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Semi-classical treatment of photon cascades in nuclei

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- based on work done with Thomas Døssing (Niels Bohr Institute, Copenhagen):

J. Randrup and Thomas Døssing, Physical Review C 109, 054613 (2024)



MOTIVATION



Such studies need accurate calculations of photon cascades

- in the context of *event-by-event* Monte-Carlo simulations FREYA, ...

The emitted photon removes angular momentum

Before emission:

The (expectation value of the) angular momentum is *directed* along the *z* direction:

$$\langle \mathcal{N} | \hat{oldsymbol{J}} | \mathcal{N}
angle \; = \; oldsymbol{\underline{J}} = J \hat{oldsymbol{z}}$$

After emission:

The (expectation value of the) angular momentum is *tilted* relative to the *z* direction:

$$\langle \mathcal{N}' | \hat{\boldsymbol{J}} | \mathcal{N}' \rangle = \underline{\boldsymbol{J}}'(\hat{\boldsymbol{\omega}}) = (\underline{J}'_{\perp}(\theta) \cos \phi, \ \underline{J}'_{\perp}(\theta) \sin \phi, \ \underline{J}'_{z}(\theta))$$

Tilting angle $\chi(\theta)$: $\tan \chi = \underline{J}'_{\perp}(\theta)/\underline{J}'_{z}(\theta)$

Approximation ("semi-classical"):

The mother state is maximally aligned along \underline{J} : $|\mathcal{N}\rangle = |\alpha; J, M = J\rangle_{\hat{z}}$ The daughter state is maximally aligned along $\underline{J}'(\hat{\omega})$: $|\mathcal{N}'(\hat{\omega})\rangle = |\alpha'; J', J'\rangle_{\hat{z}'}$

 $\hat{\boldsymbol{\omega}} = (\theta, \phi)$

E1 photon emission

J'=J-1: Stretched



E1 photon emission

\checkmark J'=J-1: Stretched

Daughter sate:

$$\begin{array}{cccc} \text{Angular distribution:} & \text{Tilting: none} \\ & \langle \hat{\omega}_1 | \mathcal{N}' \rangle_{J-1,h_1}^{E1} = |\alpha'; J-1, J-1 \rangle_{\hat{\mathcal{I}}} & P_1(\theta_1) \sim d_{1,h_1}^1(\theta_1)^2 & \underline{J}'_{\perp} = 0 \\ & \downarrow = 0 \\$$

E1 photon emission

J'=J-1: Stretched

J'=J+1: Anti-stretched

Daughter state: $\langle \hat{\omega}_{1} | \mathcal{N}' \rangle_{J+1,h_{1}}^{E1} = c_{-}(\theta_{1})e^{-i\phi_{1}} | \alpha'; J+1, J+1 \rangle - h_{1}c_{0}(\theta_{1}) | \alpha'; J+1, J \rangle + c_{+}(\theta_{1})e^{i\phi_{1}} | \alpha'; J+1, J-1 \rangle$ $c_{-} \sim \left[\frac{2J+1}{2J+3} \right]^{\frac{1}{2}} d_{-1,h_{1}}^{1}(\theta_{1}) \qquad c_{0} \sim \left[\frac{2J+1}{(2J+3)(J+1)} \right]^{\frac{1}{2}} h_{1} d_{0,h_{1}}^{1}(\theta_{1}) \qquad c_{+} \sim \left[\frac{1}{(2J+3)(J+1)} \right]^{\frac{1}{2}} d_{+1,h_{1}}^{1}(\theta_{1})$ Angular distribution: $P_{1}(\hat{\omega}_{1}) \sim \frac{2J+1}{2J+3} d_{-1,h_{1}}^{1}(\theta_{1})^{2} + \frac{(2J+1)d_{0,h_{1}}^{1}(\theta_{1})^{2}}{(J+1)(2J+3)} + \frac{d_{1,h_{1}}^{1}(\theta_{1})^{2}}{(J+1)(2J+3)}$ $\underline{J}'_{\perp}(\theta_{1}) = -h_{1}c_{0}[\sqrt{2J+2}c_{-} + \sqrt{4J+2}c_{+}] \qquad \underline{J}'_{z}(\theta_{1}) = (J+1)c_{-}^{2} + Jc_{0}^{2} + (J-1)c_{-}^{2} \implies \chi(\theta)$

*E*1-*E*1 photon cascades

	J_0	M_0	J_1	M_1	μ_1	J_2	M_2
	J_0	M_0	$J_0 - 1$	$M_0 - 1$	+1	$J_0 - 2$	$M_0 - 2$
$(\land \land c) = \sum \langle \tau \Lambda \sigma 1 \tau \tau \rangle \langle \tau \Lambda \sigma 1 \tau \Lambda \sigma \rangle \langle 1 \rangle \langle 0 \rangle \langle i \mu_1 \phi_1 \eta 1 \rangle \langle 0 \rangle \langle i \mu_2 \phi_2 \tau \Lambda \sigma \rangle$					+1	$J_0 - 1$	$M_0 - 2$
$\langle \omega_1 \omega_2 f \rangle = \sum \langle J_1 M_1 I \mu_1 J_0 J_0 \rangle \langle J_2 M_2 I \mu_2 J_1 M_1 \rangle a_{\mu_1 h_1}^* (\theta_1) e^{i \mu_1 \mu_1} a_{\mu_2 h_2}^* (\theta_2) e^{i \mu_2 \mu_2} J_2, M_2 \rangle$	>				0	$J_0 - 1$ I_0	$M_0 - 1$ $M_2 - 2$
$J_1\mu_1J_2\mu_2$					-1	J_0	$M_0 - 2$ $M_0 - 1$
					-1	$\overset{\circ}{J_0}$	\check{M}_0
If different ΔJ do not interfere: $J_0 o J_1 o J_2$	J_0	M_0	J_0	$M_0 - 1$	+1	$J_0 - 1$	$M_0 - 2$
					0	$J_0 - 1$	$M_0 - 1$
$\mu_1 = +1$					$^{+1}_{0}$	J_0	$M_0 - 2$ $M_0 - 1$
$\langle \hat{\boldsymbol{\omega}}_1 \hat{\boldsymbol{\omega}}_2 f \rangle_{J_1 J_2} = \sum \langle J_1 M_1 1 \mu_1 J_0 J_0 \rangle d^1_{\mu_1 h_1}(\theta_1) e^{i \mu_1 \phi_1}$					-1	J_0	M_0
$\mu_1 = -1$					+1	$J_0 + 1$	$M_0 - 2$
					0	$J_0 + 1$	$M_0 - 1$
$\mu_2 = +1$		Mo	I ₀	Mo	-1 +1	$J_0 + 1$ $I_0 - 1$	$\frac{M_0}{M_0-1}$
$\sum \left(\langle J_2 M_2 1 \mu_2 J_1 M_1 angle d^{ extsf{1}}_{\mu_2 h_2} (heta_2) \mathrm{e}^{i \mu_2 \phi_2} J_2, M_2 angle ight)$	50	1010	50	1010	+1	J_0	$M_0 = 1$ $M_0 - 1$
$\mu_2 = -1$					0	J_0	\check{M}_0
F*4 -					$^{+1}$	$J_0 + 1$	$M_0 - 1$
$(-,-)$: $(J-1, J-1) \to (J-2, J-2),$					-1	$J_0 + 1$ $L_0 + 1$	M_0 $M_0 \pm 1$
(-0) : $(J-1, J-1) \rightarrow (J-1, J-2) + (J-1, J-1) \rightarrow (J-1, J-1)$	$\overline{J_0}$	M_0	$J_0 + 1$	$M_0 - 1$	+1	J_0	$\frac{M_0 + 1}{M_0 - 2}$
(, 0) $(J] (J] (J]) + (J] (J]) + (J] (J]) +$	0	0		Ū	0	J_0	$M_0 - 1$
$(-,+) : (J-1, J-1) \to (J, J-2) + (J-1, J-1) \to (J, J-1) + (J-1, J-1) \to (J, J);$					-1	J_0	M_0
$(0,-) : (J,J-1) \to (J-1,J-2) + (J,J-1) \to (J-1,J-1) + (J,J) \to (J-1,J-1),$					$^{+1}_{0}$	$J_0 + 1$ $I_0 + 1$	$M_0 - 2$ $M_0 - 1$
$(0,0) : (J,J-1) \to (J,J-2) + (J,J-1) \to (J,J-1) + (J,J-1) \to (J,J)$					-1	$J_0 + 1$ $J_0 + 1$	M_0
$+ (J, J) \rightarrow (J, J-1) + (J, J) \rightarrow (J, J),$					+1	$J_0 + 2$	$M_0 - 2$
$(0 +) \cdot (I I - 1) - (I + 1 I - 2) + (I I - 1) - (I + 1 I - 1) + (I I - 1) - (I + 1 I)$					0	$J_0 + 2$	$M_0 - 1$
$(0,+) (J,J-1) \to (J+1,J-2) + (J,J-1) \to (J+1,J-1) + (J,J-1) \to (J+1,J)$	Jo	Mo	$I_0 + 1$	Mo	-1 +1	$J_0 + 2$	$\frac{M_0}{M_0-1}$
$+ (J, J) \rightarrow (J+1, J-1) + (J, J) \rightarrow (J+1, J) + (J, J) \rightarrow (J+1, J+1);$	50	1110	50 1 1	1010	0	J_0	M_0
$(+,-) : (J+1,J-1) \to (J,J-2) + (J+1,J-1) \to (J,J-1) + (J+1,J-1) \to (J,J)$					+1	$J_0 + 1$	$M_0 - 1$
$+ (J+1, J) \rightarrow (J, J-1) + (J+1, J) \rightarrow (J, J) + (J+1, J+1) \rightarrow (J, J),$					0	$J_0 + 1$	M_0
$(+ 0) \cdot (I+1 I-1) \rightarrow (I+1 I-2) + (I+1 I-1) \rightarrow (I+1 I-1) + (I+1 I-1) \rightarrow (I+1 I)$					-1 +1	$J_0 + 1$ $J_0 + 2$	$M_0 + 1$ $M_0 - 1$
$(+,0)$: $(J+1,J-1) \rightarrow (J+1,J-2) + (J+1,J-1) \rightarrow (J+1) $					0	$J_0 + 2$	M_0
$+ (J+1,J) \rightarrow (J+1,J-1) + (J+1,J) \rightarrow (J+1,J) + (J+1,J) \rightarrow (J+1,J+1)$					-1	$J_0 + 2$	$M_0 + 1$
$+ (J+1, J+1) \to (J+1, J) + (J+1, J+1) \to (J+1, J+1),$	J_0	M_0	$J_0 + 1$	$M_0 + 1$	+1	J_0	M_0
$(+,+)$: $(J+1,J-1) \rightarrow (J+2,J-2) + (J+2,J-1) \rightarrow (J+2,J-1) + (J+2,J-1) \rightarrow (J+2,J)$					$^{+1}_{0}$	$J_0 + 1$ $J_0 + 1$	M_0 $M_0 \pm 1$
$+ (I+1 \ I) \rightarrow (I+2 \ I-1) + (I+1 \ I) \rightarrow (I+2 \ I) + (I+1 \ I) \rightarrow (I+2 \ I+1)$				\rightarrow	-1	$J_0 + 1$	$M_0 + 1$ $M_0 + 2$
+ (J + 1, J) - (J + 2, J) + (J + 1, J) - (J + 2, J) + (J + 1, J) - (J + 2, J) + (J + 1, J) - (J + 2, J + 1) + (J + 1, J) + (J + 1, J) + (J + 2, J + 1) + (J + 2, J + 2)					+1	$J_0 + 2$	M_0
$+ (J+1, J+1) \rightarrow (J+2, J) + (J+1, J+1) \rightarrow (J+2, J+1) + (J+1, J+1) \rightarrow (J+2, J+2)$	•				0	$J_0 + 2$	$M_0 + 1$
					-1	$J_0 + 2$	$M_0 + 2$

E1-E1: Angular distribution of the second photon, $dN_2/d\omega_2$

J = 3



- excellent



Opening angle

$$\cos \psi_{12} = \hat{\boldsymbol{\omega}}_1 \cdot \hat{\boldsymbol{\omega}}_2 = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos \phi_{12}$$

Distribution: $P_{\psi}(\psi) = \int d^2 \hat{\boldsymbol{\omega}}_1 d^2 \hat{\boldsymbol{\omega}}_2 \delta(\psi_{12} - \psi) P_{12}(\hat{\boldsymbol{\omega}}_1, \hat{\boldsymbol{\omega}}_2)$



Opening angle

$$\cos \psi_{12} = \hat{\boldsymbol{\omega}}_1 \cdot \hat{\boldsymbol{\omega}}_2 = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos \phi_{12}$$

Distribution: $P_{\psi}(\psi) = \int d^2 \hat{\boldsymbol{\omega}}_1 d^2 \hat{\boldsymbol{\omega}}_2 \delta(\psi_{12} - \psi) P_{12}(\hat{\boldsymbol{\omega}}_1, \hat{\boldsymbol{\omega}}_2)$

Independent emission:

Correlated emission (exact):

$$\begin{split} P_{12}^{--}(\cos\psi_{12}) &= \frac{1}{2}P_0 + \frac{3}{8}h_1h_2P_1 + \frac{1}{40}P_2, \\ P_{12}^{-0}(\cos\psi_{12}) &= \frac{1}{2}P_0 + \frac{3}{8}\frac{h_1h_2}{J}P_1 - \frac{1}{40}\frac{2J^{-3}}{J}P_2, \\ P_{12}^{-+}(\cos\psi_{12}) &= \frac{1}{2}P_0 - \frac{3}{8}h_1h_2\frac{J^{-1}}{J}P_1 + \frac{1}{40}\frac{2J^2 - 5J + 3}{J(2J+1)}P_2; \\ P_{12}^{0--}(\cos\psi_{12}) &= \frac{1}{2}P_0 + \frac{3}{8}h_1h_2\frac{J^2 + J - 1}{J(J+1)^2}P_1 - \frac{1}{40}\frac{(2J^{-1})(J^2 + J - 3)}{J(J+1)^2}P_2, \\ P_{12}^{0--}(\cos\psi_{12}) &= \frac{1}{2}P_0 + \frac{3}{8}h_1h_2\frac{J^2 + J - 1}{J(J+1)^2}P_1 - \frac{1}{40}\frac{(2J^{-1})(2J^3 + J^2 - 5J - 3)}{J(J+1)^3}P_2, \\ P_{12}^{00-}(\cos\psi_{12}) &= \frac{1}{2}P_0 + \frac{3}{8}h_1h_2\frac{J^2 + J - 1}{J(J+1)^3}P_1 - \frac{1}{40}\frac{(2J^{-1})(2J^3 + J^2 - 5J - 3)}{J(J+1)^3}P_2, \\ P_{12}^{0+-}(\cos\psi_{12}) &= \frac{1}{2}P_0 - \frac{3}{8}h_1h_2\frac{J^2(J+2)}{(2J+3)(J+1)^3J}P_1 - \frac{1}{40}\frac{2J^{-1}}{2J+3}\frac{2J^3 + J^2 - 7J + 3}{(J+1)^3}P_2; \\ P_{12}^{+-}(\cos\psi_{12}) &= \frac{1}{2}P_0 - \frac{3}{8}h_1h_2\frac{J^2(J+2)}{(J+1)^3}P_1 + \frac{1}{40}\frac{J^2(J+2)(2J-1)[4J^2 + 8J - 5]}{(J+1)^3(2J+1)(2J+3)^2}P_2, \\ P_{12}^{++}(\cos\psi_{12}) &= \frac{1}{2}P_0 - \frac{3}{8}h_1h_2\frac{J(J^2+1)J+3}{(J+1)^2}P_1 + \frac{1}{40}\frac{J(J-1)[2J^4+7J^3+5J^2-4J-1]}{(J+1)^3(2J+1)(2J+3)^2}P_2, \\ P_{12}^{++}(\cos\psi_{12}) &= \frac{1}{2}P_0 - \frac{3}{8}h_1h_2\frac{J^2}{(J+1)^2}P_1 + \frac{1}{40}\frac{J^2}{(J+1)^2}\frac{(2J-1)}{(2J+1)^2}P_2. \end{split}$$



E1-E1: Opening angle distribution $P_{12}(\psi_{12})$

E2-E2: Angular distribution of the second photon , $dN_2/d\omega_2$

$$\begin{aligned} \left\langle \hat{\boldsymbol{\omega}}_{1} \hat{\boldsymbol{\omega}}_{2} \middle| f \right\rangle_{-=} &= \left[\frac{J-1}{J+1} \right]^{\frac{1}{2}} d_{1,h_{1}}^{2}(\theta_{1}) e^{i\phi_{1}} d_{2,h_{2}}^{2}(\theta_{2}) e^{2i\phi_{2}} \middle| J-3, J-3 \right\rangle \\ &- \left[\frac{2}{J+1} \frac{J}{J-1} \right]^{\frac{1}{2}} d_{2,h_{1}}^{2}(\theta_{1}) e^{2i\phi_{1}} d_{1,h_{2}}^{2}(\theta_{2}) e^{i\phi_{2}} \middle| J-3, J-3 \right\rangle \\ &- \left[\frac{2}{J+1} \frac{J-3}{J-1} \right]^{\frac{1}{2}} d_{2,h_{1}}^{2}(\theta_{1}) e^{2i\phi_{1}} d_{2,h_{2}}^{2}(\theta_{2}) e^{2i\phi_{2}} \middle| J-3, J-4 \right\rangle \end{aligned}$$

$$P_2^{-=}(\hat{\boldsymbol{\omega}}_2) = \left(\frac{5}{4\pi}\right)^2 \int |\langle \hat{\boldsymbol{\omega}}_1, \hat{\boldsymbol{\omega}}_2 | f \rangle_{-=} |^2 d^2 \hat{\boldsymbol{\omega}}_1 = \frac{5}{4\pi} \left[\frac{J^2 - 5}{J^2 - 1} d_{2,h_2}^2 (\theta_2)^2 + \frac{4}{J^2 - 1} d_{1,h_2}^2 (\theta_2)^2\right]$$

E2-E2: Angular distribution of the second photon , $dN_2/d\omega_2$

$$\begin{aligned} \langle \hat{\boldsymbol{\omega}}_{1} \hat{\boldsymbol{\omega}}_{2} | f \rangle_{-=} &= \left[\frac{J-1}{J+1} \right]^{\frac{1}{2}} d_{1,h_{1}}^{2}(\theta_{1}) e^{i\phi_{1}} d_{2,h_{2}}^{2}(\theta_{2}) e^{2i\phi_{2}} | J-3, J-3 \rangle \\ &- \left[\frac{2}{J+1} \frac{2}{J-1} \right]^{\frac{1}{2}} d_{2,h_{1}}^{2}(\theta_{1}) e^{2i\phi_{1}} d_{1,h_{2}}^{2}(\theta_{2}) e^{i\phi_{2}} | J-3, J-3 \rangle \\ &- \left[\frac{2}{J+1} \frac{J-3}{J-1} \right]^{\frac{1}{2}} d_{2,h_{1}}^{2}(\theta_{1}) e^{2i\phi_{1}} d_{2,h_{2}}^{2}(\theta_{2}) e^{2i\phi_{2}} | J-3, J-4 \rangle \end{aligned}$$

$$P_2^{-=}(\hat{\omega}_2) = \left(\frac{5}{4\pi}\right)^2 \int |\langle \hat{\omega}_1, \hat{\omega}_2 | f \rangle_{-=} |^2 d^2 \hat{\omega}_1 = \frac{5}{4\pi} \left[\frac{J^2 - 5}{J^2 - 1} d_{2,h_2}^2(\theta_2)^2 + \frac{4}{J^2 - 1} d_{1,h_2}^2(\theta_2)^2\right]$$

$$\begin{split} \left\langle \hat{\boldsymbol{\omega}}_{1} \hat{\boldsymbol{\omega}}_{2} \middle| \boldsymbol{f} \right\rangle_{--} &= \left[\frac{J-1}{J+1} \right]^{\frac{1}{2}} \left[\frac{J-2}{J} \right]^{\frac{1}{2}} d_{1,h_{1}}^{2}(\theta_{1}) e^{i\phi_{1}} d_{1,h_{2}}^{2}(\theta_{2}) e^{i\phi_{2}} \middle| J-2, J-2 \right\rangle \\ &- \left[\frac{J-1}{J+1} \right]^{\frac{1}{2}} \left[\frac{2}{J} \right]^{\frac{1}{2}} d_{1,h_{1}}^{2}(\theta_{1}) e^{i\phi_{1}} d_{2,h_{2}}^{2}(\theta_{2}) e^{2i\phi_{2}} \middle| J-2, J-3 \right\rangle \\ &- \left[\frac{2}{J+1} \right]^{\frac{1}{2}} \left[\frac{3(J-2)}{J(J-1)} \right]^{\frac{1}{2}} d_{2,h_{1}}^{2}(\theta_{1}) e^{2i\phi_{1}} d_{0,h_{2}}^{2}(\theta_{2}) \left| J-2, J-2 \right\rangle \\ &- \left[\frac{2}{J+1} \right]^{\frac{1}{2}} \left[\frac{J-4}{J(J-1)} \right]^{\frac{1}{2}} d_{2,h_{1}}^{2}(\theta_{1}) e^{2i\phi_{1}} d_{1,h_{2}}^{2}(\theta_{2}) e^{i\phi_{2}} \middle| J-2, J-3 \right\rangle \\ &+ \left[\frac{2}{J+1} \right]^{\frac{1}{2}} \left[\frac{2(2J-5)}{J(J-1)} \right]^{\frac{1}{2}} d_{2,h_{1}}^{2}(\theta_{1}) e^{2i\phi_{1}} d_{2,h_{2}}^{2}(\theta_{2}) e^{i\phi_{2}} \middle| J-2, J-4 \right\rangle \end{split}$$

$$P_2^{--}(\hat{\boldsymbol{\omega}}_2) = \frac{5}{4\pi} \left[6(J-2)d_{0,h_2}^2(\theta_2)^2 + (J^3 - 2J^2 - 11J + 30)d_{1,h_2}^2(\theta_2)^2 + 2(J^2 + 2J - 9)d_{2,h_2}^2(\theta_2)^2 \right] \frac{1}{J(J^2 - 1)}$$

E2: Angular distribution of the second photon

J = 4



 $dN_2/d\omega_2$

E2: Angular distribution of the second photon



- reasonable

E2-E2: Opening angle distribution $P_{12}(\psi_{12})$

J = 4



- excellent

Semi-classical treatment of photon cascades in nuclei

The mother state is maximally aligned along < \mathcal{N} | J | \mathcal{N} >

The daughter state is maximally aligned along < $\mathcal{N}' \mid \mathbf{J} \mid \mathcal{N}' >$



E1 cascades:

Excellent for angular distributions *Very good* for angular correlations

E2 cascades:

Reasonable for angular distributions *Excellent* for angular correlations

Easy to implement into simulation codes (and fast) May be useful when using photon angular correlations to learn about fission fragment angular momenta



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- based on work done with Thomas Døssing (Niels Bohr Institute, Copenhagen):

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