R-MATRIX AND THREE-BODY PROBLEMS

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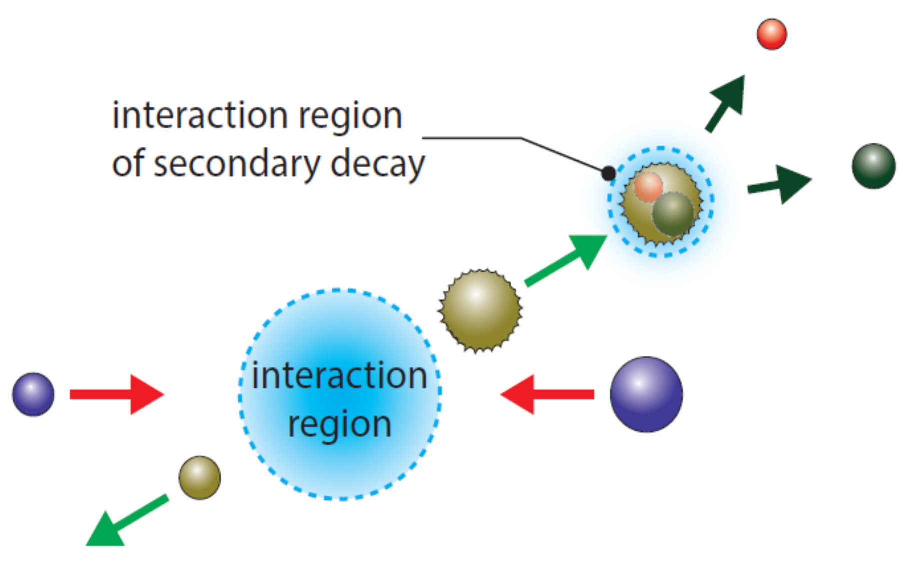
Background and Motivation

R-matrix theory is an elegant tool for the description of cross sections in the resolved resonance regime. It is not microscopically based but satisfies the conservation rules and provides a consistent set of reaction cross sections. The R-matrix description is particularly useful for reactions in light nuclear systems for which microscopic models albeit very involved do not satisfy the required quantitative accuracy. For light nuclear systems R-matrix analyses yield an excellent description for binary channels. However, in light nuclear system also dominant breakup channels may occur which cannot be described by standard R-matrix theory because of its limitation to two-particle channels. Frequently a sequential approach is applied [1], where on assumes breakup to be divided into two two-particle processes. Alternatively combinations with other methods, e.g hypershperical co-ordinates, are applied. In this contribution we report on the status of developments of extensions of the R-matrix concept to three-body channels. Two approaches are considered:

a) An improvement of the R-matrix formalism of Raab [4] for three-body formalism which is based on proposal of Glöckle [2].

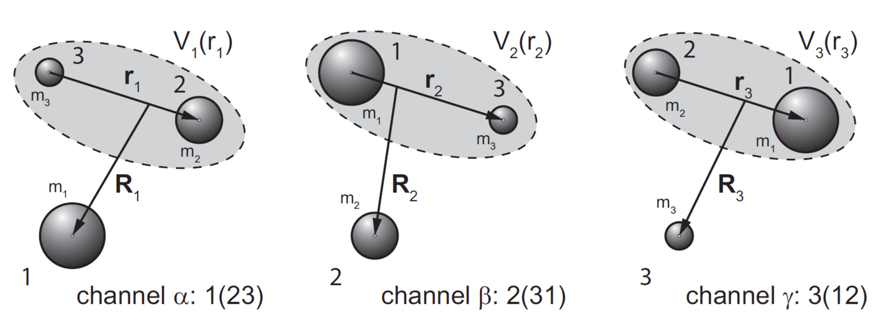
b) First development of the R-Matrix Faddeev formalism suggested by our group.

The sequential approach was first presented in the seminal work of Lane and Thomas [1] to treat the breakup reaction as two successive two-body processes.



R-Matrix Formulation of Glöckle Type

Glöckle [2] proposed an R-Matrix formulation based on the Faddeev equations for three equal masses and s-states which was generalized to arbitrary particle masses and interactions by **Raab** [3]. This provided a full quantum mechanical treatment of three-body processes in the frame of R-matrix theory. It was successfully elaborated and numerically implemented by Raab on the neutron+deuteron and neutron+^{9}Be system [4]. However, the issue of **discontinuity in the first derivative** of the wave functions at matching radii remained. Bloch operators proposed by **Baye** [5] offered a solution to this problem.



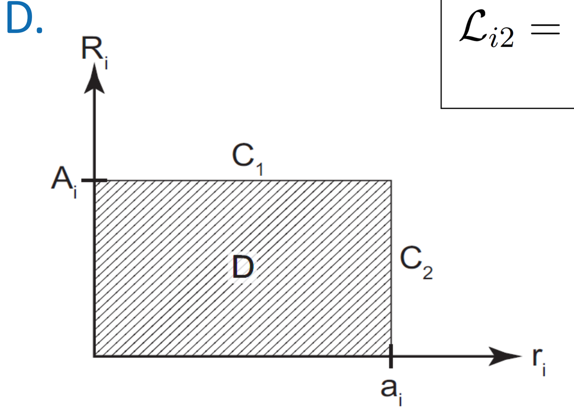
Asymptotic wave functions

$$u_i(r_i,R_i) \simeq u_i^b(r_i)\sin(QR_i) - 2\mu_{i(jn)}u_i^b(r_i)\mathrm{e}^{\mathrm{i}QR_i}T_i^b - \frac{4}{\pi}\mu_{i(jn)}\int_0^{\sqrt{2\mu_{jn}E}} \mathrm{d}k\; u_k^{(-)}(r_i)\mathrm{e}^{\mathrm{i}Q_kR_i}T_i(k) \quad R_i \to \infty \text{ and } r_i \text{ fixed}$$

$$u_i(r_i,R_i) \simeq -\frac{4\mu_{i(jn)}}{\pi}\int_0^{\sqrt{2\mu_{jn}E}} \mathrm{d}k\;\sin(Q_kR_i)\mathrm{e}^{\mathrm{i}kr_i}T_i(k) + O\left(\frac{1}{r_i^2}\right) \quad r_i \to \infty \text{ and } R_i \text{ fixed}$$

Division of jacobi coordinate space into internal (interaction) **and external** (asymptotic) **region** divided at matching radii. **Bloch operators** were introduced by Baye [5], making the matrix Hamiltonian Hermitian and establishing a connection between the partial derivatives of the wave functions on the contour of D.

$$\mathcal{L}_{i1} = \frac{1}{2\mu_{i(jn)}}\,\delta(R_i - A_i)\frac{\partial}{\partial R_i}$$
$$\mathcal{L}_{i2} = \frac{1}{2\mu_{jn}}\,\delta(r_i - a_i)\frac{\partial}{\partial r_i}.$$



Bloch-Faddeev equations

$$(H_i + \mathcal{L}_{i1} + \mathcal{L}_{i2} - E)u_i^{\mathrm{int}}(r_i,R_i) - \sum_{j=1,j\neq i}^{3} \Lambda_i^j u_j^{\mathrm{int}}(r_j,R_j) = (\mathcal{L}_{i1} + \mathcal{L}_{i2})u_i^{\mathrm{ext}}(r_i,R_i)$$

The continuity of the wave functions on the borders of D leads to **relations of the T Amplitudes** appearing in the asymptotic expressions while the R-kernels with matrix A contain the whole information of the interaction region [5].

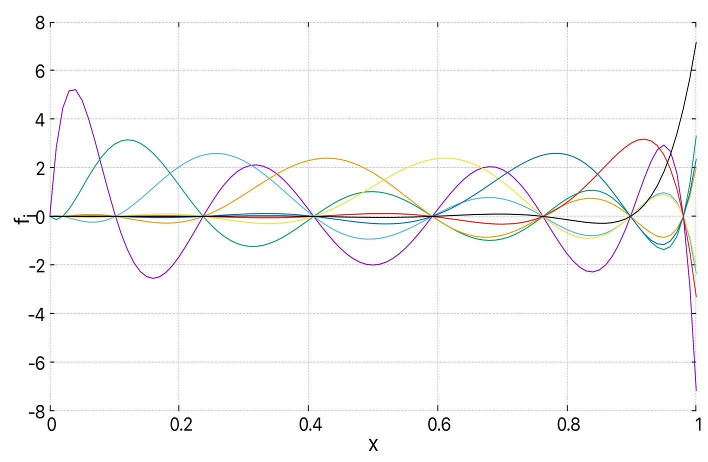
$$u_i^{\mathrm{int}}(r_i,R_i) = \sum_{\mu\mu' i'} (\boldsymbol{A}^{-1})_{\mu\mu'}^{(ii')} \langle \varphi_{\mu'}^{(i')} | \mathcal{L}_{i'1} + \mathcal{L}_{i'2} | u_{i'}^{\mathrm{ext}} \rangle \varphi_{\mu}^{(i')}(r_{i'},R_{i'})$$

$$u_i^{\mathrm{ext}}(r_i,A_i) = \int_0^{a_i} \mathrm{d}r_i'\,\mathcal{R}_i^{(1)}(r_i,r_i')\frac{\partial}{\partial A_i}u_i^{\mathrm{ext}}(r_i',A_i)$$
$$u_i^{\mathrm{ext}}(a_i,R_i) = \int_0^{A_i} \mathrm{d}R_i'\,\mathcal{R}_i^{(2)}(R_i,R_i')\frac{\partial}{\partial a_i}u_i^{\mathrm{ext}}(a_i,R_i')$$

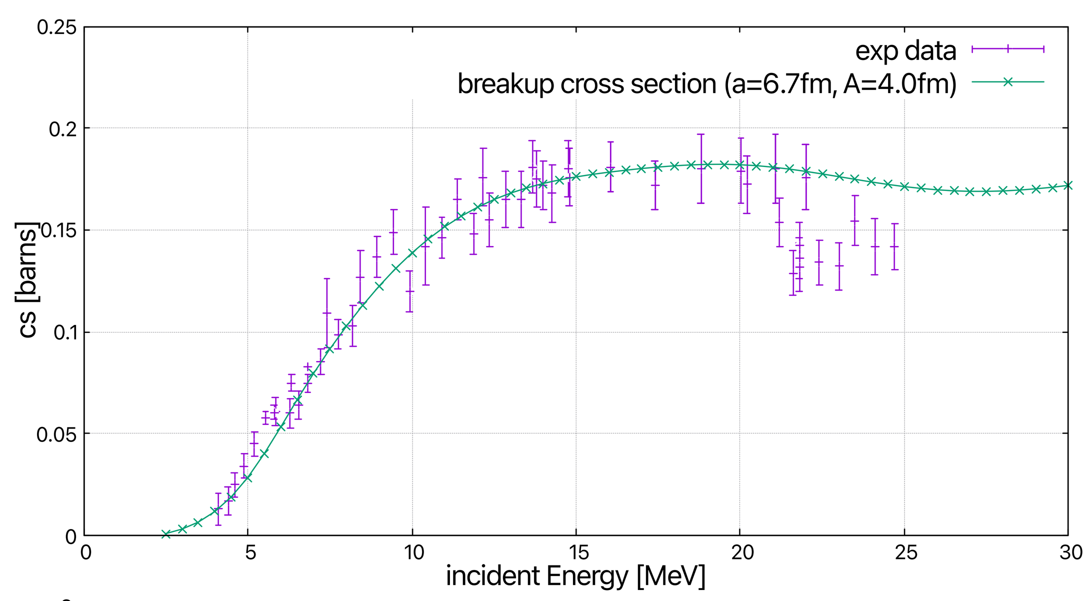
Lagrange-Legendre functions [6] were used as basis functions.

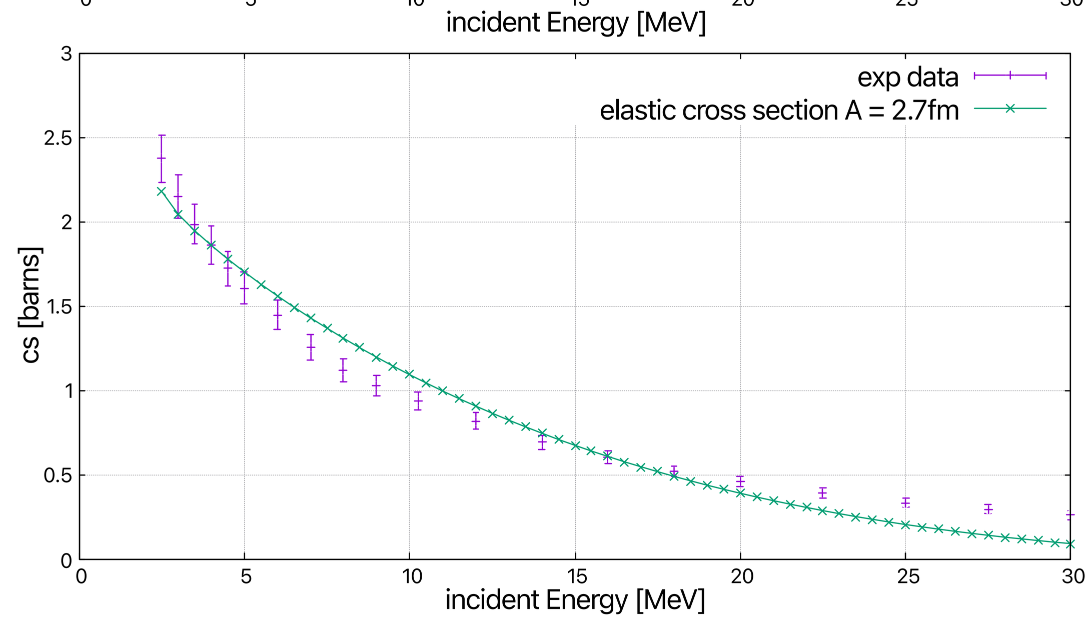
$$\varphi_\mu^{(i)}(r,R) = (a_iA_i)^{-1/2} f_{\mu_1}(r/a_i) f_{\mu_2}(R/A_i)$$

$$f_\mu(x) = (-1)^{N-\mu}\sqrt{\frac{1-x_\mu}{x_\mu}}\,\frac{xP_N(2x-1)}{x-x_\mu}$$



Results of neutron-deuteron scattering





R-Matrix Faddeev Method

The starting point of this approach is the solution of the Faddeev-equations for separable two-body potentials. Thus the component T-Matrix elements are also separable and allow the separation of momentum dependence and energy dependence, necessary for the description of breakup. The energy dependence can be described by standard R-matrix parametrization associated with the corresponding two-body potentials. This feature promises to be combined with standard R-matrix analyses for binary channels. At present the numerical implementation of the method is in progress. Especially, we are currently working on the solution of the set of integral eqations for the three-body collision matrix. The main difficulty of these equations is the handling of the singularities of the integral kernel. The first results are expected in autumn 2024.

Conclusion

The introduction of Bloch operators offered an elegant way to relate the asymptotic wave functions at the boundaries of the internal region, while encapsulating the entire information of the scattering process.

The method provides accurate representations of the breakup and elastic cross sections in three-body scattering. However, the dependence of these observables on the matching radius A indicates that there is still room for improvement.

References

[1] A.M. Lane, R.G. Thomas, Rev.Modern Phys. 30, 257 (1958)
[2] W. Glöckle, Z. Phys. 271, 31 (1974)
[3] B. Raab, A Faddeev based R-matrix method, (Master thesis, TU Wien, 2017)
[4] B. Raab, A novel R-matrix formalism for three-body channels, EPJ Web of Conferences 284, 03018 (2023)
[5] D. Baye, private communication (2022)
[6] D. Baye, The Lagrange-mesh method, Phys. Rep. 565 (2015)