



# Nuclear reaction mechanisms for incident nucleons and light composite particles

B. V. Carlson

*Instituto Tecnológico de Aeronáutica, São José dos Campos SP, Brazil.*

with lots of help from

R. Capote, E. V. Chimanski, M. Sin, and F. Torabi

*NAPC-Nuclear Data Section, International Atomic Energy Agency, 1400 Vienna, Austria*

*National Nuclear Data Center, Brookhaven National Laboratory, Brookhaven NY, USA*

*Faculty of Physics, University of Bucharest, P.O. Box MG-11, 70709 Bucharest-Magurele,  
Romania*

## Nucleon-induced reactions

At low incident energy, a nucleon can be absorbed directly by a target to form a compound nucleus.

As the incident energy increases, several nucleon-nucleon collisions may become necessary for the incident nucleon and its collision partners to be bound.

At even higher energies, the incident nucleon or one or several of its collision partners may be emitted before all are bound – a pre-equilibrium emission.

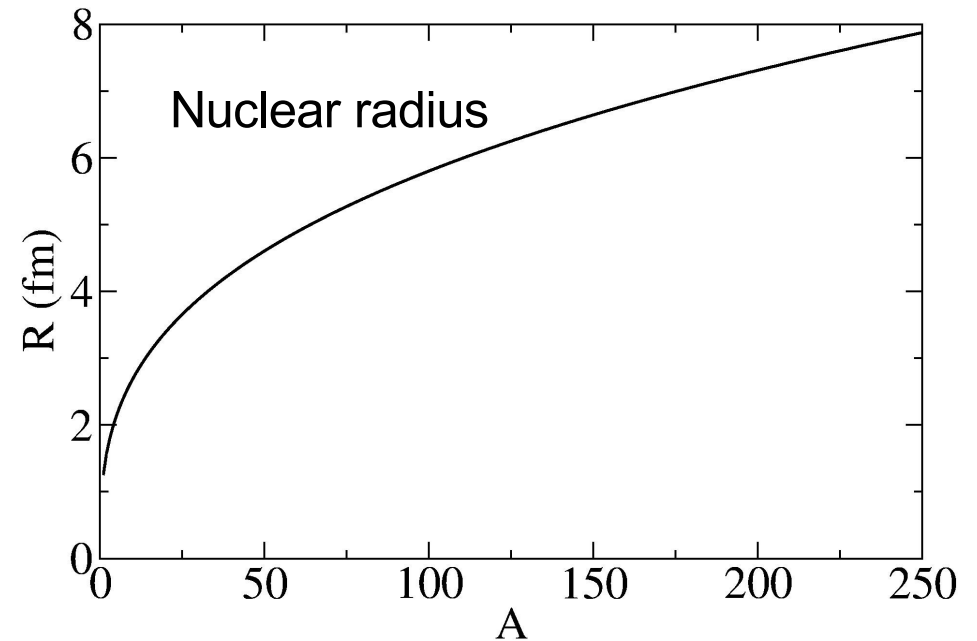
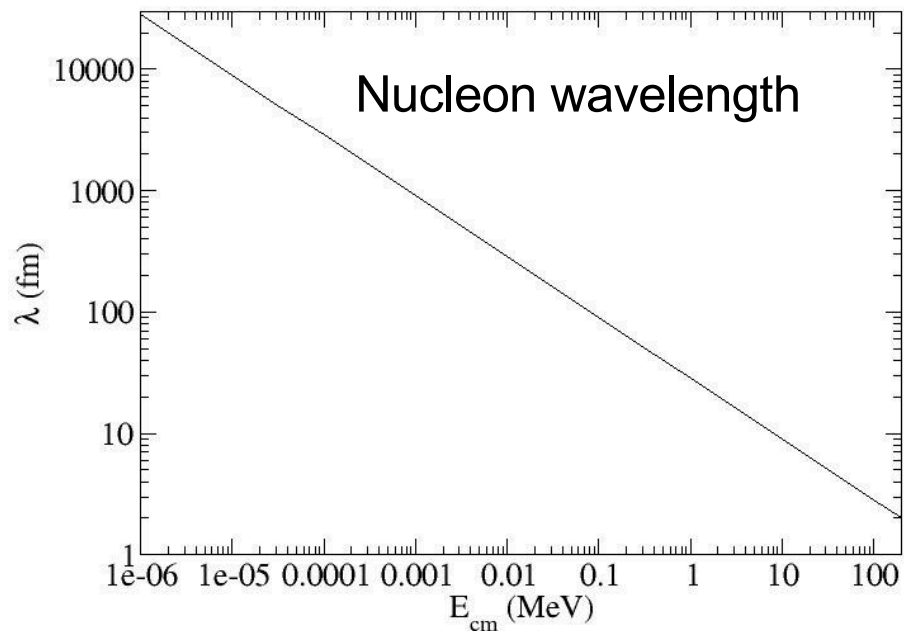
We want to discuss here the characteristics of this process to determine what to require of a model that could describe it faithfully.

## Quantum vs classical

The wave-like nature of a scattering particle can be neglected if its wavelength is much smaller than the length scale on which the scattering system varies.

For nuclear scattering, the appropriate length scale would be at most the size of the nucleus and should probably be of the size of the nuclear surface – about 0.5 to 1.0 fm.

Thus the wavelike nature of the nucleon should be taken into account.



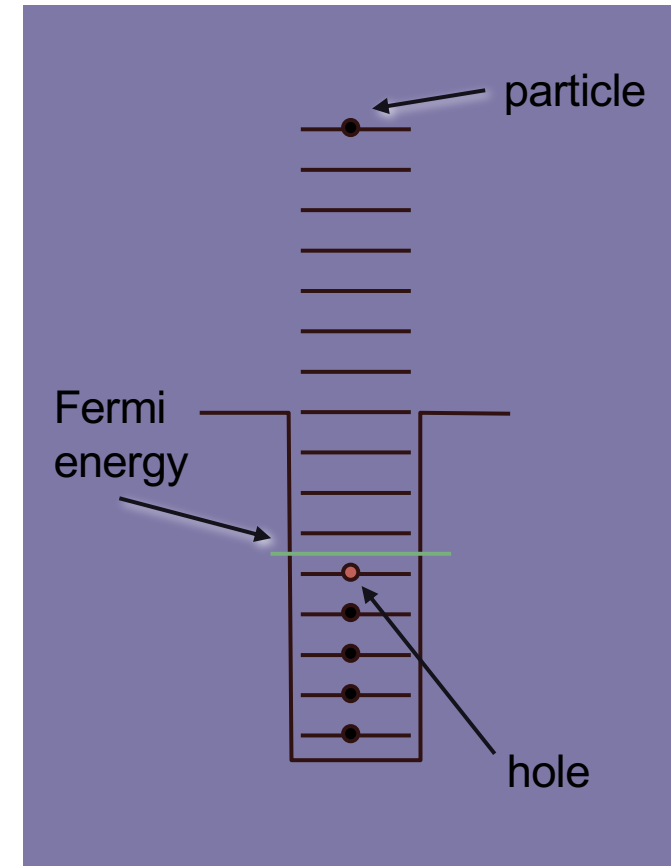
# The particle-hole model of nuclear structure

An important part of any nuclear reaction is the nuclear structure that underlies it.

The simplest description of the structure important here is given in terms of particle-hole states

- Nucleons occupy single-particle states;
- The Fermi energy is between the last occupied and first unoccupied level of the target ground state;
- Particles are nucleons above the Fermi energy and holes are unoccupied states below it. Both are called excitons.
- We model a pre-equilibrium reaction as emission from a process going through a sequence of states,

$1p \rightarrow 2p-1h \rightarrow 3p-2h \rightarrow \dots$



J.J. Griffin, Phys. Lett. 17 (1966) 478.

# Occupation of particle-hole configurations

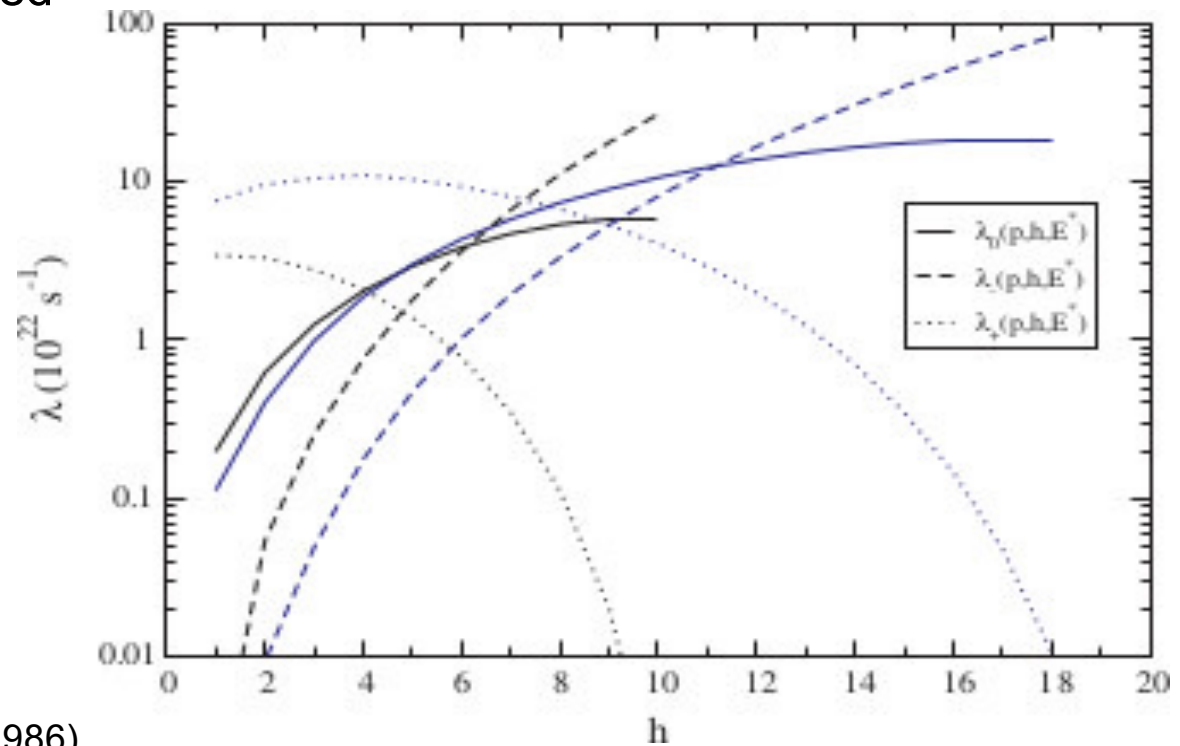
J. Bisplinghoff:

- Only the 2p-1h exciton states are populated uniformly.
- Equilibrium cannot be assumed for higher configurations.

The rate of transitions between states of the same configuration,  $\lambda_0$ , is much smaller than the rate at which states of higher exciton number are created,  $\lambda_+$ .

There is no time for equilibration to occur.

System of 40 nucleons at 25 and 100 MeV. The  $(n+1)$  particle,  $n$  hole configurations are labelled by the number of holes.



## Bound and unbound nucleons

An energy-momentum conserving collision of a particle in a Fermi gas takes

$$1p \rightarrow 2p - 1h.$$

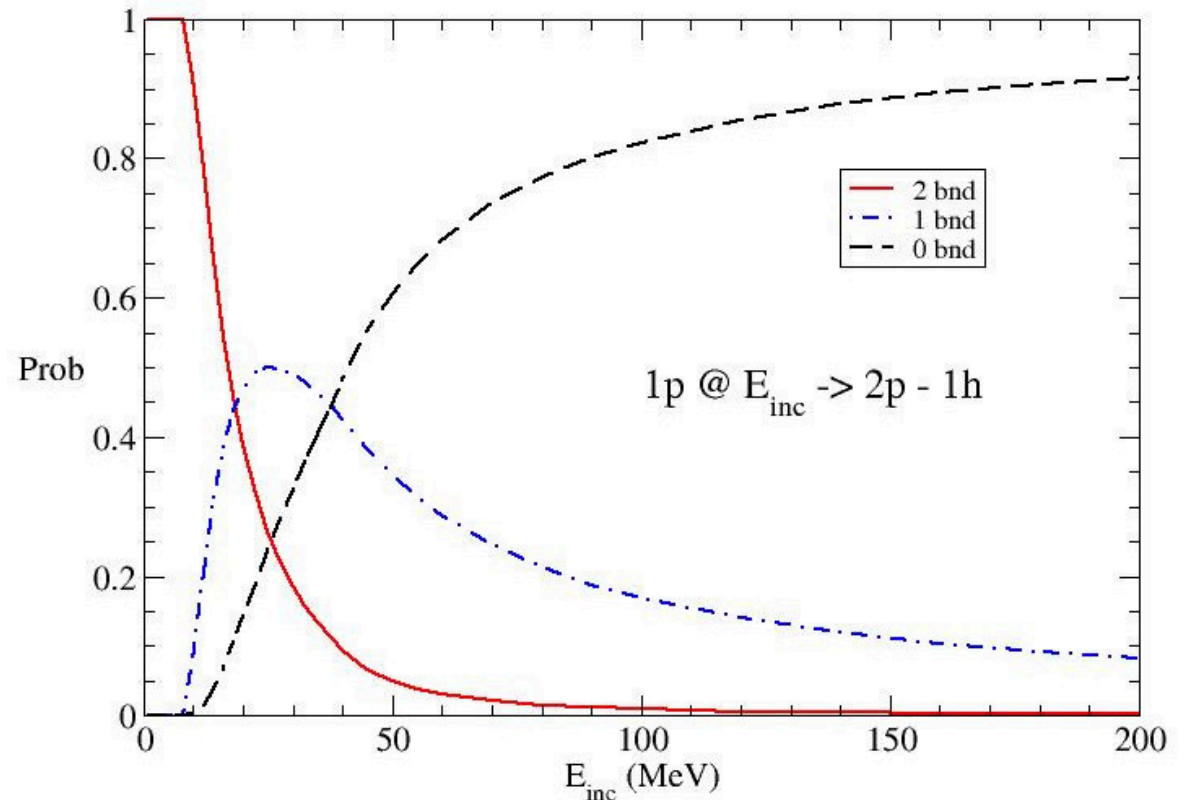
With a Fermi energy of  $E = 35$  MeV, consider a particle with

$$E > 8 \text{ MeV}$$

above the Fermi energy as unbound.

Final states with one bound and one unbound nucleon peak at about 25 MeV.

Final states with two unbound nucleons dominate above 40 MeV.



K. Kikuchi and M. Kawai, Nuclear matter and nuclear reactions (Amsterdam: North-Holland, 1968)

# Nucleon knockout

Knockout = emission of both colliding nucleons in first collision.

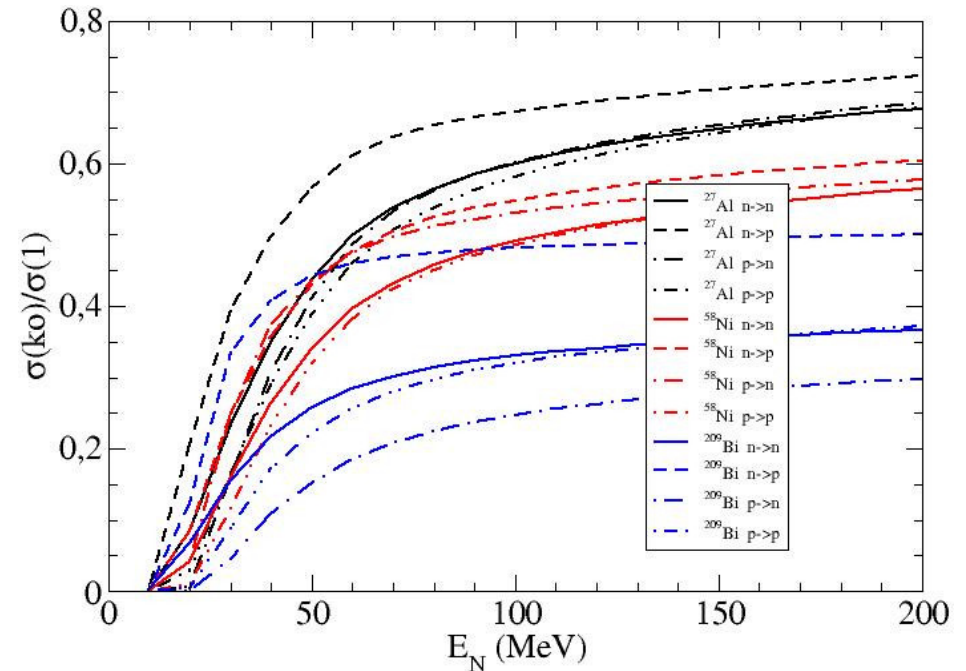
The ratio of knockout to all first collisions increases with energy as does the probability of two unbound nucleons.

With increasing mass, the probability of a second collision before emission grows, reducing the knockout probability.

Knockout can be roughly associated with peripheral scattering in the outermost layer (in impact parameter) of

$$\Delta R \approx 1.5 \text{ fm}$$

of the target density.



At high incident energy, the ratio tends to saturate at about

$$^{27}\text{Al} : \quad \sigma(\text{ko})/\sigma(1) \approx 0.70$$

$$^{58}\text{Ni} : \quad \sigma(\text{ko})/\sigma(1) \approx 0.50$$

$$^{209}\text{Bi} : \quad \sigma(\text{ko})/\sigma(1) \approx 0.35$$

# Number of primary collisions

Primary collisions are those that involve at least one unbound nucleon.

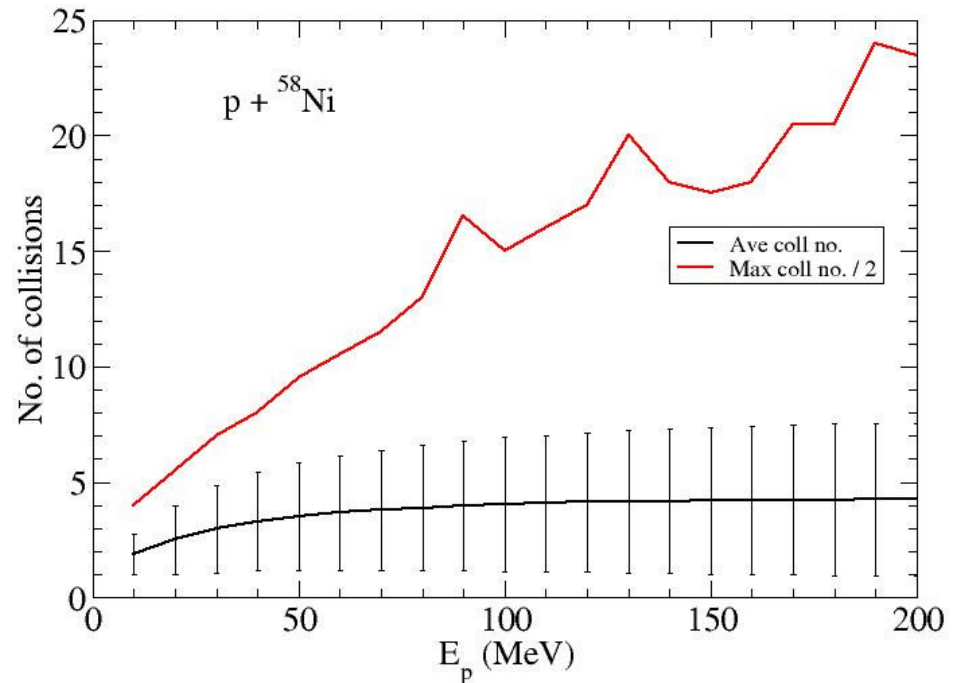
The primary / pre-equilibrium multi-step direct process ends when all remaining nucleons are bound.

Although the mean value saturates at a value close to 4, the mean value minus variance is fixed at 1, the knockout value.

The maximum number of primary collisions increases roughly as

$$3 \cdot (E_p / (1 \text{ MeV}))^{1/2}$$

Calculated using the HMS-0 model



M. Blann, Phys. Rev. C 54, 1341 (1996).



## A physically motivated model

A model consistent with the physical processes involved would:

- Be quantal;
- Include both bound and continuum particle states;
- Treat  $(n+1)$  particle,  $n$  hole states individually;
- Permit an indefinite number of nucleon-nucleon collisions;
- Permit multiple emissions.

The only quantum mechanical models proposed to date are the multistep direct/compound ones. Although successful in describing the high-energy region of pre-equilibrium emission, they are still limited to a small number of collisions and a single nucleon in the continuum.

H. Feshbach, A. Kerman, S. Koonin, *Ann. Phys. (N.Y.)* 125 (1980) 429.

T. Tamura, T. Udagawa, H. Lenske, *Phys. Rev. C* 26 (1982) 379.

H. Nishioka, H. A. Weidenmüller, S. Yoshida, *Ann. Phys. (N.Y.)* 183 (1988) 166.

M. Dupuis, T. Kawano, J. P. Delaroche, E. Bauge, *Phys. Rev. C* 83 (2011) 014602; M. Dupuis, E. Bauge, S. Hilaire, S. F. Lechaftois, S. Péru, N. Pillet, C. Robin, *Eur. Phys. J. A* 51 (2015) 168.

## Multistep / multiple pre-equilibrium emission models

**Exciton model** –  $(n+1)$  particle –  $n$  hole transition rates; CN-like emission rates;

Code – TALYS;

A. J. Koning, S. Hilaire, and S. Goriely, Eur. Phys. J. A 59, 131 (2023); A. J. Koning and M. Duijvestijn, Nucl. Phys. A 744, 14 (2004).

**HMS-0** – 2 particle – 1 hole transition rates; hybrid model emission rate;

CODE – EMPIRE3/DDHMS;

M. Herman et al., NDS 108, 2655 (2007); M. Blann and M. Chadwick, Phys. Rev. C 57, 233 (1998).

**HMS-2** – Fermi gas transition rates; hybrid model emission rate;

Code – EMPIRE3/DDHMS;

M. Herman et al., NDS 108, 2655 (2007); B. V. Carlson and D. F. Mega, EPJ WOC 21, 09001 (2012).

**Intra-Nuclear Cascade** – Fermi gas transition rates; no holes; trajectory propagation;

Code – INCL++/ABLA07;

D. Mancusi, A. Boudard, J. Cugnon, J.-C. David, P. Kaitaniemi and S. Leray, Phys. Rev. C90, 054602 (2014).

# Differential cross sections vs experiment

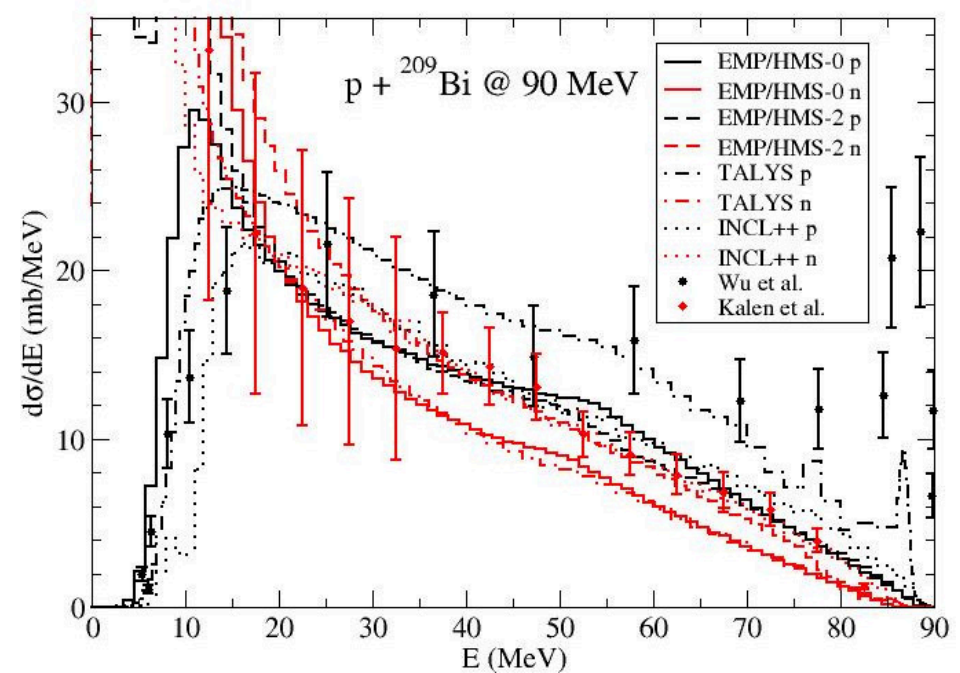
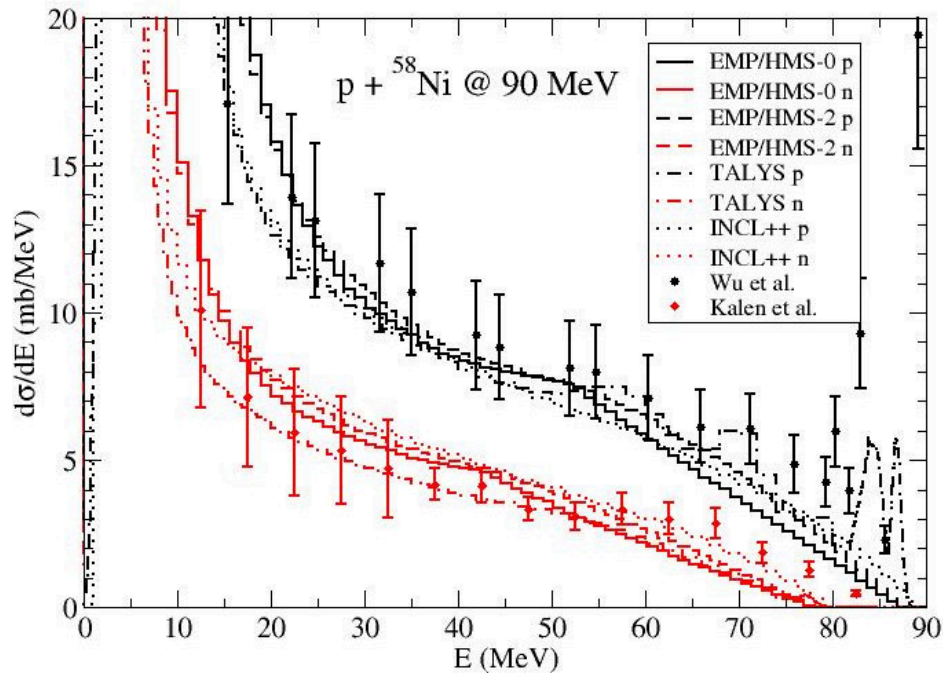
All of the models describe the  $^{58}\text{Ni}$  spectra well.

The agreement is not as good for  $^{209}\text{Bi}$ .

Data:

**(p,p')** - J. R. Wu, C. C. Chang, H. D. Holmgren, Phys. Rev. C19 (1973) 698.

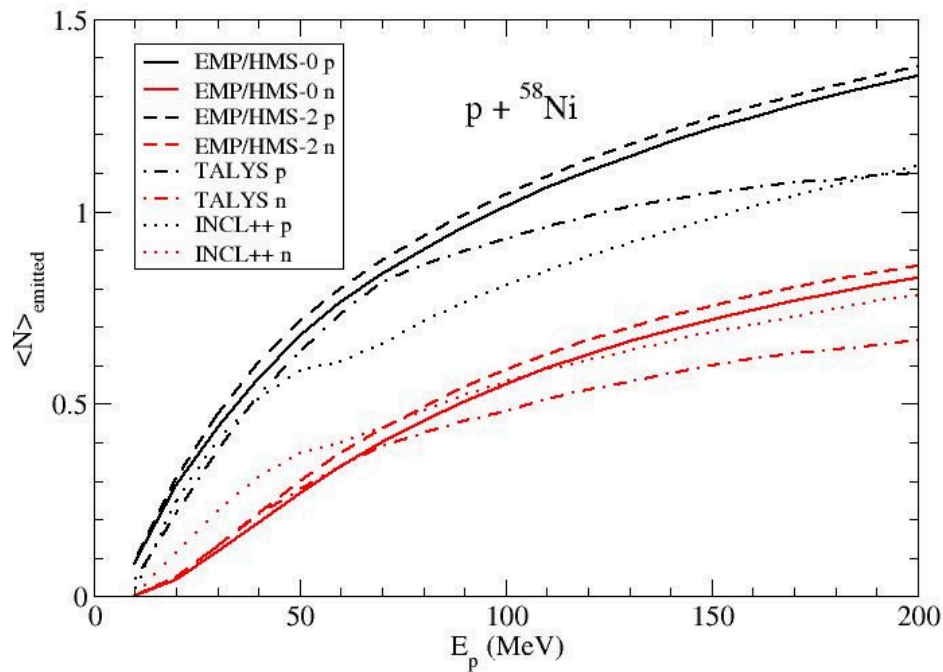
**(p,n)** - A. M. Kalend et al., Phys. Rev. C28 (1983) 105.



# Primary nucleon emission

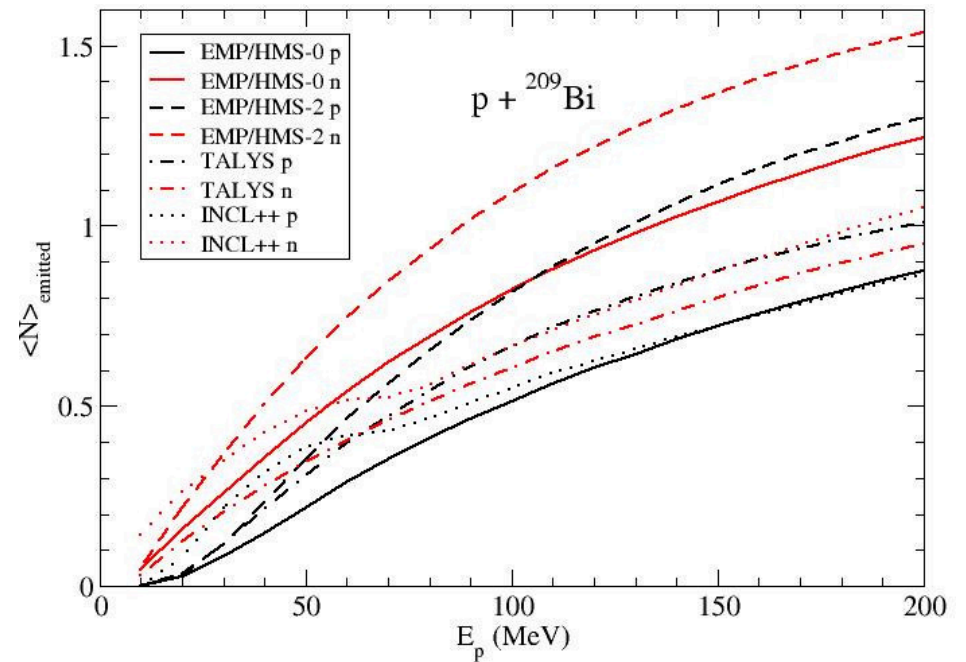
HMS calculations only emit nucleons.

About 10% of particle emission in TALYS and INCL++ is composite emission.



The neutron excess in  ${}^{209}\text{Bi}$  is reflected in the higher neutron yields.

Variances in emission number are on the order of  $\langle N/2 \rangle^{1/2}$ .

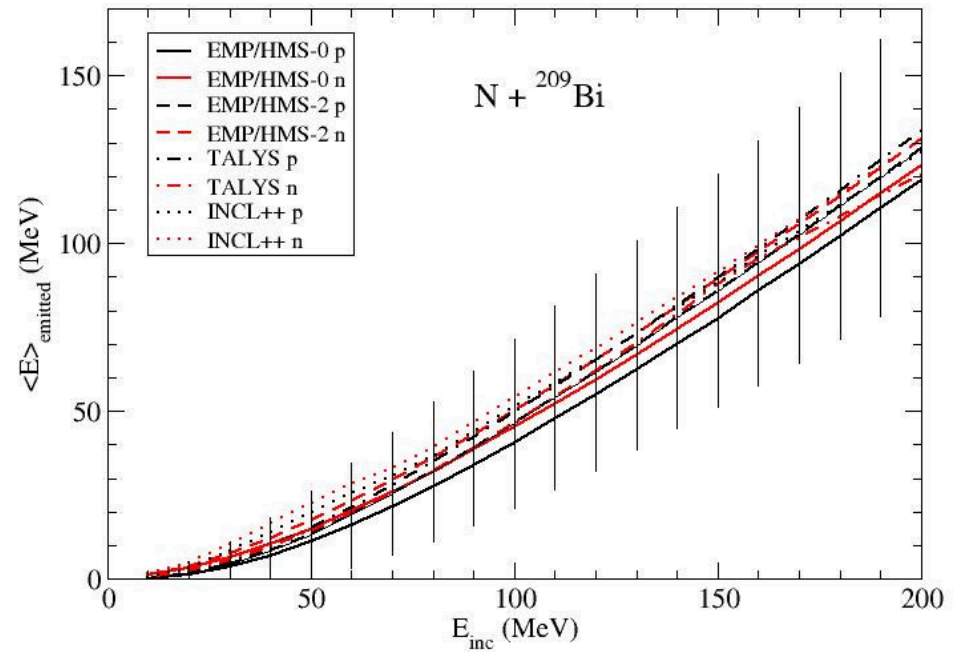
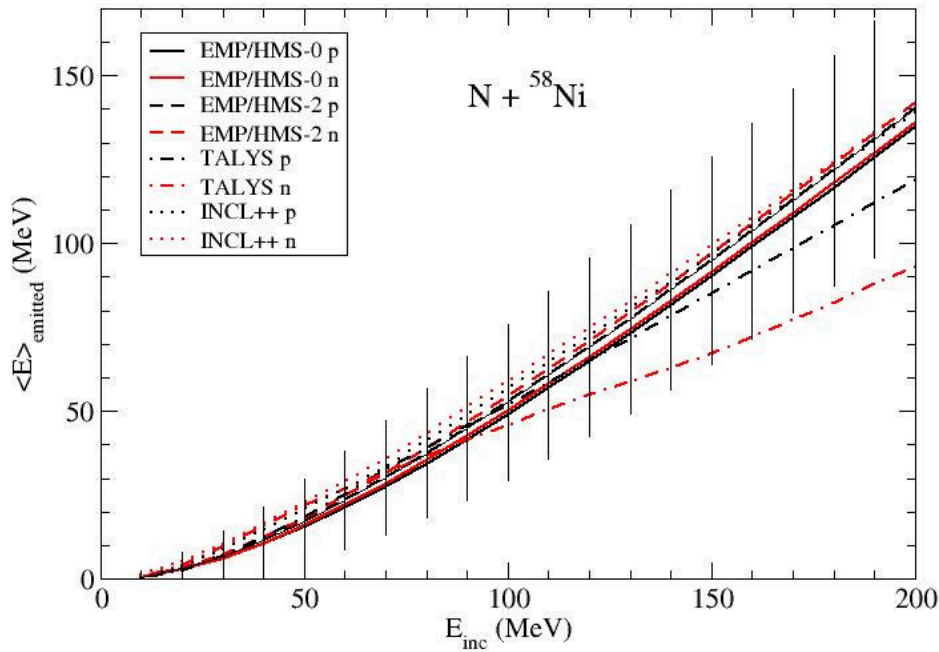


# Primary energy emission

- At 100 MeV, about 1/2 of the energy is carried away by primaries.
- At 200 MeV, about 2/3 of the energy is emitted.

Black – proton-induced  
Red – neutron induced

The variance is about  
 $2*(E*(1 \text{ MeV}))^{1/2}$ .



## Average values and fluctuations

- In spite of their differences, all of the models can be calibrated to provide reasonable emission spectra.
- The isospin dependence of the interaction seems better described by TALYS.
- All models agree that few nucleons are emitted on average in the primary cascade, although there are slight differences in the numbers.
- All models agree surprisingly well that a large fraction of the incident energy is carried away by the particles emitted in the primary cascade.
- The fluctuations in particle and energy emission are large – about half the average number for particle emission and about one third of the average energy emitted.
- Similar fluctuations also appear in the average number of primary collisions, which saturate at about  $4 \pm 3$  above 100 MeV incident energy for  $p + {}^{58}\text{Ni}$ .
- The fluctuations in number of emissions and number of primary collisions are essential to a physical multistep emission model.

# Pre-equilibrium emission of composite nuclei – d, t, h, and $\alpha$

About 10 to 15% of the particles emitted in a pre-equilibrium reaction are composites.

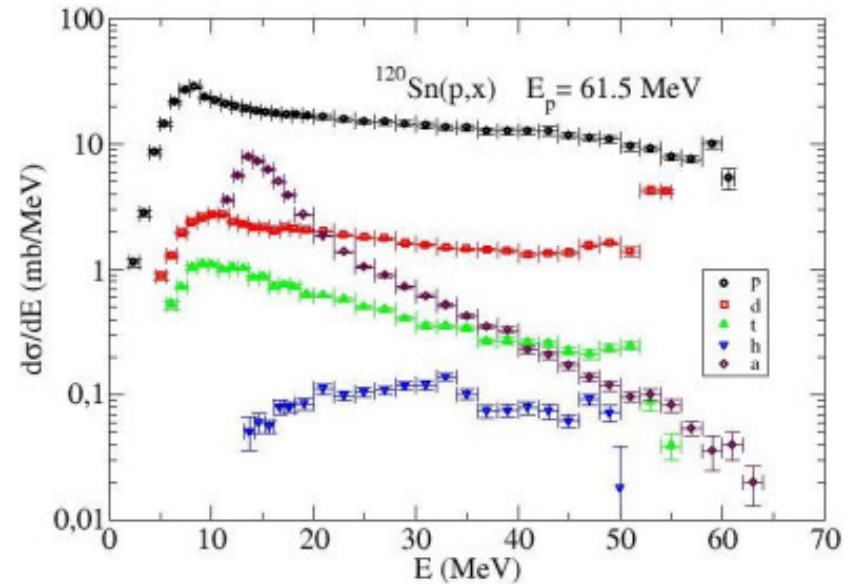
A 'physical' model would include these in the MSD chain through a matrix element such as

$$\langle \psi_d A-1 | V | \psi_p A \rangle,$$

for a deuteron pickup reaction. Another possibility would be a knockout reaction, with matrix element

$$\langle \psi_d \psi_p A-1 | V | \psi_p A \rangle,$$

These are incorporated in the TALYS and INCL++ codes using simpler approximations.



F. Bertrand and R. Peelle, Phys. Rev. C 8, 1045 (1973).

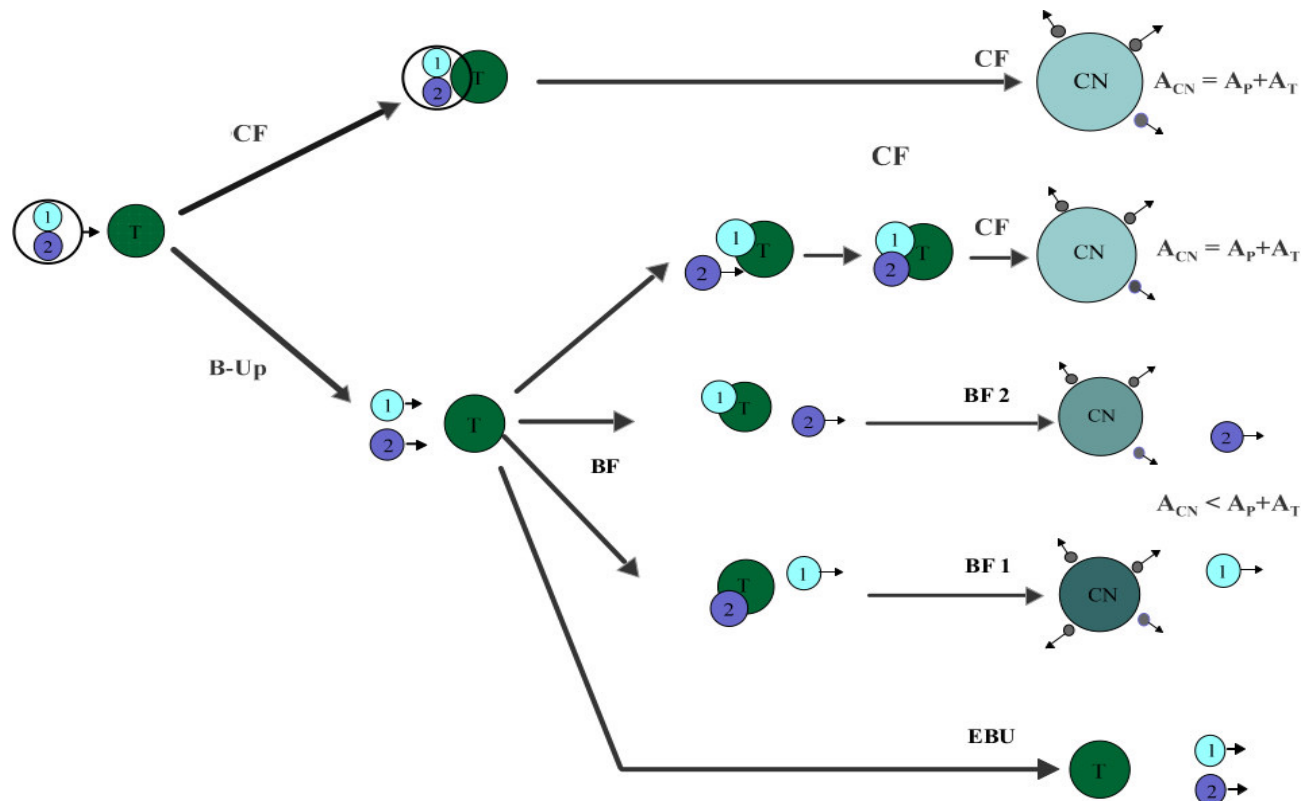
TALYS: C. Kalbach, Phys. Rev. C 71, 034506 (2005).

INCL++: A. Boudard, J. Cugnon, J.-C. David, S. Leray and D. Mancusi, Phys. Rev. C 87, 014606 (2013).

K. Sato, A. Iwamoto and K. Harada K, Phys. Rev. C 28 1527 (1983).

# Breakup and stripping reactions

$$\sigma_{\text{reac}} = \sigma_{\text{bu}} + \sigma_{\text{bf},n} + \sigma_{\text{bf},p} + \sigma_{\text{cf}}$$





## The IAV formalism

The inclusive proton breakup cross section

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{d^2\sigma^{EBU}}{d\Omega_p dE_p} + \frac{d^2\sigma^{NEB}}{d\Omega_p dE_p}$$

is the sum of the elastic breakup cross section (EBU),

$$\frac{d^2\sigma^{EBU}}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho_p(E_p) \int |T(\vec{k}_p, \vec{k}_n; \vec{k}_d)|^2 \delta(E_d + \varepsilon_d - E_p - E_n) d\vec{k}_n$$

defined in terms of the post-form DWBA matrix element,

$$T(\vec{k}_p, \vec{k}_n; \vec{k}_d) = \left\langle \tilde{\chi}_p^{(-)}(\vec{k}_p, \vec{r}_p) \tilde{\chi}_n^{(-)}(\vec{k}_n, \vec{r}_n) |v_{np}(\vec{r})| \chi_d^{(+)}(\vec{k}_d, \vec{R}) \phi_d(\vec{r}) \right\rangle$$

## The IAV formalism

and the nonelastic breakup cross section (NEB or BF,p),

$$\frac{d^2\sigma^{NEB}}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d} \rho_b(E_B) \left\langle \Psi_n(\vec{k}_p, \vec{r}_n; \vec{k}_d) \left| W_n(\vec{r}_n) \right| \Psi_n(\vec{k}_p, \vec{r}_n; \vec{k}_d) \right\rangle$$

which can be interpreted as the generalized absorption cross section of the breakup neutron,

$$\left| \Psi_n(\vec{k}_p, \vec{r}_n; \vec{k}_d) \right\rangle = \left( \tilde{\chi}_p^{(-)}(\vec{k}_p, \vec{r}_p) G_n^{(+)}(\vec{r}_n, \vec{r}'_n) |v_{pn}(\vec{r})| \chi_d^{(+)}(\vec{k}_d, \vec{R}) \phi_d(\vec{r}) \right)$$

The theory is old. Systematic calculations are more recent.

M. Ichimura, N. Austern, and C. M. Vincent, Phys. Rev. C 32, 431 (1985).

N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher, M. Yahiro, Phys. Rep. 154, 125 (1987).

Jin Lei and A. M. Moro, Phys. Rev. C 92, 044616 (2015).

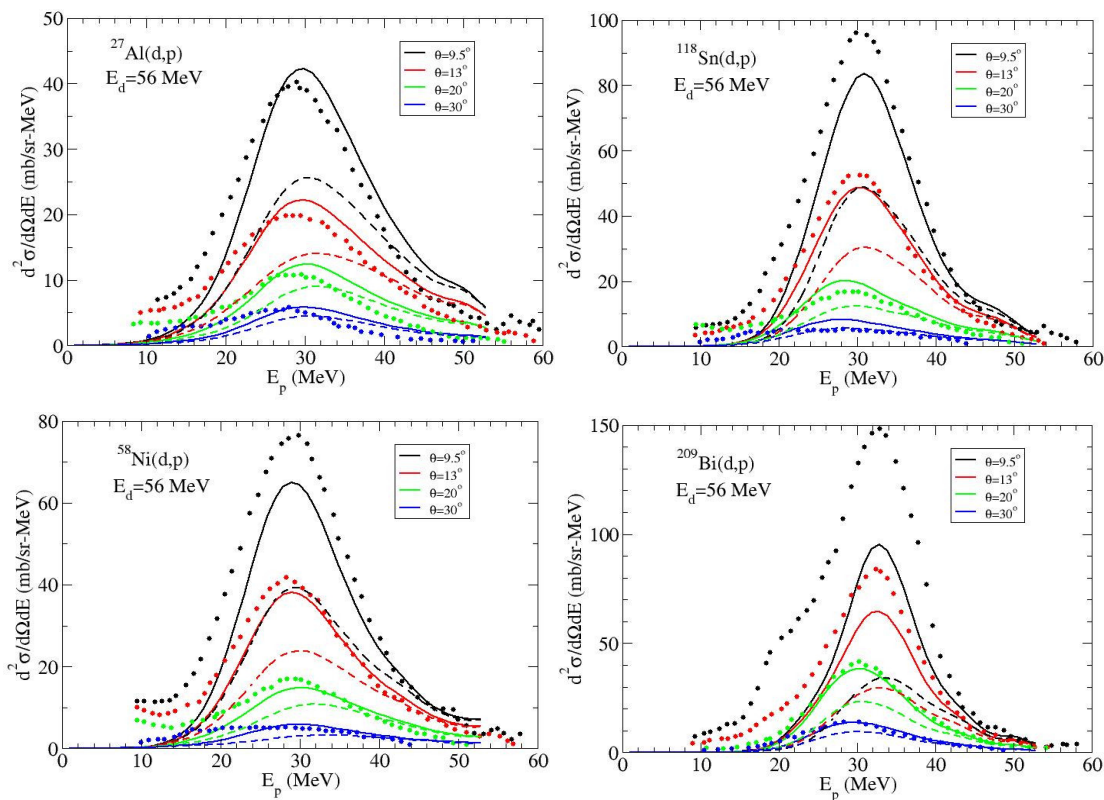
B. V. Carlson, R. Capote, M. Sin, Few-Body Syst. 57, 307 (2016).

G. Potel et al, Eur. Phys. J. A 53, 178 (2017).

F. Torabi and B. V. Carlson, J. Phys. G 50, 045107 (2023).

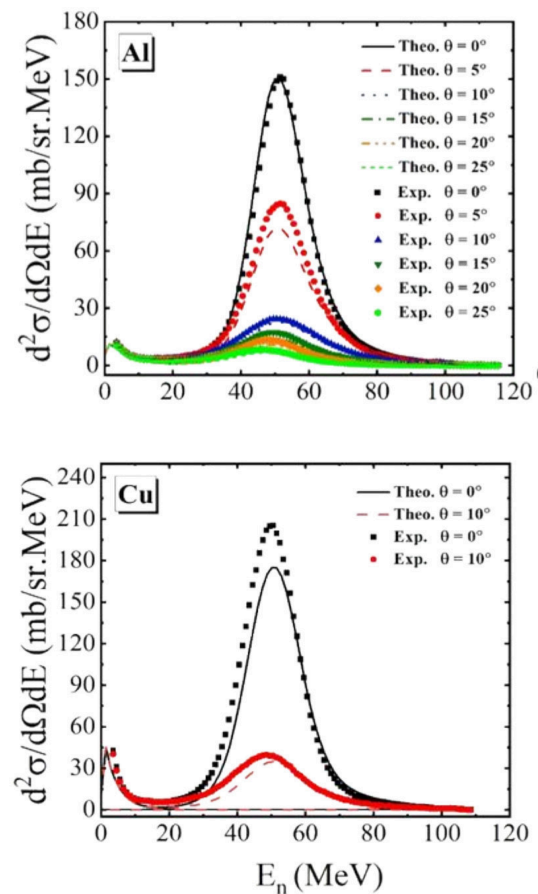
# Inclusive double differential cross sections

56 MeV (d,p)



N. Matsuoka, Nucl. Phys. A 345, 1 (1980).

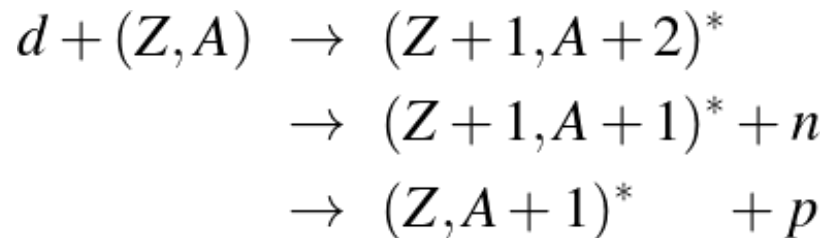
102 MeV (d,n)



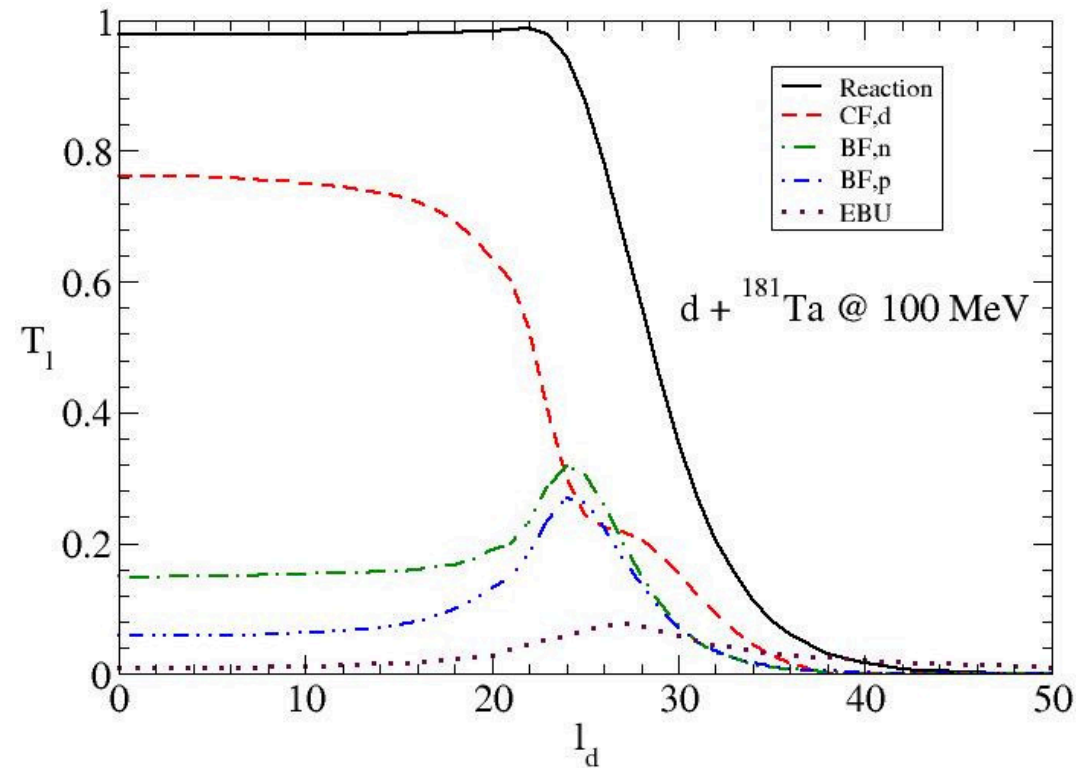
S. Araki, Methods Phys. Res. A, 842, 62, (2017).

## EMPIRE3 reaction code

- Calculates pre-equilibrium and equilibrium emission from the compound nucleus
- Takes into account emission of  $\gamma$ ,  $n$ ,  $p$ ,  $d$ ,  $t$ ,  ${}^3\text{He}$  and  $\alpha$
- The IAV formalism is integrated into the code
- Deuteron-induced reactions form three compound nuclei



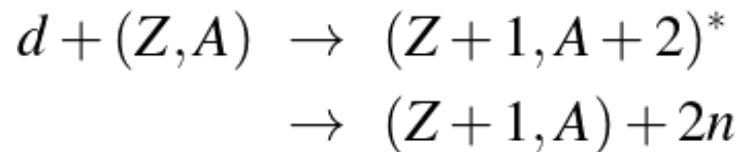
$$\sigma_x = \frac{\pi}{k_d^2} \sum_l (2l + 1) T_{l,x}$$



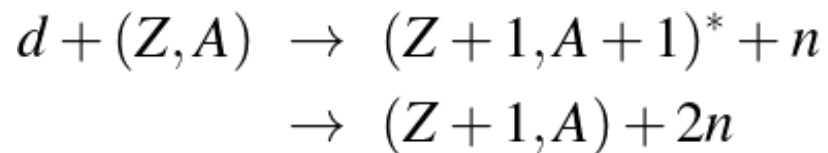
M. Herman et al., Nucl. Data Sheets 108, 2655 (2007).

# The (d,2n) reaction

Complete fusion

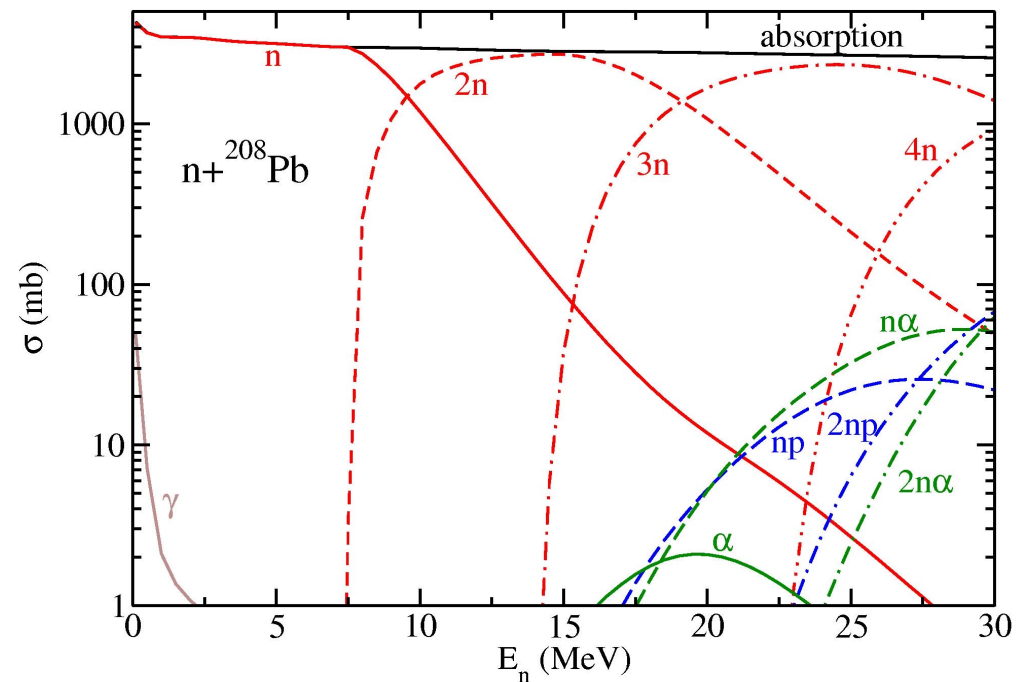


Breakup-fusion

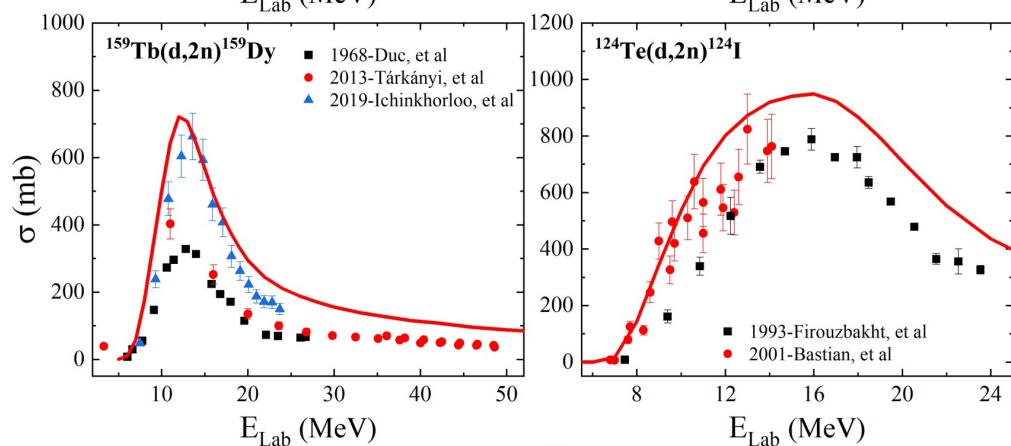
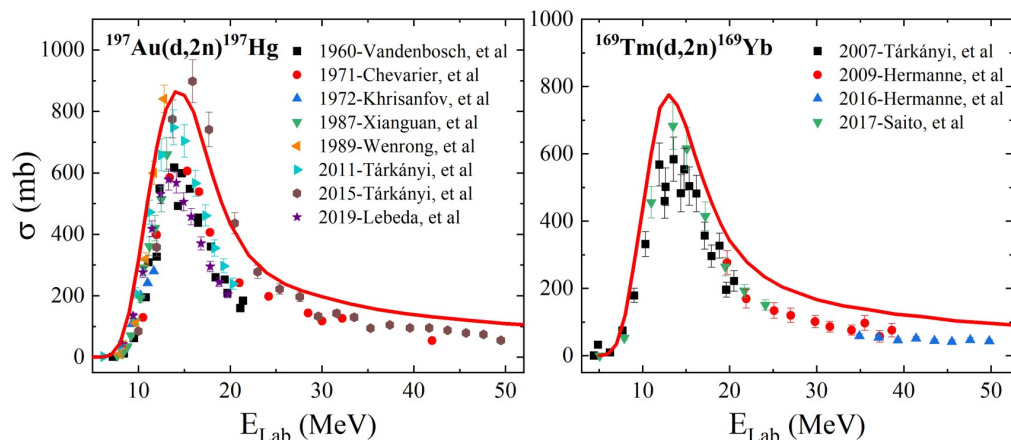


- The NEB cross section can include inelastic breakup as well as breakup fusion
- How important is the inelastic breakup (IBU)?

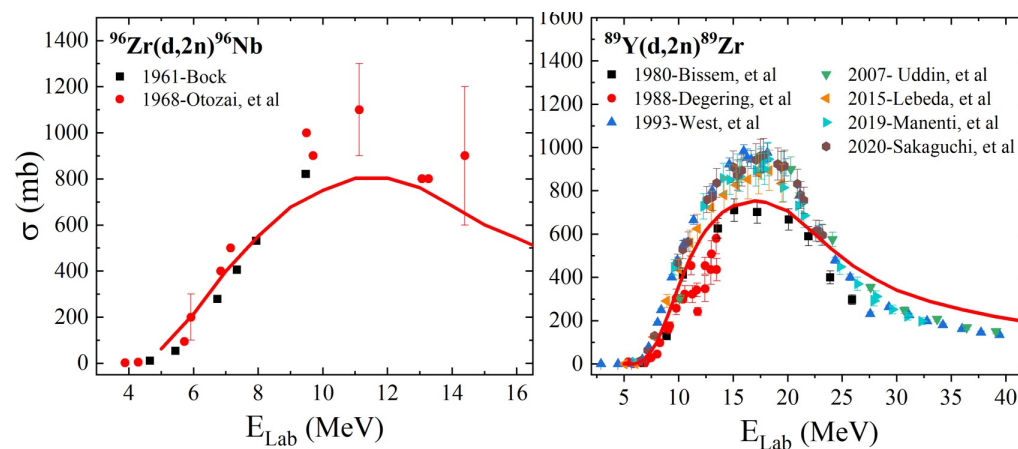
- Look at heavy nuclei, where charged particle emission is suppressed.
- Only neutrons are emitted.



# (d,2n) activation cross sections



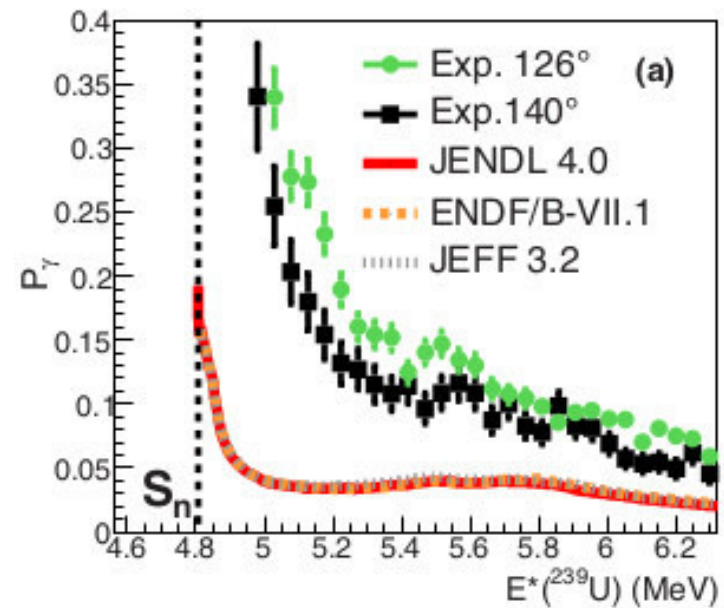
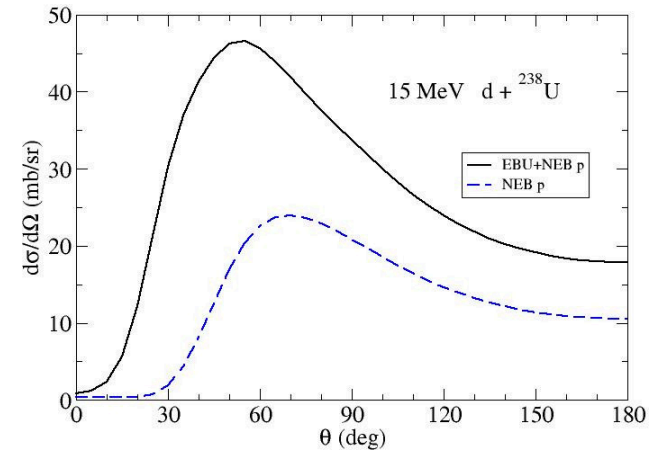
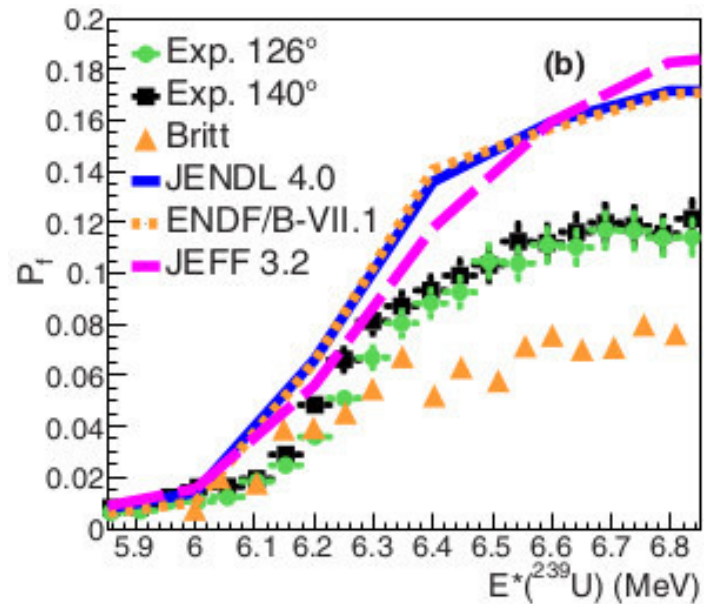
	$\beta_2$	DD-ME2	FRDM	Sk-HFB	Exptl
$^{89}\text{Y}$	0.000	0.035	0.000	-	-
$^{96}\text{Zr}$	0.254	0.217	-0.220	0.080	0.080
$^{124}\text{Te}$	0.217	0.208	0.260	0.183	0.183
$^{159}\text{Tb}$	0.322	0.271	0.350	-	-
$^{168}\text{Tm}$	0.346	0.295	0.360	-	-
$^{197}\text{Au}$	-0.131	-0.130	-0.140	-	-
$^{238}\text{U}$	0.276	0.215	0.270	0.278	0.278



Experimental data taken from the EXFOR library.

# Exclusive (d,pg) and (d,pf)

$$P_x = \frac{N(x \text{ and } p)}{N(p) \varepsilon(x)}$$

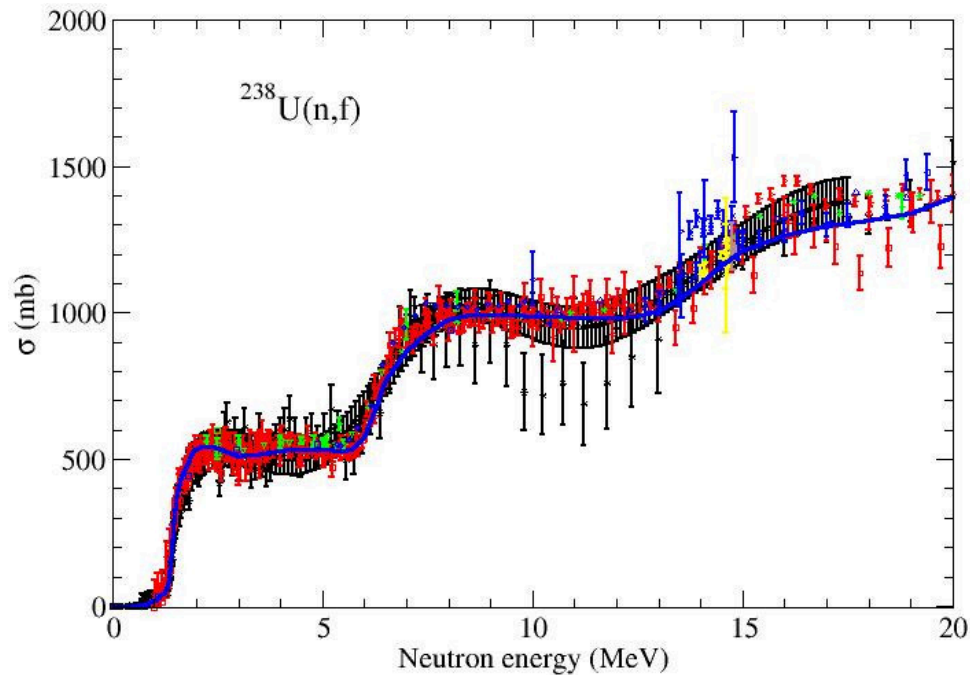


15 MeV - Q. Ducasse et al., Phys. Rev. C 94, 024614 (2016).

18 MeV - H.C. Britt and J.D. Cramer, Phys. Rev. C 2, 1758 (1970).

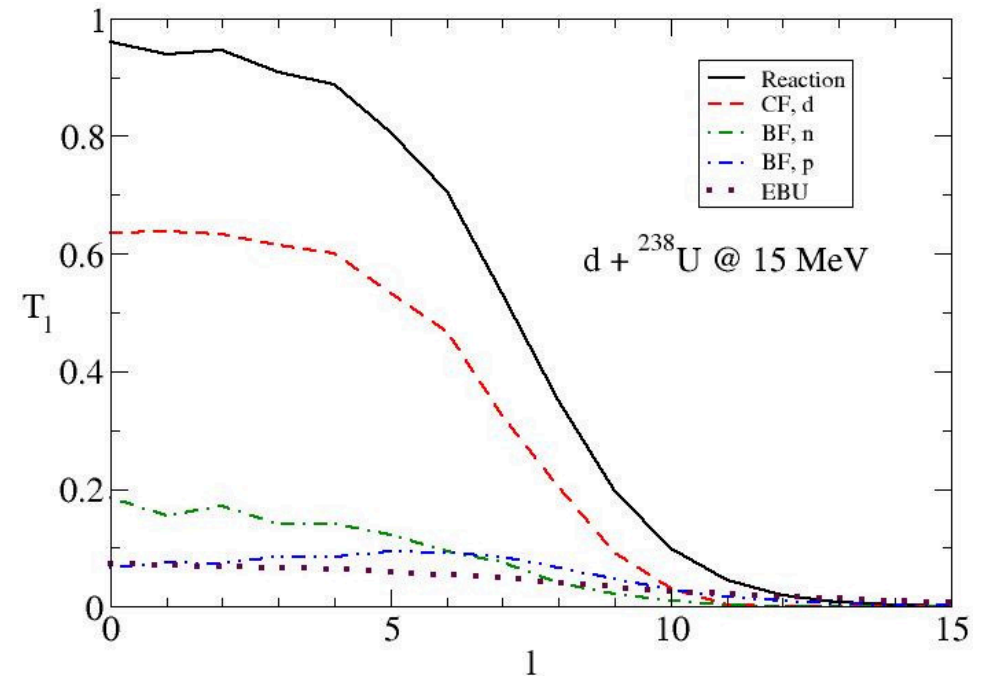
# The (n,f) cross section

The fission barriers and densities of the CIELO evaluation were used to calculate the  $^{238}\text{U}$  fission cross section.



R. Capote et al., NDS 148, 254 (2018).

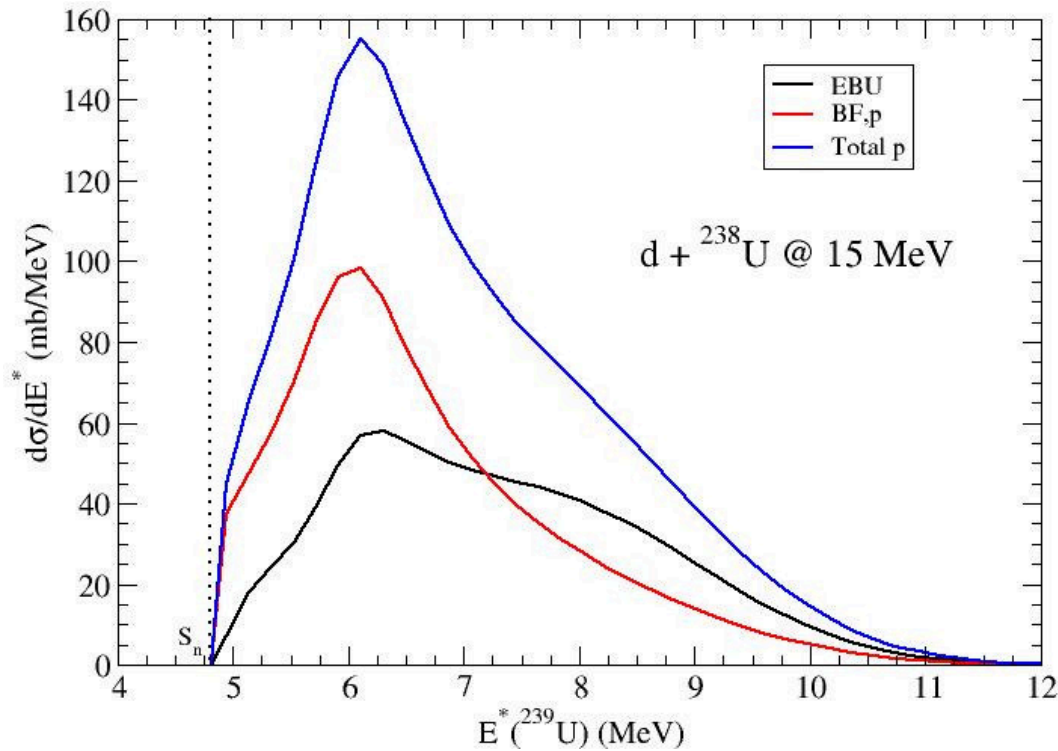
The  $^{238}\text{U}$  CN (BF,p) cross section corresponds to about 10% of the total CN cross section at  $E = 15$  MeV.





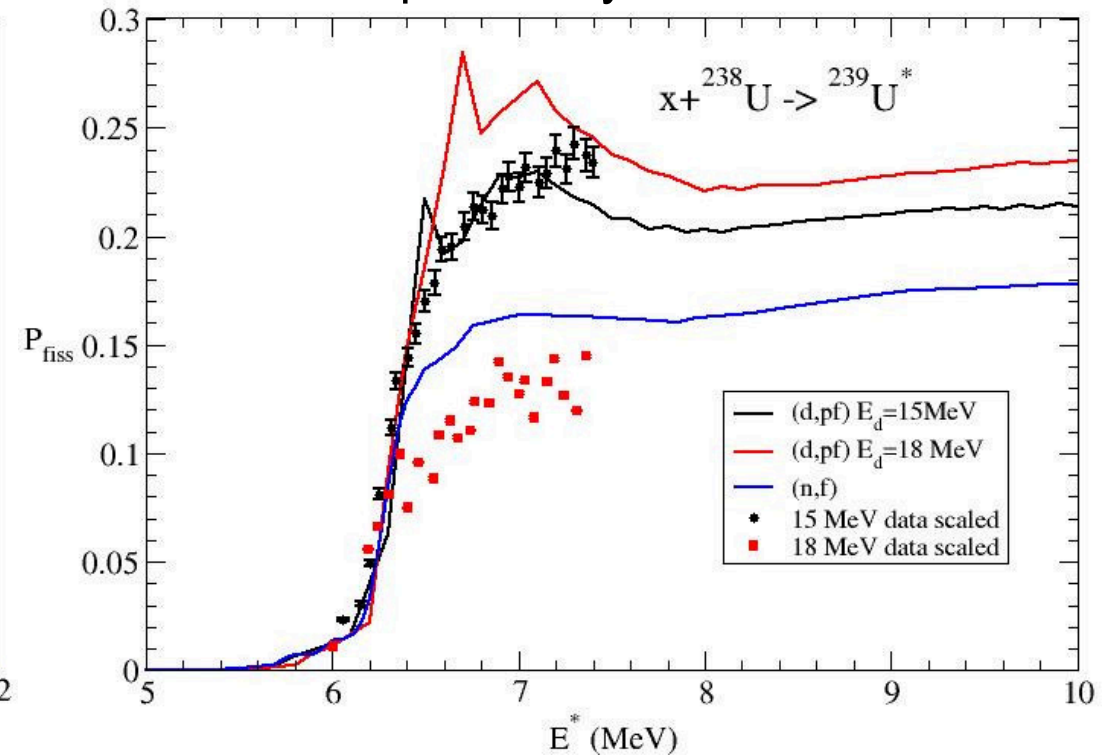
# Fission probability

The experimental (d,pf) fission probability includes all protons – both EBU and BF,p ones in the denominator.

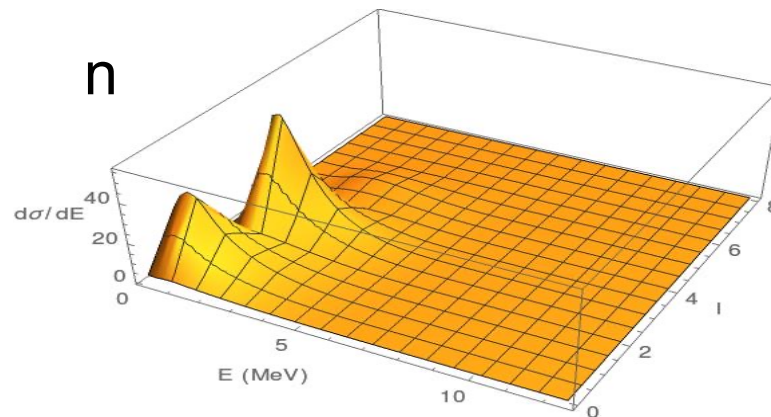
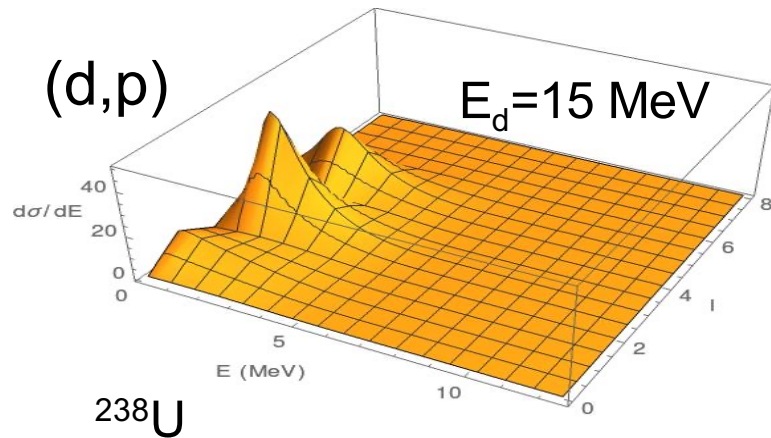


But only BF,p protons contribute to the  ${}^{238}\text{U}$  excitation cross section.

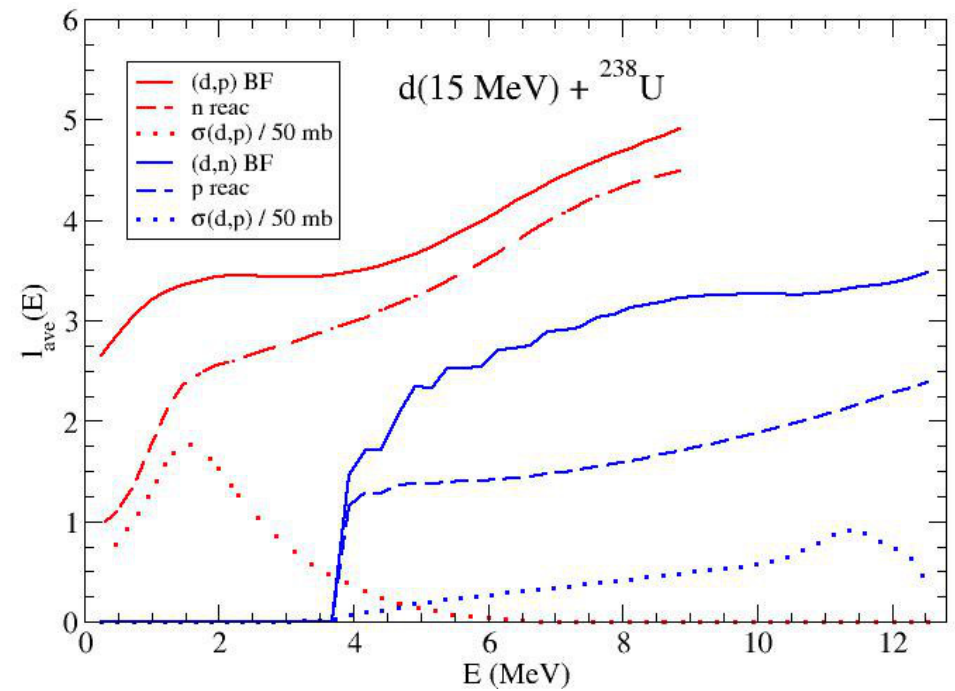
The fission probability must be rescaled.



# Angular momentum transfer

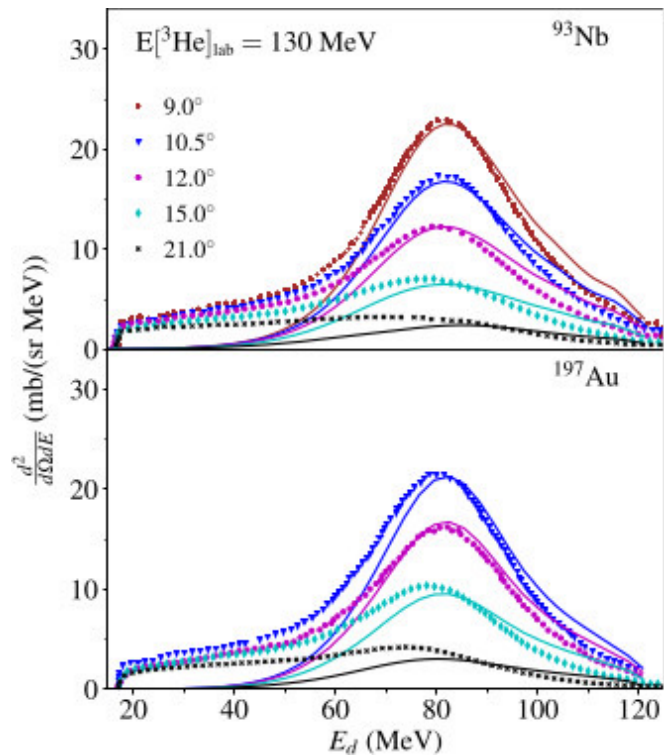


The (d,p) reaction transfers more angular momentum to the compound nucleus than the neutron-induced reaction.

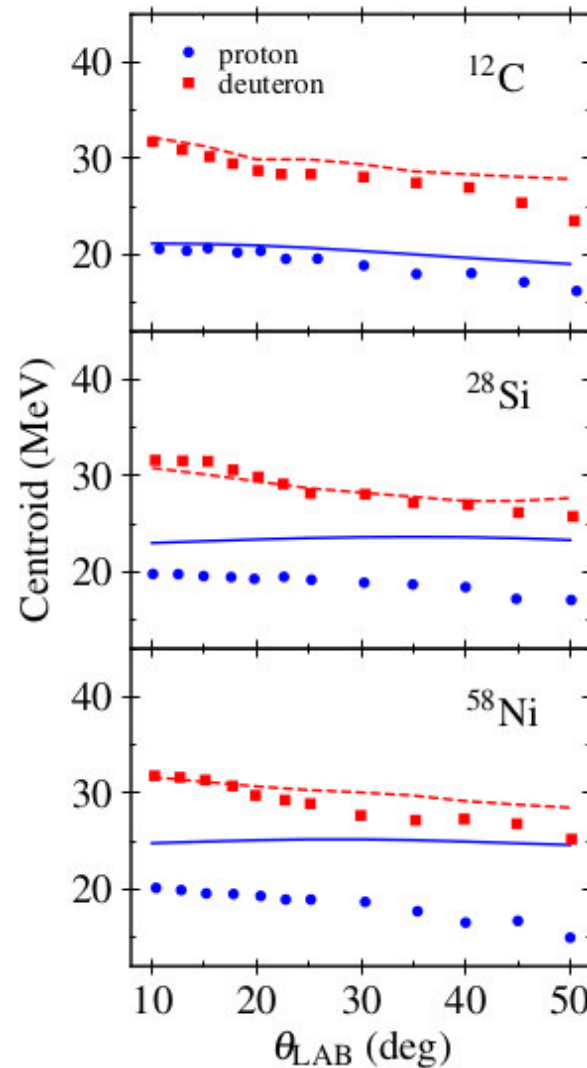


# Breakup of $^3\text{He}$

$(^3\text{He},d)$



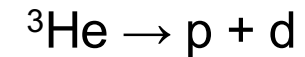
A. Djaloeis et al., Phys. Rev. C 27, 2389 (1983).



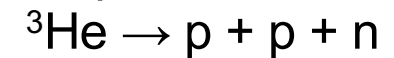
The  $(^3\text{He},d)$  DDX's can be described reasonably well.

The  $(^3\text{He},p)$  DDX's cannot.

As well as the



channel, the three-body breakup channel



must also be taken into account.

E.V. Chimanski, L.A. Souza and B.V. Carlson, Braz. J. Phys. 51, 323 (2021).

## Conclusions

- The multistep / multiple emission pre-equilibrium model codes available today all have their deficiencies but still agree reasonably well with data.
- In all of the models, a large fraction of the initial incident energy is removed by pre-equilibrium particle emission of a relatively small number of particles that have undergone a greatly varying number of collisions.
- More work will be needed to develop a quantum model capable of describing multiple emission and emission of composites.
- The IAV + CN formalism provides a good description of inclusive nucleon production in deuteron-induced reactions.
- The (d,2n) calculations show no consistent effect of inelastic breakup.
- The (d,pf) reaction illustrates the importance of the additional angular momentum transferred to the compound nucleus when compared to the neutron-induced reaction.

# Thank you!

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