Microscopic calculations with noniterative finite amplitude methods and the application to neutron radiative captures and inelastic scatterings

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1. Introduction

Progress of microscopic theories for nuclear many-body systems is continuously required for basic science and various practical applications. Stellar nucleosynthesis such as the rapid neutron capture process (*r*–process) is a typical research subject where the nuclear theory is indispensable due to the missing experimental data for many radioactive unstable nuclei in stars.

We derive the fully self-consistent quasiparticle random-phase approximation (QRPA) equations with noniterative finite amplitude methods (FAMs) and calculate the transition strengths of giant resonances [1]. Then, we apply the QRPA results to neutron radiative capture calculations based on the statistical Hauser-Feshbach theory [2]. Finally, we show preliminary results of inelastic scattering calculations based on QRPA plus distorted-wave Born approximation (DWBA).

2. Finite Amplitude Method (FAM)

FAM is an efficient calculation method to solve the residual interaction and the linear response equations [3]. FAM was applied to calculate photoabsorptions, β-decays, and spontaneous fissions.

Linear response of the time-dependent Hartree Fock (HF) equations

$$\omega|X_i(\omega)\rangle = (h_0 - \epsilon_i)|X_i(\omega)\rangle + \hat{Q}\{V_{\rm ext}(\omega) + \delta h(\omega)\}|\phi_i\rangle$$

$$\omega\langle Y_i(\omega)| = -\langle Y_i(\omega)|(h_0 - \epsilon_i) - \langle\phi_i|\{V_{\rm ext}(\omega) + \delta h(\omega)\}\hat{Q}$$

$$|\psi_i\rangle = |\phi_i\rangle + \eta|X_i(\omega)\rangle$$

$$\langle\psi'_i| = \langle\phi_i| + \eta\langle Y_i(\omega)|$$

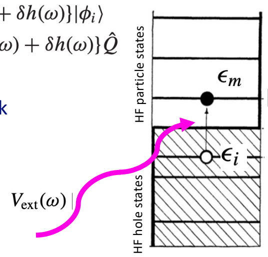
Feedback

Residual interaction

$$\delta h(\omega) = \frac{1}{\eta}\left(h[\langle\psi'|, |\psi\rangle] - h[\langle\phi|, |\phi\rangle]\right)$$

η : Small parameter

ω : Frequency



1p-1h transition induced by the weak external field $V_{\rm ext}(\omega)$

3. Noniterative FAM/(Q)RPA

The explicit linearization of the residual interaction avoids iterative procedure used in other conventional FAM and fully self-consistent (Q)RPA equations are derived from the linear response equations with Skyrme forces [1,2]. The transition strength and cross sections are calculated with the forward and backward amplitudes, X and Y obtained from the (Q)RPA equations.

Explicit linearization

$$\lim_{\eta\to 0}\delta h = \sum_{q'}\sum_{nj\in q'} X_{nj}^{q'} \left.\frac{\partial h}{\partial(\eta X_{nj}^{q'})}\right|_{\eta=0} + \sum_{q'}\sum_{nj\in q'} Y_{nj}^{q'} \left.\frac{\partial h}{\partial(\eta Y_{nj}^{q'})}\right|_{\eta=0}$$

(Q)RPA equations

$$\left\{\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} - \omega\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\}\begin{pmatrix} X_{nj}^{q'} \\ Y_{nj}^{q} \end{pmatrix} = -\begin{pmatrix} f_{mi}^{q} \\ f_{im}^{q} \end{pmatrix}$$

$$A_{mi,nj}^{q,q'} = (\epsilon_m - \epsilon_i)\delta_{mn}\delta_{ij} + \int d^3r\, \phi_m^{q*}\left(\frac{\partial h_q}{\partial(\eta X_{nj}^{q'})}\right)_{\eta=0}\phi_i^q$$

$$B_{mi,nj}^{q,q'} = \int d^3r\, \phi_m^{q*}\left(\frac{\partial h_q}{\partial(\eta Y_{nj}^{q'})}\right)_{\eta=0}\phi_i^q$$

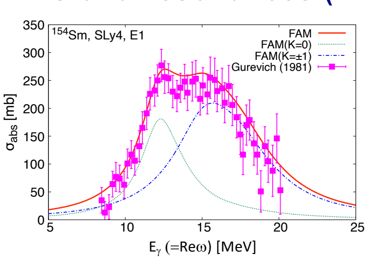
Transition strength

$$\frac{dB(E;V_{\rm ext})}{dE} = -\frac{1}{\pi}{\rm Im}\sum_q\sum_{m,i\in q}\left(f_{mi}^{q*}X_{mi}^q + f_{im}^{q*}Y_{mi}^q\right)$$

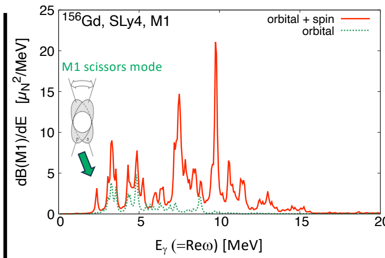
$$f_{mi}^q = \int d^3r\, \phi_m^{q*}V_{\rm ext}\phi_i^q$$

q.. neutron,proton

4. Giant Resonances (E1 and M1)



The electric giant dipole resonance (E1) is calculated by substituting the E1 operator for the external field $V_{\rm ext}(\omega)$ [1]. Our FAM/(Q)RPA calculation reproduces well the resonance energies for heavy nuclei without any adjustment parameter. The energy-weighted sum rule m_1 computed from both HF ground-state density and the integration of $\sigma_{\rm abs}$ is consistent within a few percent.



The M1 transition is calculated by employing the M1 operator for the external field $V_{\rm ext}(\omega)$ [2].

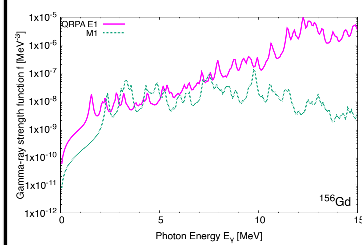
M1 operator

$$M_K = \mu_N\sum_{i=1}^{A}\left(\underbrace{g_s^{(i)}\frac{\vec{\sigma}_i}{2}}_{\text{spin}} + \underbrace{g_l^{(i)}\vec{l}_i}_{\text{orbital}}\right)\cdot\nabla(r_iY_{1K}(\theta_i,\varphi_i))$$

Low energy transitions (E_γ <4MeV) are associated with the orbital operator that can be seen as M1 scissors modes.

Our FAM/QRPA calculation overestimates the total strength for the scissors mode ($\Sigma B(M1)_{\rm exp} \sim 3\mu_N^2$), which can be improved by introducing the quench of the $g^{(i)}_s$.

5. Neutron radiative captures

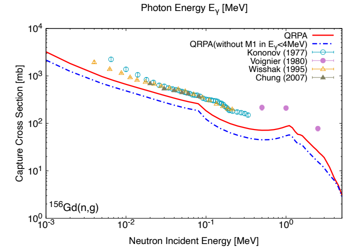


The QRPA results for photoabsorption cross sections $\sigma_{\rm abs}$ (E_γ,XL) (XL=E1,M1) are used to calculate γ-ray strength functions [2].

γ-ray strength function

$$f_{XL}(E_\gamma) = \frac{\sigma_{\rm abs}(E_\gamma;XL)}{(2L+1)(\pi\hbar c)^2E_\gamma^{2L-1}}$$

(XL=E1,M1)



We apply the γ-ray strength function to calculations of a transmission coefficient T_γ and neutron capture cross sections based on the statistical Hauser-Feshbach theory with CoH$_3$ code [2].

Capture cross section

$$\sigma_{n\gamma}(E_n) = \frac{\pi}{k_n^2}\sum_{J\Pi} g_c\frac{T_n^{J\Pi}T_\gamma^{J\Pi}}{T_n^{J\Pi}+T_\gamma^{J\Pi}}W_{n\gamma}^{J\Pi}$$

The strength of low energy M1 transition (f_{M1} (E_γ < 4 MeV)) contributes to about half of the total calculated capture cross section (red solid line). The underestimation of the cross section could be improved by uncertainties of the low energy E1 transition neglected in our QRPA calculation.

6. Neutron-induced inelastic scatterings

Our FAM/QRPA is applicable to DWBA calculations for neutron-induced inelastic scatterings by using a nuclear force as the external field instead of E1 and M1 operators. We demonstrate ^{208}Pb (n,inl) as in M. Dupuis et al. [4]. We multiply a factor $(\alpha+2)(\alpha+1)/2$ by a three-body term in the Skyrme [5] where α is the density dependence of the Skyrme force. We also consider the Perey effect (β=0.85).

External field

$$V_{\rm ext}(\omega) = \chi_{\Sigma_\alpha}^{(-)\dagger}v_{12}\chi_{\Sigma_\beta}^{(+)}$$

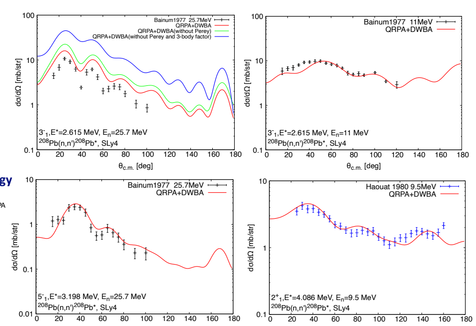
Incoming distorted wave: $\chi_{\Sigma_\beta}^{(+)}$

Outgoing distorted wave: $\chi_{\Sigma_\alpha}^{(-)}$

Nuclear force (Skyrme SLy4): v_{12}

Transition strength & excitation energy

J_k^Π	$B^{\rm exp}$(EL)	$B^{\rm QRPA}$(EL)	$E^{*\rm exp}$	$E^{*\rm QRPA}$
	[$10^{2L}e^2$ fm^{2L}]		[MeV]	
3_1^-	0.611	0.656	2.615	3.625
5_1^-	0.0447	0.0415	3.198	4.625
2_1^+	0.318	0.298	4.086	4.938



7. References

[1] H. Sasaki, T. Kawano, I. Stetcu, PRC105, 044311(2022)
[2] H. Sasaki, T. Kawano, I. Stetcu, PRC107, 054312(2023)
[3] T. Nakatsukasa, T. Inakura, K. Yabana, PRC76, 024318(2007)
[4] M. Dupuis et al., PRC100, 044607(2019)
[5] K. T. R. Davies, G. R. Satchler, Nucl. Phys. A222, 13 (1974)