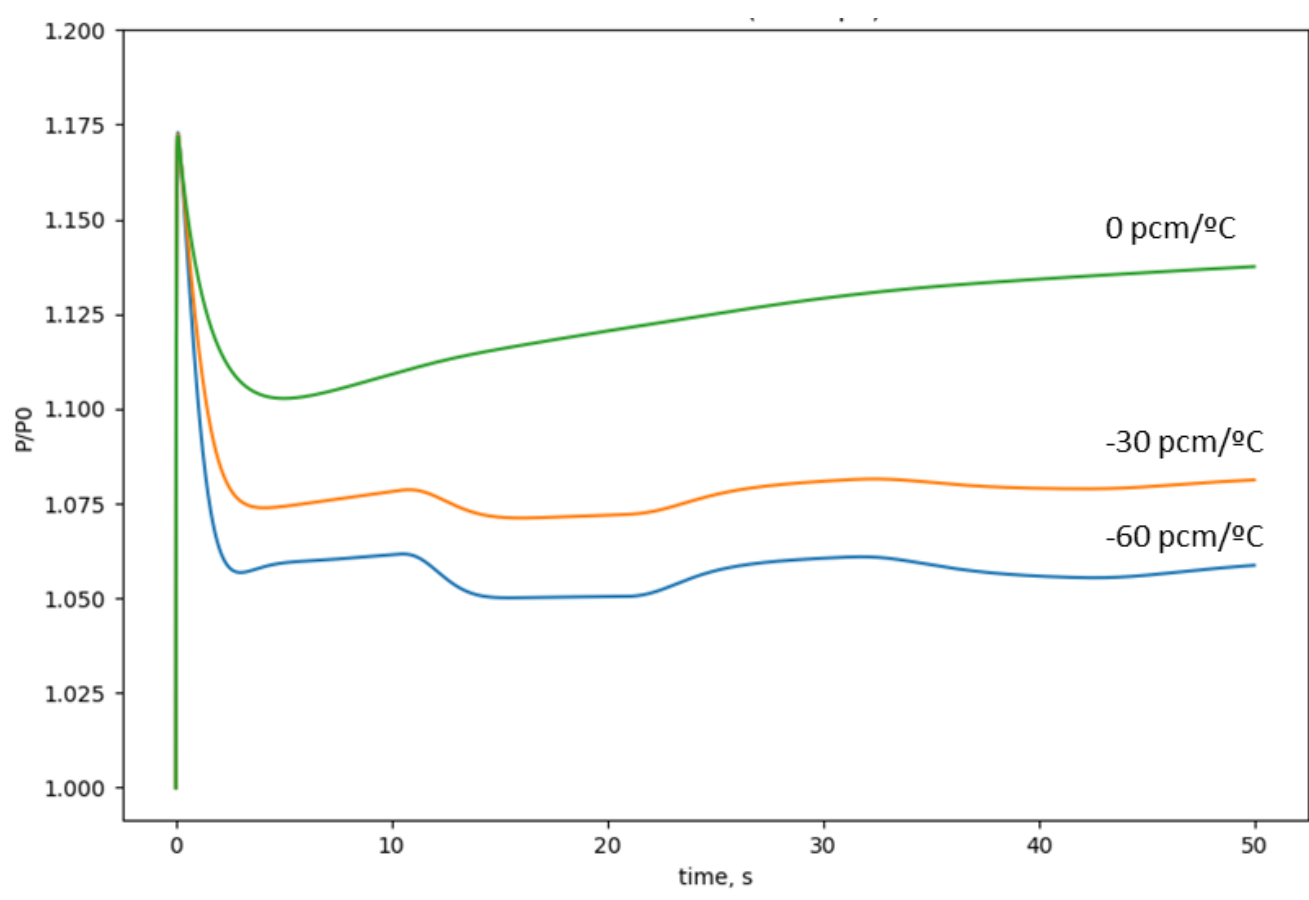
$$I \frac{dW_w}{dt} = \frac{gL\beta_v \rho_w Q_{cw}}{c p_w W_w(t - \tau)} - \xi W_w^2$$

Simplified Modeling

- Point kinetics is valid for relatively small reactivity perturbations. Besides, lumped parameter thermohydraulics allow for the qualitative description of transient behavior.
- The small size and simplicity of SMRs enable modeling their behavior with few equations and variables, allowing the use of Lyapunov's theorems for stability analysis.

ODE11

- 11 equations
 - Response to an arbitrary step of reactivity?
- = Stable and smooth response with some minor oscillations
- ∴ **stable behaviour!**



© Smooth and stable response to a reactivity step

ODE5

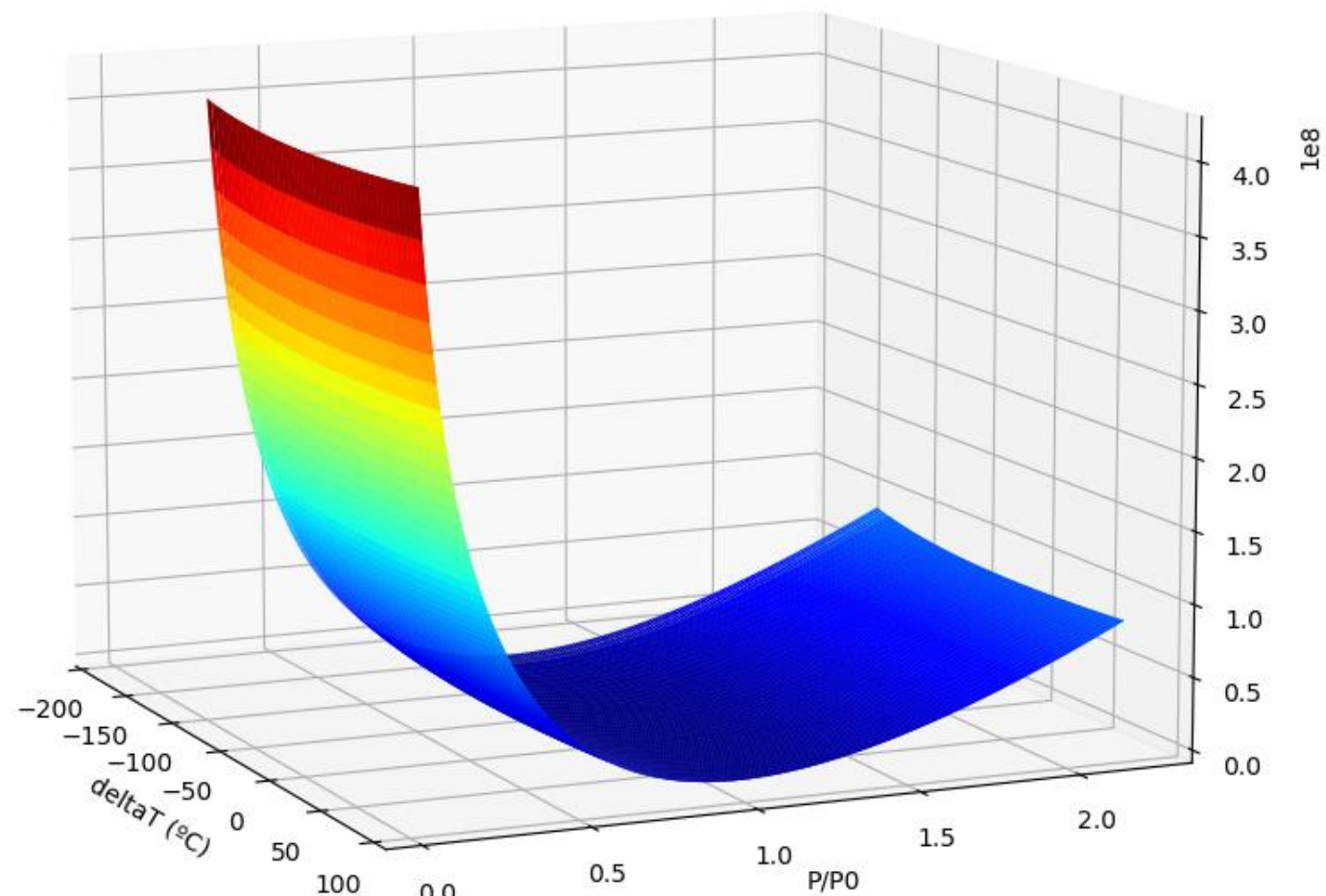
- 5 equations: 1group PKE + Fuel HT + Coolant HT + Flow momentum
 - First Method of Lyapunov (eigenvalues of Jacobian)
- = All eigenvalues are real-negative
- ∴ **locally asymptotically stable!**

HFP	HZP (~ 1%)
$\alpha\omega = 0 \text{ pcm/}^\circ\text{C}$	$\alpha\omega = +5 \text{ pcm/}^\circ\text{C}$
-64.7550	-65.0734
-0.0422	-1.0743
-0.5492	-0.00012
-1.4711	-0.1521
-1.5	-1.5
$\alpha\omega = -60 \text{ pcm/}^\circ\text{C}$	$\alpha\omega = -15 \text{ pcm/}^\circ\text{C}$
(-64.7724+0j)	-65.0735
(-0.9909+0.9351j)	-1.0677
(-0.9909-0.9351j)	-0.0031
(-0.0634+0j)	-0.1557
(-1.5+0j)	-1.5

© All eigenvalues are real-negative

ODE2

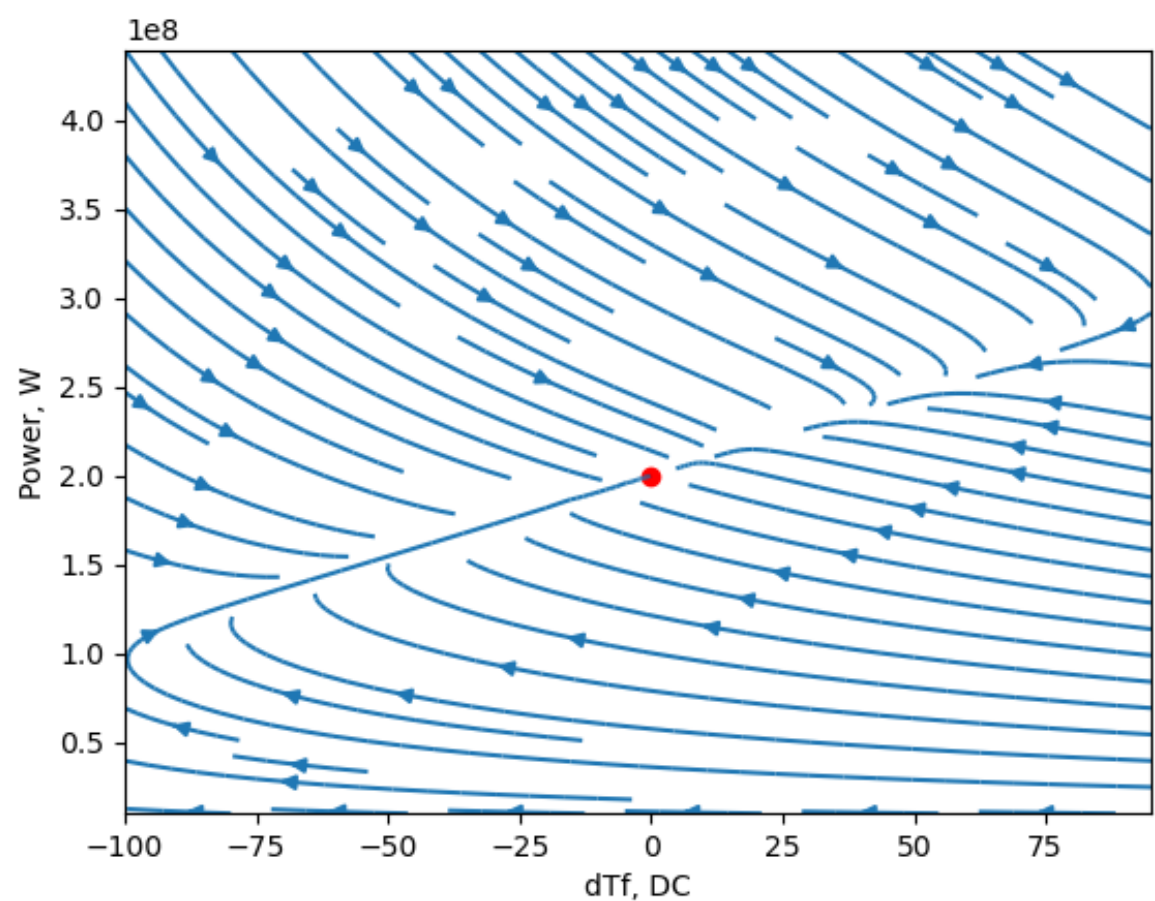
- 2 equations: Prompt-jump point kinetics + Newton Law of Cooling
 - Second Method of Lyapunov (surrogate of potential energy)
- = V function is found so that V > 0, and dV/dt < 0 along solutions of the system
- ∴ **Regional asymptotic stability!**



© A Lyapunov V function

Conclusions:

- **Lyapunov's Theorems** enable the characterization of stability zones and behaviors around nodes, complementing current methods in the frequency and time domains.
- **Result and extension of the Method:** This work concludes N-TH stability of these reactors and is consistent with previous studies. The method can be extended to other operational states and to other SMRs such as molten salt reactors or lead-cooled fast reactors, by using the appropriate equations.



© A stability valley and equilibrium saddle