The Optical Potential in direct and compound nucleus reactions

Gregory Potel Aguilar

Vienna, July 8 2024

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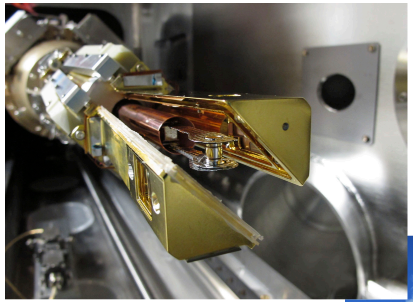
Part 1

Introduction: nuclear reactions and the Optical Potential

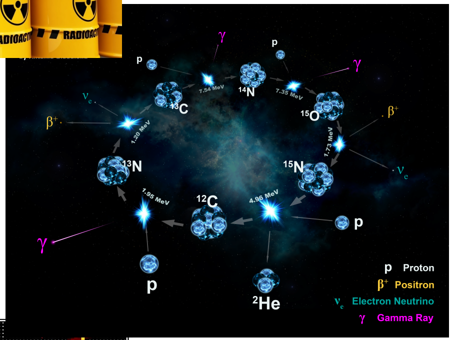
Nuclear reactions: why do we care? Nuclear reactions around us

inertial confinement fusion

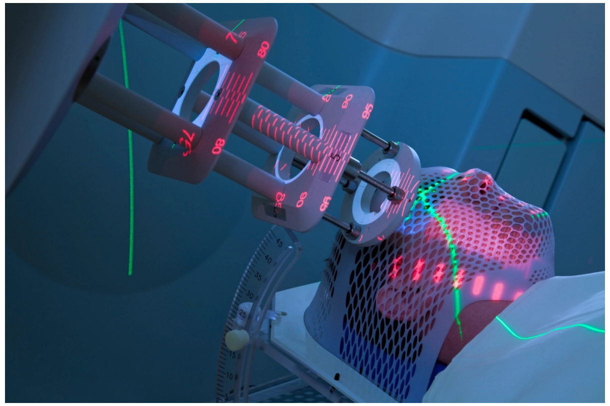


waste management

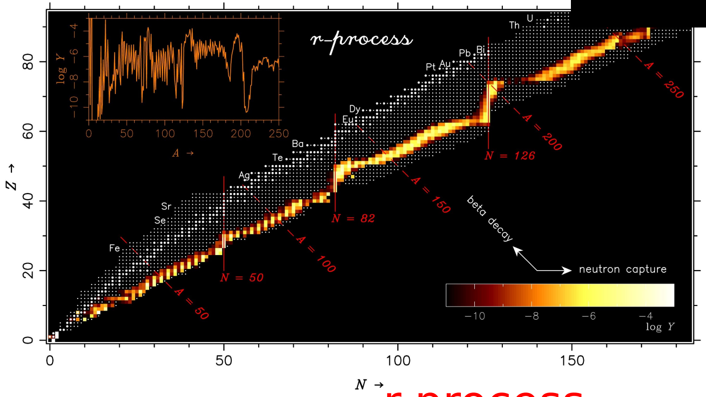
nuclear reactors



reactions of astrophysical interest



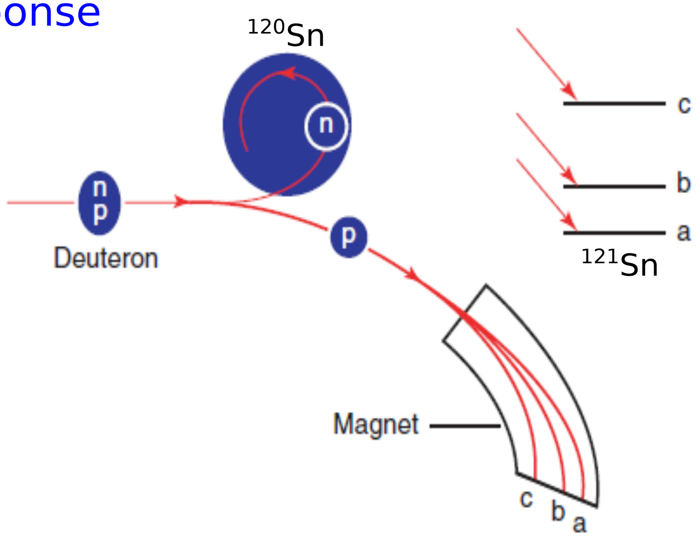
nuclear medicine



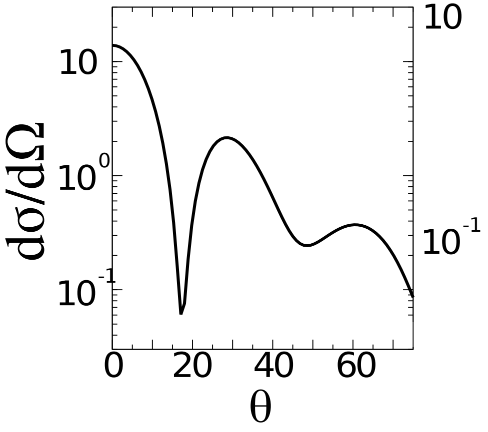
r-process

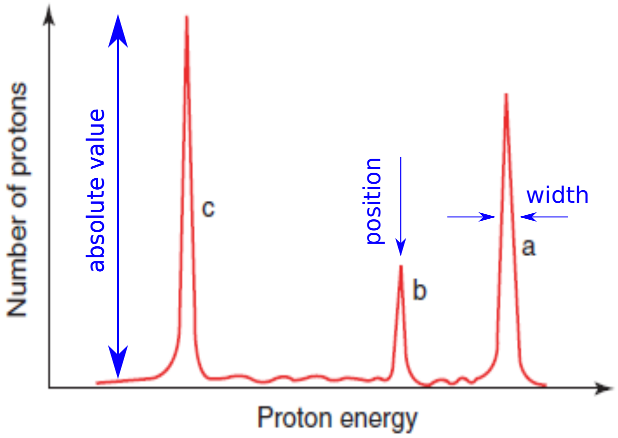
Why do we care? Nuclear reactions as an experimental tool

- transfer reactions probe nuclear response to the addition of a nucleon
- a variety of observables provide rich information about nuclear structure:
  - angular differential cross section
  - absolute value
  - position
  - width (when in the continuum)

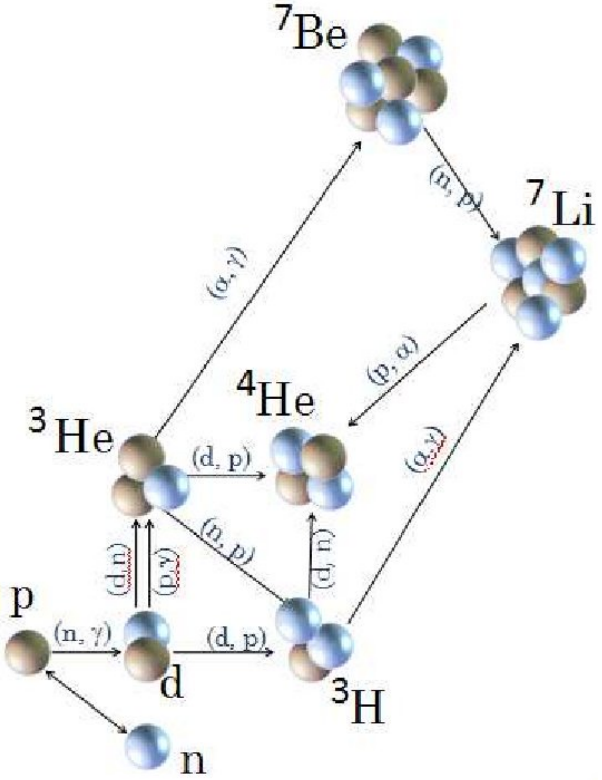


angular differential cross section

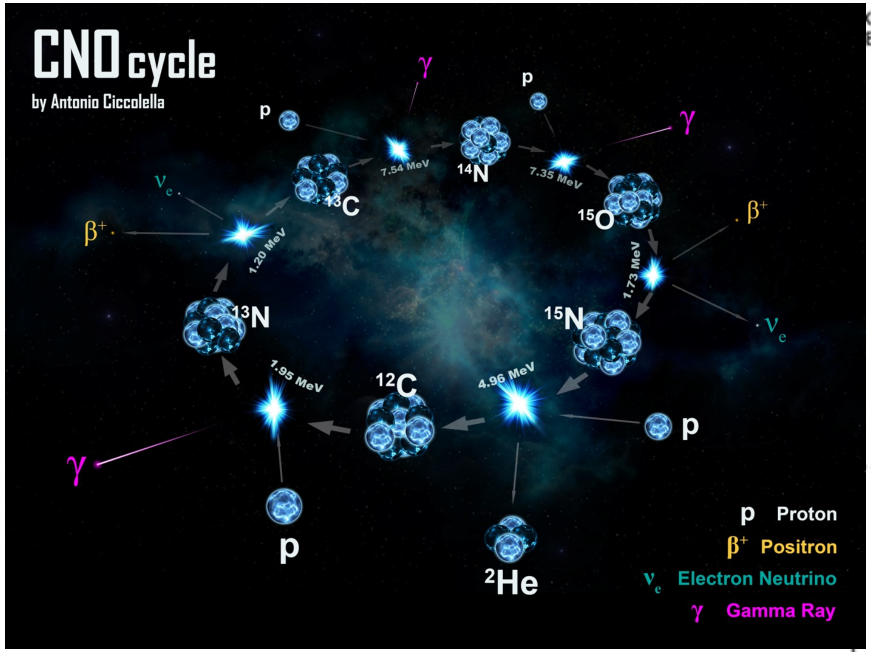


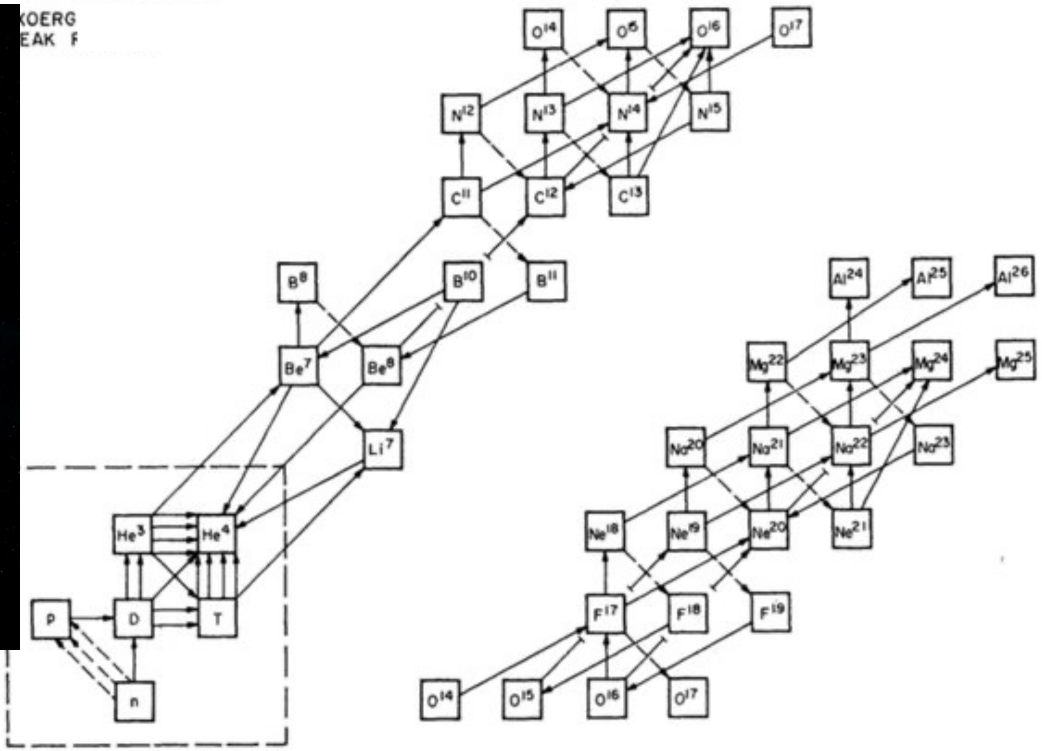


Nuclear reactions of astrophysical interest (light elements)



Big Bang nucleosynthesis





reaction networks

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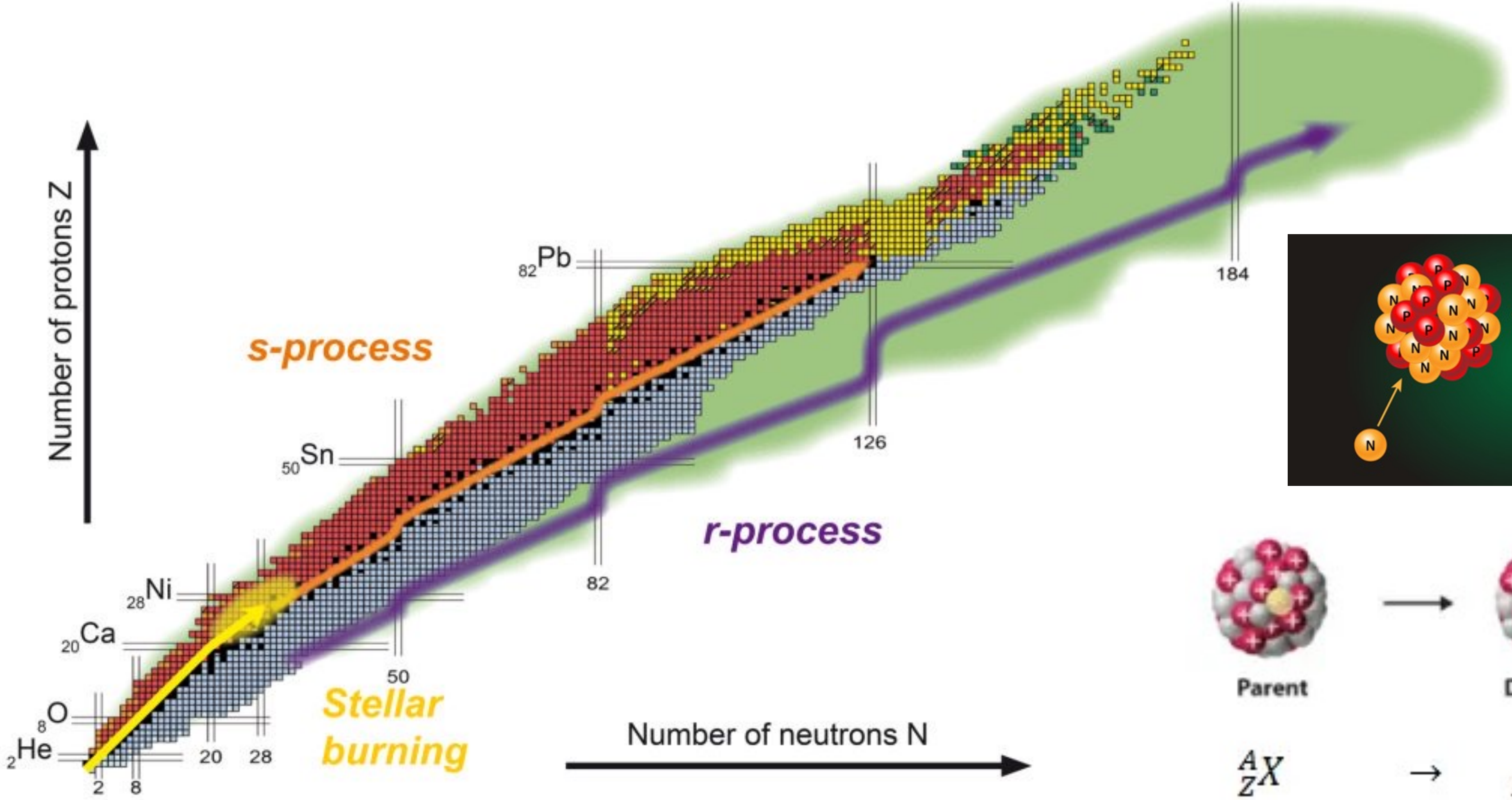
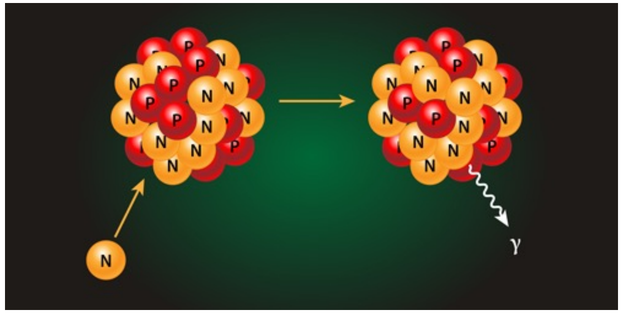
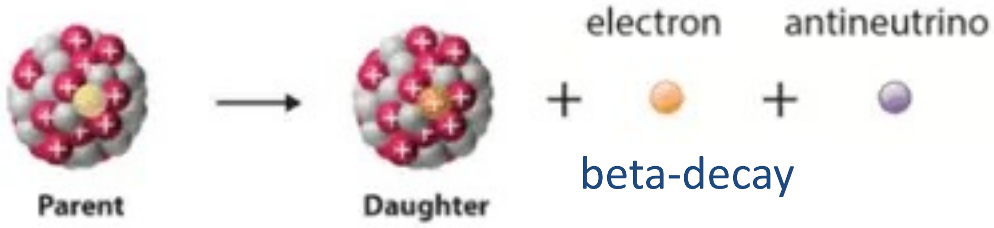


neutron capture

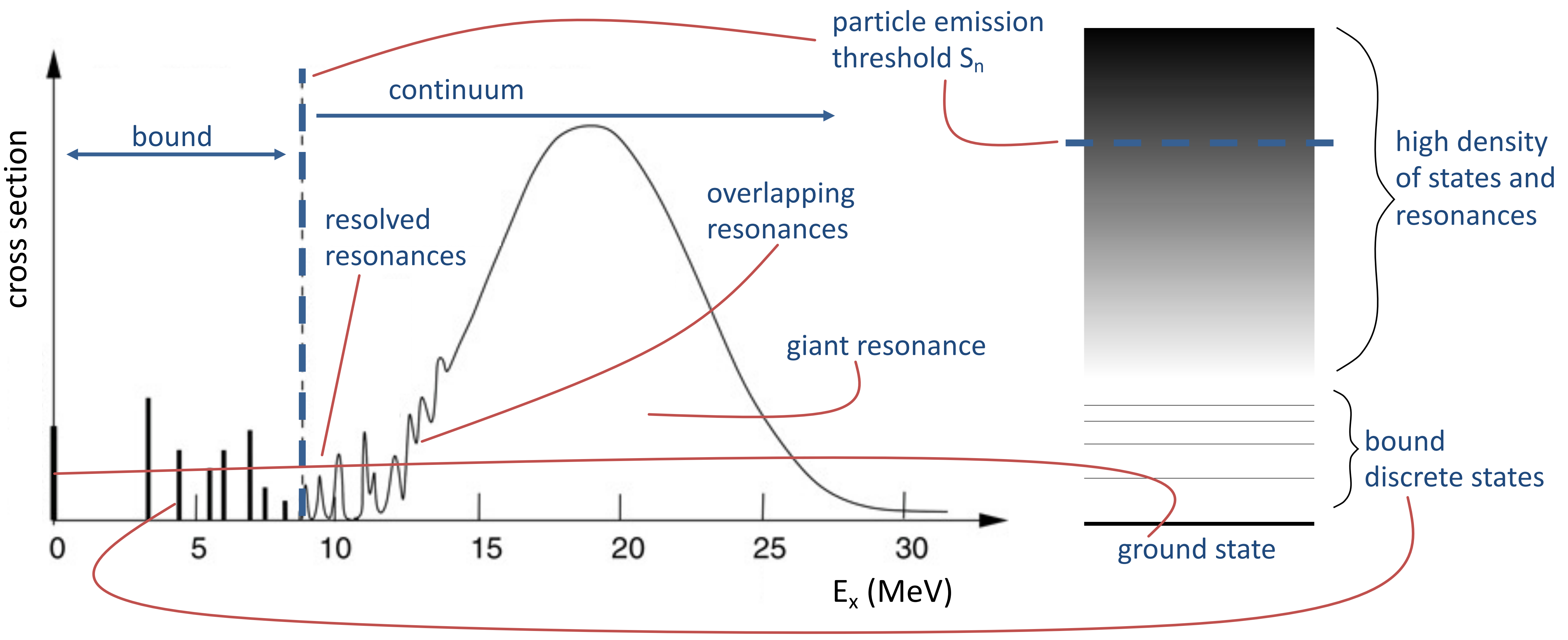




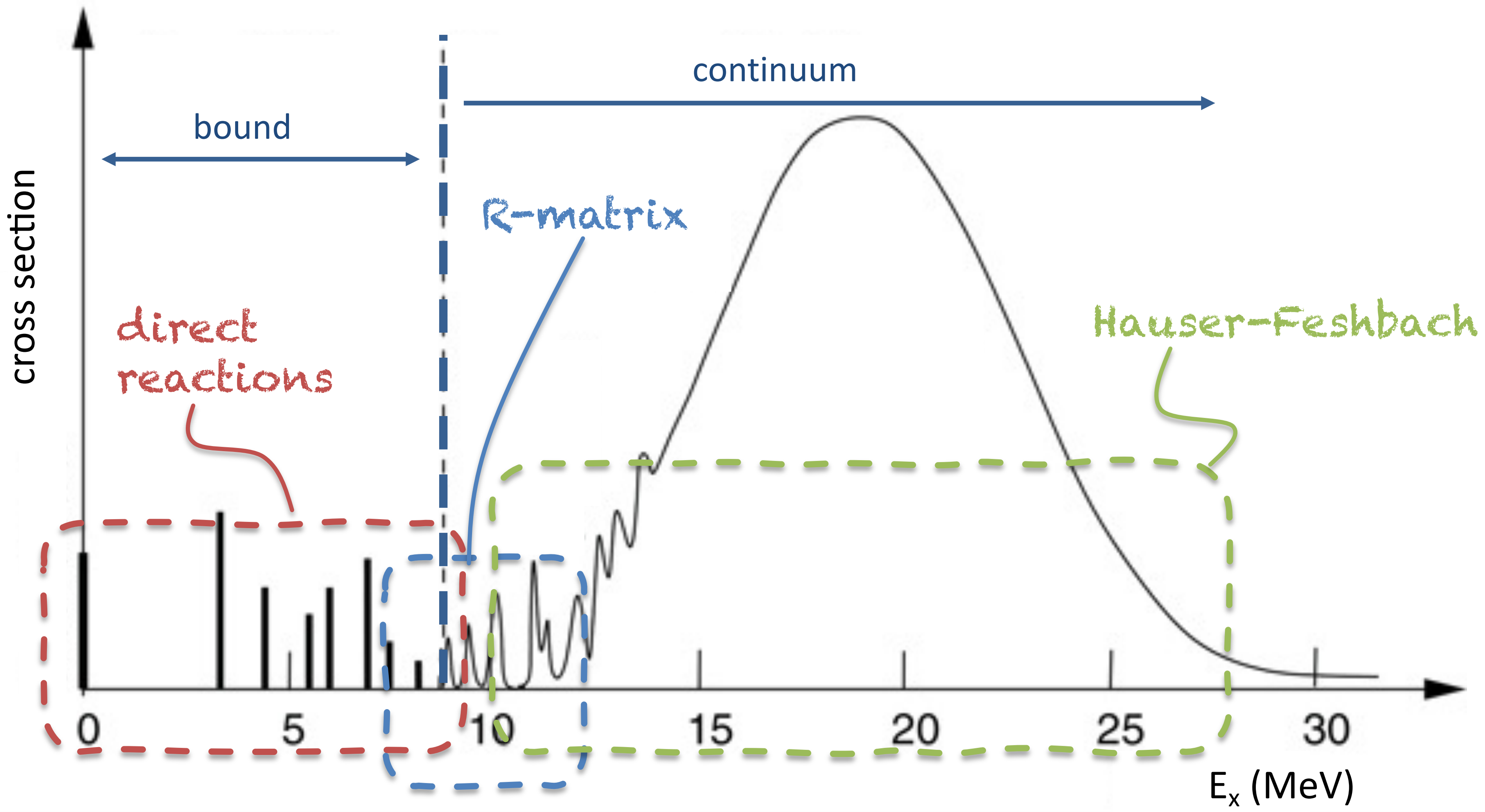
beta-decay

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + \beta^{-} + \bar{\nu}_e$$

Features of nuclear spectra probed by nuclear reactions



Which reaction theories, and where?

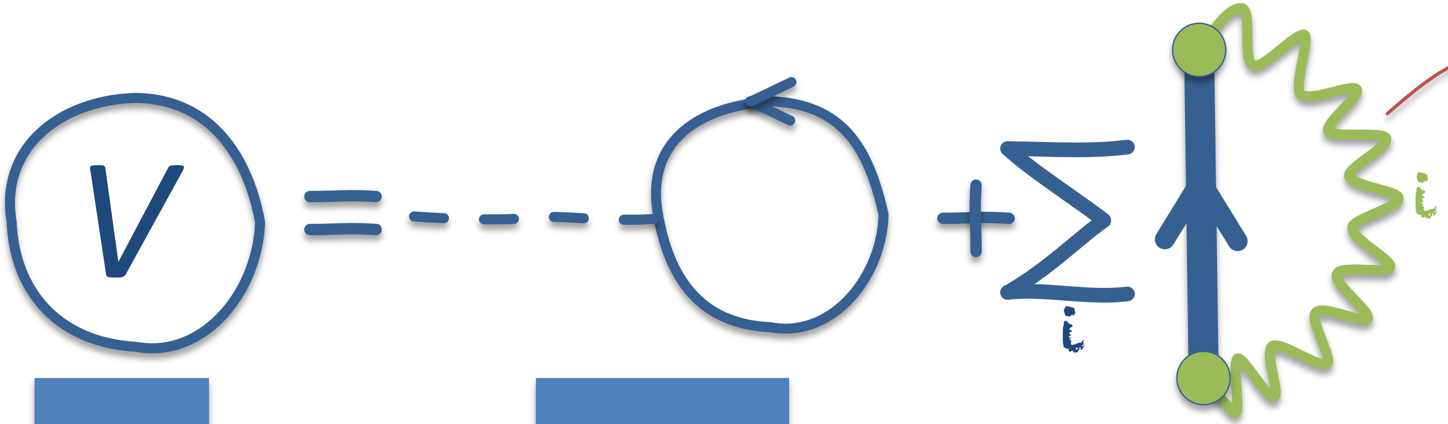


The Optical Potential is a projection of the many-body Hamiltonian on the elastic channel

$$\begin{bmatrix} T+V_{00} & V_{01} & V_{02} & \cdots \\ V_{10} & T+V_{11} & V_{12} & \cdots \\ V_{20} & V_{21} & T+V_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \Rightarrow [T+V]$$

- The "optical reduction" transforms a many-body operator into a one-body operator
- It is a well-defined, in principle exact, mathematical operation

The OP accounts for the composite nature of the target nucleus



excited states of the composite system

full optical potential (OP)

static, real, energy independent (mean field) potential

energy-dependent, complex, dynamical polarization potential

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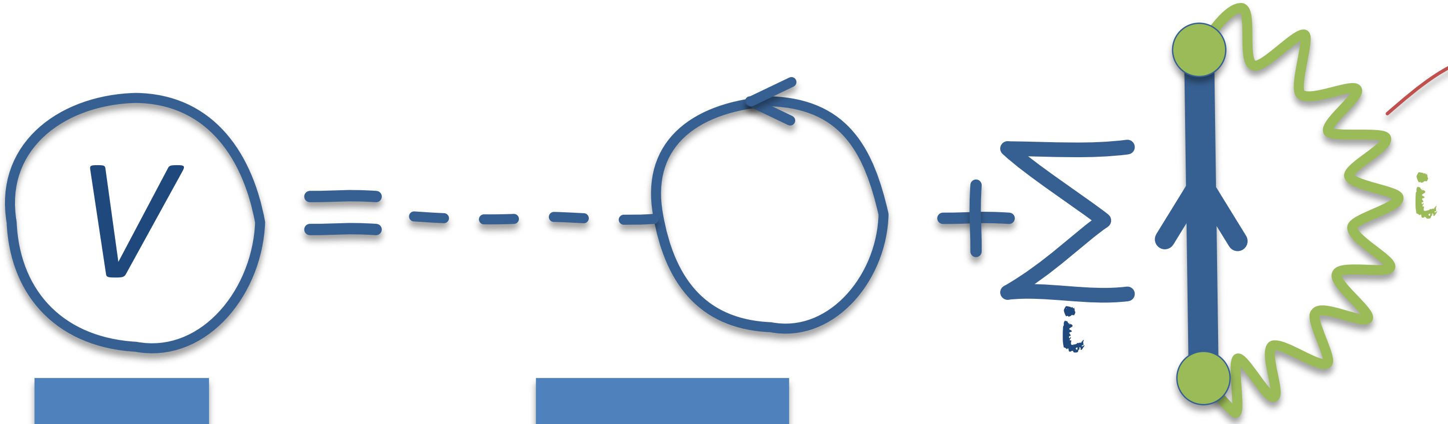

Goal: connect the OP with the underlying microscopic structure, encoded in the spectrum of the levels i

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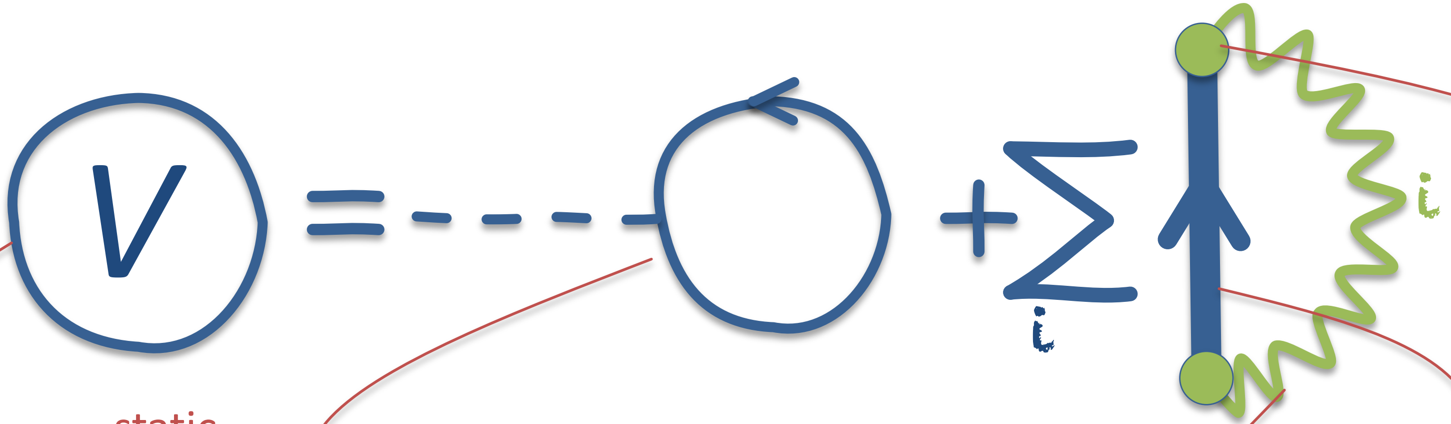


OP

static potential

excited states

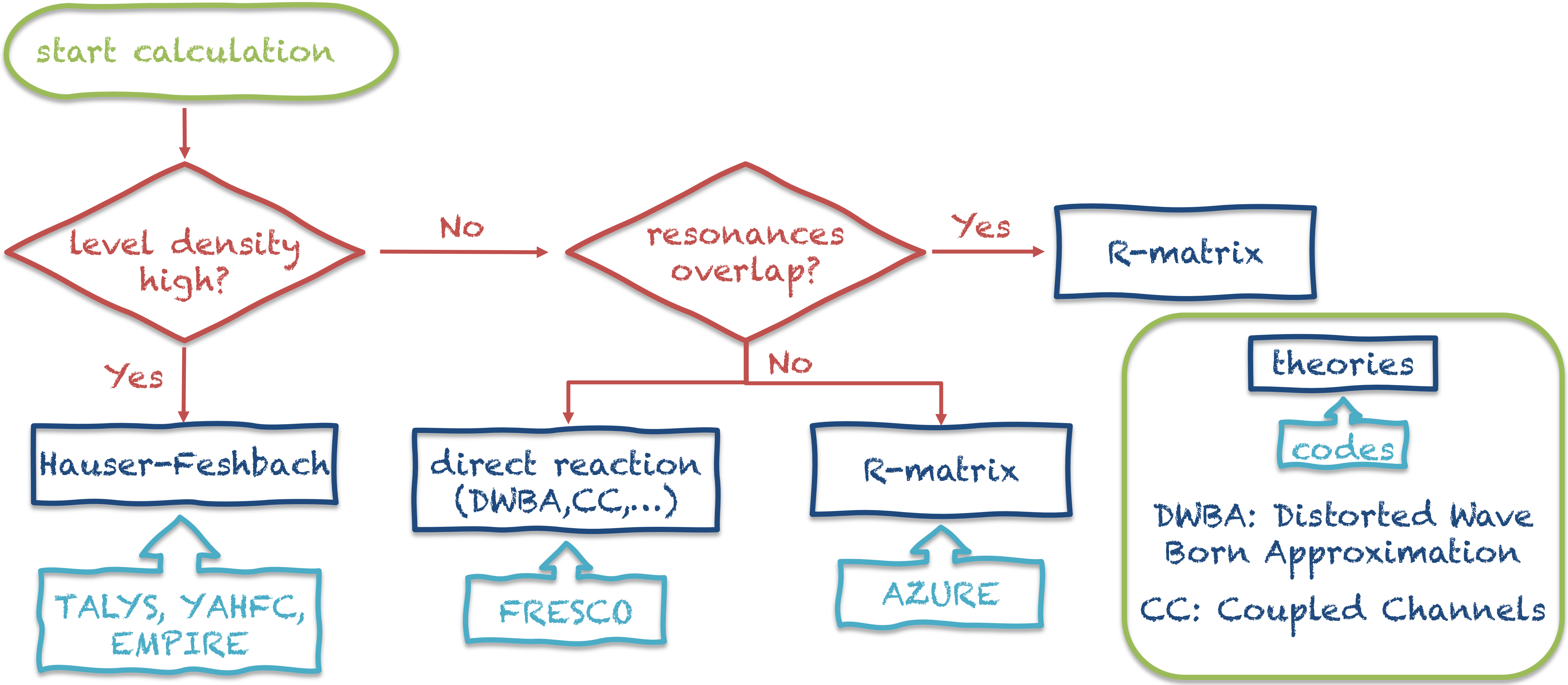
Green's function

coupling vertex

$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r})G(E-E_i,\mathbf{r},\mathbf{r}')U_{i0}(\mathbf{r}')$$

- The computed OP is energy dependent, non-local, complex, and dispersive
- The OP verifies the Kramers-Kronig dispersion relations between the real and the imaginary part

Nuclear reaction theorist's roadmap



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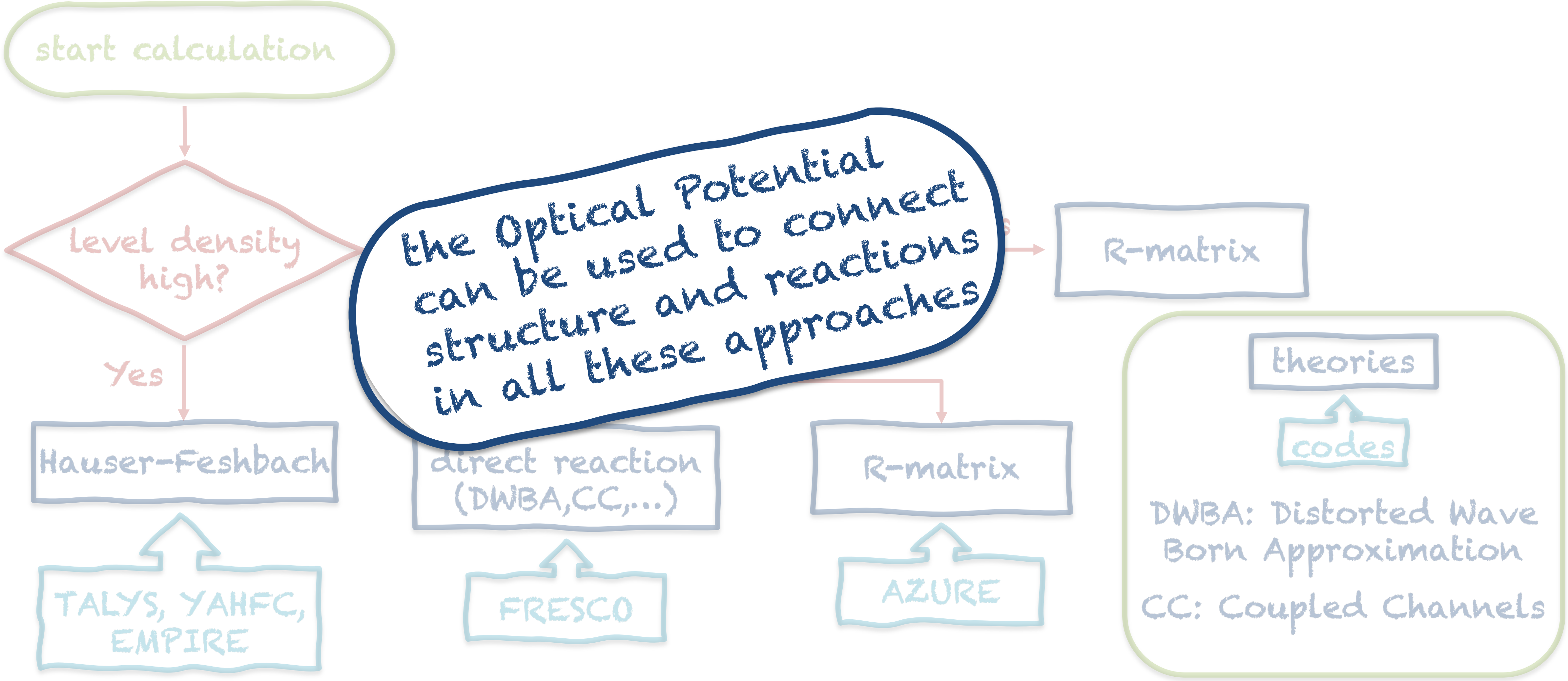
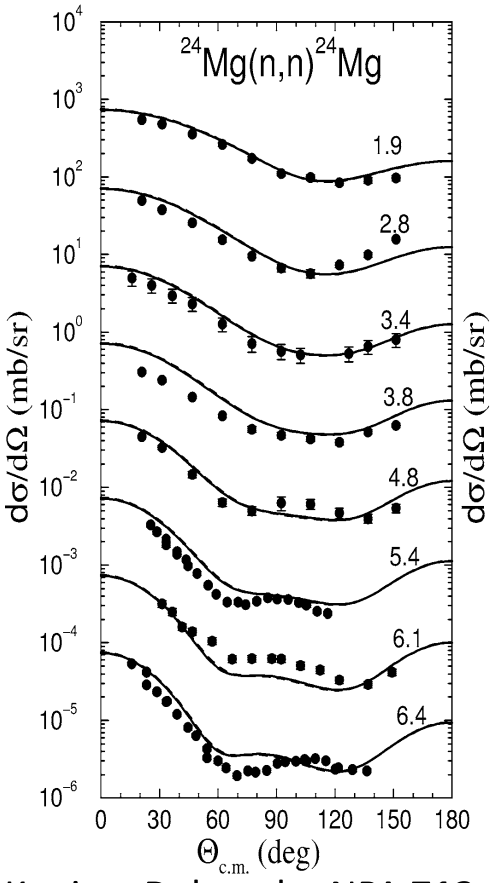


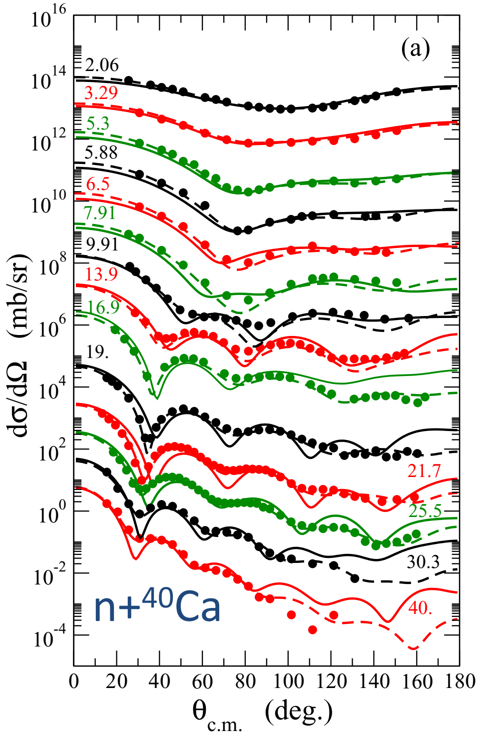
There are different strategies for calculating the nucleon-nucleus optical potential (OP)

phenomenological fit



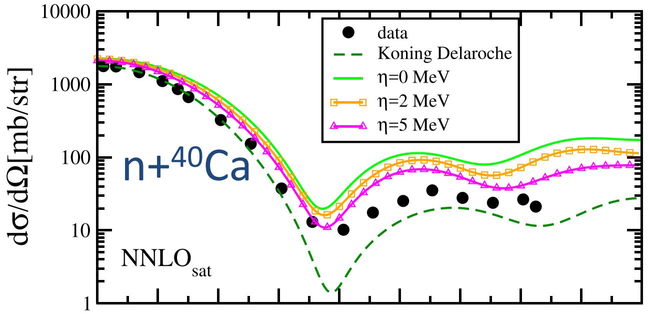
Koning, Delaroche NPA **713** (2003) 231

RPA calculation with added imaginary part



Blanchon *et al*. PRC **91** 014612 (2015)

coupled-cluster ab initio with non-zero η parameter



Rotureau *et al*. PRC **98** 044625 (2018)

- Phenomenological fits are widely used, but are disconnected from the structure and extrapolation away from stability is risky
- Microscopic theories often struggle to get absorption right
- Ab-initio approaches are only feasible for light nuclei

Topical program recently held at FRIB to assess the status and needs associated with Optical Potentials

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Journal of Physics G: Nuclear and Particle Physics
J. Phys. G: Nucl. Part. Phys. **50** (2023) 060501 (58pp)
https://doi.org/10.1088/1361-6471/acc348

Optical potentials for the rare-isotope beam era

C Hebborn[1,2], F M Nunes[1,3], G Potel[2], W H Dickhoff[4], J W Holt[5], M C Atkinson[2,6], R B Baker[7], C Barbieri[8,9], G Blanchon[10,11], M Burrows[12], R Capote[13], P Danielewicz[1,3], M Dupuis[10,11], Ch Elster[7], J E Escher[2], L Hlophe[2], A Idini[14], H Jayatissa[15], B P Kay[15], K Kravvaris[2], J J Manfredi[16], A Mercenne[17], B Morillon[10,11], G Perdikakis[18], C D Pruitt[2], G H Sargsyan[2], I J Thompson[2], M Vorabbi[19,20] and T R Whitehead[1]

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6 authors are attending this conference

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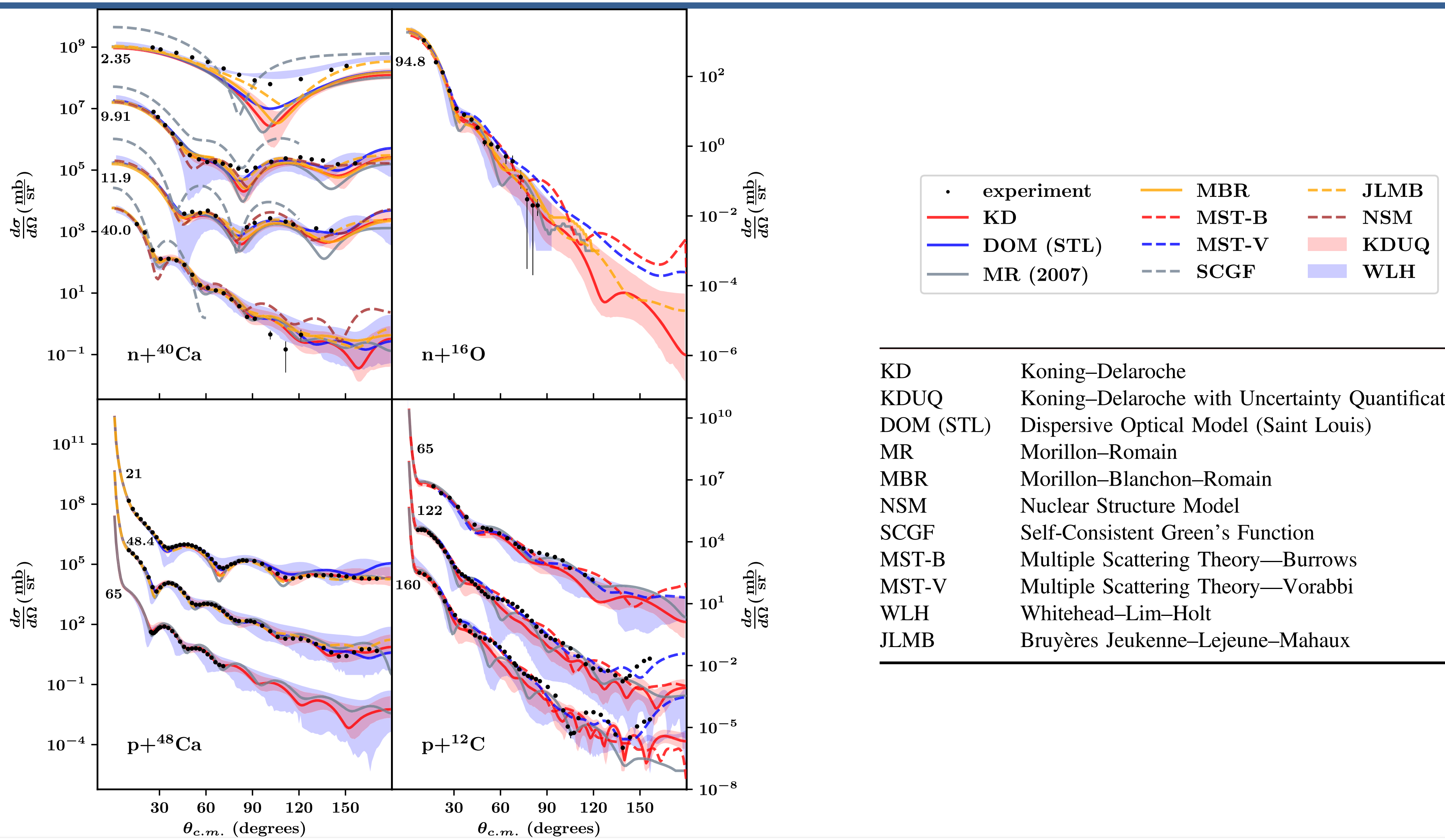

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- Include UQ
- Efforts in both ab-initio and beyond mean field structure calculations for improved collectivity and level density

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KDUQ	Koning–Delaroche with Uncertainty Quantification
DOM (STL)	Dispersive Optical Model (Saint Louis)
MR	Morillon–Romain
MBR	Morillon–Blanchon–Romain
NSM	Nuclear Structure Model
SCGF	Self-Consistent Green's Function
MST-B	Multiple Scattering Theory—Burrows
MST-V	Multiple Scattering Theory—Vorabbi
WLH	Whitehead–Lim–Holt
JLMB	Bruyères Jeukenne–Lejeune–Mahaux

11 potentials were surveyed in this work

- 5 dispersive potentials
- 6 microscopic (based on nuclear structure)
- 3 include uncertainty quantification (UQ)

Recommendations

- Theory-experiment collaboration to address isospin dependence away from stability
- Include UQ
- Efforts in both ab-initio and beyond mean field structure calculations for improved collectivity and level density
- Integrate dispersivity and non-locality

Topical program recently held at FRIB to assess the status and needs associated with Optical Potentials

	Mass	Energy	D.	Mic.	UQ
KD	$24 \leqslant A \leqslant 209$	1 keV $\leqslant E \leqslant$ 200 MeV	×	×	×
KDUQ	$24 \leqslant A \leqslant 209$	1 keV $\leqslant E \leqslant$ 200 MeV	×	×	✓
DOM (STL)	C, O, Ca, Ni, Sn, Pb isotopes	$-\infty < E <$ 200 MeV	✓	×	✓
MR	$12 < Z < 83$	$E <$ 200 MeV	✓	×	×
MBR	$12 < Z < 83$	$E <$ 200 MeV	✓	×	×
NSM	^{40}Ca, ^{48}Ca, ^{208}Pb	$E <$ 40 MeV	✓	✓	×
SCGF	O, Ca, Ni isotopes	$E <$ 100 MeV	✓	✓	×
MST-B	$A \leqslant 20$	$E \gtrsim$ 70 MeV	×	✓	×
MST-V	$4 \leqslant A \leqslant 16$	$E \gtrsim$ 60 MeV	×	✓	×
WLH	$12 \leqslant A \leqslant 242$	$0 \leqslant E \leqslant$ 150 MeV	×	✓	✓
JLMB	$A > 30$	1 keV $< E <$ 340 MeV	×	✓	×

KD	Koning–Delaroche
KDUQ	Koning–Delaroche with Uncertainty Quantification
DOM (STL)	Dispersive Optical Model (Saint Louis)
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11 potentials were surveyed in this work

- 5 dispersive potentials
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- 3 include uncertainty quantification (UQ)

Recommendations

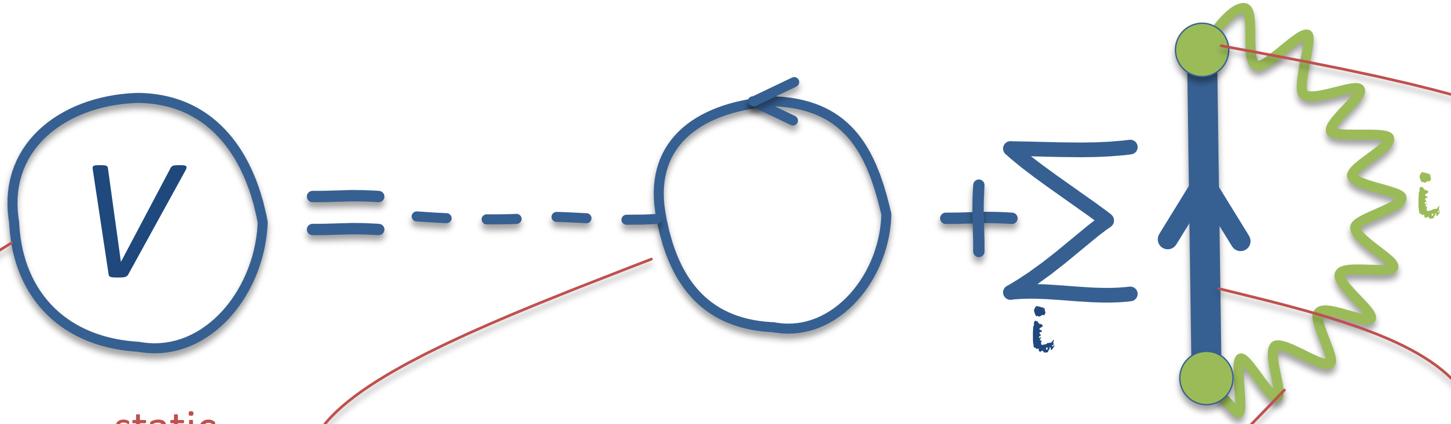
- Theory-experiment collaboration to address isospin dependence away from stability
- Include UQ
- Efforts in both ab-initio and beyond mean field structure calculations for improved collectivity and level density
- Integrate dispersivity and non-locality
- Extension to nucleus-nucleus OP

Part 2

How we* do it

* **G. Sargsyan** (MSU, FRIB Theory Fellow), J. Escher, K. Kravvaris, GP (LLNL)

We calculate the OP by coupling the system to all excited states



coupling
vertex

OP

static
potential

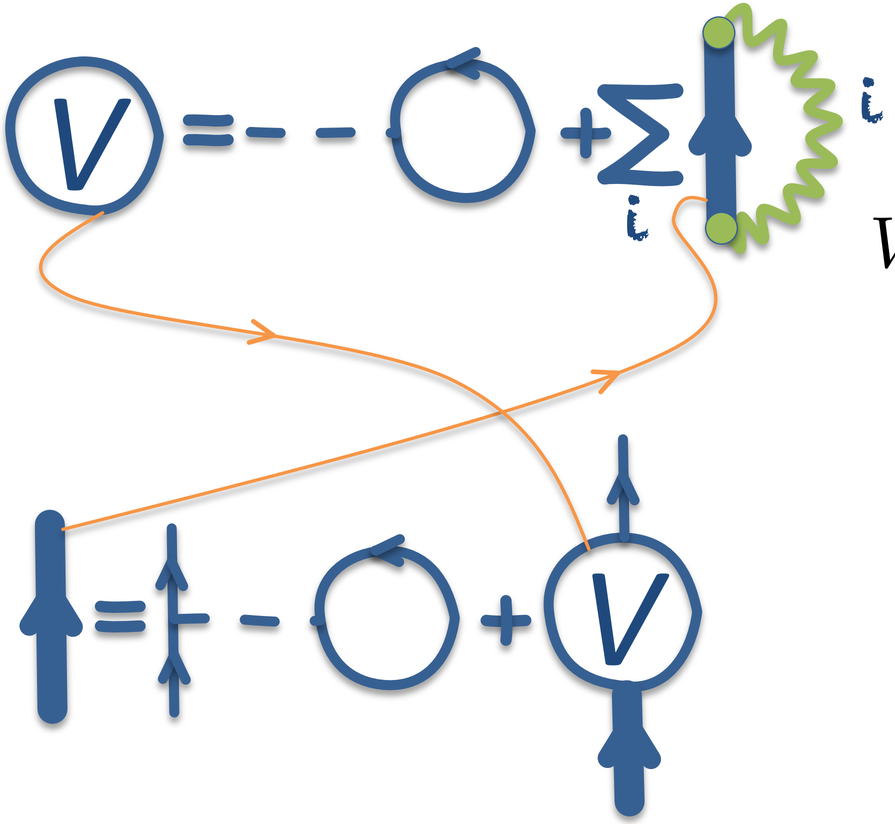
excited
states

Green's
function

$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r})G(E-E_i,\mathbf{r},\mathbf{r}')U_{i0}(\mathbf{r}')$$

- Excited states, static potential, and couplings, come from structure theory input
- But how do we get the Green's function?

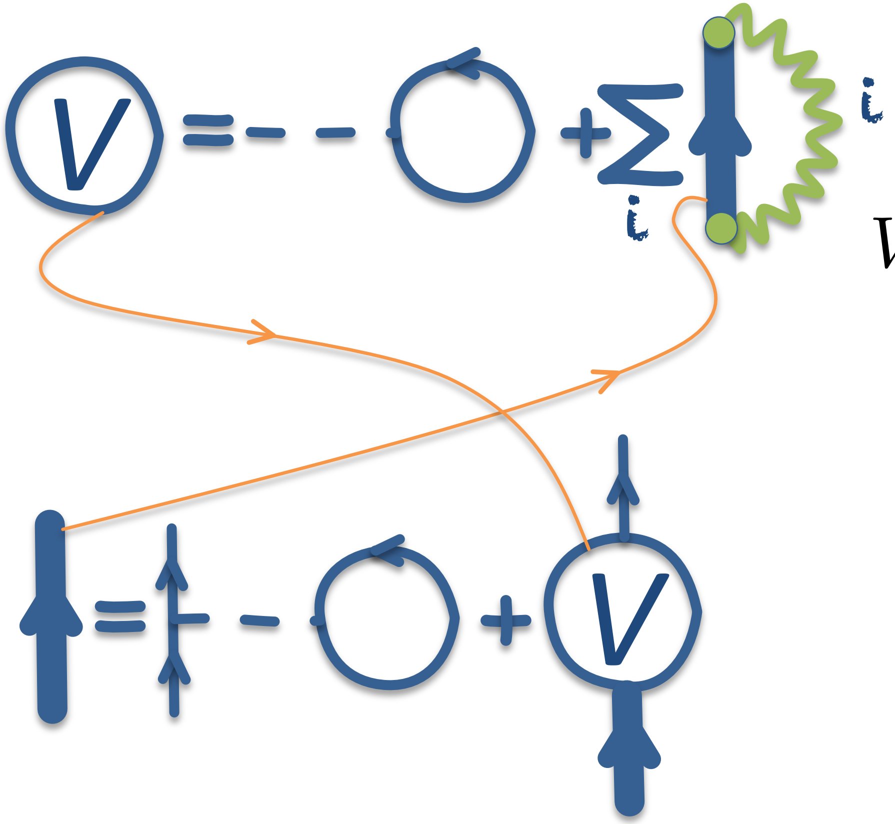
The solution strategy is based on an iterative procedure



$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r})G(E-E_i,\mathbf{r},\mathbf{r}')U_{i0}(\mathbf{r}')$$

$$G(\mathbf{r},\mathbf{r}',E) = \lim_{\eta\to 0}\left(E - T - V(\mathbf{r},\mathbf{r}';E) + i\eta\right)^{-1}$$

The solution strategy is based on an iterative procedure



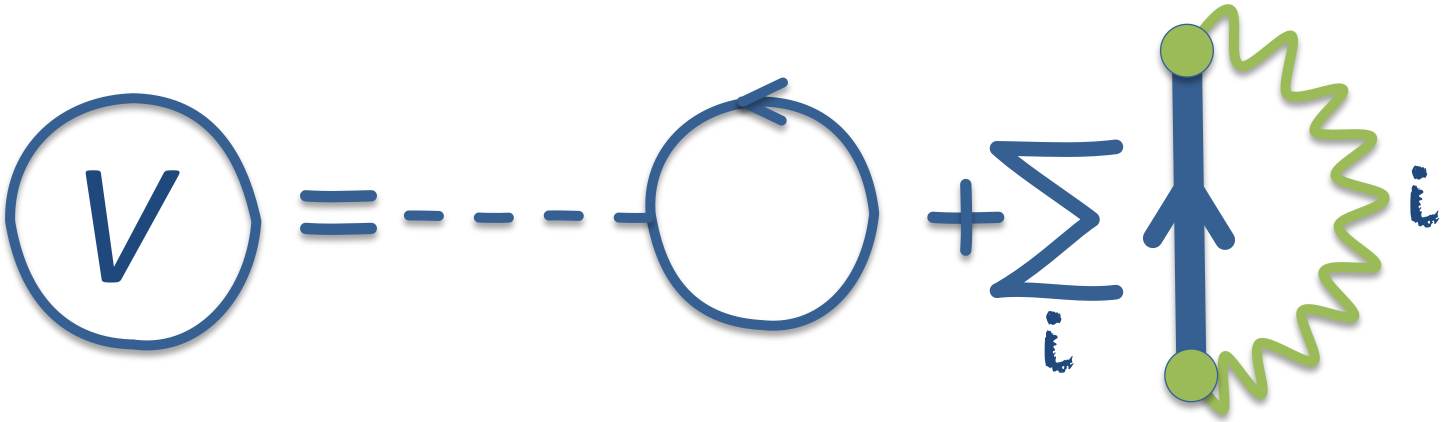
$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r}) G(E - E_i, \mathbf{r}, \mathbf{r}') U_{i0}(\mathbf{r}')$$

$$G(\mathbf{r},\mathbf{r}',E) = \lim_{\eta \to 0} \left(E - T - V(\mathbf{r},\mathbf{r}';E) + i\eta\right)^{-1}$$

Dyson equation verified

$$G = G_0 + G_0 V G$$

Both elastic and absorption cross sections can be calculated from the OP

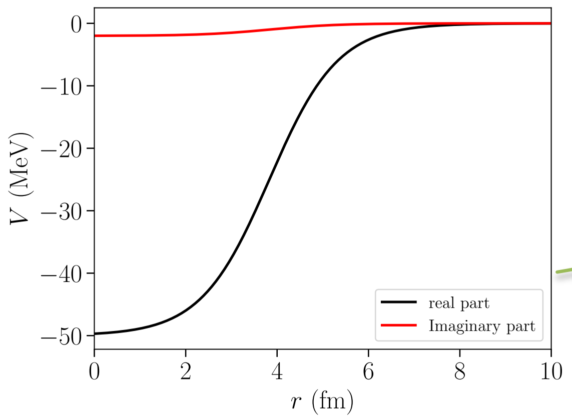


$$\left(E - T - V(\mathbf{r},\mathbf{r}';E)\right)\phi = 0$$ ⇨ elastic scattering from phase shifts

⇨ absorption from imaginary part of polarization potential

^{40}Ca OP calculated in a weak coupling, collective model approximation

$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r})G(E-E_i,\mathbf{r},\mathbf{r}')U_{i0}(\mathbf{r}')$$



- the static potential is a simple Woods-Saxon
- a small imaginary part *W* is included to account for the lack of absorption of the model
- this is a consequence of the over-simplification of the spectrum
- The small imaginary part spoils dispersivity

From Rao, Reeves, and Satchler, NPA **207** (1973) 182

^{40}Ca OP calculated in a weak coupling, collective model approximation

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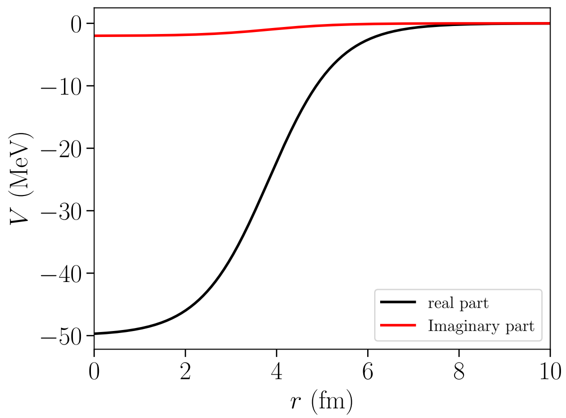

- the spectrum of ^{40}Ca is approximated by 6 collective vibrational states
- the deformation parameters β_λ are constrained by the experimental inelastic scattering cross section

$$\left|\Phi(^{41}\mathrm{Ca})\right\rangle_i \approx \left|\Phi(^{40}\mathrm{Ca})\right\rangle_i \otimes |\chi(n)\rangle_i$$

λ_n^π	1^-	2^+	2^+	3^-	4^+	5^-
E_n (MeV)	18.0	3.9	8.0	3.73	8.0	4.48
$\beta_\lambda(n)$	0.087	0.143	0.309	0.354	0.254	0.192
σ_A (mb)	17	43	176	164	78	37

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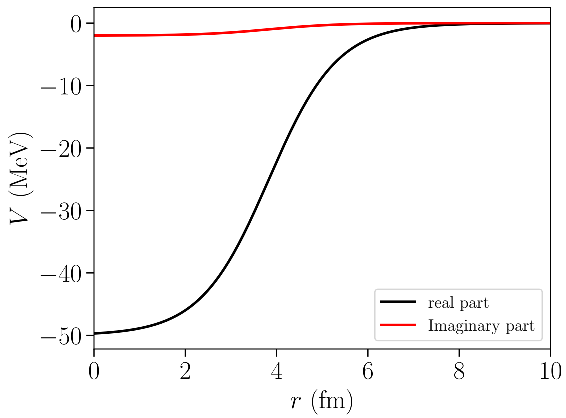
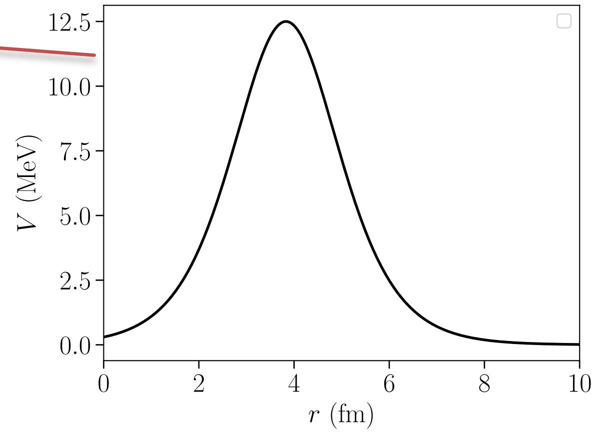
^{40}Ca OP calculated in a weak coupling, collective model approximation

$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r}) G(E - E_i, \mathbf{r}, \mathbf{r}') U_{i0}(\mathbf{r}')$$



- the couplings are surface peaked
- by construction, the experimental contribution of each vibrational state to the absorption cross section is reproduced



$$U_{0i}(\mathbf{r}) \sim \beta_i \frac{dU(r)}{dr} Y^{\lambda_i}(\hat{r})$$

$\lambda_n{}^\pi$	1^-	2^+	2^+	3^-	4^+	5^-
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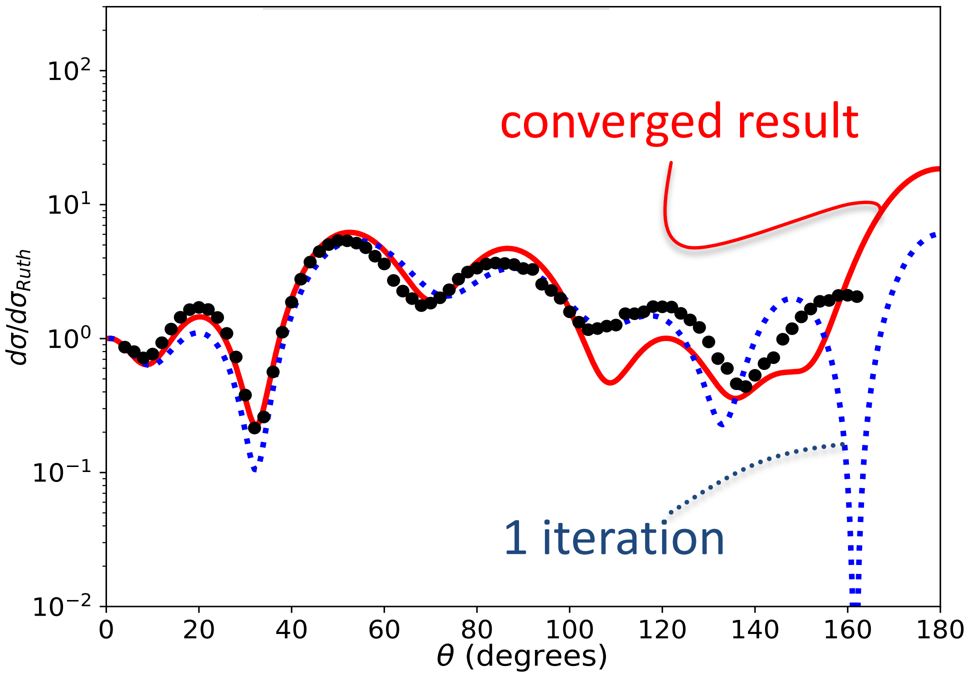
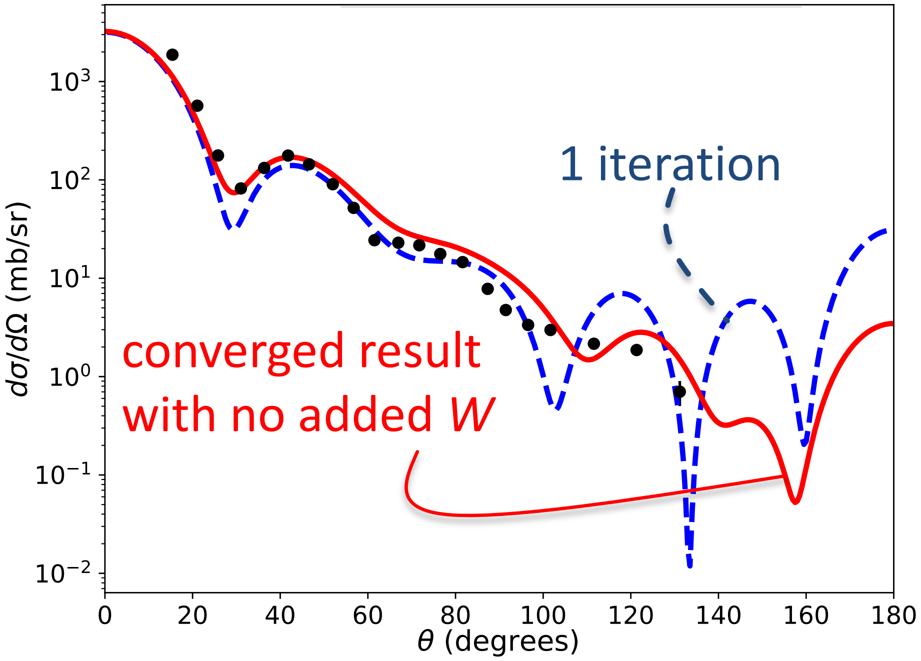
We benchmark our results against Rao *et al.*, and look at the effect of iterations

p+^{40}Ca elastic scattering at 30.3 MeV



n+^{40}Ca elastic scattering at 30.3 MeV



- 1 iteration calculation agrees with Rao et al. (not shown)
- Converged result different at large angles

- Good result for neutrons just by removing Coulomb
- Added non-dispersive imaginary part W not needed for the converged result

^{24}Mg+n with valence shell model

excitation energy E_i

angular momentum

parity

0	0	2	1	0.584066
0.701	0	0	1	-0.716831
1.169	0	2	1	0.488705
2.033	0	2	1	-0.288311
2.529	0	0	1	0.318332
2.701	0	2	1	0.542416
3.859	0	2	1	0.0495903
3.926	0	1	-1	-0.0298132
4.118	0	1	-1	-0.584623
4.226	0	3	-1	-0.651056
4.46	0	2	1	0.100777
4.816	0	2	1	0.0975601
4.945	0	1	-1	0.516883
5.065	0	1	-1	-0.0511968
5.416	0	0	1	-0.283037
5.638	0	3	-1	-0.219064
5.785	0	2	1	-0.0103979
5.792	0	2	1	0.132477
5.935	0	3	-1	0.069005
5.936	0	2	1	0.094658
6.033	0	1	-1	0.0353684
6.12	0	1	-1	-0.159821
6.243	0	2	1	-0.174362
6.35	0	3	-1	0.122727
6.385	0	0	1	0.182001
6.417	0	2	1	0.115995
6.609	0	2	1	0.100457
6.739	0	3	-1	0.157325
6.771	0	1	-1	0.419452
6.801	0	3	-1	0.160889

step 1: ^{25}Mg shell-model calculation

spectroscopic factor S_i

~600 states from E_i=0 to E_i=14.6 MeV

Shell model calculations by K. Kravvaris with PSDPF interaction M Bouhelal, *et al.*, Nucl. Phys. A **864** (2011)

^{24}Mg+n with valence shell model

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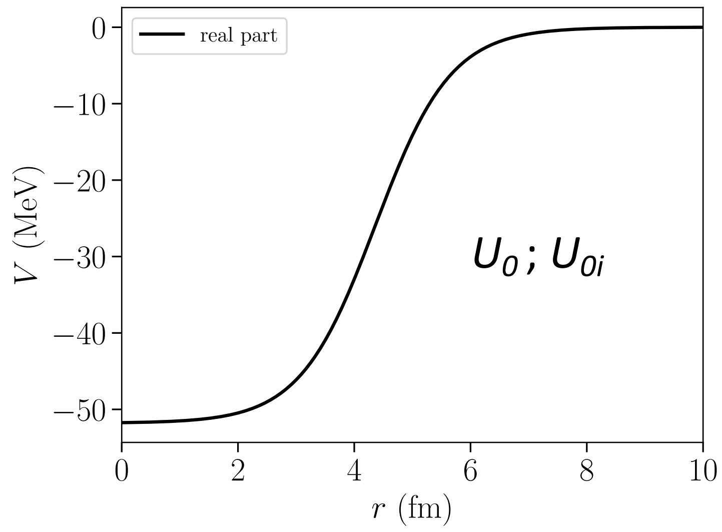
spectroscopic factor S_i

step 2: Static potential and couplings

$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r}) G(E-E_i,\mathbf{r},\mathbf{r}') U_{i0}(\mathbf{r}')$$



- static potential U_0 : real, local Woods-Saxon adjusted to reproduce binding energy of ^{25}Mg
- couplings U_{0i} : same Woods-Saxon, but adjusted to each E_i and multiplied by spectroscopic factor S_i

^{24}Mg+n with valence shell model

excitation energy E_i

angular momentum

parity

spectroscopic factor S_i

0	0	2	1	0.584066
0.701	0	0	1	-0.716831
1.169	0	2	1	0.488705
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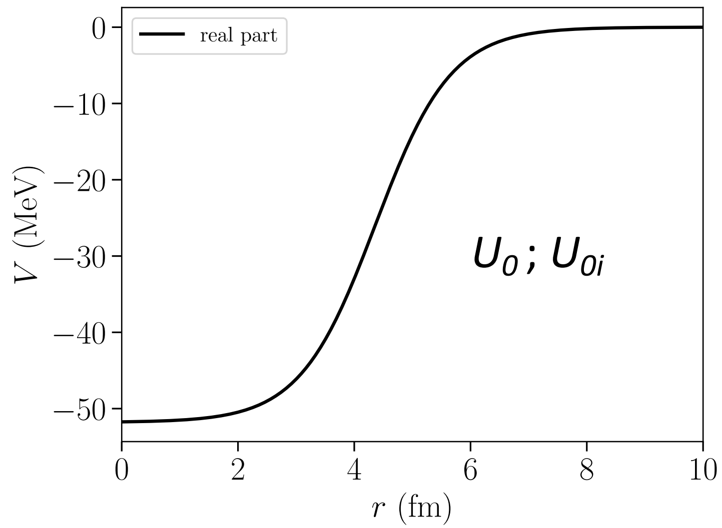


step 3: Iterative procedure

$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r}) G(E-E_i,\mathbf{r},\mathbf{r}') U_{i0}(\mathbf{r}')$$

$$G(\mathbf{r},\mathbf{r}',E) = (E - T - V(\mathbf{r},\mathbf{r}';E))^{-1}$$

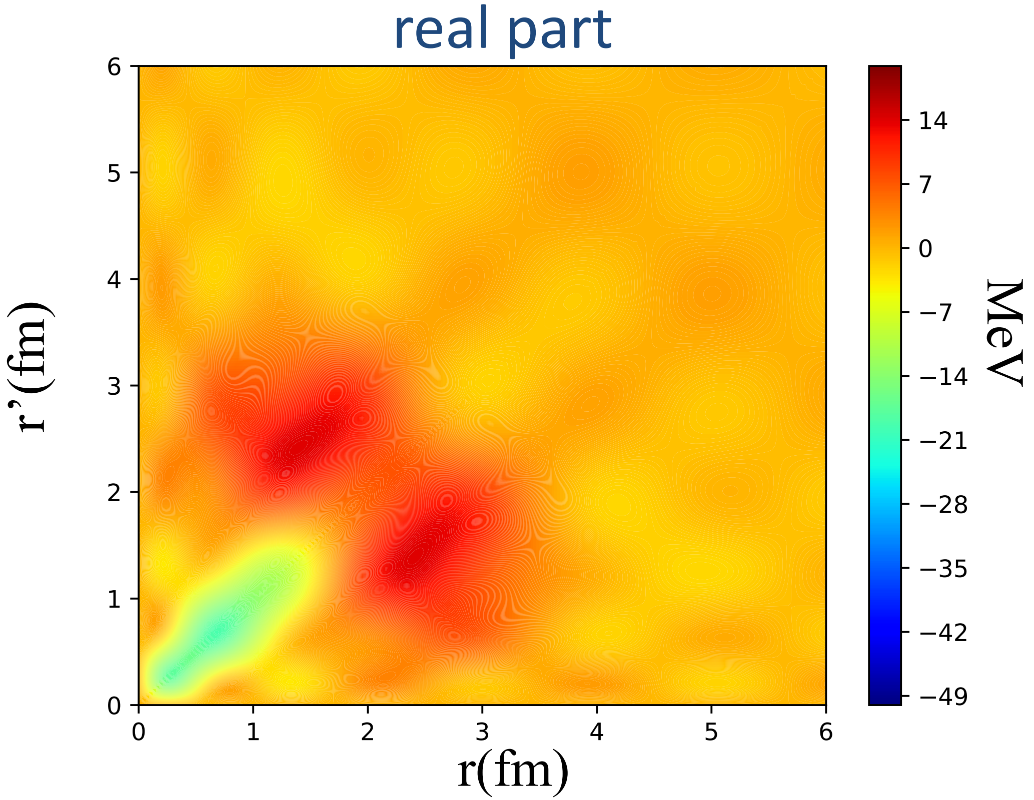
- Iterate until convergence is achieved
- Consistency between potential and Green's function is achieved, as expressed by Dyson's equation:

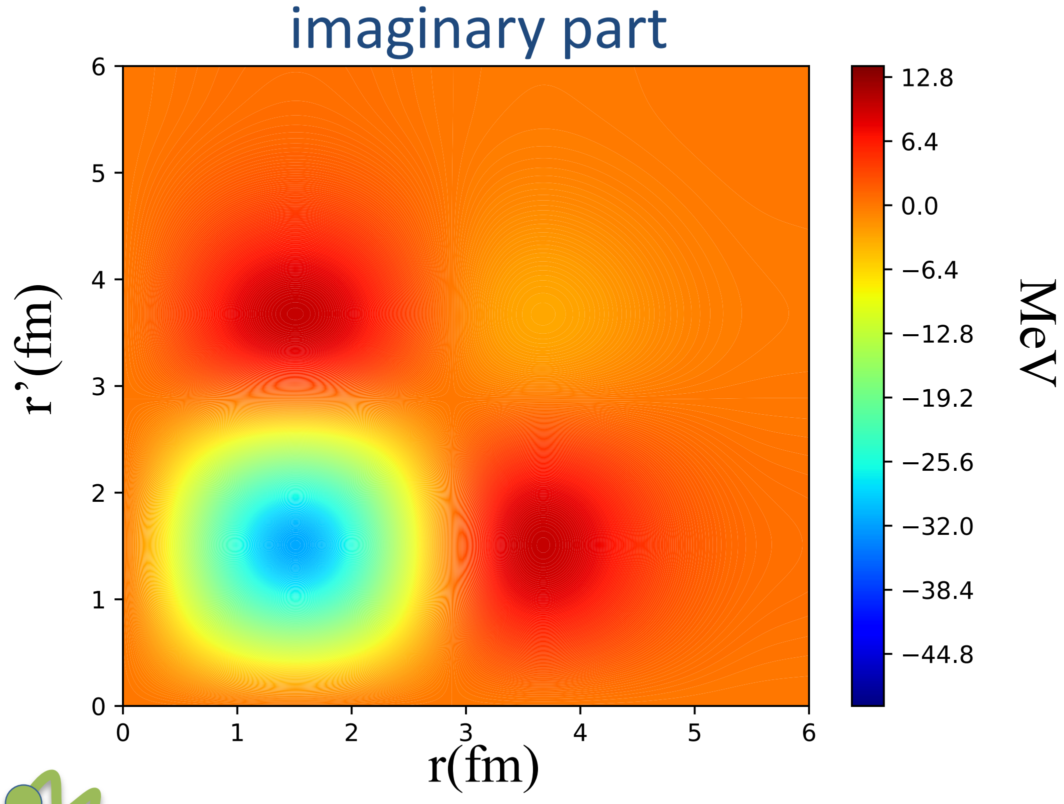
$$G(\mathbf{r},\mathbf{r}';E) = G_0(\mathbf{r},\mathbf{r}';E) + G_0(\mathbf{r},\mathbf{r}';E)V(\mathbf{r},\mathbf{r}';E)G(\mathbf{r},\mathbf{r}';E)$$

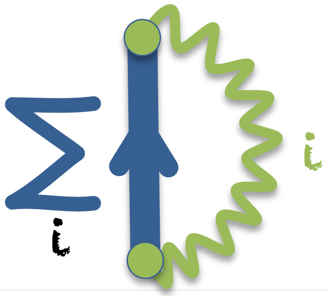
$$G_0(\mathbf{r},\mathbf{r}';E) = (E - T - U_0(r))^{-1}$$

As a bonus, we obtain the Green's function

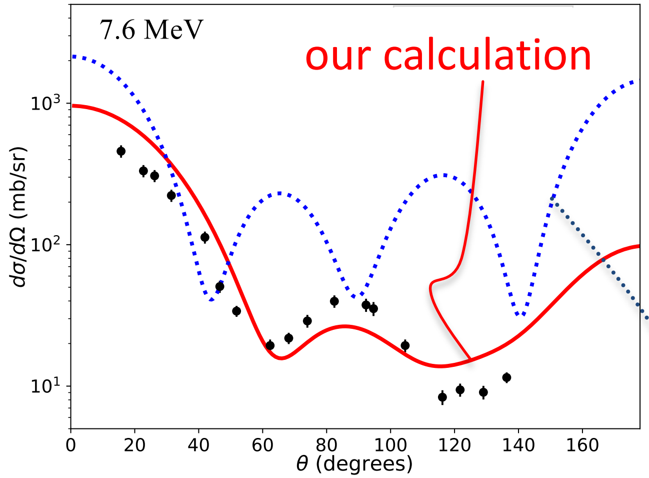
The dynamical polarization potential is complex, energy-dependent, dispersive, and non-local





V_{pol}(r,r’;E)=  ; for E=1MeV

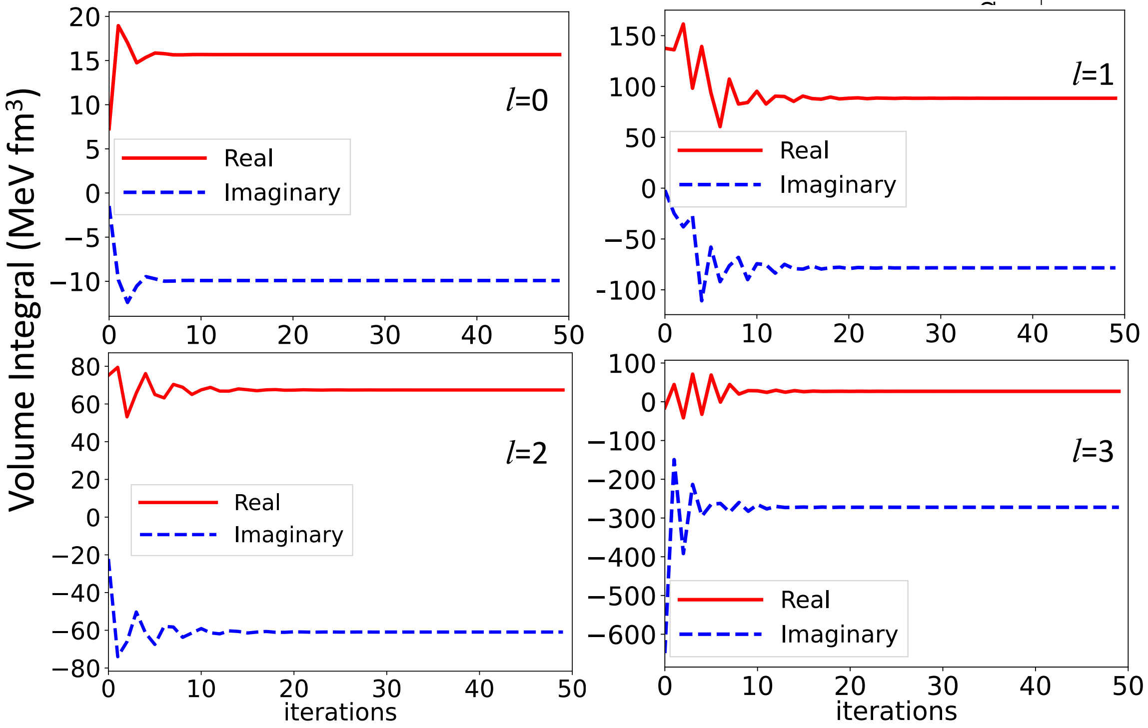
Our ^{24}Mg calculation compares well with experiment

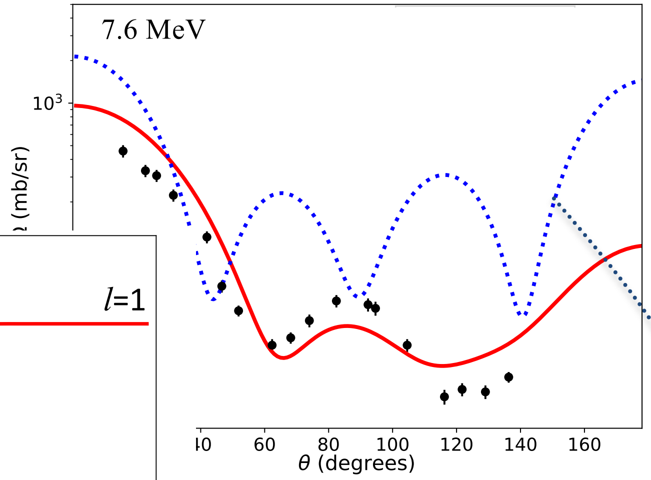


not surprisingly, the static potential alone gives a very wrong result!

Our ^{24}Mg calculation compares well with experiment

we check for convergence by looking at the volume integrals as a function of the iteration





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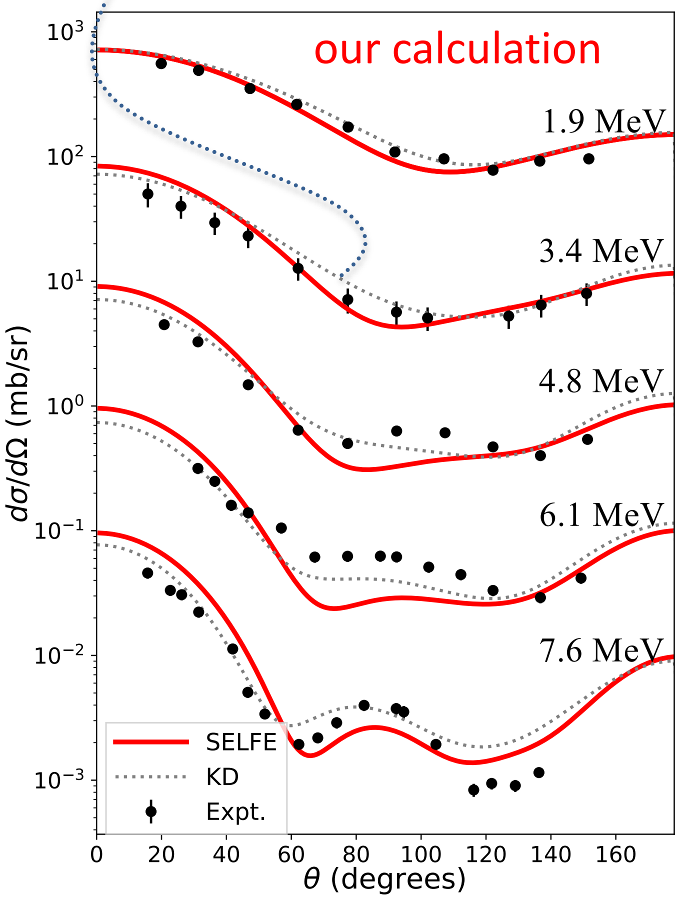
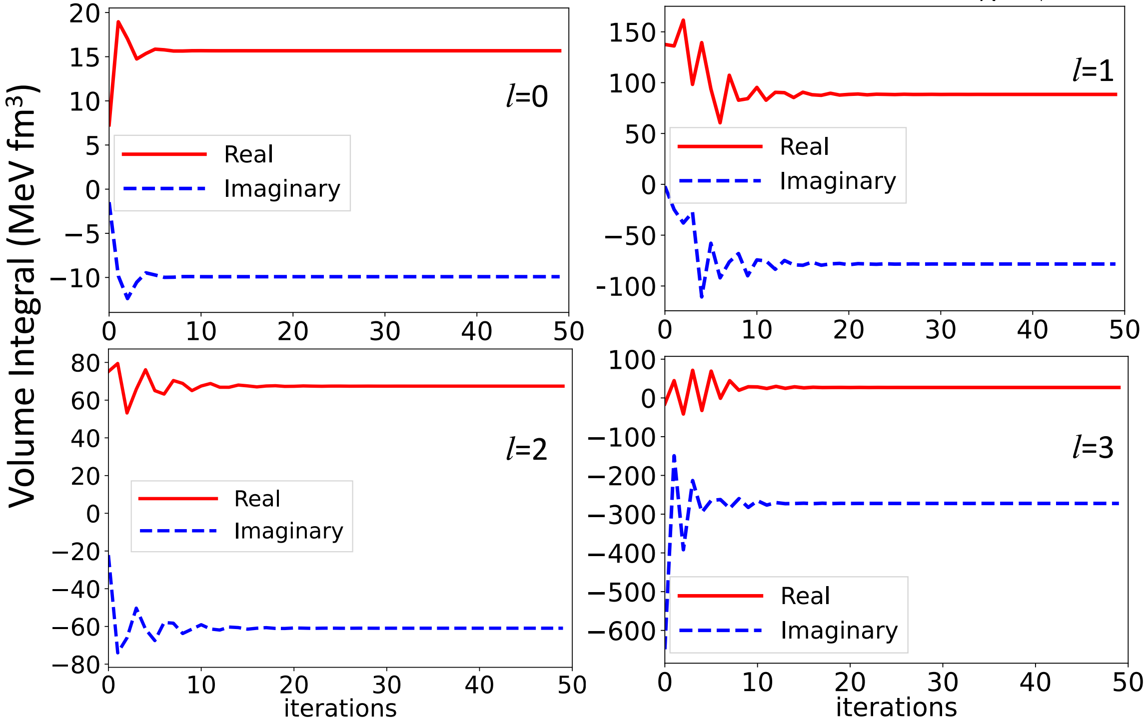
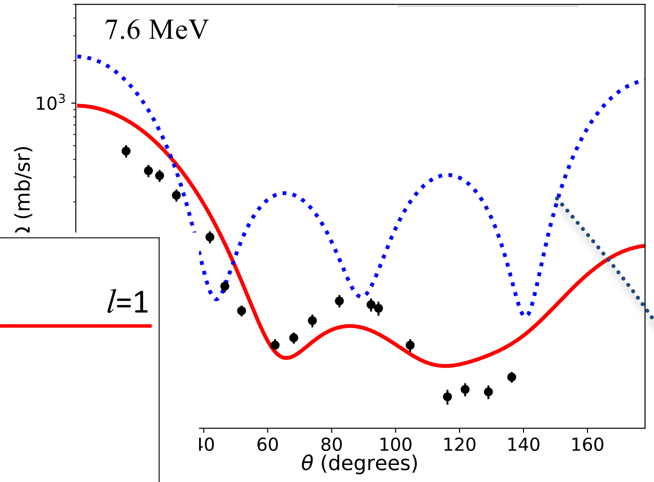
Our ^{24}Mg calculation compares well with experiment

Koning-Delaroche



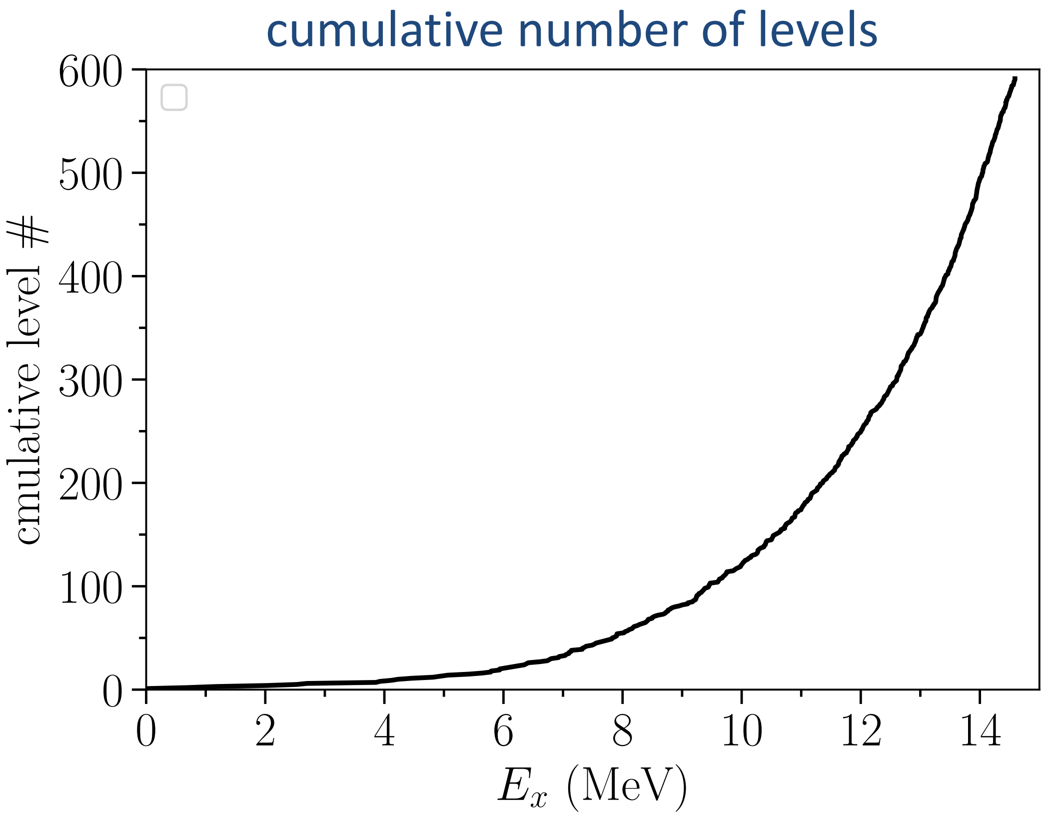
we check for convergence by looking at the volume integrals as a function of the iteration

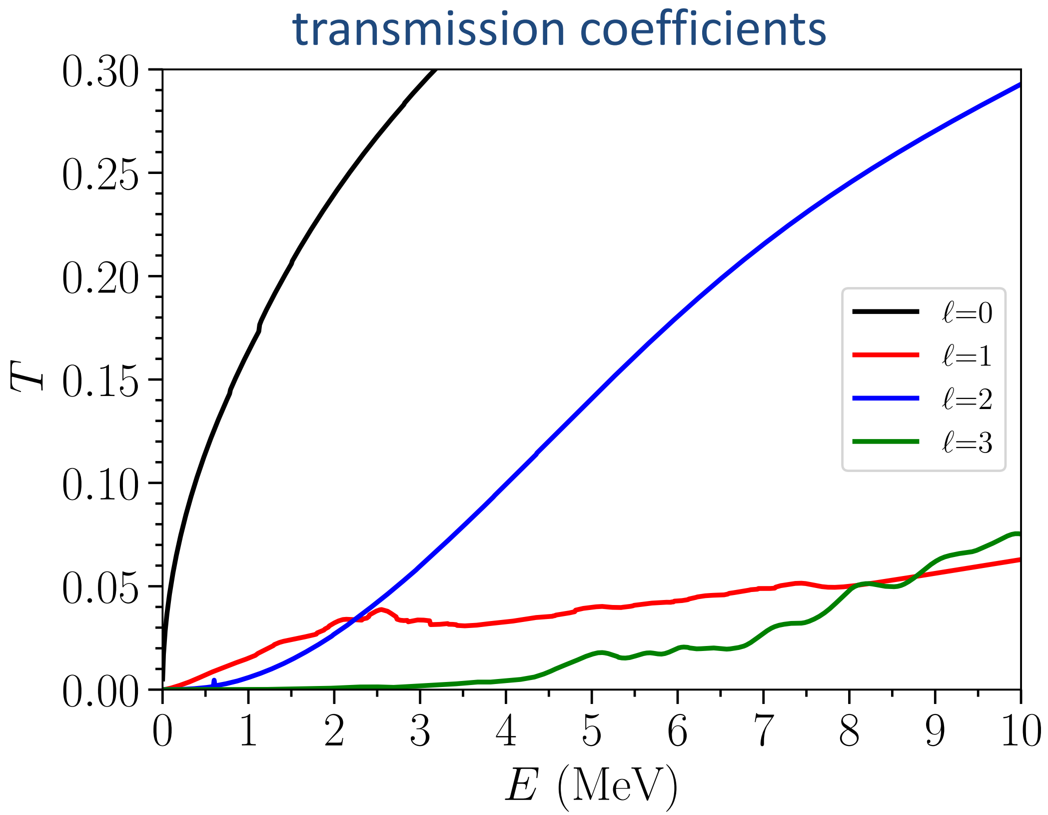




not surprisingly, the static potential alone gives a very wrong result!

The OP, the level density, and the γ strength function are connected through the same underlying physics





We can explicitly check the limits of the statistical model (Hauser-Feshbach approach)

Part 3

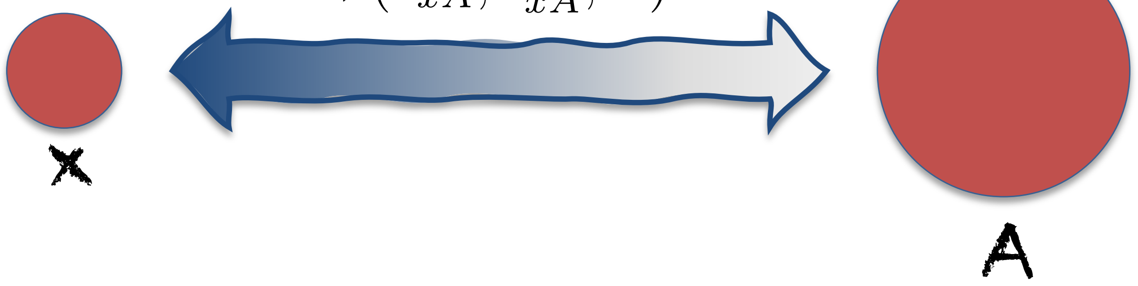
Extending the scope: Green's Function Transfer (GFT)

The Green’s Function Transfer (GFT) formalism

elastic scattering between 2 nuclei x and A

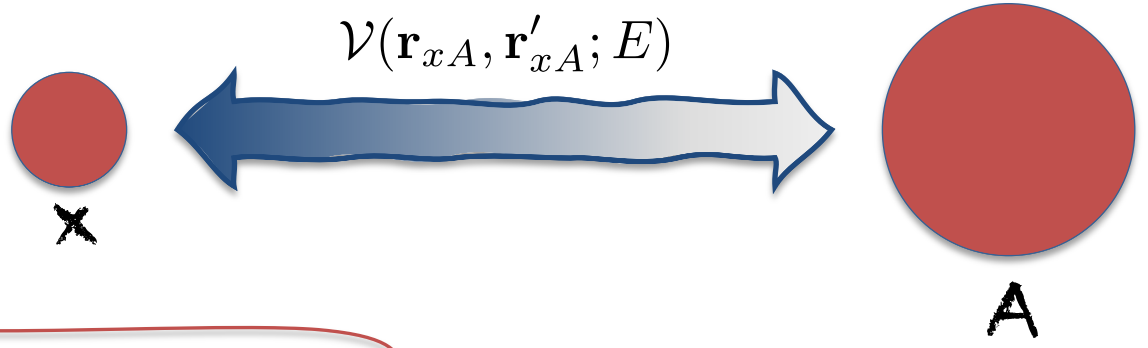
$\mathcal{V}(\mathbf{r}_{xA}, \mathbf{r}'_{xA}; E)$

optical potential



The Green's Function Transfer (GFT) formalism

elastic scattering between 2 nuclei x and A



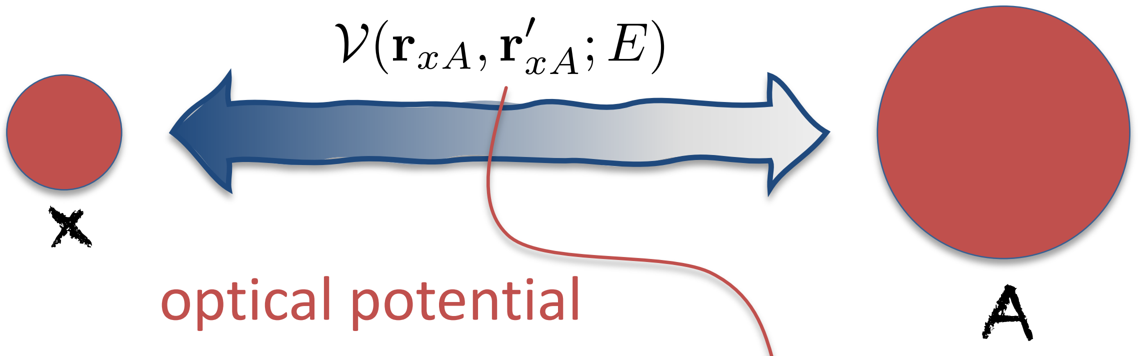
free wave

$$\psi_0(r_{xA}, E) = F(r_{xA}) + G(E)\,\mathcal{V}(E)\,F(r_{xA})$$

Lippmann-Schwinger equation

The Green’s Function Transfer (GFT) formalism

elastic scattering between 2 nuclei x and A



$$\psi_0(r_{xA}, E) = F(r_{xA}) + G(E)\,\mathcal{V}(E)\,F(r_{xA})$$

The Green’s Function Transfer (GFT) formalism

elastic scattering between 2 nuclei x and A

$$\mathcal{V}(\mathbf{r}_{xA}, \mathbf{r}'_{xA}; E)$$

$$\psi_0(r_{xA}, E) = F(r_{xA}) + G(E)\,\mathcal{V}(E)\,F(r_{xA})$$

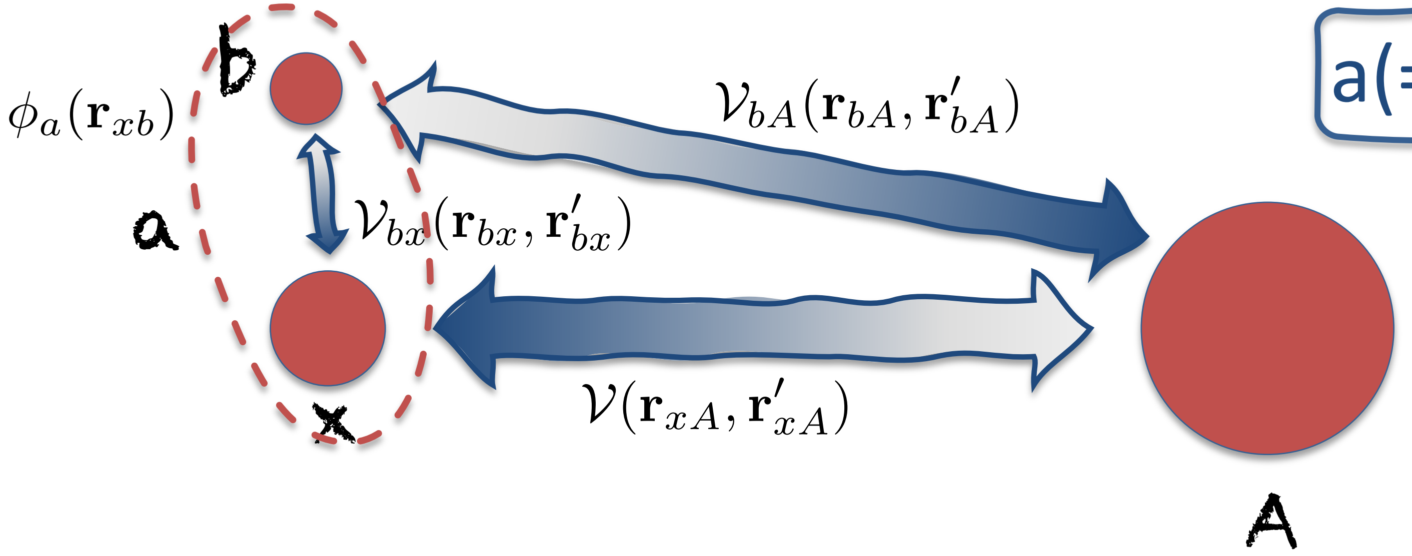
Green’s function

$$G(E) = (E - T_x - \mathcal{V}(E))^{-1}$$

reaction cross section

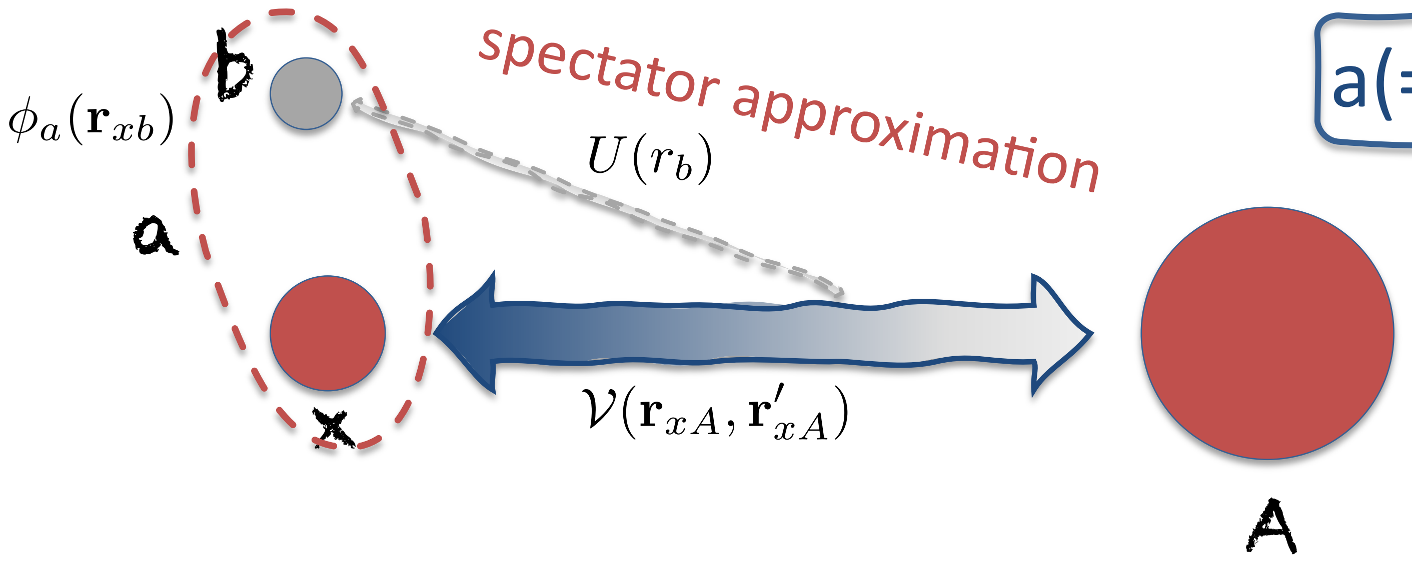
$$\sigma_R = \frac{2\mu}{\hbar k_x}\langle\psi_0|\mathrm{Im}\mathcal{V}|\psi_0\rangle$$

The Green's Function Transfer (GFT) formalism



a(=b+x)+A $\rightarrow$ b+B(=A+x)

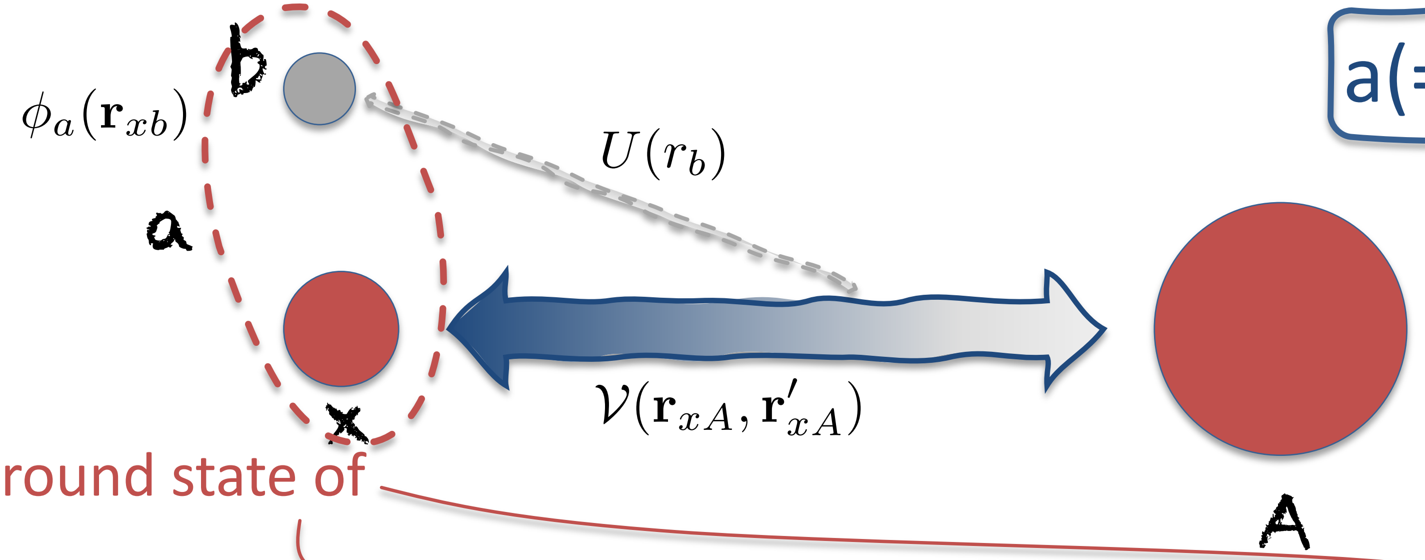
The Green's Function Transfer (GFT) formalism



a(=b+x)+A ⟶ b+B(=A+x)

The Green's Function Transfer (GFT) formalism

$a(=b+x)+A \longrightarrow b+B(=A+x)$

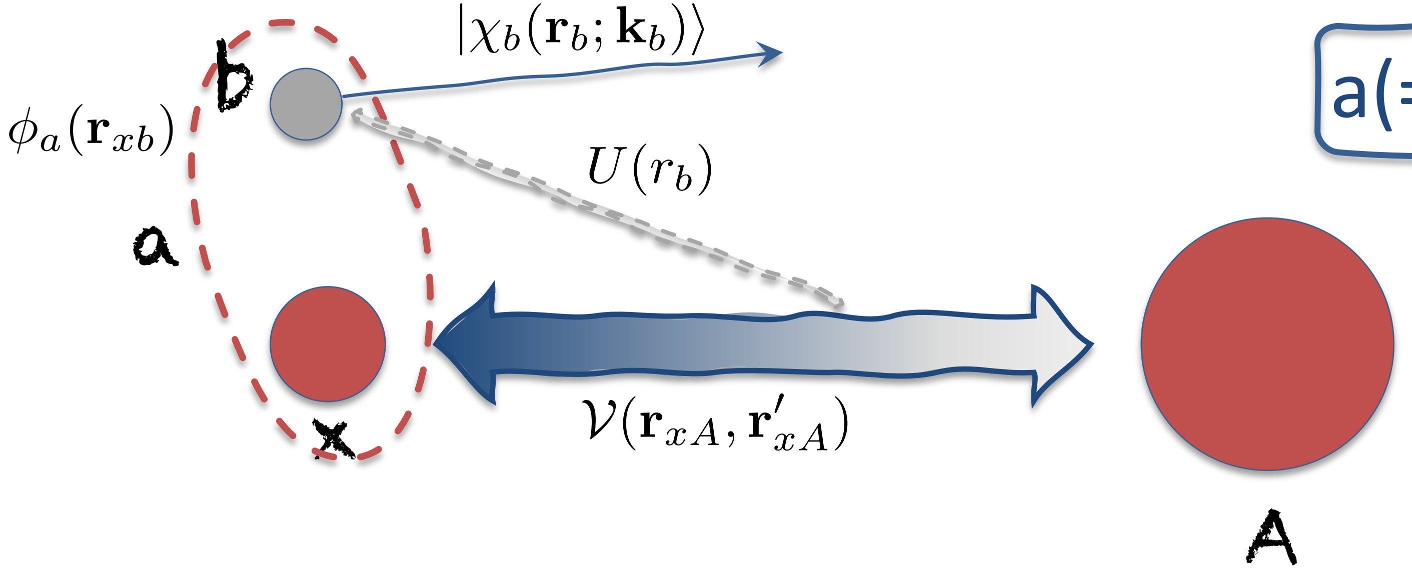


intrinsic ground state of nucleus a

$$\Psi_0(\mathbf{r}_{xA}, \mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E - E_b)\mathcal{P}(\mathbf{r}_b)\left[\mathcal{V}(E - E_b) + U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

free wave for nucleus a

The Green’s Function Transfer (GFT) formalism



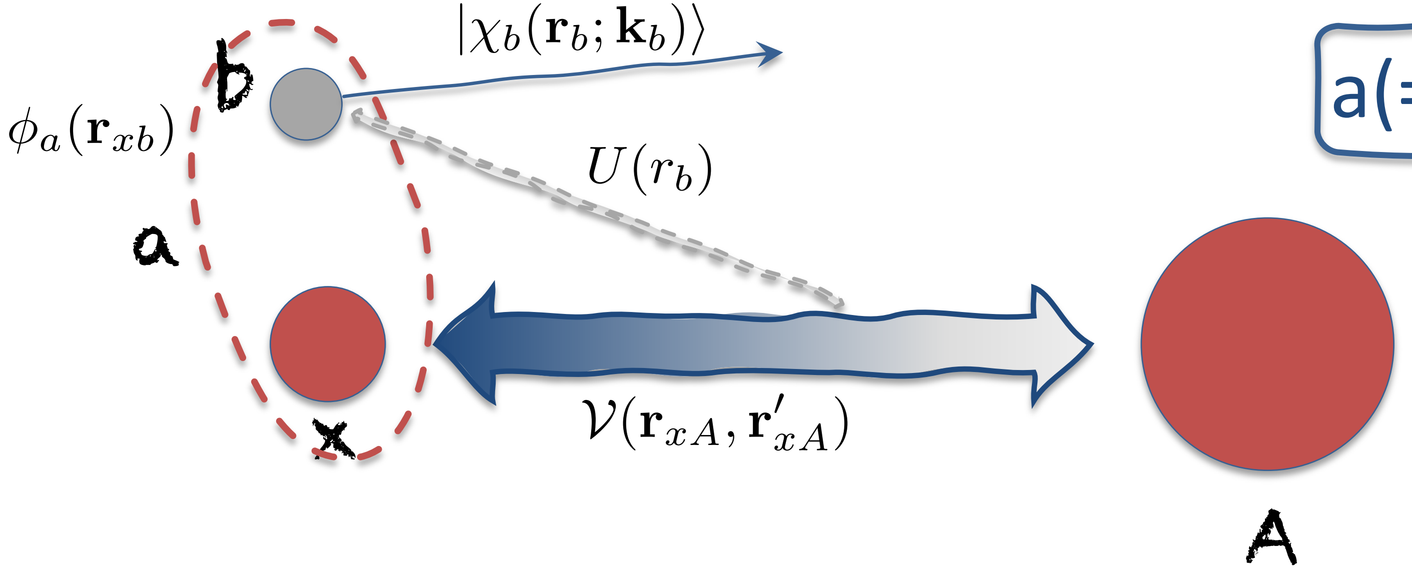
a(=b+x)+A ➡ b+B(=A+x)

$$\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E-E_b)\mathcal{P}(\mathbf{r}_b)\big[\mathcal{V}(E-E_b) + U_b(r_b)\big]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

projector over b states $\mathcal{P}(\mathbf{r}_b) = \int |\chi_b(\mathbf{r}_b;\mathbf{k}_b)\rangle\langle\chi_b(\mathbf{r}_b;\mathbf{k}_b)|\, d\mathbf{k}_b$

$$E_b = \frac{\hbar^2 k_b^2}{2\mu_b}$$

The Green's Function Transfer (GFT) formalism



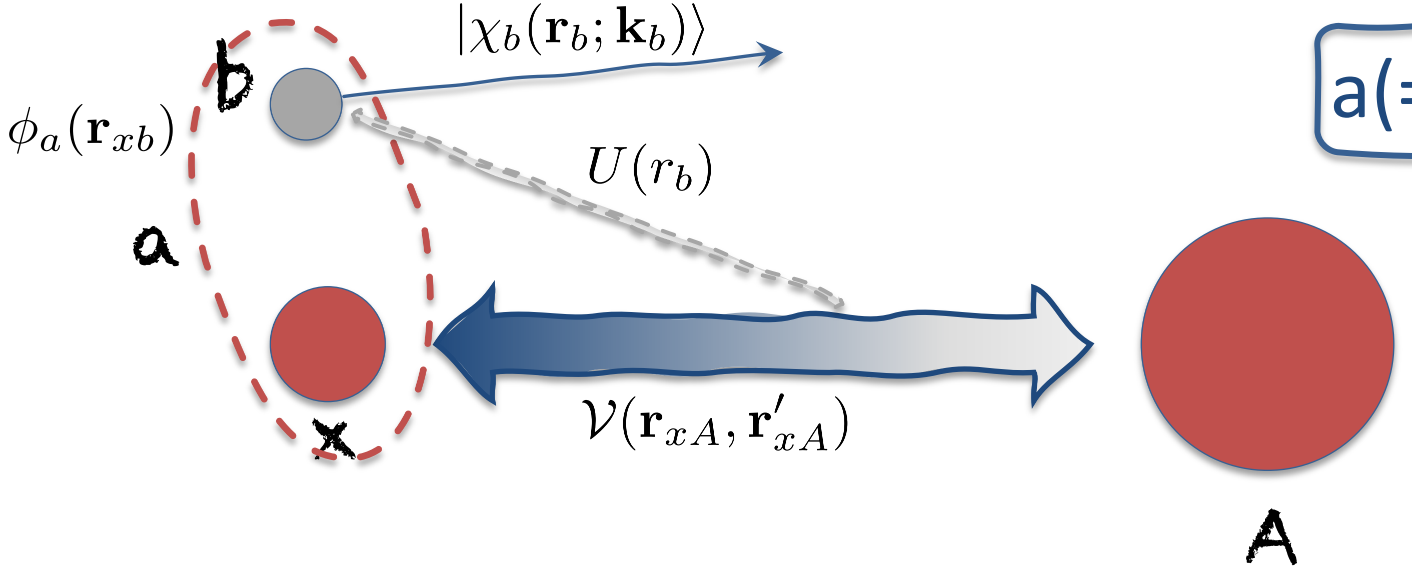
a(=b+x)+A ➡ b+B(=A+x)

$$\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E-E_b)\mathcal{P}(\mathbf{r}_b)\left[\mathcal{V}(E-E_b) + U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

Green's function $G(E) = (E - T_x - \mathcal{V}(E))^{-1}$

Same as for x-A scattering!

The Green's Function Transfer (GFT) formalism

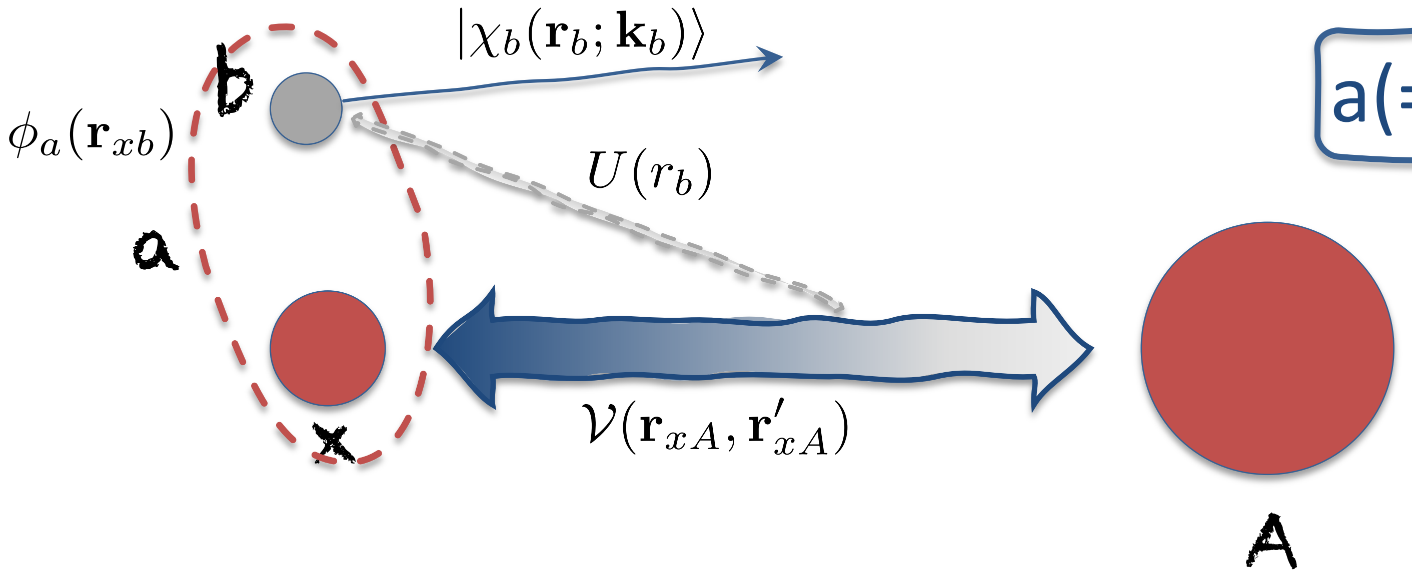


a(=b+x)+A ⟶ b+B(=A+x)

$$\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + \boxed{G(E-E_b)\mathcal{P}(\mathbf{r}_b)}\left[\mathcal{V}(E-E_b)+U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

factorized propagator

The Green's Function Transfer (GFT) formalism

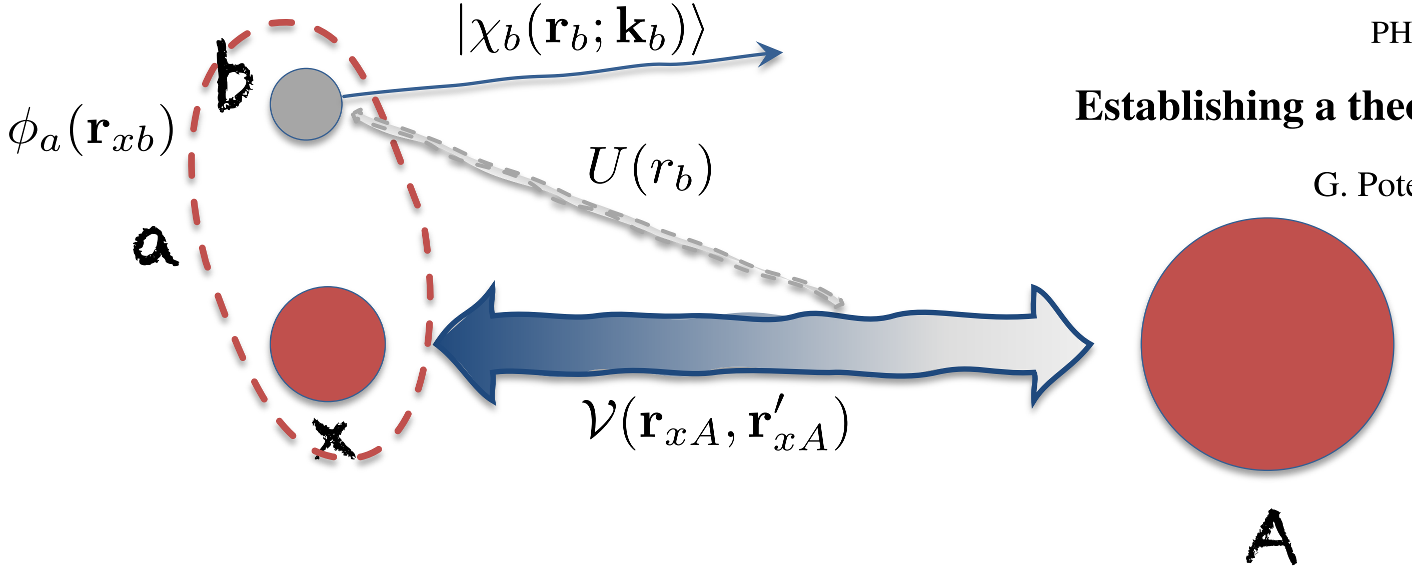


a(=b+x)+A ⟶ b+B(=A+x)

$$\langle\chi_b(\mathbf{r}_b;\mathbf{k}_b)|\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = \langle\chi_b(\mathbf{r}_b;\mathbf{k}_b)|\left(F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E-E_b)\mathcal{P}(\mathbf{r}_b)\left[\mathcal{V}(E-E_b)+U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})\right)$$

project over b state to get x-A wavefunction

The Green's Function Transfer (GFT) formalism



PHYSICAL REVIEW C **92**, 034611 (2015)

Establishing a theory for deuteron-induced surrogate reactions

G. Potel,[1,2] F. M. Nunes,[1,3] and I. J. Thompson[2]

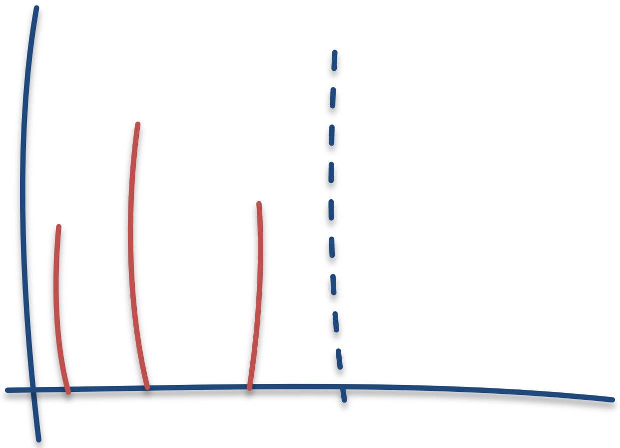
$$\psi_0^I(\mathbf{r}_{xA},E) = \psi^{HM}(\mathbf{r}_a) + G(E-E_b)\left[\mathcal{V}(E-E_b)\psi^{HM} + \langle\chi_b|U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

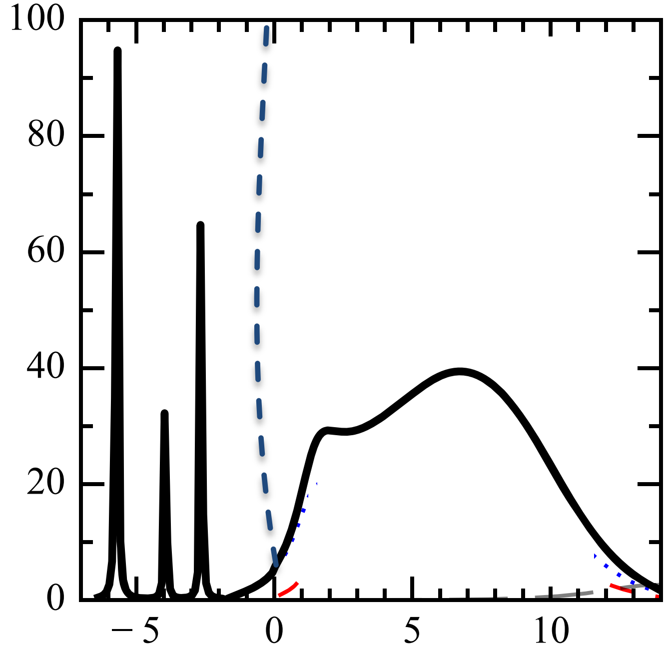
$$\psi^{HM}(\mathbf{r}_{xA}) = \int \chi_b^*(\mathbf{r}_{bB},\mathbf{k}_b)\phi_a(\mathbf{r}_{xb})F(\mathbf{r}_{aA})\,d\mathbf{r}_{xb}$$ Hussein-McVoy term

$$\sigma_R^I(E,E_b) = \frac{2\mu}{\hbar k_x}\langle\psi_0^I|\mathrm{Im}\mathcal{V}(E-E_b)|\psi_0^I\rangle$$ inclusive x–A cross section

DWBA vs GFT

$$\sigma_{i0}^{DWBA} \sim |\langle \psi_i | V | \psi_0 \rangle|^2 \qquad \sigma_R^{GFT}(E) \sim \langle G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM} | \mathrm{Im}\mathcal{V}(E) | G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM} \rangle$$

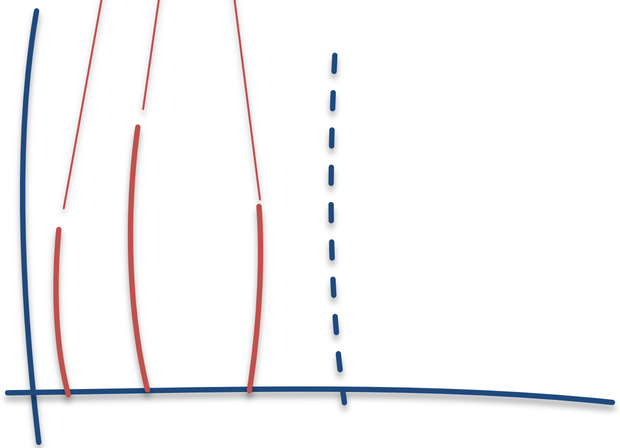




DWBA vs GFT

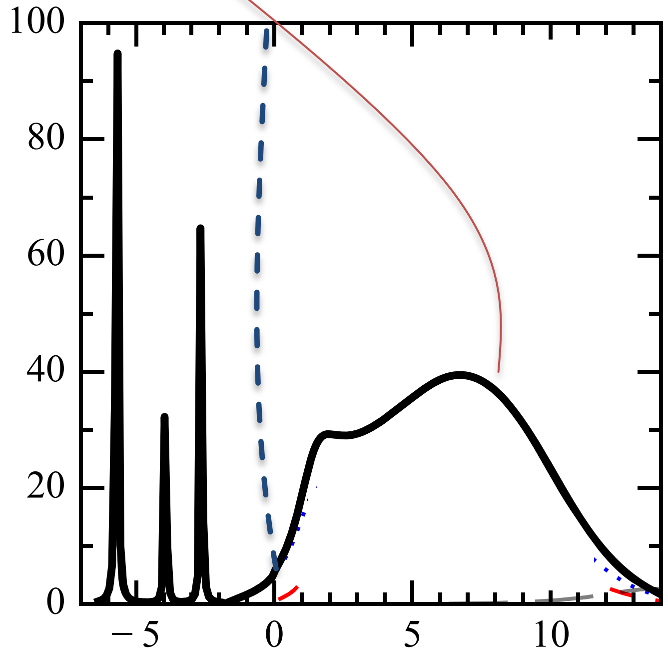
$$\sigma_{i0}^{DWBA} \sim |\langle\psi_i|V|\psi_0\rangle|^2$$

discrete final states



$$\sigma_R^{GFT}(E) \sim \langle G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM}|\mathrm{Im}\mathcal{V}(E)|G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM}\rangle$$

continuous function of E



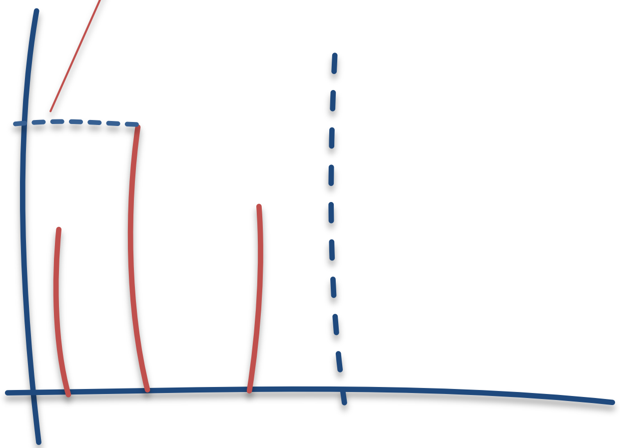
DWBA vs GFT

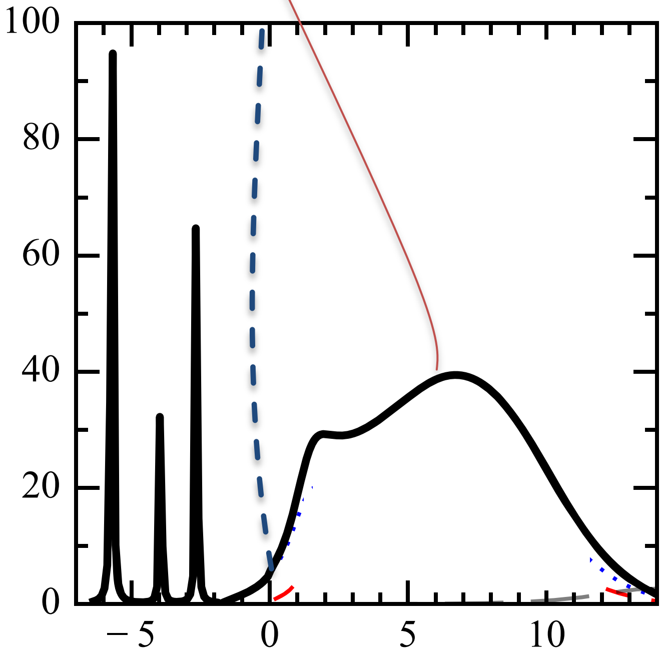
$$\sigma_{i0}^{DWBA} \sim |\langle\psi_i|V|\psi_0\rangle|^2$$

$$\sigma_R^{GFT}(E) \sim \langle G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM}|\mathrm{Im}\mathcal{V}(E)|G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM}\rangle$$

consistent normalization

extracted S=σ_{exp}/σ_{th}

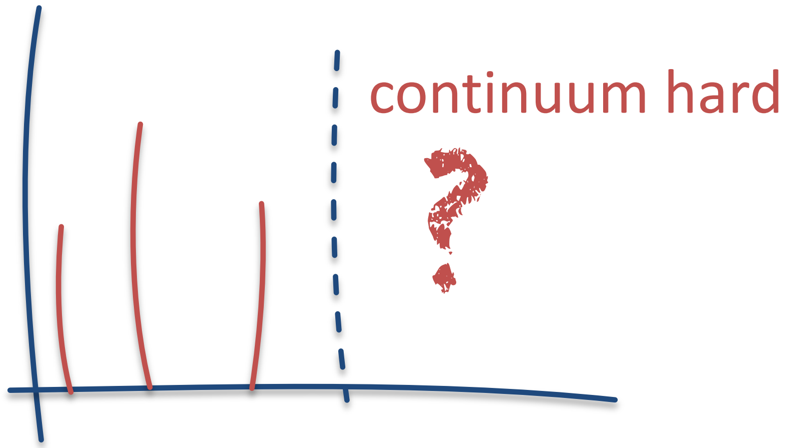


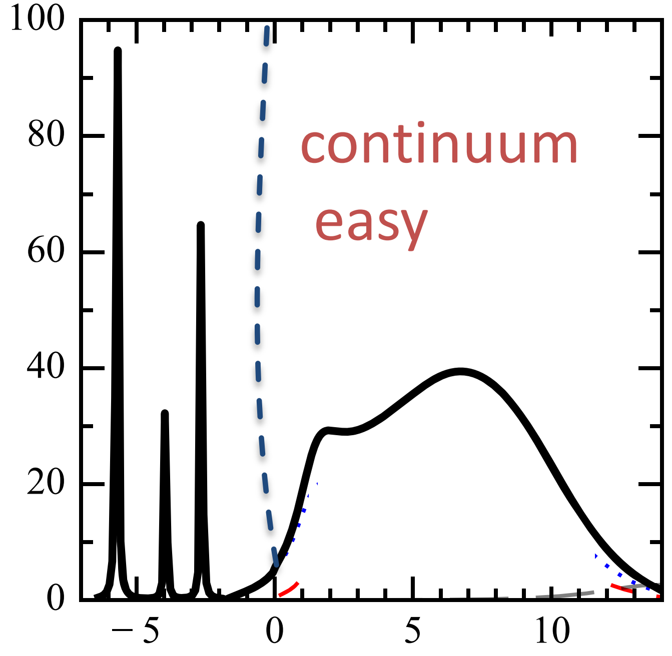


DWBA vs GFT

$$\sigma_{i0}^{DWBA} \sim |\langle\psi_i|V|\psi_0\rangle|^2$$

$$\sigma_R^{GFT}(E) \sim \langle G(E)\left(\mathcal{V}(E)+U_b\right)\psi^{HM}|\mathrm{Im}\mathcal{V}(E)|G(E)\left(\mathcal{V}(E)+U_b\right)\psi^{HM}\rangle$$

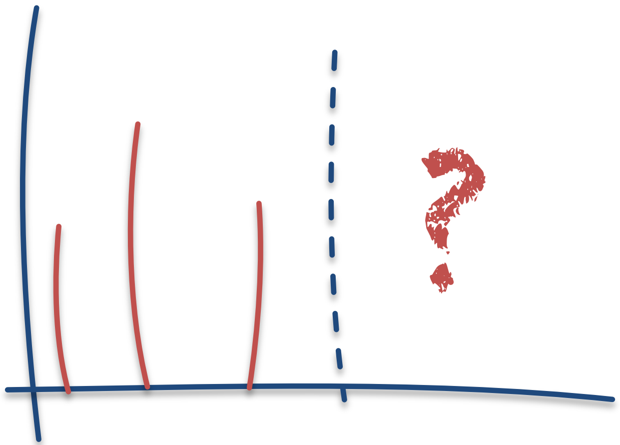




DWBA vs GFT

$$\sigma_{i0}^{DWBA} \sim |\langle \psi_i | V | \psi_0 \rangle|^2$$

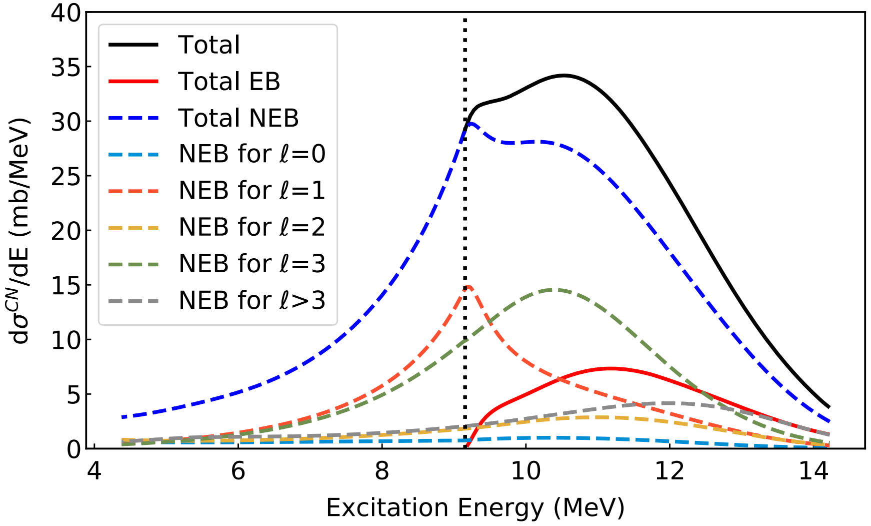
$$\sigma_R^{GFT}(E) \sim \langle G(E)\left(\mathcal{V}(E)+U_b\right)\psi^{HM} | \mathrm{Im}\mathcal{V}(E) | G(E)\left(\mathcal{V}(E)+U_b\right)\psi^{HM} \rangle$$



$$G(E) = (E - T - \mathcal{V}(E))^{-1}$$

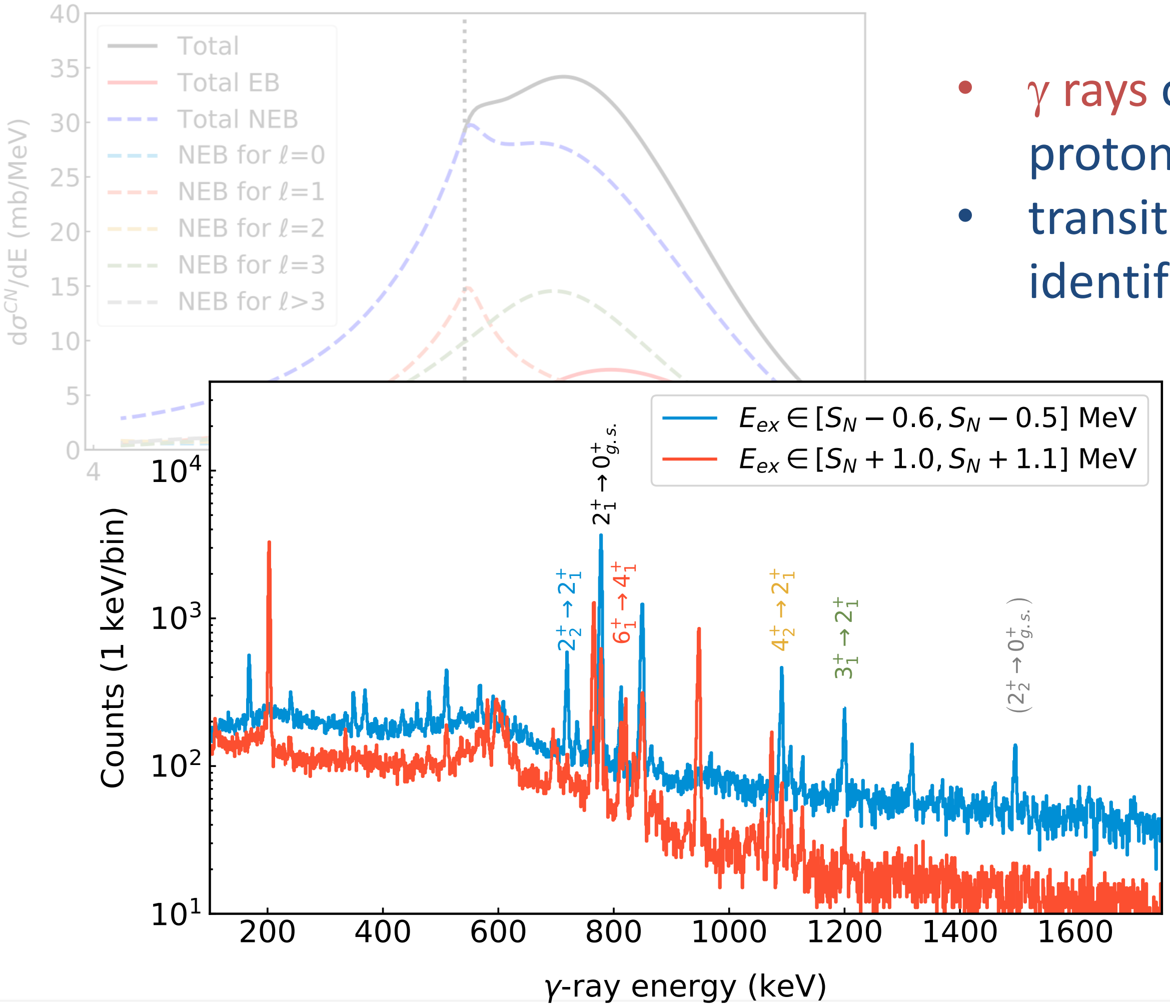
- Consistency between structure and reactions
- Same ingredients as x-A scattering
- Need for tools for inverting Hamiltonians with non-local potentials

Compound nucleus reaction: ^{95}Mo(d,pγ) with Koning-Delaroche OP



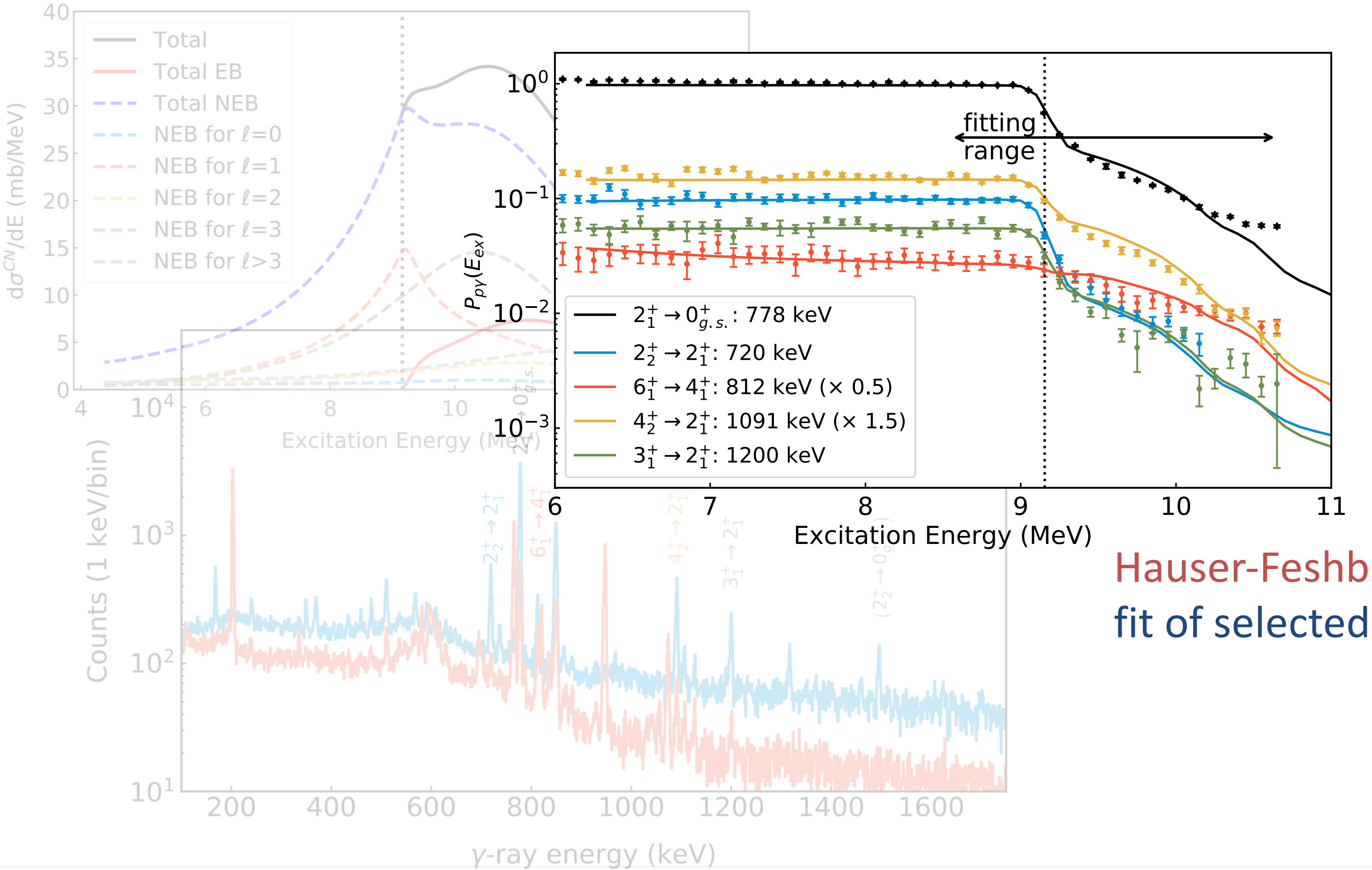
- Absorption of the neutron as a function of excitation energy and spin computed with GFT formalism
- We used the phenomenological Koning-Delaroche OP

Compound nucleus reaction: ^{95}Mo(d,pγ) with Koning-Delaroche OP



- γ rays observed in coincidence with protons
- transitions from both ^{95}Mo and ^{96}Mo are identified

Compound nucleus reaction: ^{95}Mo(d,pγ) with Koning-Delaroche OP



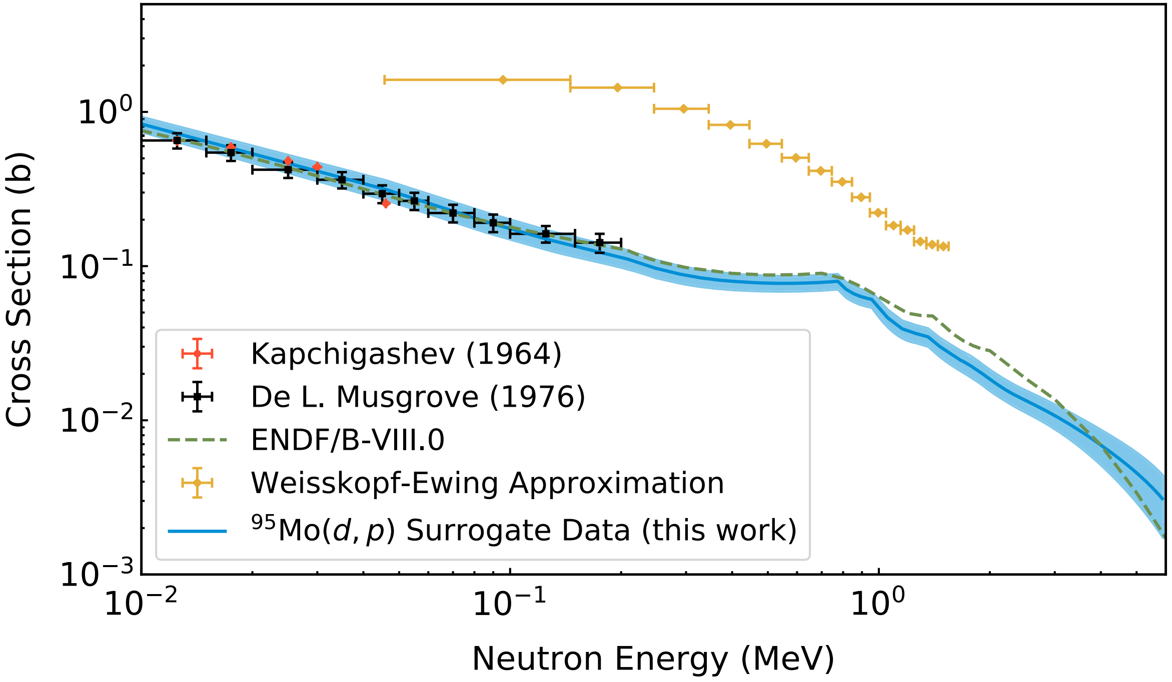
Hauser-Feshbach parameters determined by fit of selected γ lines (J. Escher)

Compound nucleus reaction: ^{95}Mo(d,pγ) with Koning-Delaroche OP

PHYSICAL REVIEW LETTERS **122,** 052502 (2019)

Towards Neutron Capture on Exotic Nuclei: Demonstrating $(d,p\gamma)$ as a Surrogate Reaction for (n,γ)

A. Ratkiewicz,[1,2,*] J. A. Cizewski,[2] J. E. Escher,[1] G. Potel,[3,4] J. T. Burke,[1] R. J. Casperson,[1] M. McCleskey,[5] R. A. E. Austin,[6] S. Burcher,[2] R. O. Hughes,[1,7] B. Manning,[2] S. D. Pain,[8] W. A. Peters,[9] S. Rice,[2] T. J. Ross,[7] N. D. Scielzo,[1] C. Shand,[2,10] and K. Smith[11]



- The obtained Hauser-Feshbach parameters are used to calculate (n,γ)
- We found an excellent agreement with the direct measurement.

Compound nucleus reaction: ^{95}Mo(d,pγ) with Koning-Delaroche OP

PHYSICAL REVIEW LETTERS **122,** 052502 (2019)

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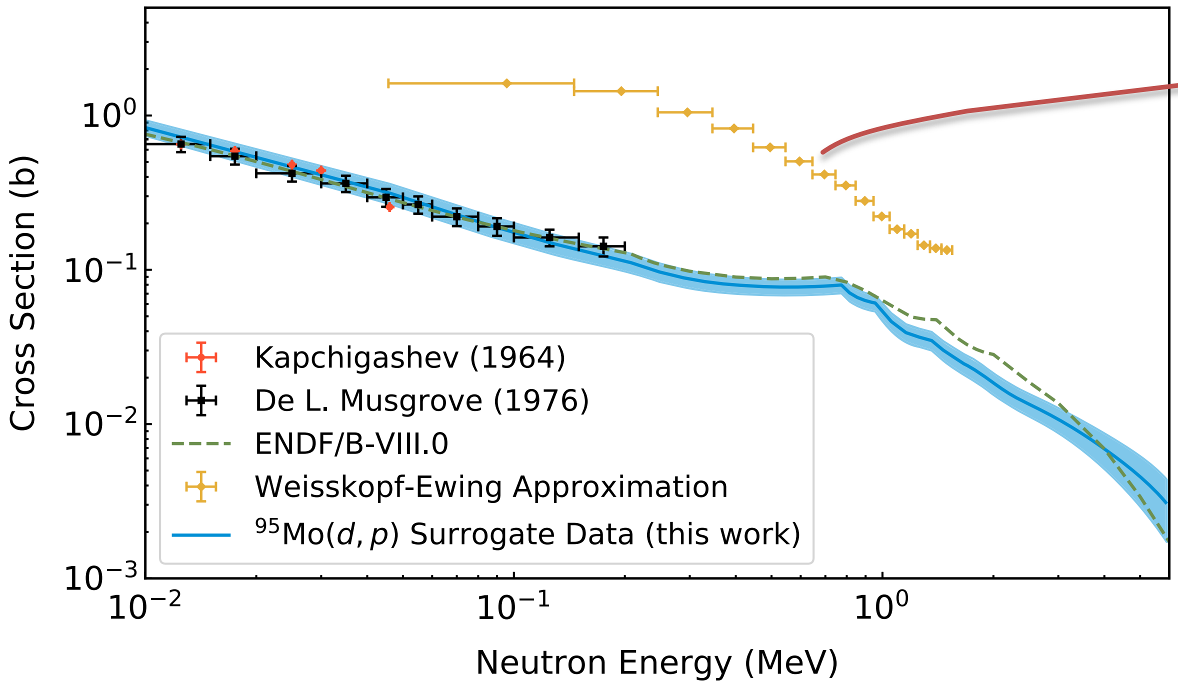
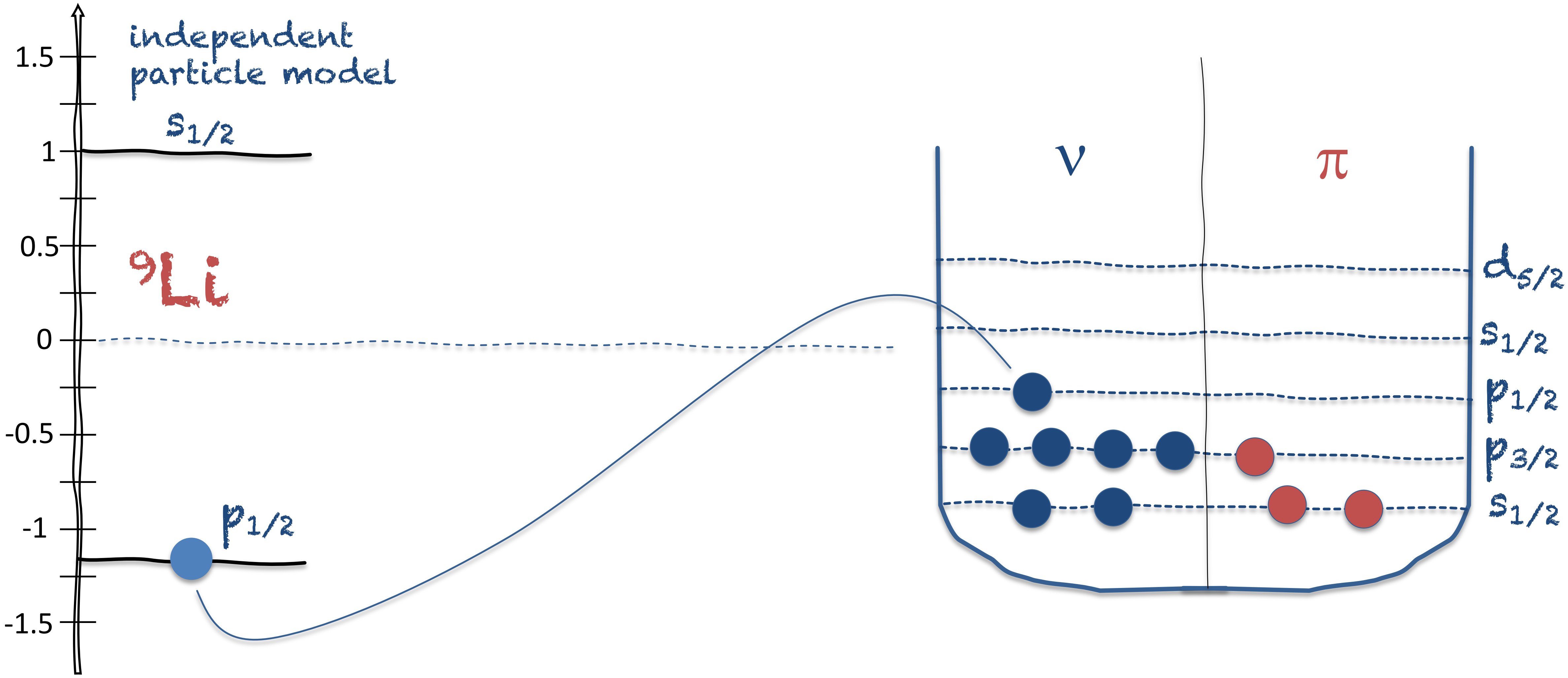


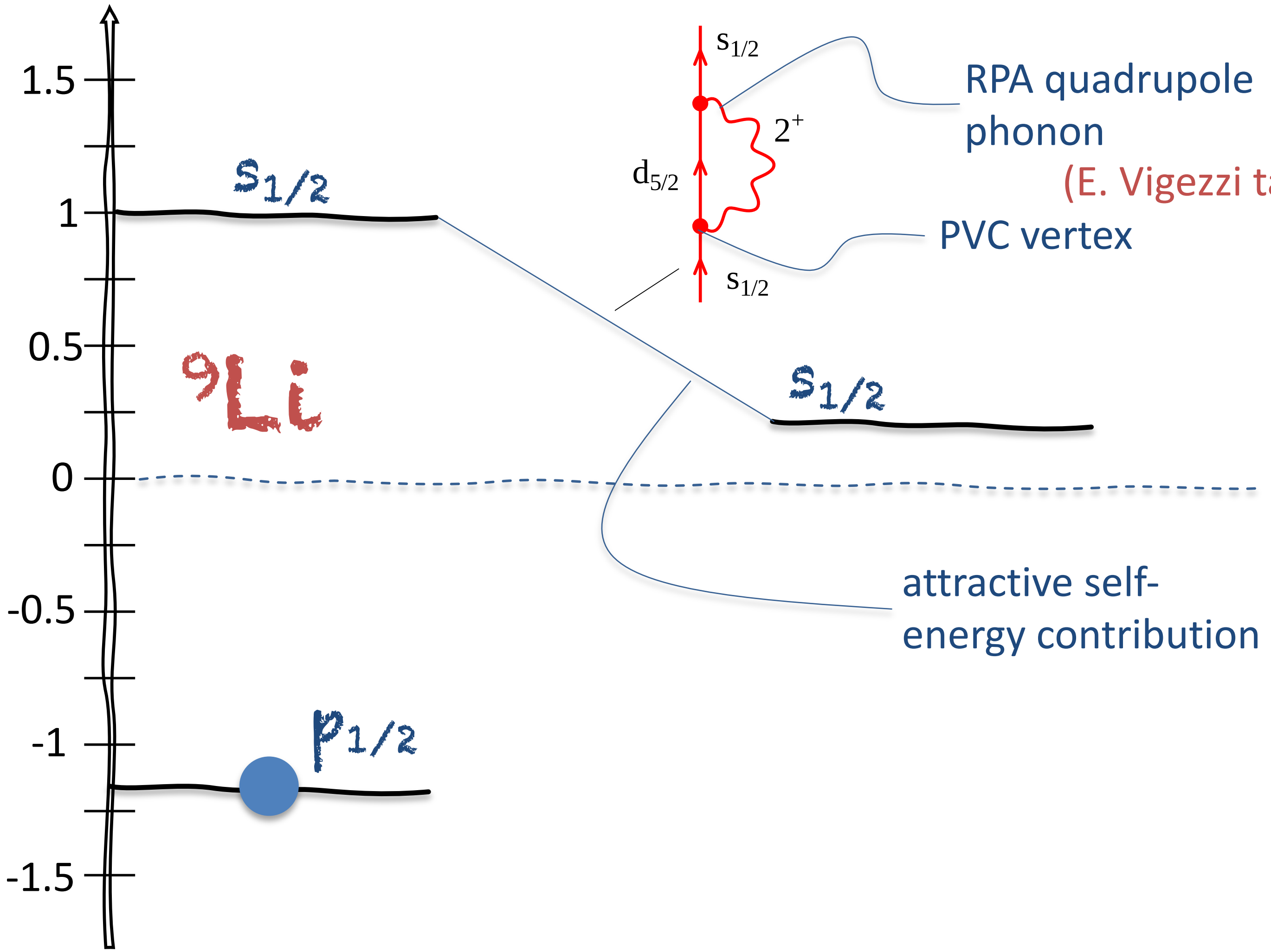
a failure to account for the initial spin distribution (Weisskopf-Ewing approximation) leads to poor results!

- The obtained Hauser-Feshbach parameters are used to calculate (n,γ)
- We found an excellent agreement with the direct measurement.

Direct reaction: ^{9}Li(d,p) with Nuclear Field Theory (NFT)



Direct reaction: ^{9}Li(d,p) with Nuclear Field Theory (NFT)



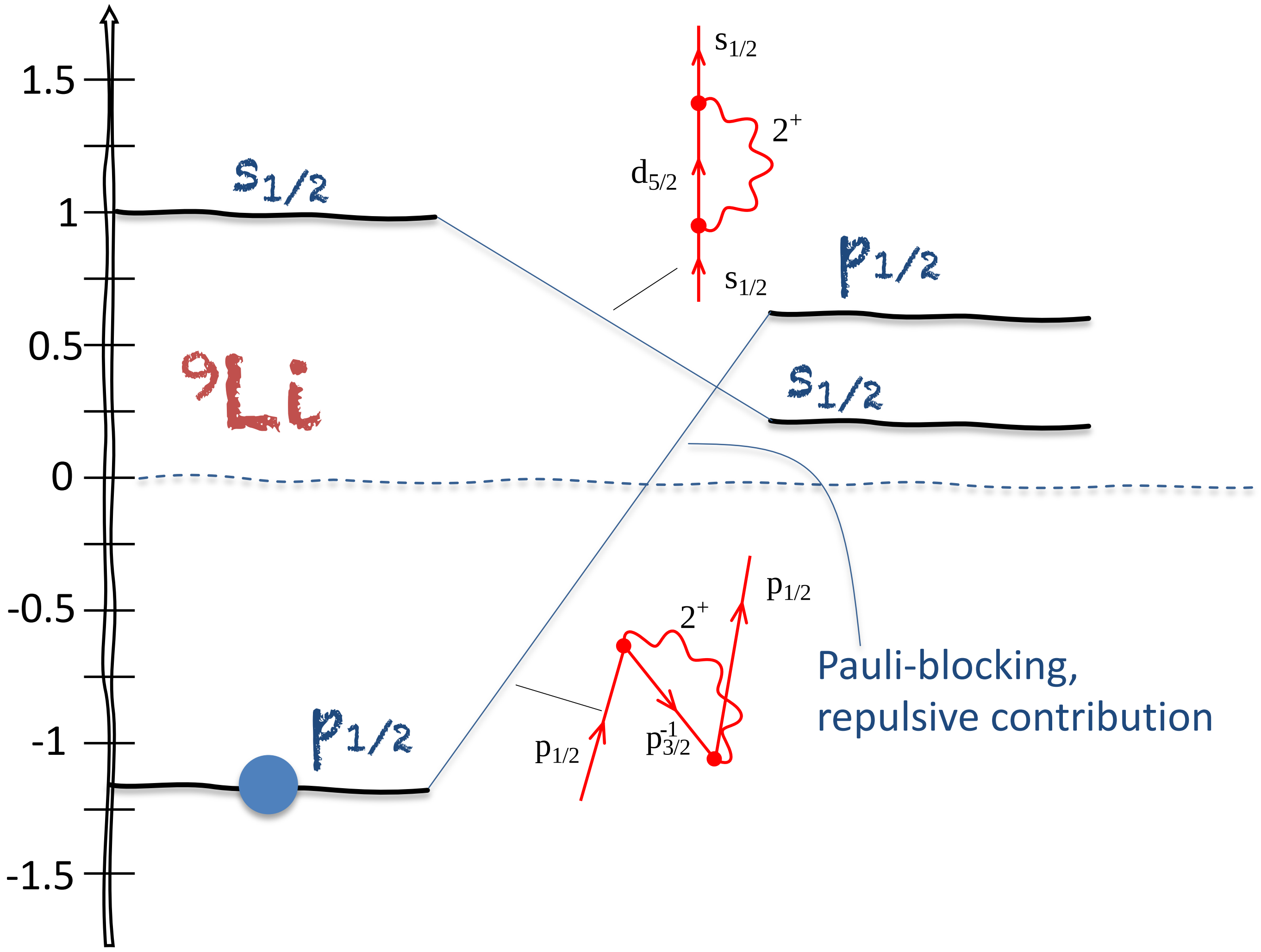
RPA quadrupole phonon

(E. Vigezzi talk yesterday)

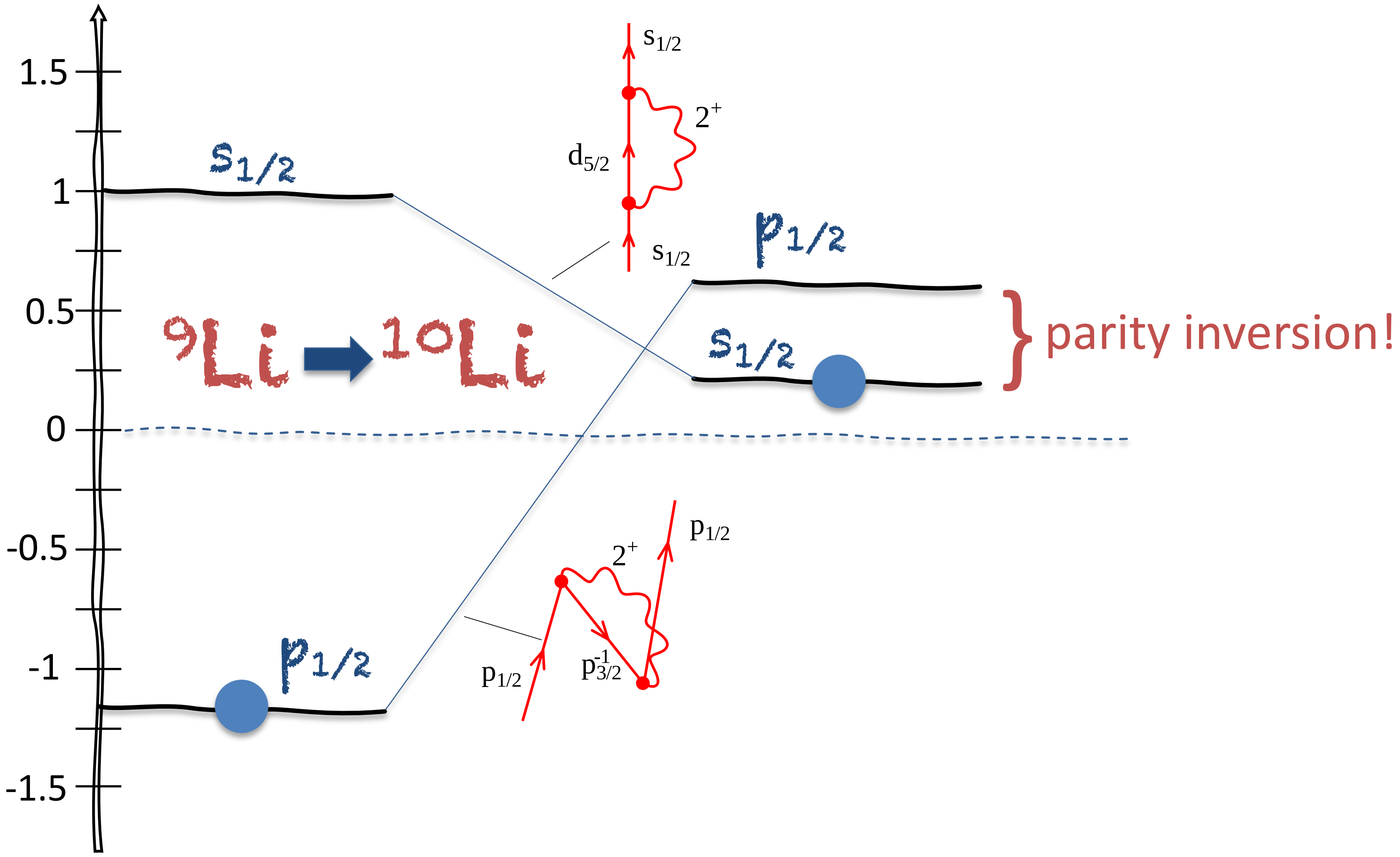
PVC vertex

attractive self-energy contribution

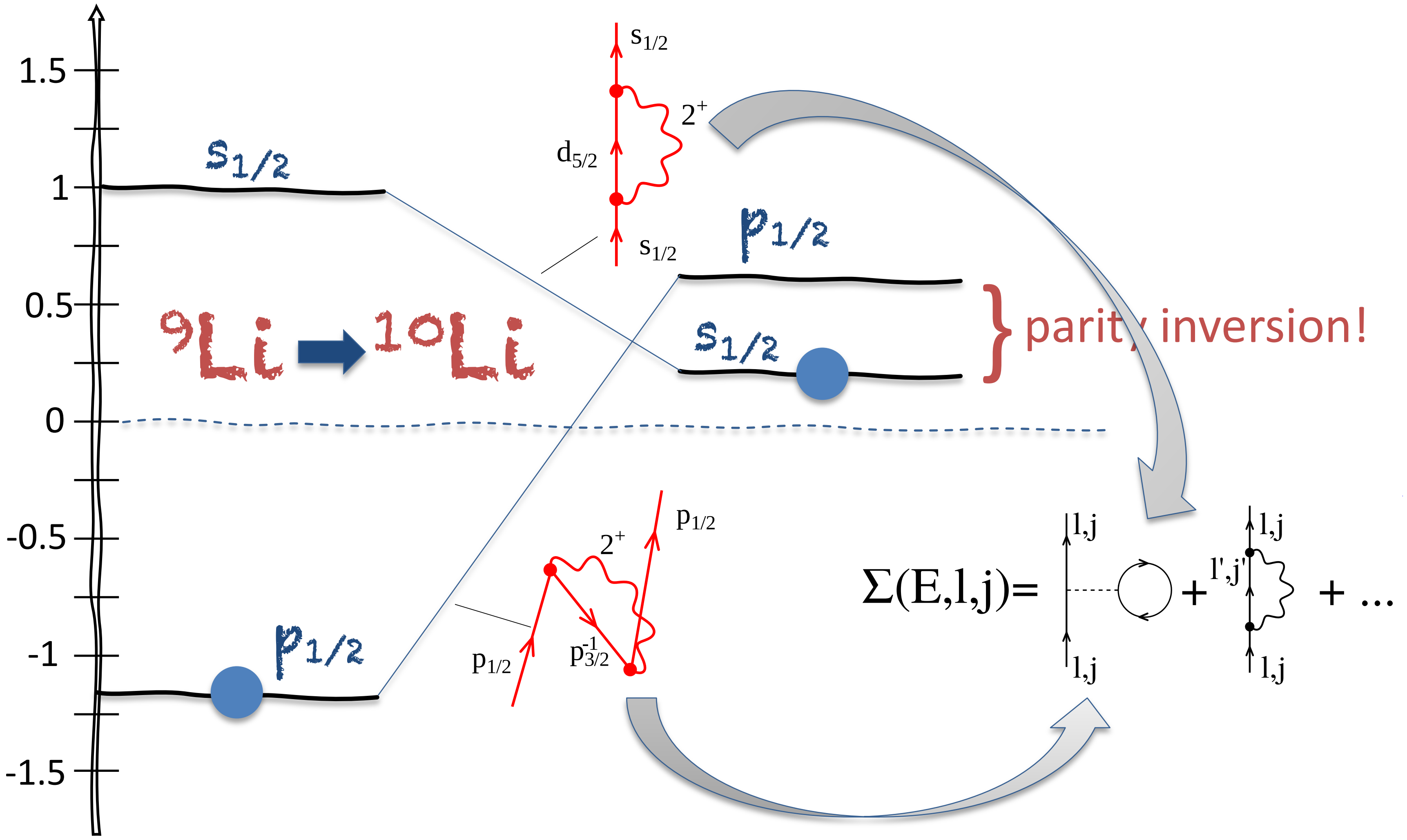
Direct reaction: ^{9}Li(d,p) with Nuclear Field Theory (NFT)



Direct reaction: ^{9}Li(d,p) with Nuclear Field Theory (NFT)

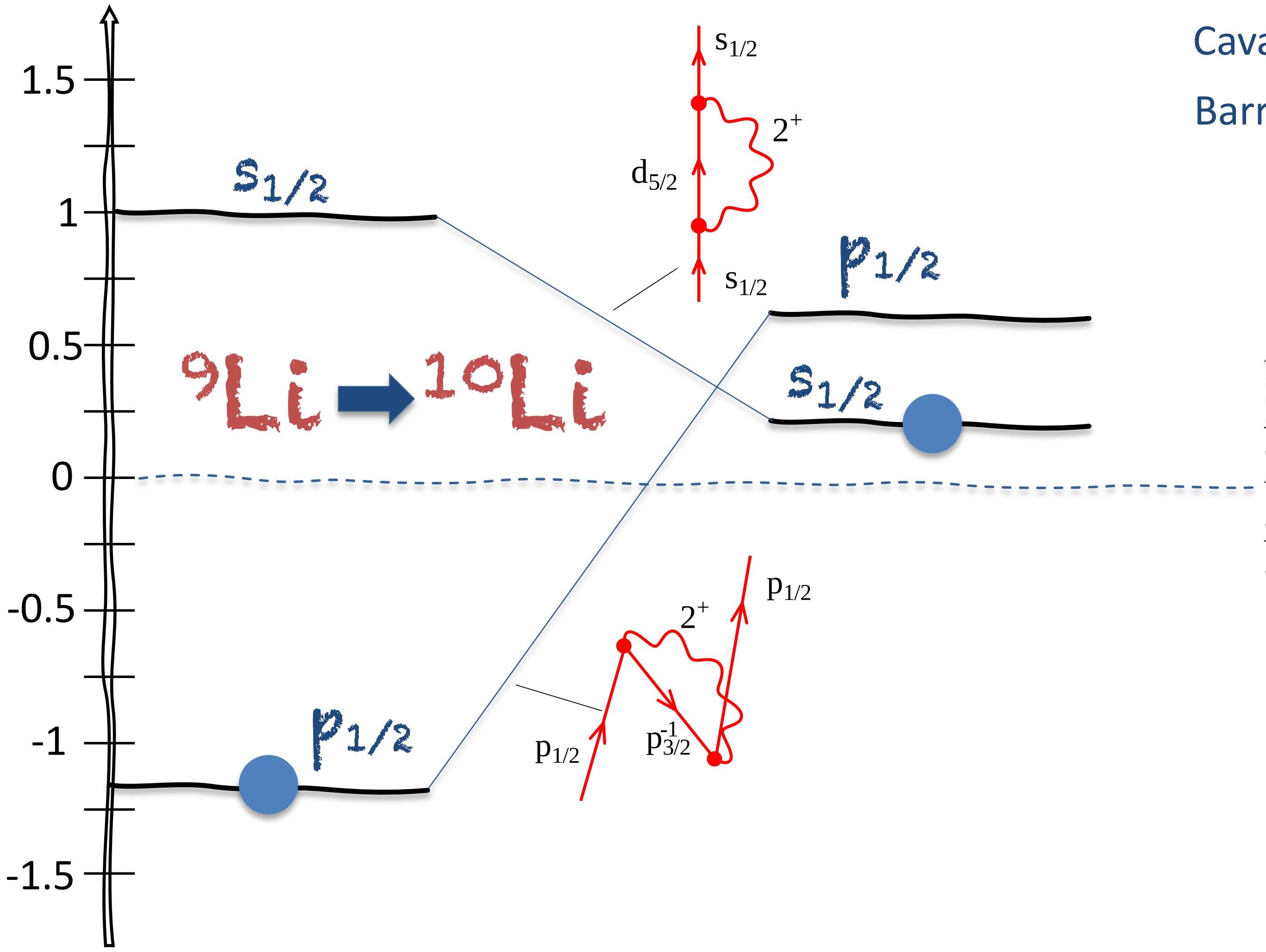


Direct reaction: ^{9}Li(d,p) with Nuclear Field Theory (NFT)



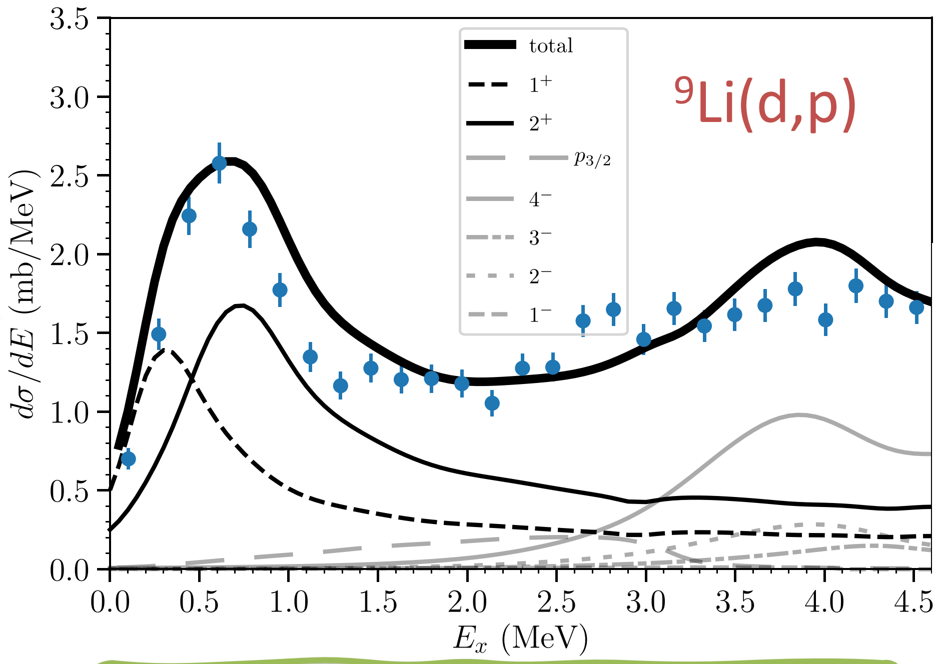
$$\Sigma(E,l,j)= \ldots + \ldots + \ldots$$

Direct reaction: ^{9}Li(d,p) with Nuclear Field Theory (NFT)



Cavallaro *et al.*, PRL **118**, 012701 (2017)

Barranco, GP, Vigezzi, Broglia PRC **101**, 031305(R) (2020)



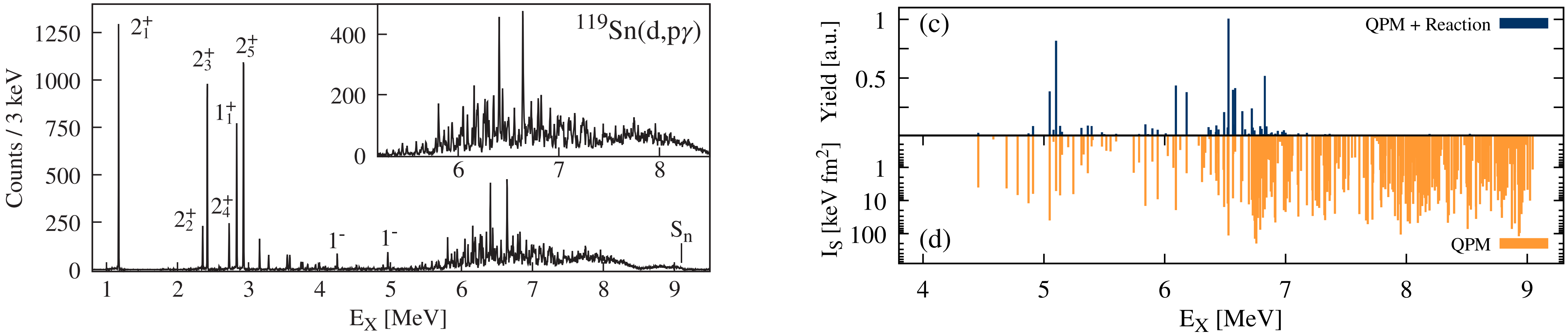
theoretical description validated by experiment

Applications: population of low-lying dipole states in $^{120}Sn(1^-)$ with (d,p) reactions

PHYSICAL REVIEW LETTERS **127,** 242501 (2021)

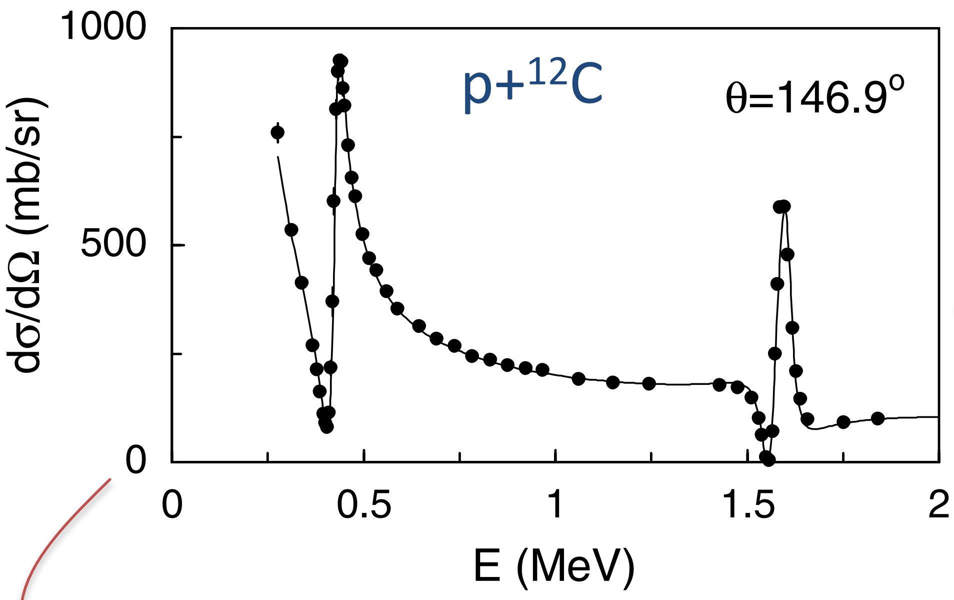
Microscopic Structure of the Low-Energy Electric Dipole Response of ^{120}Sn

M. Weinert,[1,*] M. Spieker,[2] G. Potel,[3] N. Tsoneva,[4] M. Müscher,[1] J. Wilhelmy,[1] and A. Zilges[1]



Structure of ^{120}Sn calculated within the Quasiparticle Phonon Model (QPM) by N. Tsoneva

The GFT formalism suggests an *R*-matrix parametrization for the indirect cross section



$$\frac{d\sigma(E)}{d\sigma} \propto |T_{i0}(E)|^2$$

$$T_{i0}(E) = \sqrt{P_i(E)P_0(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))}$$

T-matrix partial widths and energy parameters fitted from data

The GFT formalism suggests an *R*-matrix parametrization for the indirect cross section

connection between direct and indirect R-matrix parameters

example:

- direct: α scattering ($T_{i0}(E)$)
- indirect: (^{6}Li,d). ($T^I_{i0}(E)$)

The GFT formalism suggests an *R*-matrix parametrization for the indirect cross section

connection between direct and indirect R-matrix parameters

example:

- direct: α scattering ($T_{i0}(E)$)
- indirect: (^{6}Li,d). ($T^I_{i0}(E)$)

indirect T-matrix

$$T^I_{i0}(E) = \int T_{i0}(E_k)\, g(\mathbf{k}; E)\, d\mathbf{k}.$$

direct T-matrix

The GFT formalism suggests an *R*-matrix parametrization for the indirect cross section

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example:

- direct: α scattering ($T_{i0}(E)$)
- indirect: (^{6}Li,d). ($T^I_{i0}(E)$)

indirect T-matrix

$$E_k = \frac{\hbar^2 k^2}{2\mu}$$

$$T^I_{i0}(E) = \int T_{i0}(E_k)\, g(\mathbf{k}; E)\, d\mathbf{k}.$$

direct T-matrix

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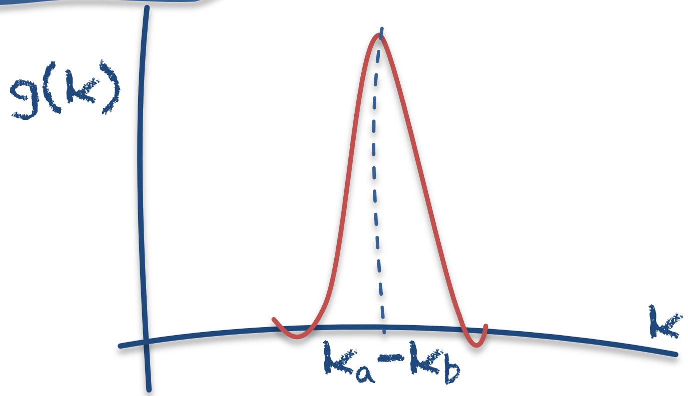

indirect T-matrix

$$T^I_{i0}(E) = \int T_{i0}(E_k)\, g(\mathbf{k}; E)\, d\mathbf{k}.$$

direct T-matrix

broadening factor

$$g(\mathbf{k}; E) = \int \psi^{HM}(\mathbf{r}_{xA}; E)\, F^*(\mathbf{r}_{xA}, \mathbf{k})\, d\mathbf{r}_{xA}$$

The GFT formalism suggests an *R*-matrix parametrization for the indirect cross section

connection between direct and indirect R-matrix parameters

example:

- direct: α scattering
- indirect: (^{6}Li,d)

indirect T-matrix

$$T^I_{i0} = \int \sqrt{P_i(E_k)P_0(E_k)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E_k)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E_k) + iP_c(E_k))}\, g(\mathbf{k})\, d\mathbf{k}.$$

$$E_k = \frac{\hbar^2 k^2}{2\mu}$$

broadening factor

$$g(\mathbf{k}) = \int \psi^{HM}(\mathbf{r}_{xA})\, F^*(\mathbf{r}_{xA}, \mathbf{k})\, d\mathbf{r}_{xA}$$

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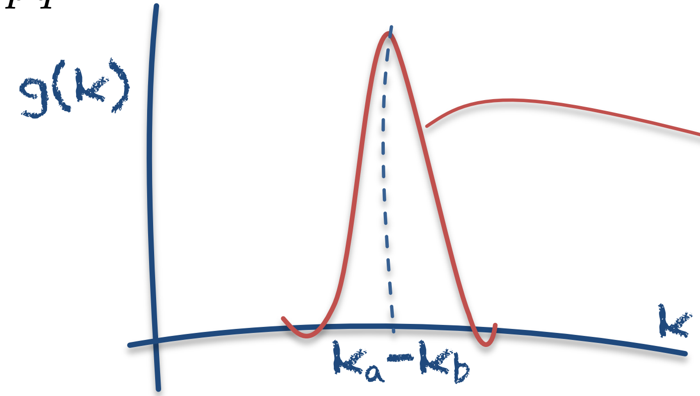


$$g(\mathbf{k}) = \int \psi^{HM}(\mathbf{r}_{xA})\, F^*(\mathbf{r}_{xA}, \mathbf{k})\, d\mathbf{r}_{xA}$$

The GFT formalism suggests an *R*-matrix parametrization for the indirect cross section

- If the broadening distribution is narrow, the T-matrix can be evaluated at the peak
- This is essentially the approximation made by Barker in *Aust. J. Phys*. **20** (341) 1967 for isolated resonances

$$T^I_{i0} = \int \sqrt{P_i(E_k)P_0(E_k)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E_k)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E_k) + iP_c(E_k))} \, g(\mathbf{k})\, d\mathbf{k}.$$

$$\Downarrow$$

$$T^I_{i0} \approx \sqrt{P_i(E_k^{max})P_0(E_k^{max})} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E_k^{max})\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E_k^{max}) + iP_c(E_k^{max}))} \int g(\mathbf{k})\, d\mathbf{k}.$$

Conclusions and some perspectives

- The calculation of the OP provides a flexible and versatile connection between structure and reactions, including 3-body reactions (with GFT)
- It can be used across different regimes (compound vs direct; bound vs unbound…)

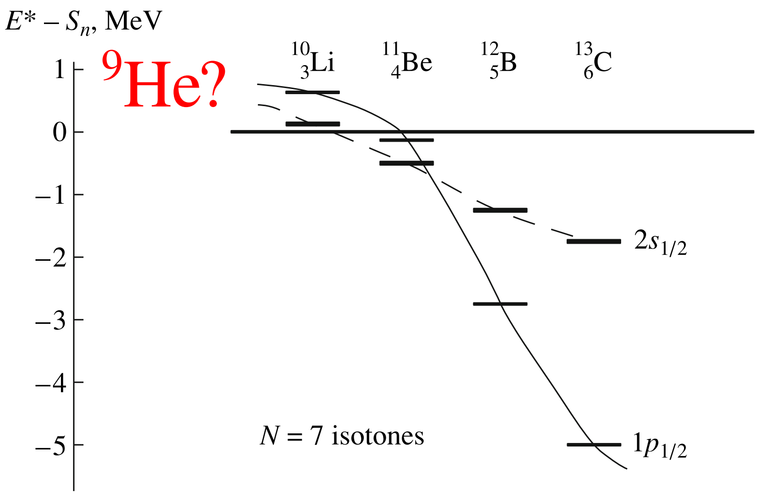
what's next?

- Implement the recommendations of the community.
- Disentangle direct, pre-equilibrium, and compound reactions.
- Explore the limits of validity of the statistical model.
- Systematic implementation for deformed nuclei

Thank you!

Lawrence Livermore National Laboratory

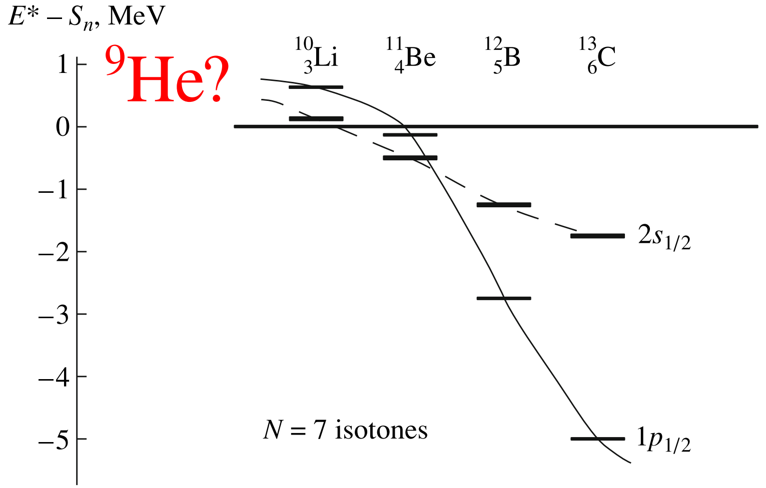
Teaser: ^{9}He structure with ^{8}He(d,p)



is ^{9}He parity-inverted?

the parity of the unbound ^{9}He ground state is still controversial

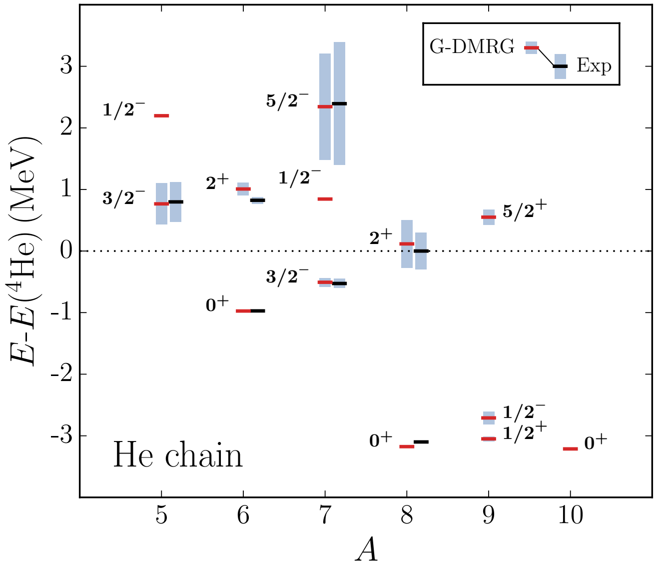
Teaser: ^{9}He structure with ^{8}He(d,p)



is ^{9}He parity-inverted?

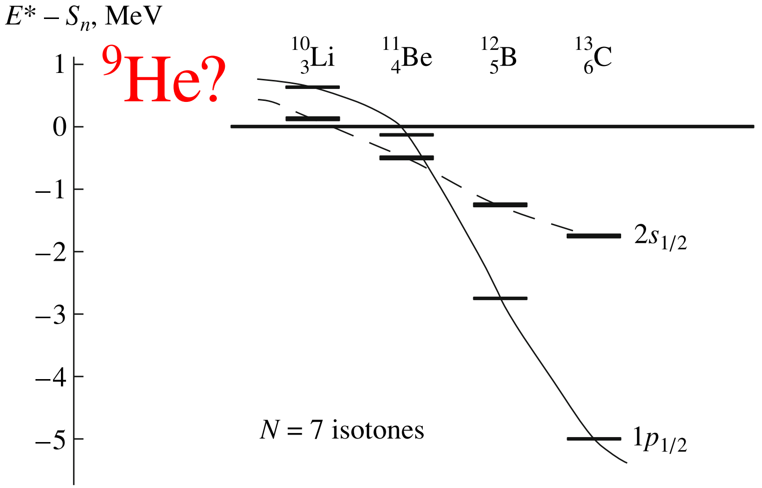
the parity of the unbound ^{9}He ground state is still controversial

K. Fossez, J. Rotureau, W. Nazarewicz.
Phys. Rev. C **98** (2018) 061302(R)



we use a structure approach with explicit inclusion of the continuum (K. Fossez)

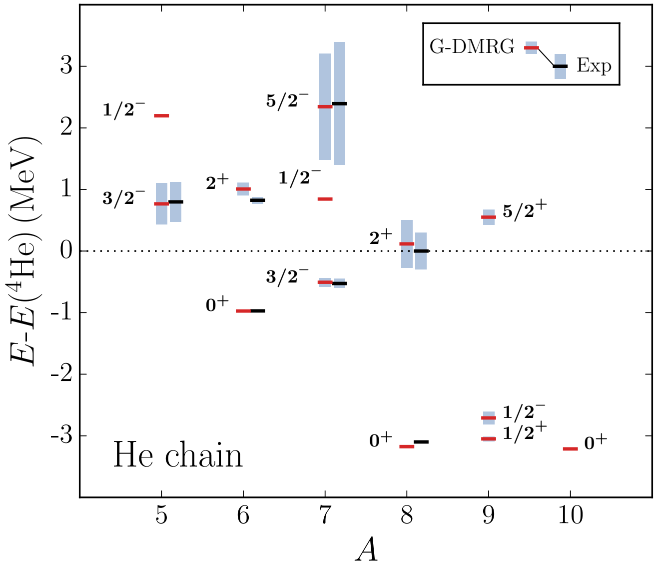
Teaser: ^{9}He structure with ^{8}He(d,p)



is ^{9}He parity-inverted?

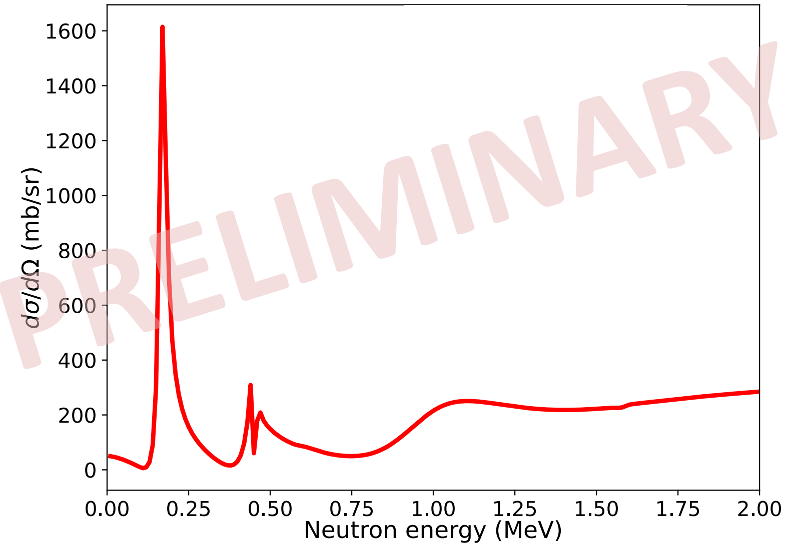
the parity of the unbound ^{9}He ground state is still controversial

K. Fossez, J. Rotureau, W. Nazarewicz.
Phys. Rev. C **98** (2018) 061302(R)



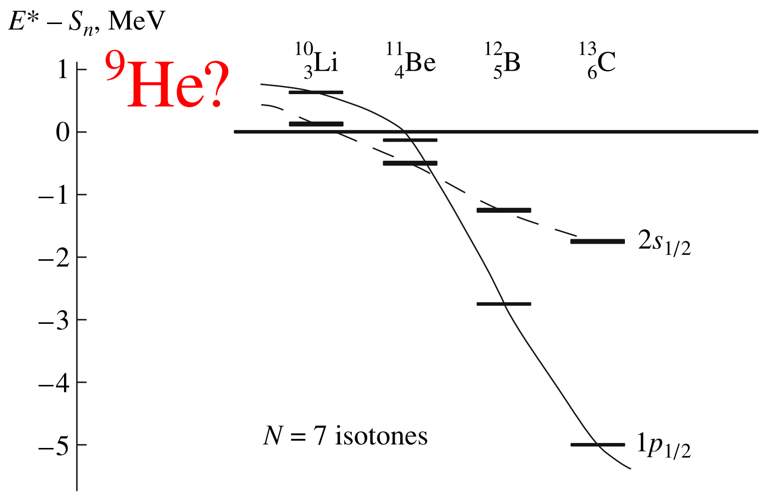
we use a structure approach with explicit inclusion of the continuum (K. Fossez)

n+^{8}He elastic scattering



with the GFT formalism we can ”translate” neutron elastic scattering into (d,p) cross sections

Teaser: ^{9}He structure with ^{8}He(d,p)



is ^{9}He parity-inverted?

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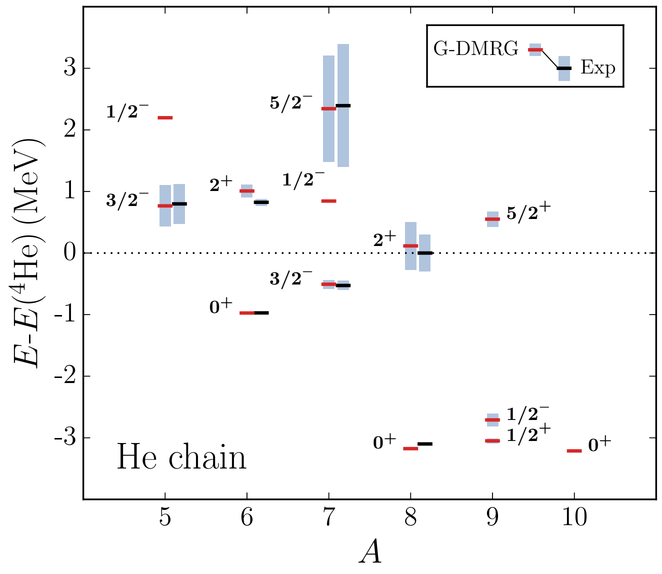
Phys. Rev. C **98** (2018) 061302(R)



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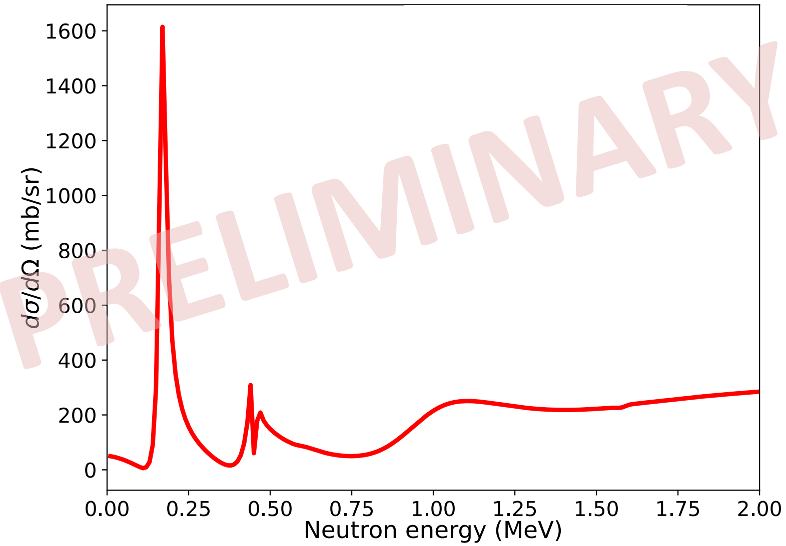


we are partnering with experimentalists Y. Ayyad and A. Macchiavelli to prepare a proposal for the ^{8}He(d,p)^{9}He measurement at FRIB

Thank you for your attention!

Thanks to my collaborators:

Grigor Sargsyan

Theory Alliance
FACILITY FOR RARE ISOTOPE BEAMS

Jutta Escher, Kostas Kravvaris

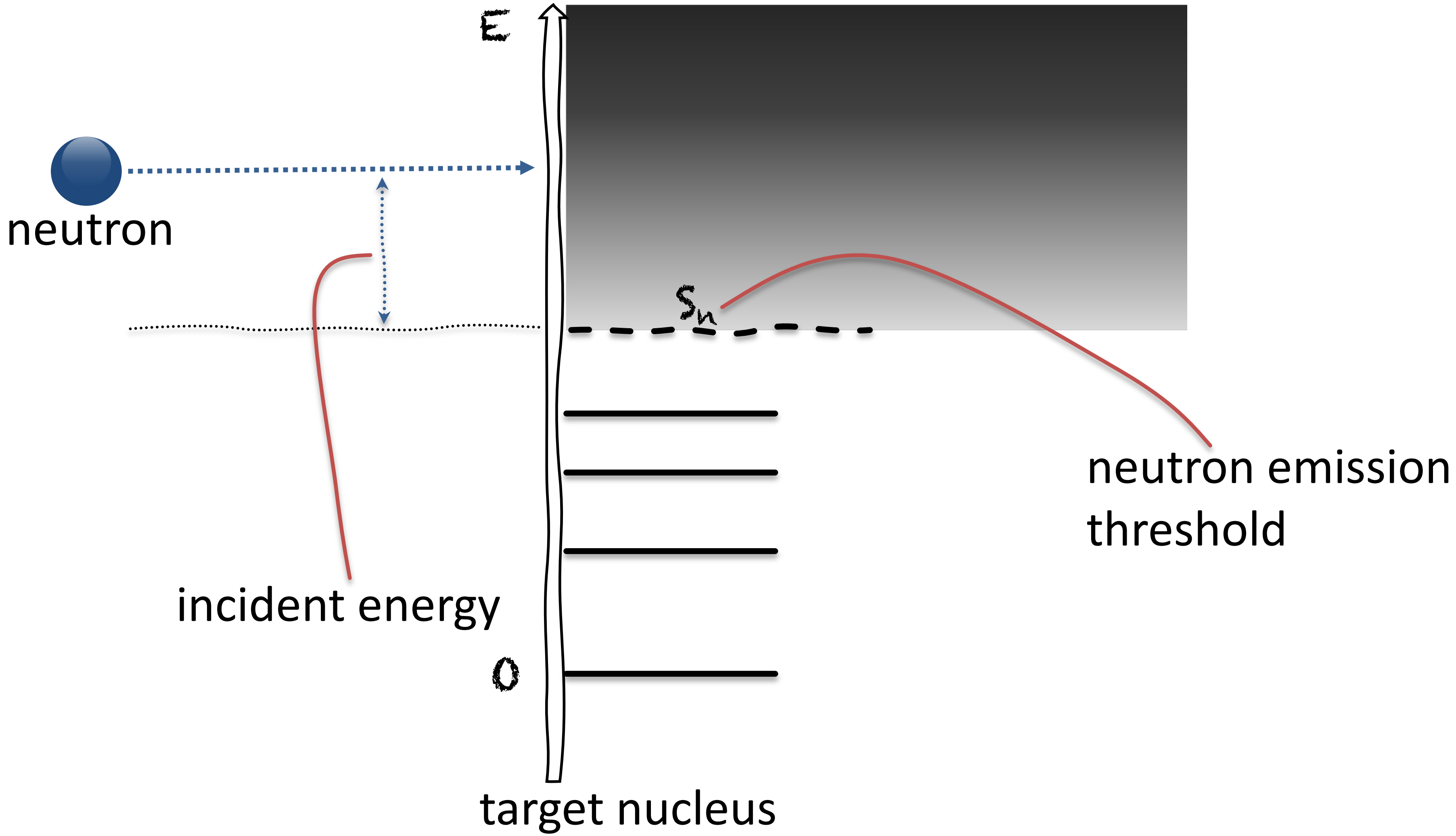
Lawrence Livermore National Laboratory

K. Fossez (FSU)
A. O. Macchiavelli (ORNL)
Y. Ayyad (U of Santiago de Compostela)

^{95}Mo: A. Ratkiewicz, J. Escher, J. Burke, R. Casperson, R. Hughes, N. Scielzo (LLNL), J. Cizewski, S. Burcher, B. Manning, S. Rice, C. Shand (Rutgers), M. McCleskey (TAMU), R. Austin (St Mary's), S. Pain (ORNL), W. Peters (U of Tennessee), T. Ross (U of Richmond) and K. Smith (LANL).

^{120}Sn: M. Weinert, M. Müscher, J. Wilhelmy, A. Zilges (U of Cologne), M. Spieker (FSU), N. Tsoneva (ELI-NP).

Neutron capture process (n,γ)



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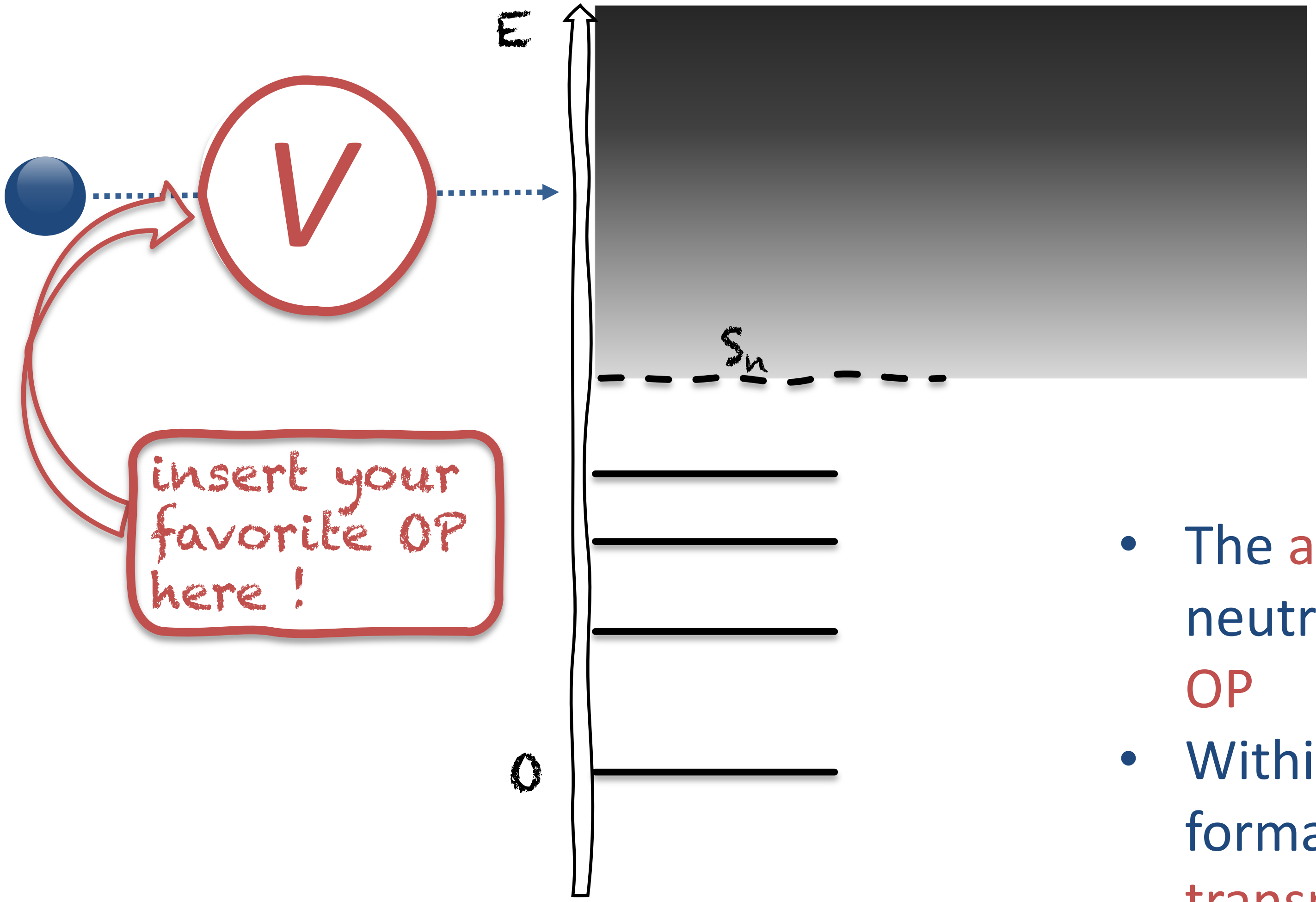
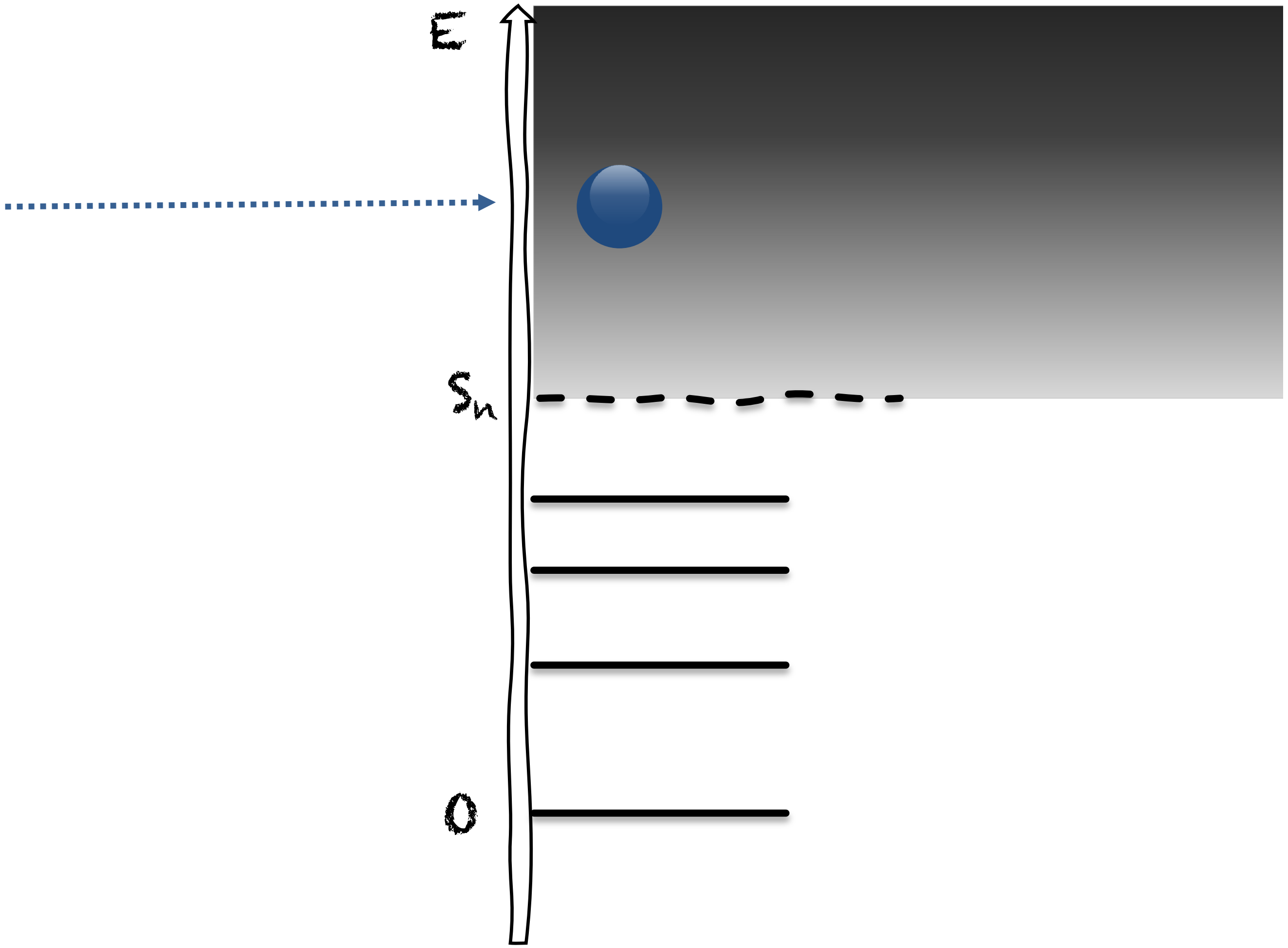
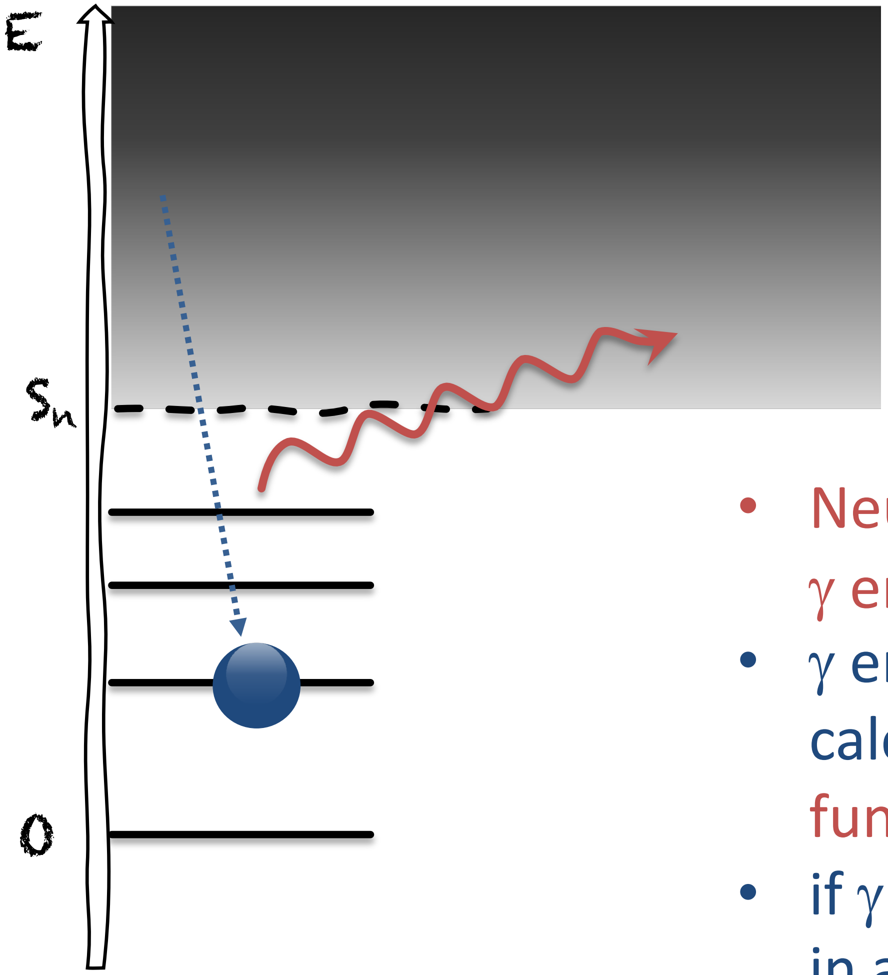


- The absorption probability of the neutron is calculated with the OP
- Within the Hauser-Feshbach formalism, it is encoded in the transmission coefficients

Neutron capture process (n,γ)

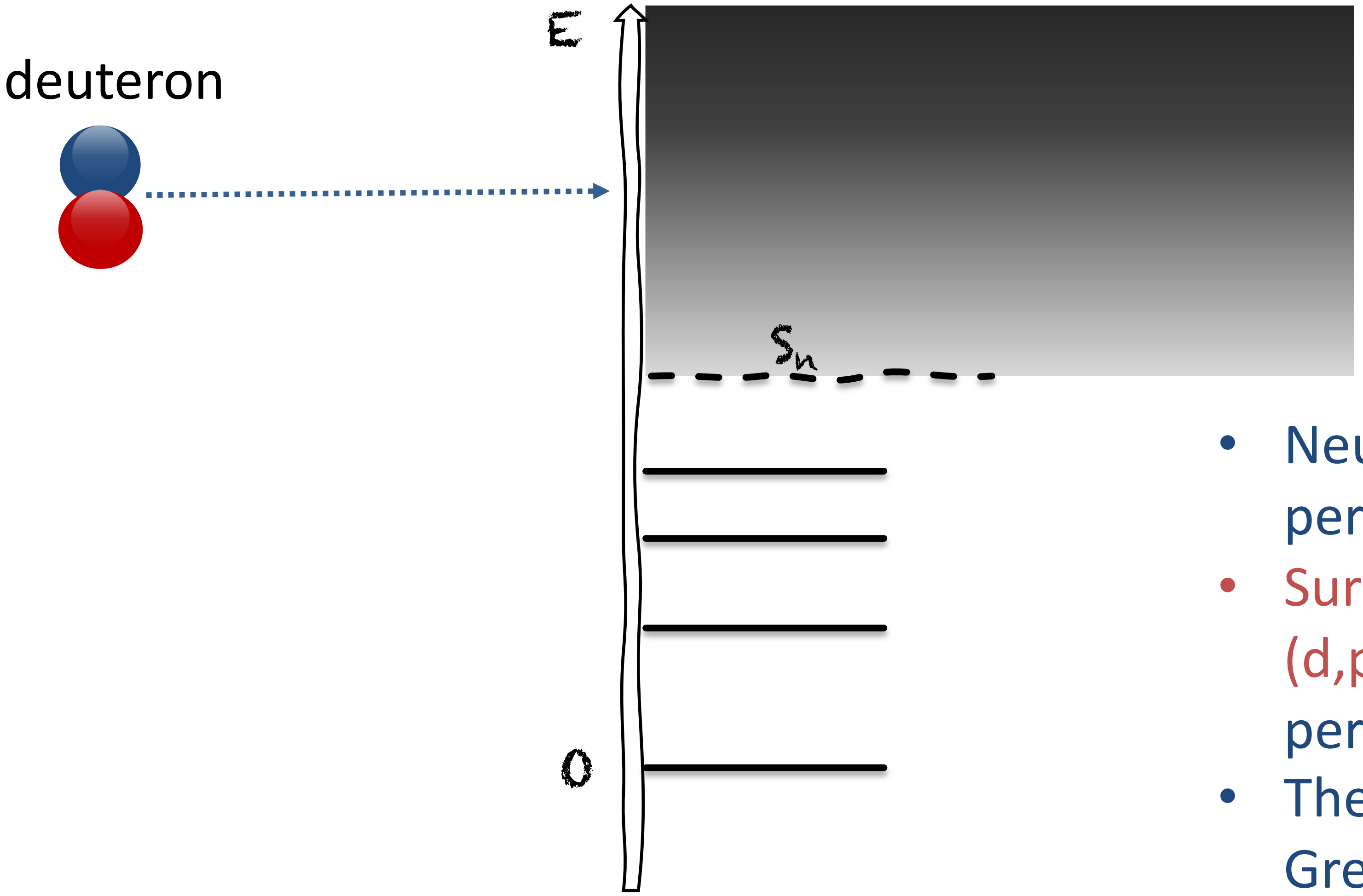


Neutron capture process (n,γ)



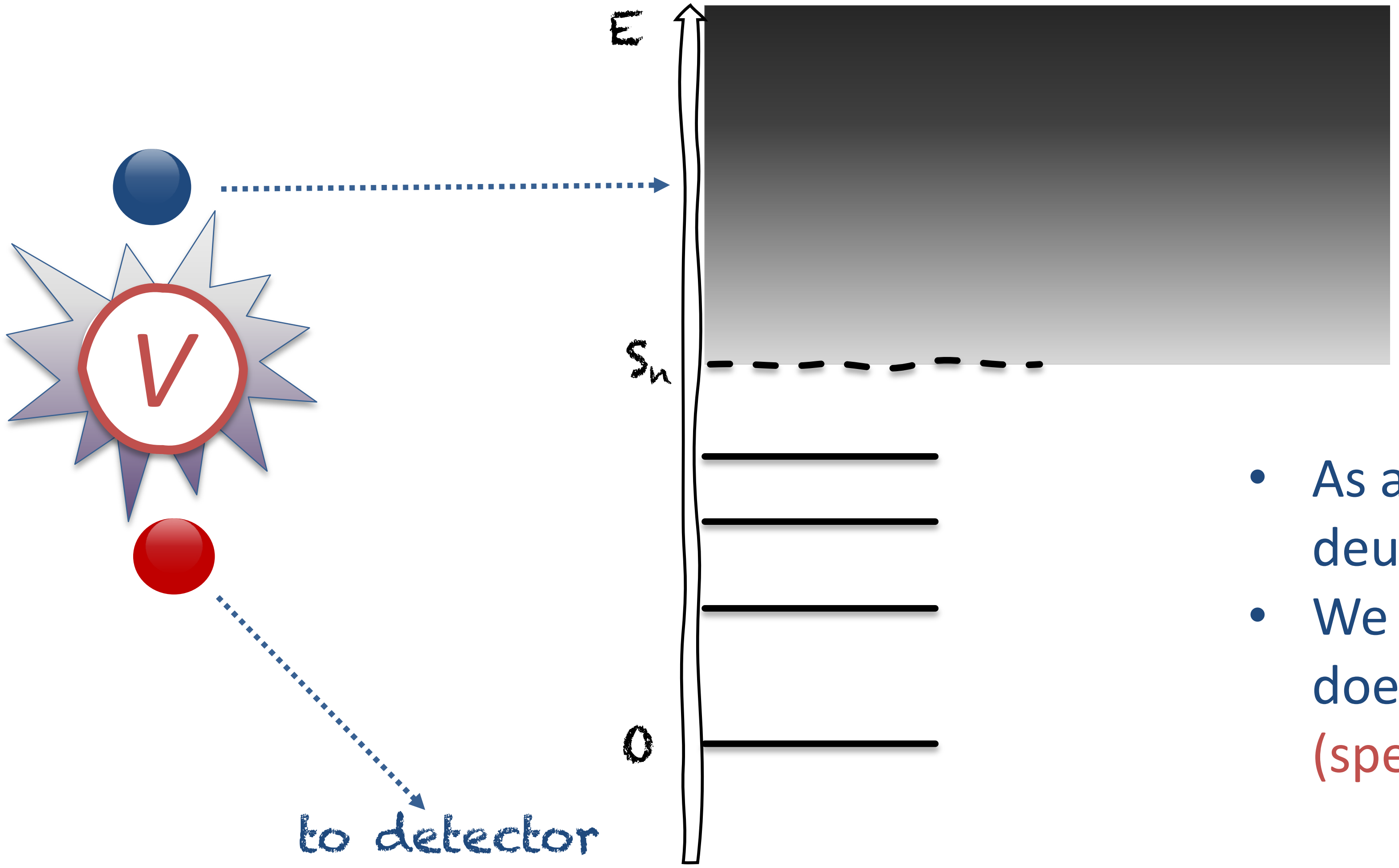
- Neutron emission competes with γ emission
- γ emission probability is calculated from the γ strength function
- if γ wins, the neutron is captured in a bound state

A surrogate for (n,γ): (d,pγ) with the GFT formalism



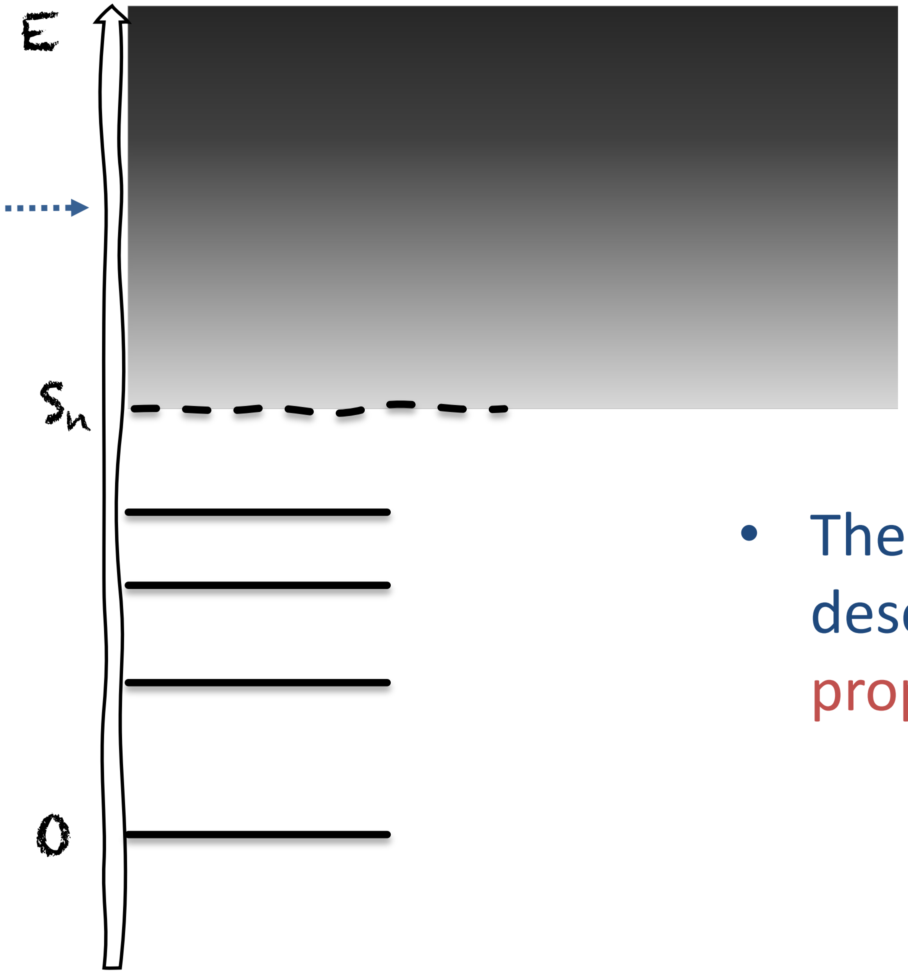
- Neutron scattering cannot be performed on radioactive nuclei
- Surrogate inverse kinematics (d,pγ) reactions can be performed instead
- The process is described within Green's Function Transfer (GFT) formalism

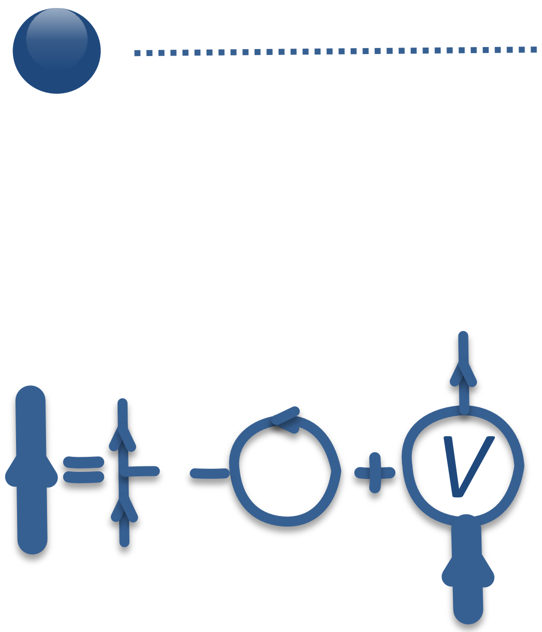
A surrogate for (n,γ): (d,pγ) with the GFT formalism



- As a first step, the OP breaks the deuteron
- We assume that the proton doesn't play any subsequent role (spectator approximation)

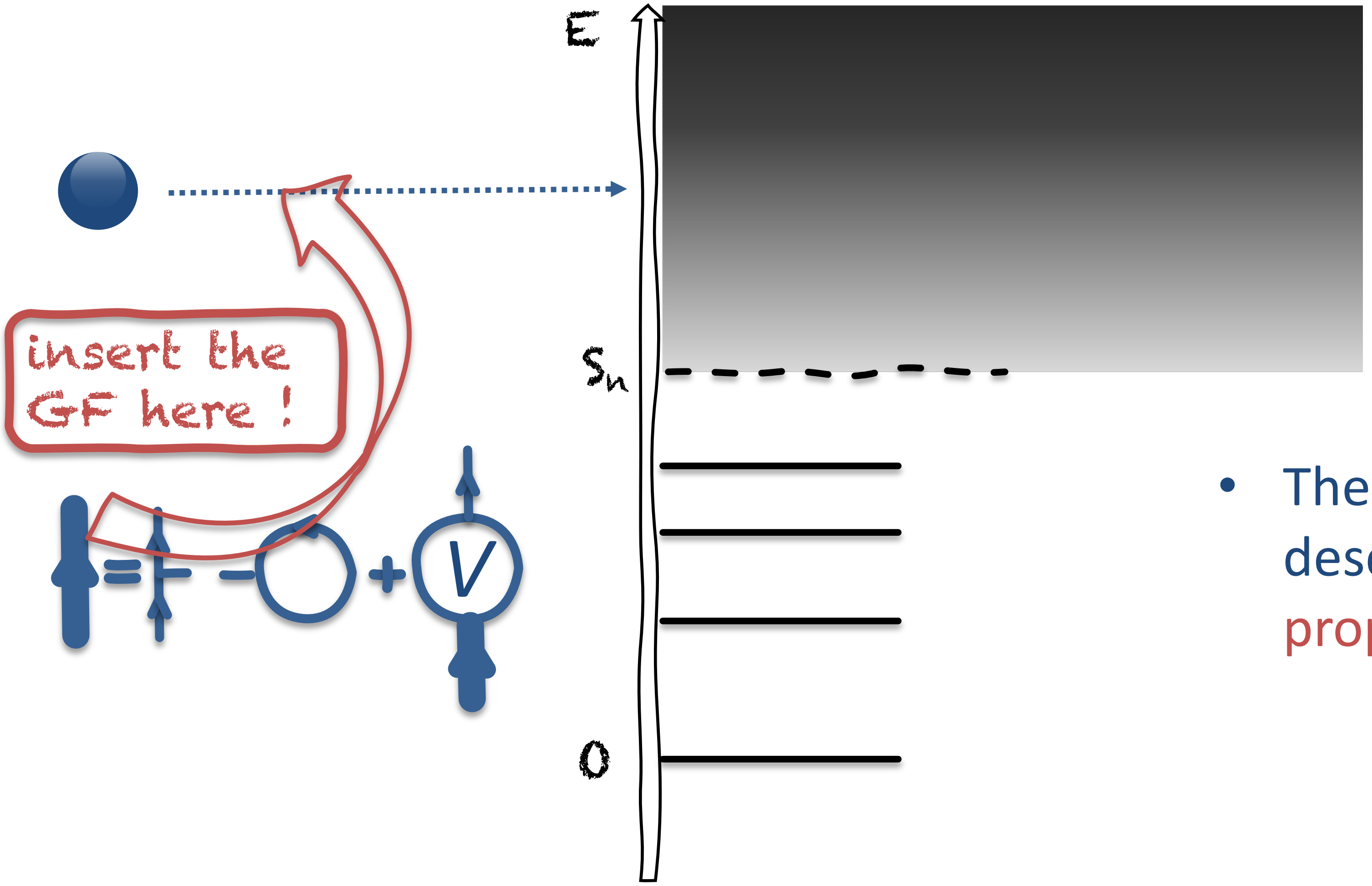
A surrogate for (n,γ): (d,pγ) with the GFT formalism





- The Green's function is used to describe the neutron-nucleus propagation

A surrogate for (n,γ): (d,pγ) with the GFT formalism



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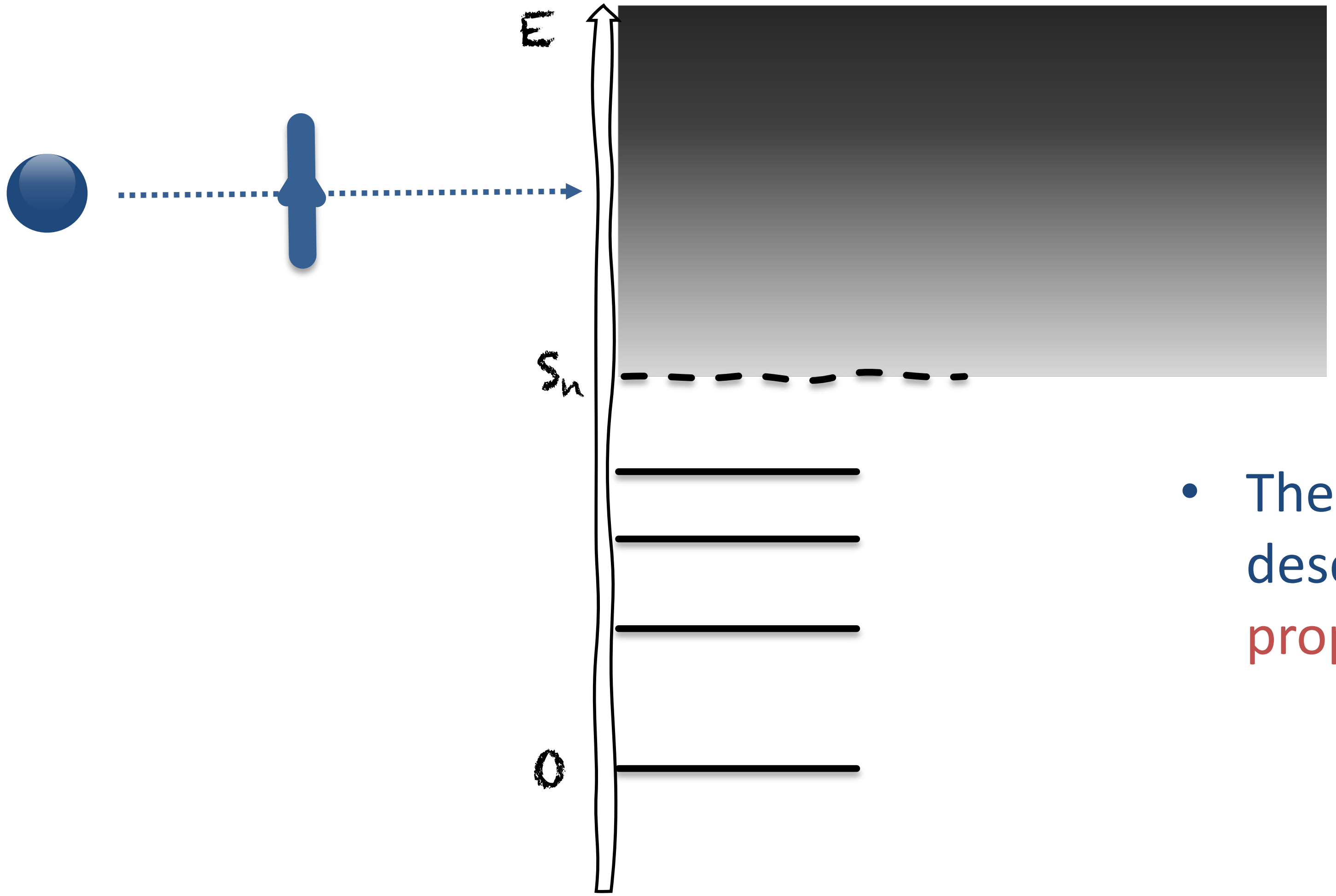

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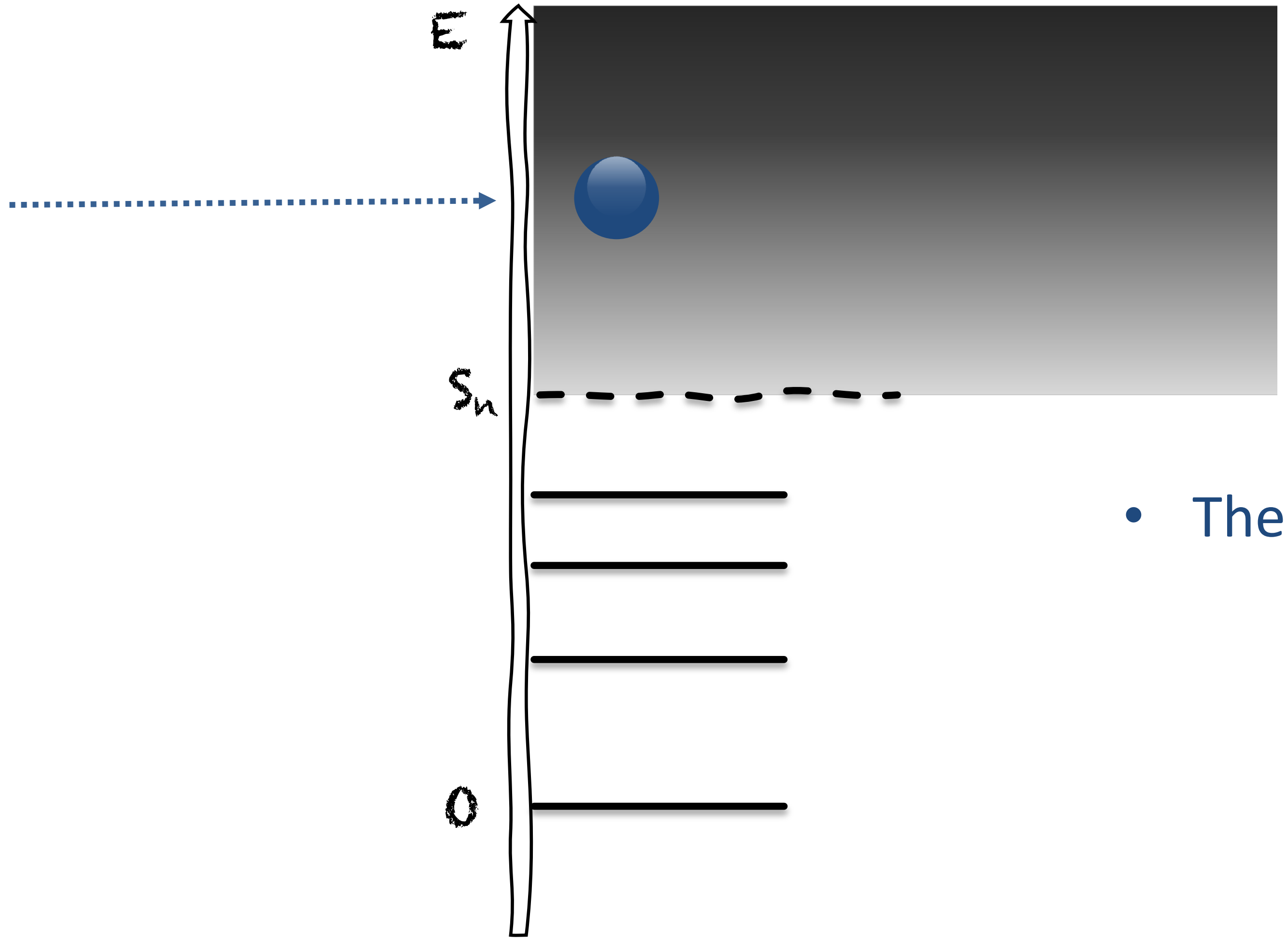
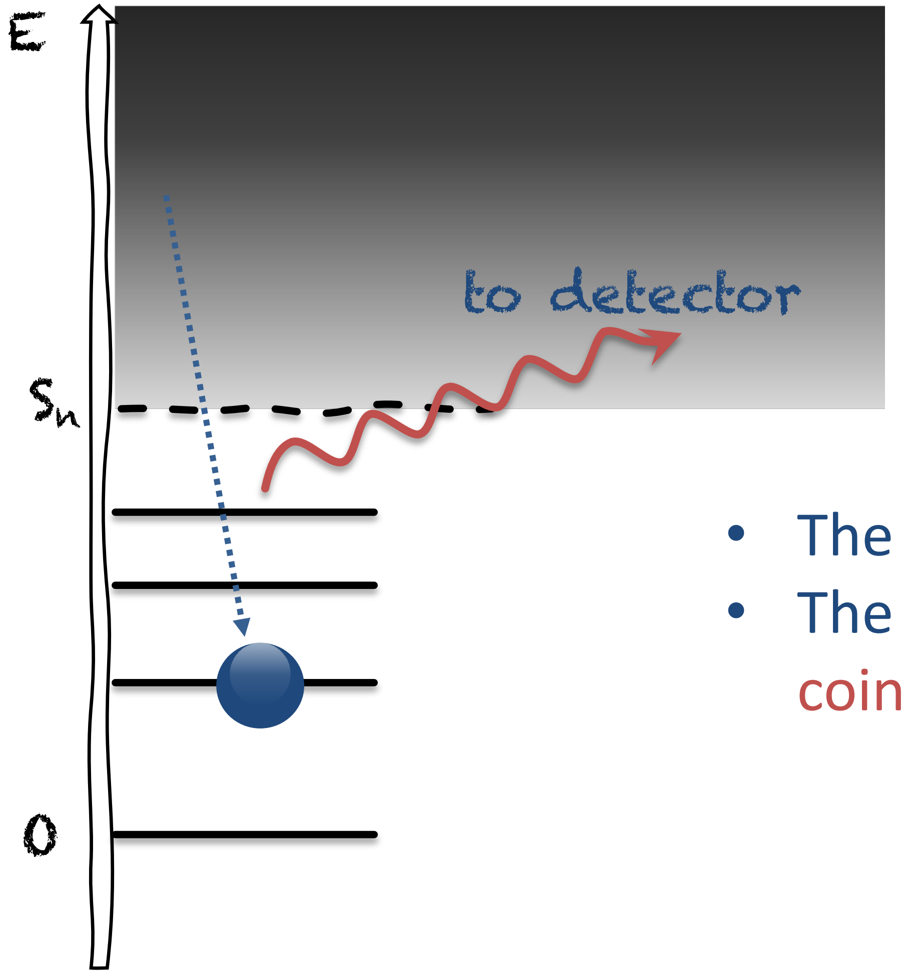


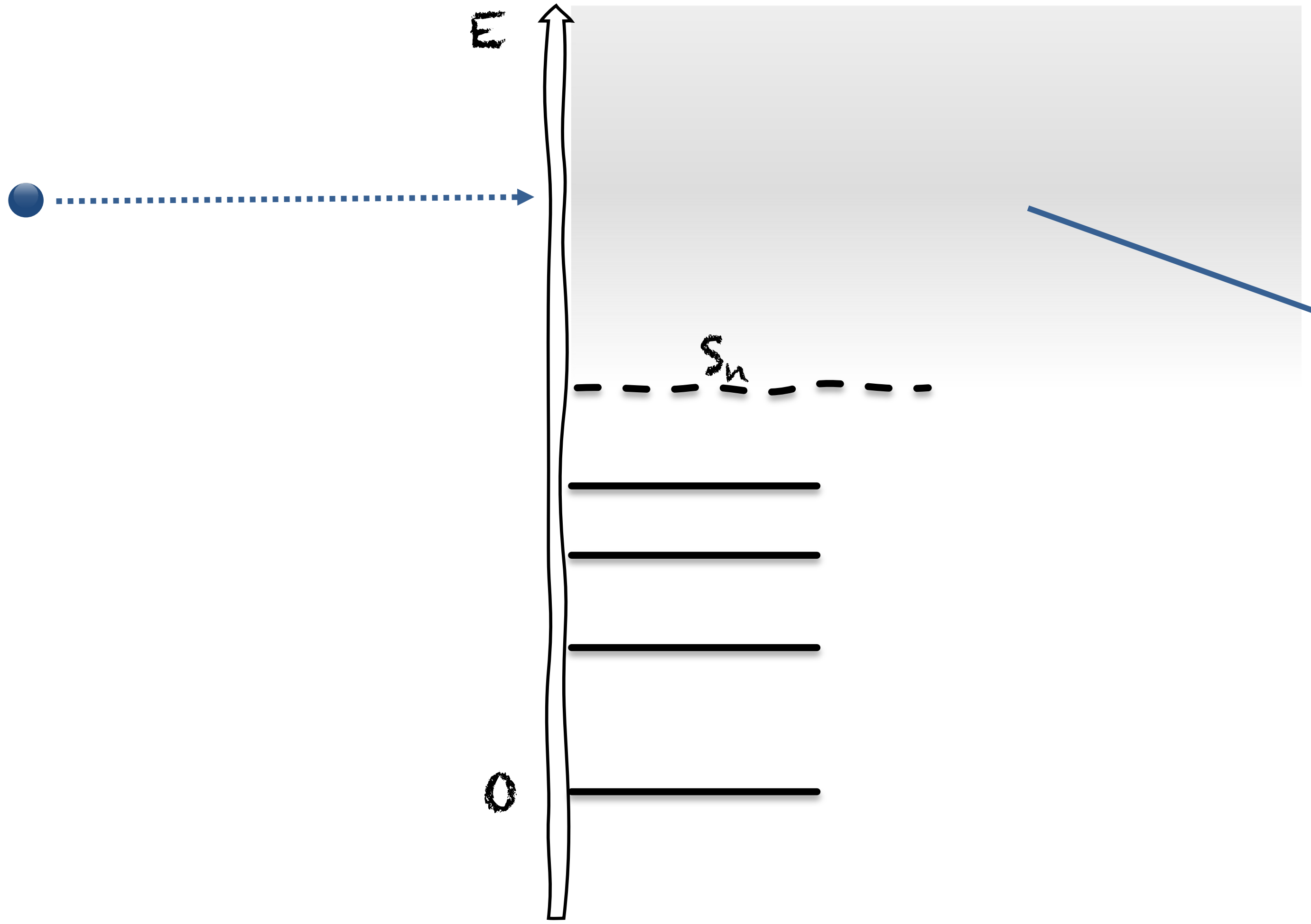
- The rest is history!

A surrogate for (n,γ): (d,pγ) with the GFT formalism



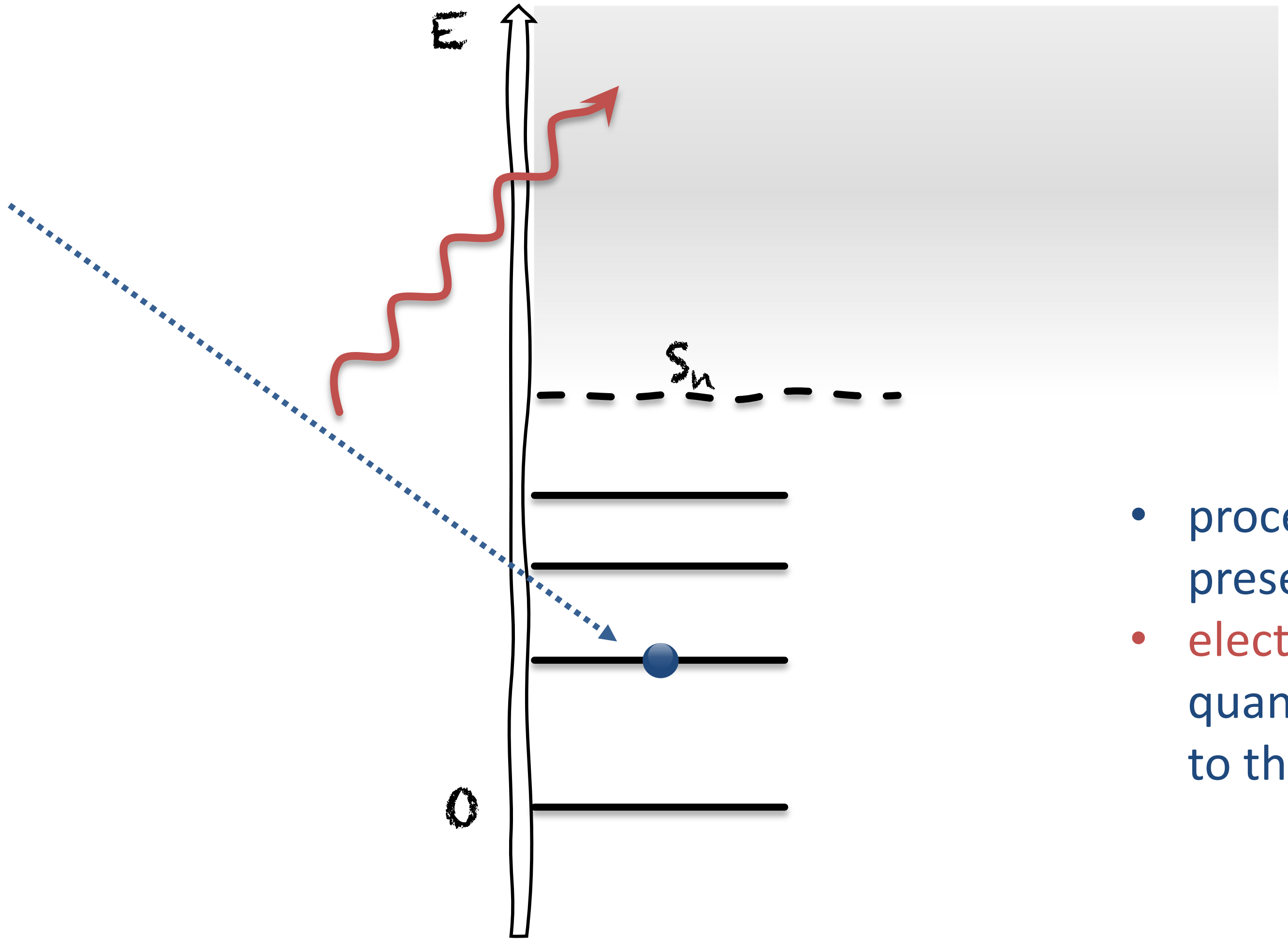
- The rest is history!
- The photon is detected in coincidence with the proton

Capture processes: direct capture



continuum with no resonances (or too weak)

Capture processes: direct capture



$$T_{if}^{d} = \langle \phi_i^s | A | \phi_f \rangle$$

- process favored if strong resonances are not present
- electromagnetic T-matrix accounts for the quantum amplitude from the scattering state to the final bound state

Self energy in a nutshell

nucleon in vacuum with external field *U*

$$(E - T - U)\,\chi_0 = 0$$

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$$\Psi = \phi_0 \chi_0 + \phi_1 \chi_1$$

Self energy in a nutshell

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$$\Psi = \phi_0\chi_0 + \phi_1\chi_1$$

the nucleon couples to the medium

$$(E - T - U)\,\chi_0 = U_{01}\chi_1$$

$$(E - \epsilon_1 - T - U)\,\chi_1 = U^*_{01}\chi_0$$

Self energy in a nutshell

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$$(E - \epsilon_1 - T - U)\,\chi_1 = U_{01}^{*}\chi_0$$

we manipulate the second equation

$$\chi_1 = \lim_{\eta\to 0}\left(E - \epsilon_1 - T - U + i\eta\right)^{-1} U_{01}^{*}\chi_0 = G(E - \epsilon_1)U_{01}^{*}\chi_0$$

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where we have defined the Green's function

$$G(E) = \lim_{\eta\to 0}\left(E - T - U + i\eta\right)^{-1}$$

Self energy in a nutshell

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$$(E - \epsilon_1 - T - U)\,\chi_1 = U_{01}^{*}\chi_0$$

substituting in the first equation

$$(E - T - U)\,\chi_0 = U_{01}G(E - \epsilon_1)U_{01}^{*}\chi_0$$

Self energy in a nutshell

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$$(E - T - U)\,\chi_0 = U_{01}G(E - \epsilon_1)U_{01}^{*}\chi_0$$

we define the optical potential

$$\boxed{\mathcal{V}(E) = U + U_{01}G(E - \epsilon_1)U_{01}^{*}}$$

$$(E - T - \mathcal{V}(E))\,\chi_0 = 0$$

Self energy in a nutshell

the nucleon couples to the medium

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a.k.a the self energy!

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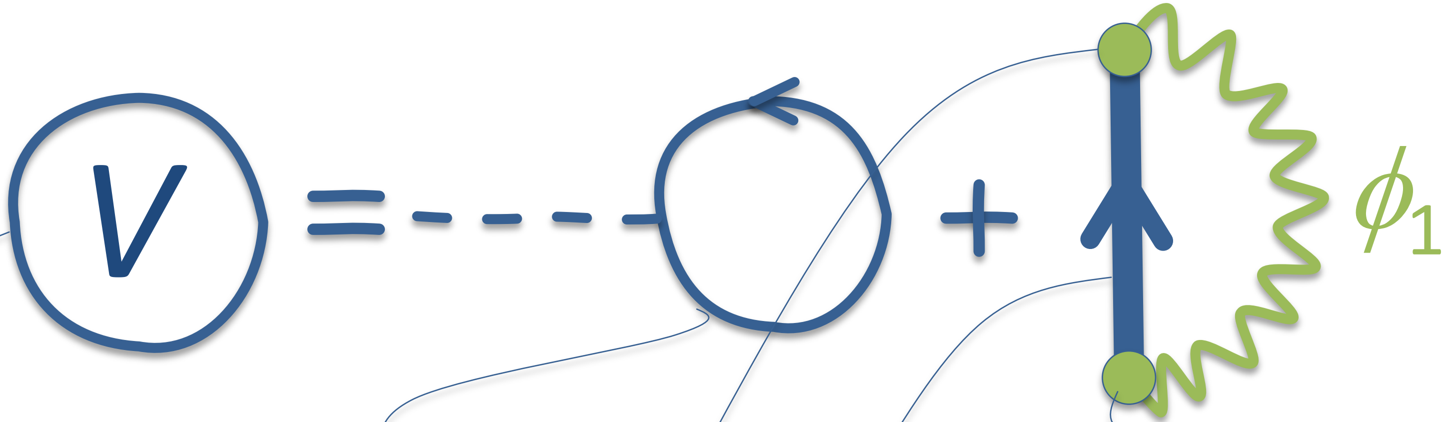
Self energy in a nutshell



$$\mathcal{V}(E) = U + U_{01}G(E-\epsilon_1)U_{01}^*$$

Dispersion (Kramers-Kronig) relations

$$\mathcal{V}(E) = \lim_{\eta \to 0} \frac{U_{01}^*(r)U_{01}(r')}{E - T - U + i\eta} = \lim_{\eta \to 0} \frac{U_{01}^*(r)U_{01}(r')(E - T - U - i\eta)}{(E - T - U)^2 + \eta^2}$$

$$\mathrm{Re}\mathcal{V}(E) = \frac{U_{01}^*(r)U_{01}(r')}{(E - T - U)}; \quad \mathrm{Im}\mathcal{V}(E) = -\pi U_{01}^*(r)U_{01}(r')\delta(E - T - U)$$

$$\mathrm{Re}\mathcal{V}(E) = -\frac{1}{\pi}\int \frac{\mathrm{Im}\mathcal{V}(E')}{(E - T - U)}\, dE'$$

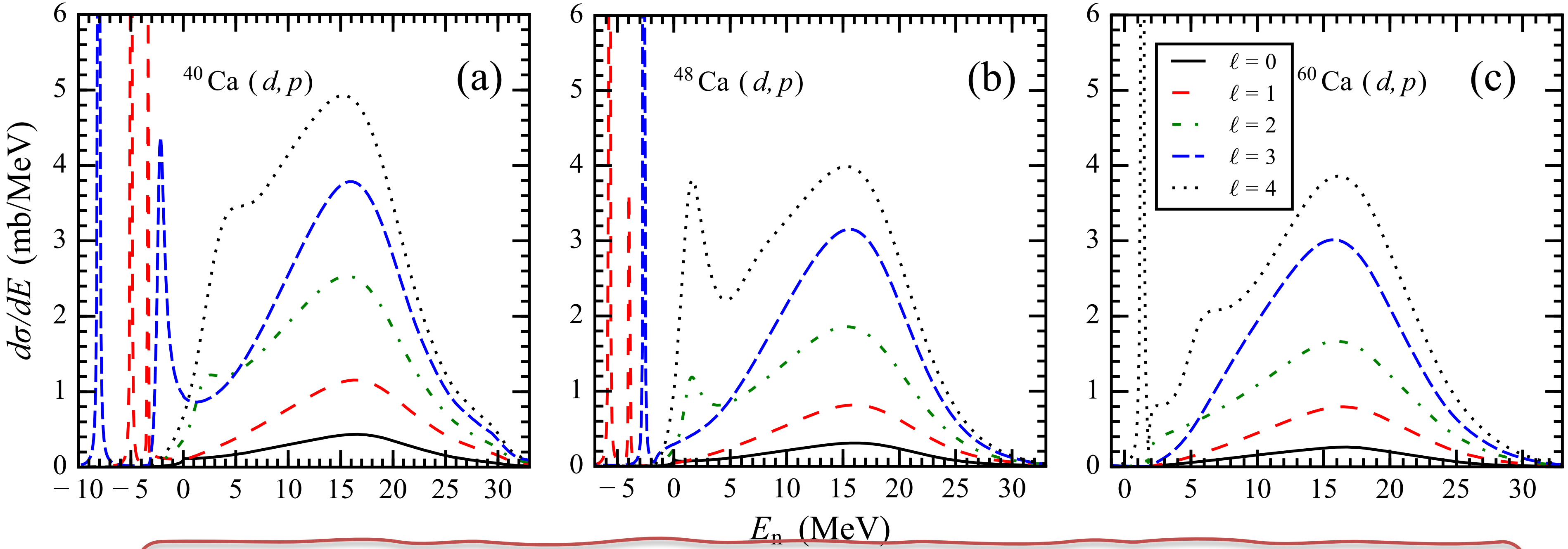
Applications: Dispersive Optical Model (DOM); Ca(d,p)

Eur. Phys. J. A (2017) **53**: 178

THE EUROPEAN PHYSICAL JOURNAL A

Toward a complete theory for predicting inclusive deuteron breakup away from stability

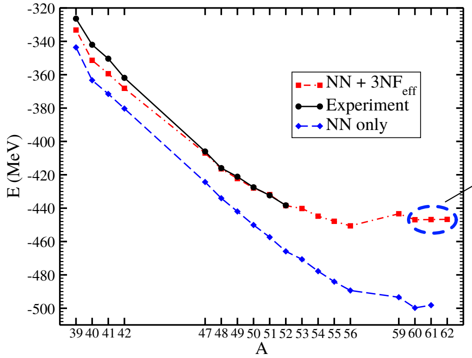
G. Potel[1,a], G. Perdikakis[1,2,3,b], B.V. Carlson[4,c], M.C. Atkinson[5], W.H. Dickhoff[5], J.E. Escher[6], M.S. Hussein[4,7,8], J. Lei[9,d], W. Li[1], A.O. Macchiavelli[10], A.M. Moro[9], F.M. Nunes[1,11], S.D. Pain[12], and J. Rotureau[1]



- Dispersive: reproduction of positive and negative energy cross section
- Controlled extrapolation to exotic nuclei

Applications: Dispersive Optical Model (DOM); Ca(d,p)

is ^{61}Ca bound?

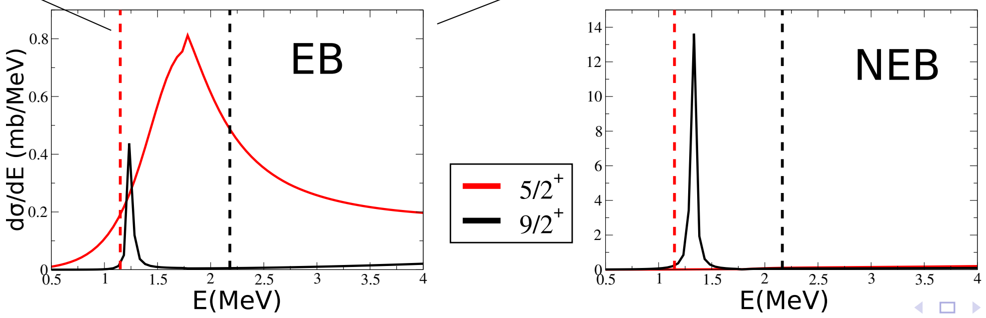


NO (but too close to be conclusive)

	^{53}Ca		^{55}Ca		^{61}Ca	
J^{π}	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

coupled cluster
Hagen et al., PRL **109**, 032502 (2012)

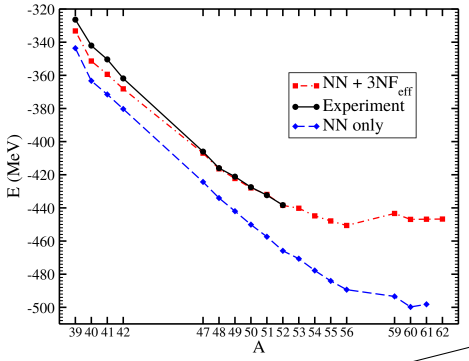
NO



^{60}Ca(d,p) with DOM

Applications: Dispersive Optical Model (DOM); Ca(d,p)

inversion of 9/2+ and 5/2+ orbitals?



YES

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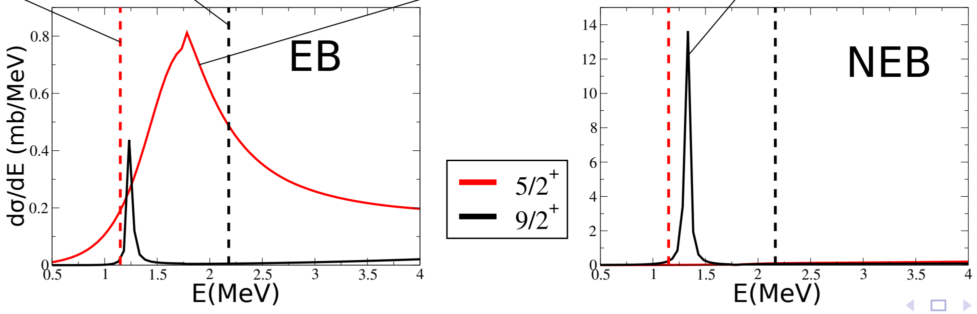

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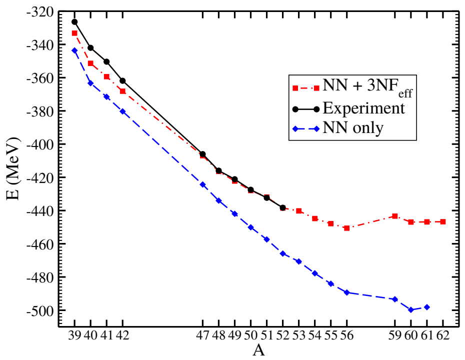
Applications: Dispersive Optical Model (DOM); Ca(d,p)



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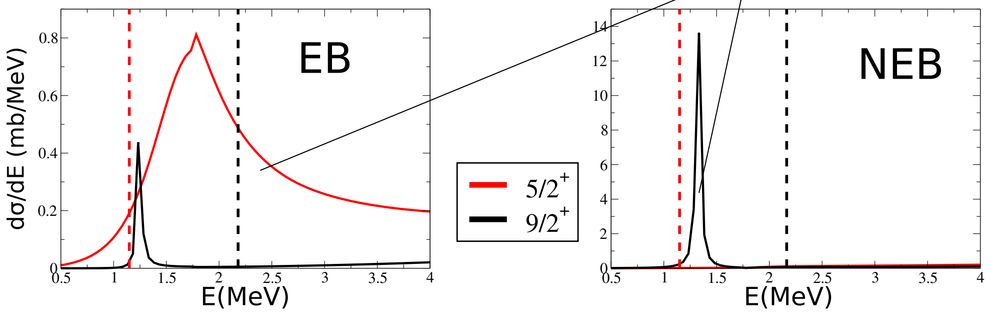
transfer strength function informs about widths

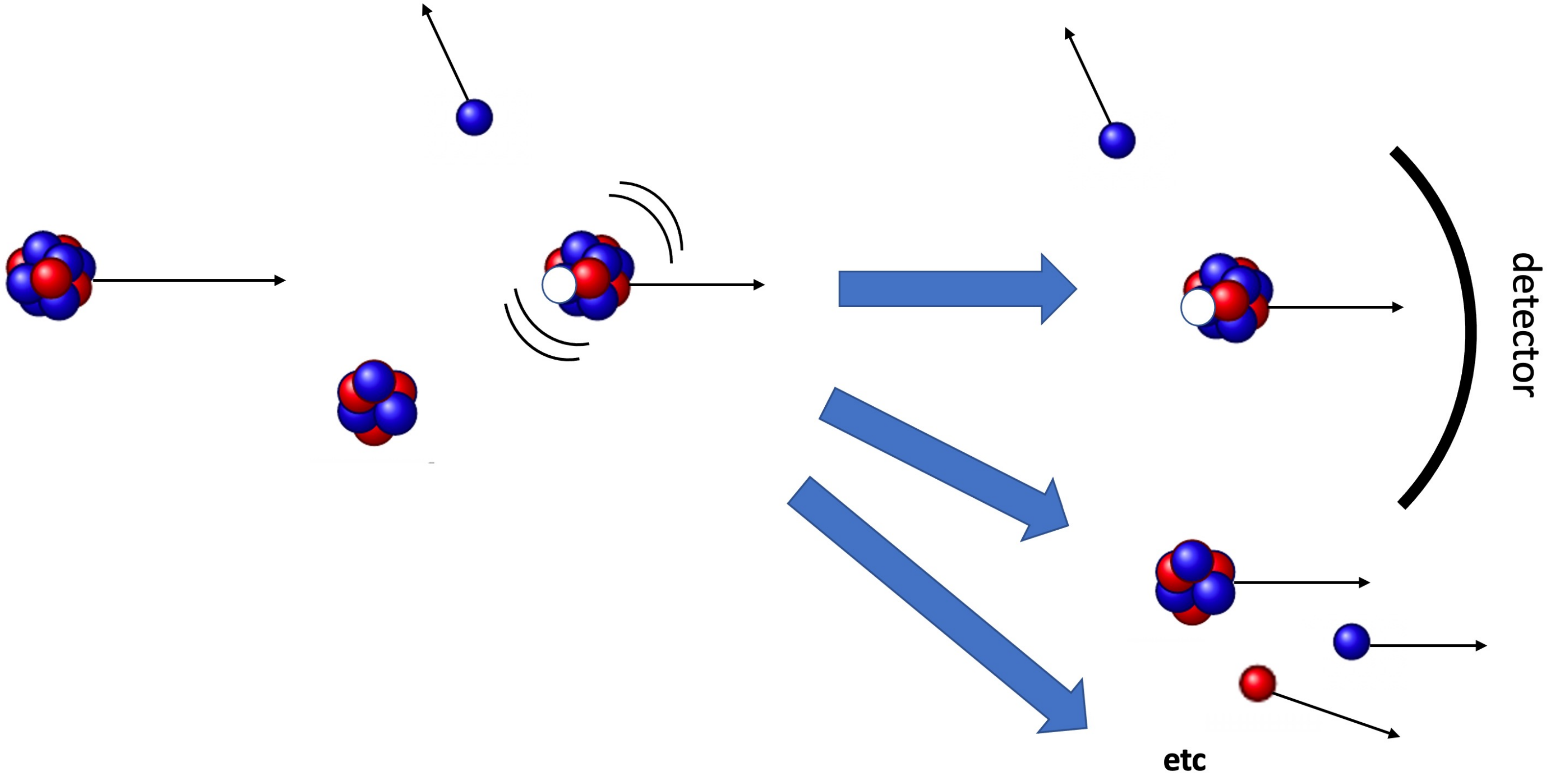
coupled cluster
Hagen et al., PRL **109**, 032502 (2012)



^{60}Ca(d,p) with DOM

An extension to holes: The Green's function knockout (GFK) and the asymmetry plot

(talk by J. Gómez Camacho)



An extension to holes: The Green's function knockout (GFK) and the asymmetry plot

PHYSICAL REVIEW C **107**, 014607 (2023)

Green's function knockout formalism

C. Hebborn[1,2,*] and G. Potel[2,†]

(talk by J. Gómez Camacho)

$$\frac{d\sigma}{dE_{f_{cT}}\,d\Omega} = -\frac{2\mu_{PT}}{\hbar^2 k_{PT}}\,\rho(E_{f_{cT}}) \times \sum_{E_h=-S_x^{(c)}-S_N^{(P)}}^{-S_N^{(P)}} \left\langle \phi_h^{(f_{NT})} \middle| \mathrm{Im}\,\hat{U}_h(E_h) \middle| \phi_h^{(f_{NT})} \right\rangle$$

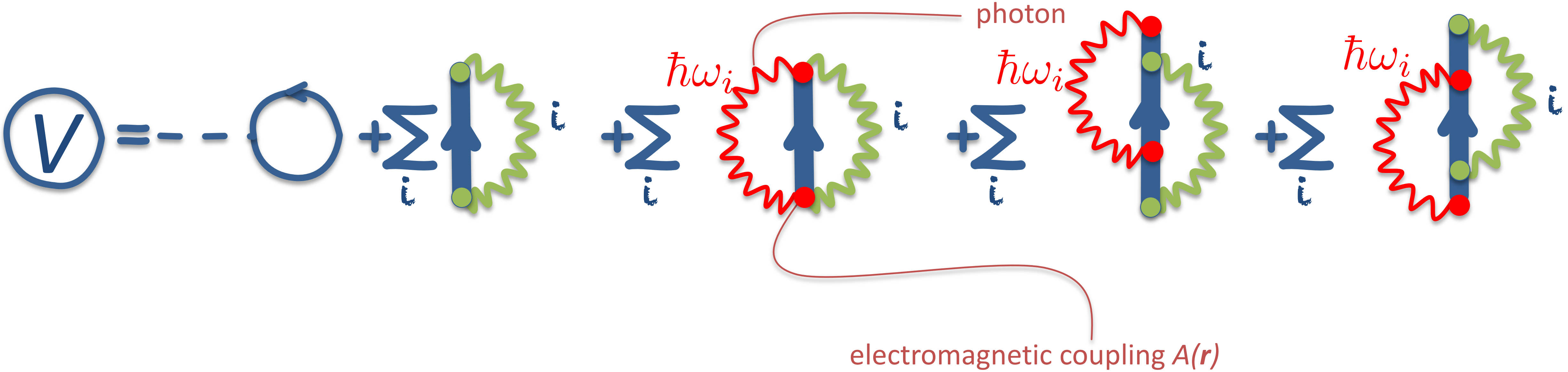
hole optical potential

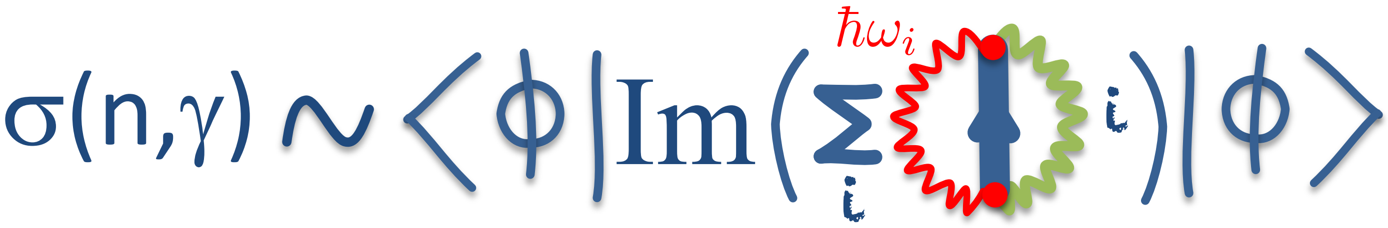
sum over energies below particle threshold

$$\phi_h^{(f_{NT})}(\boldsymbol{r}) = \hat{G}_h^{\mathrm{opt}}(E_h)\,\rho_h^{(f_{NT})}(\boldsymbol{r}).$$

hole Green's function

When we include the coupling to the electromagnetic field, we can compute the (*n*,γ) cross section





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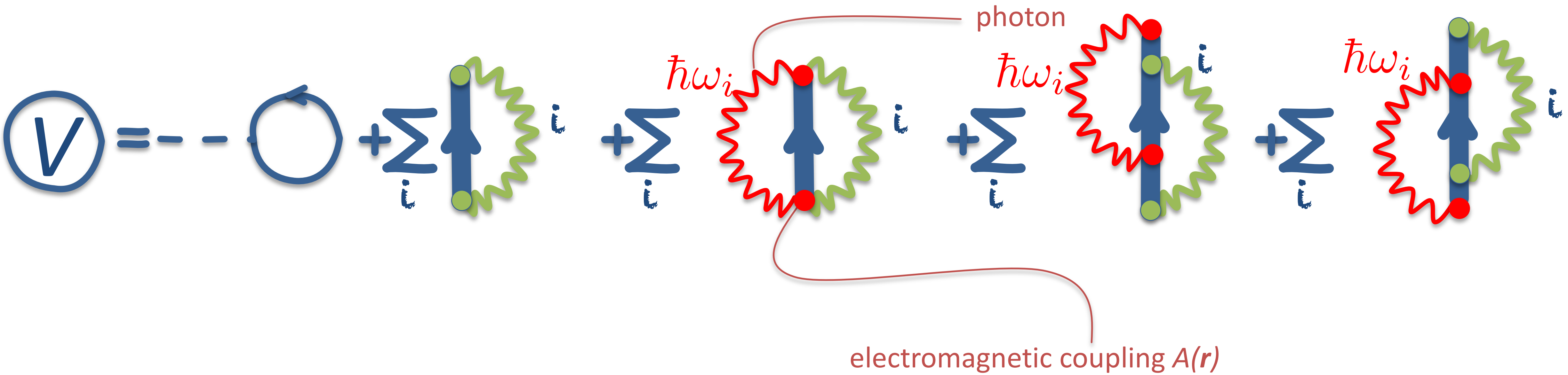
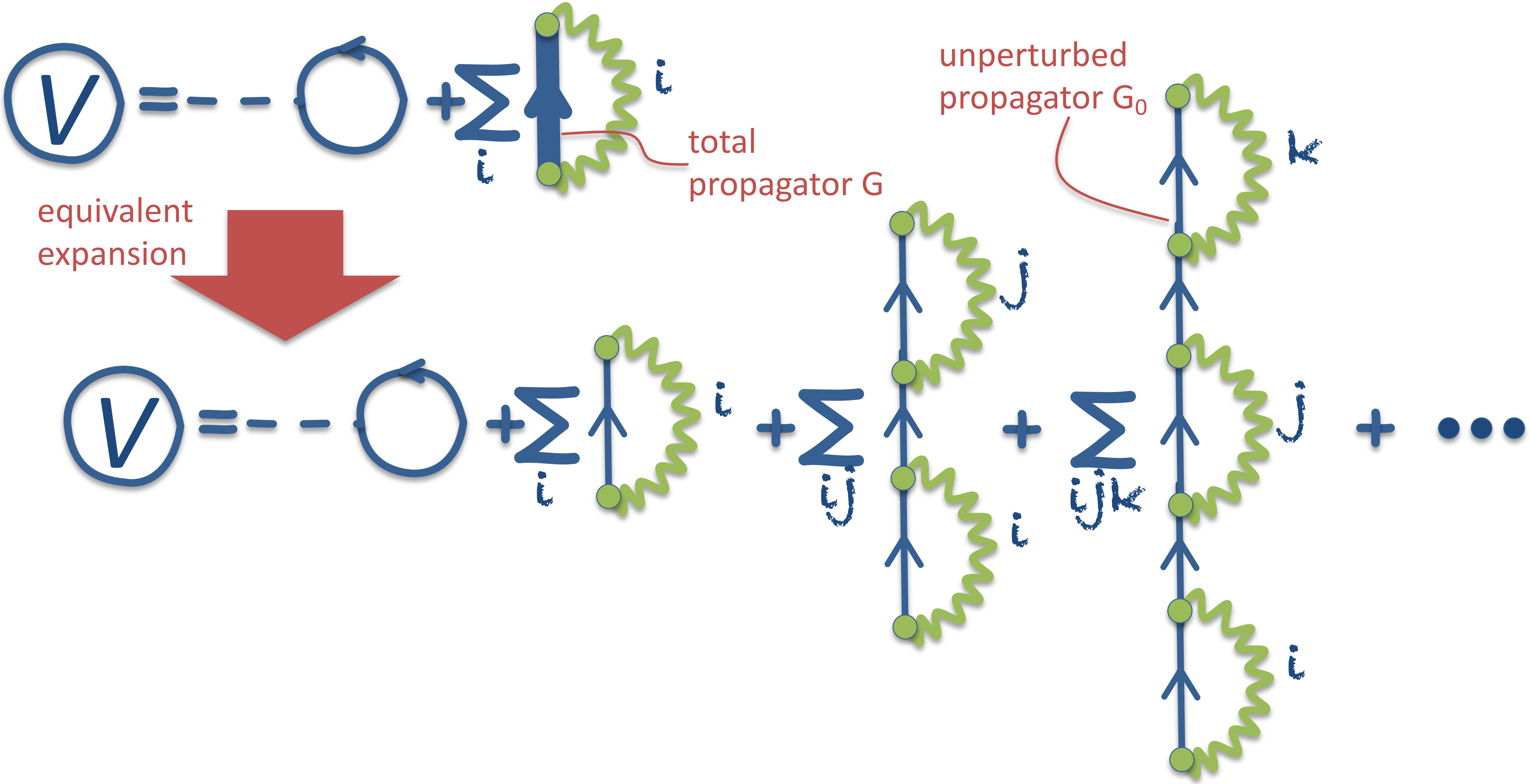


The coupling of the system with an electric photon of multipolarity λ is

$$A_i(\mathbf{r}) = r^{\lambda} Y^{\lambda}(\hat{r}) T_{i,\lambda}$$

where $T_{i,\lambda}$ is the partial γ width, calculated with, e.g., the shell model

An equivalent expansion in powers of the couplings can shed light on direct, preequilibrium, and compound processes



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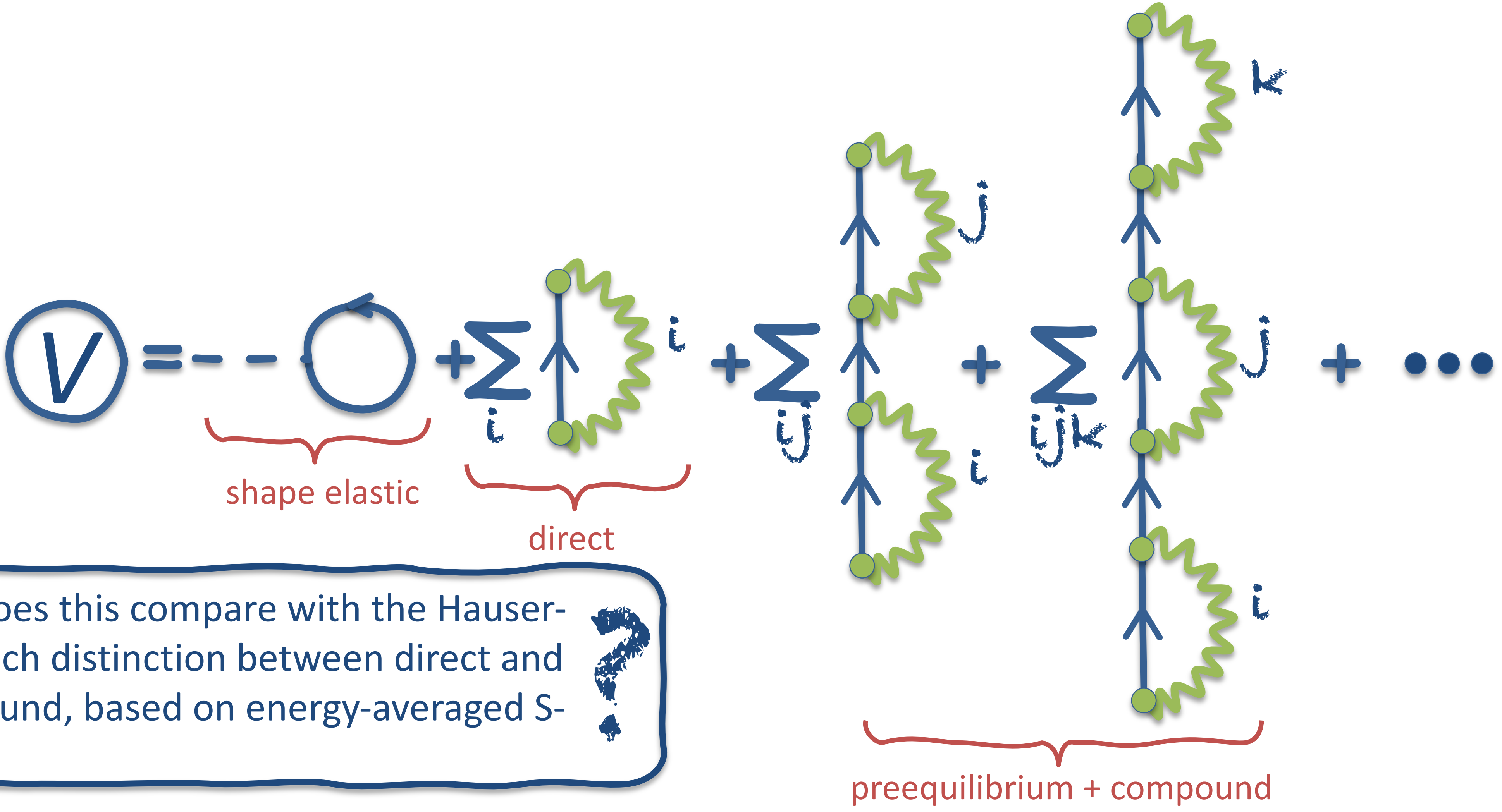
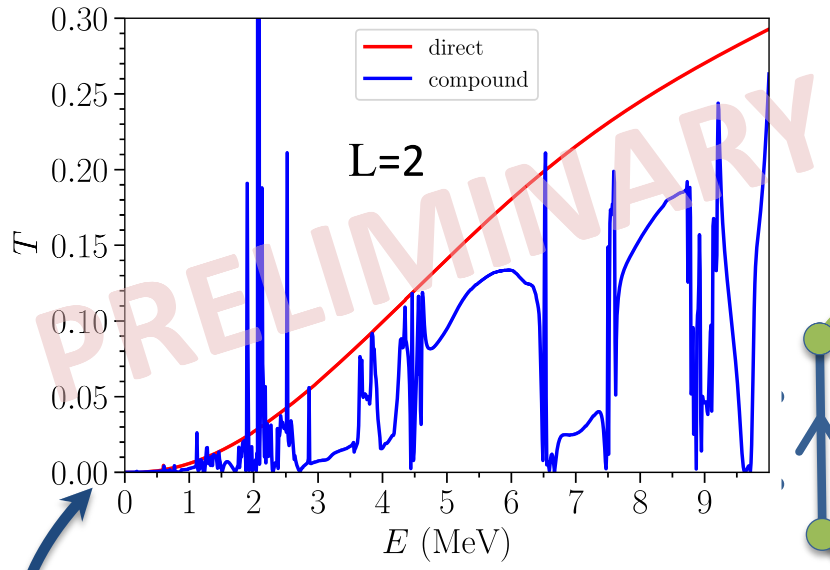
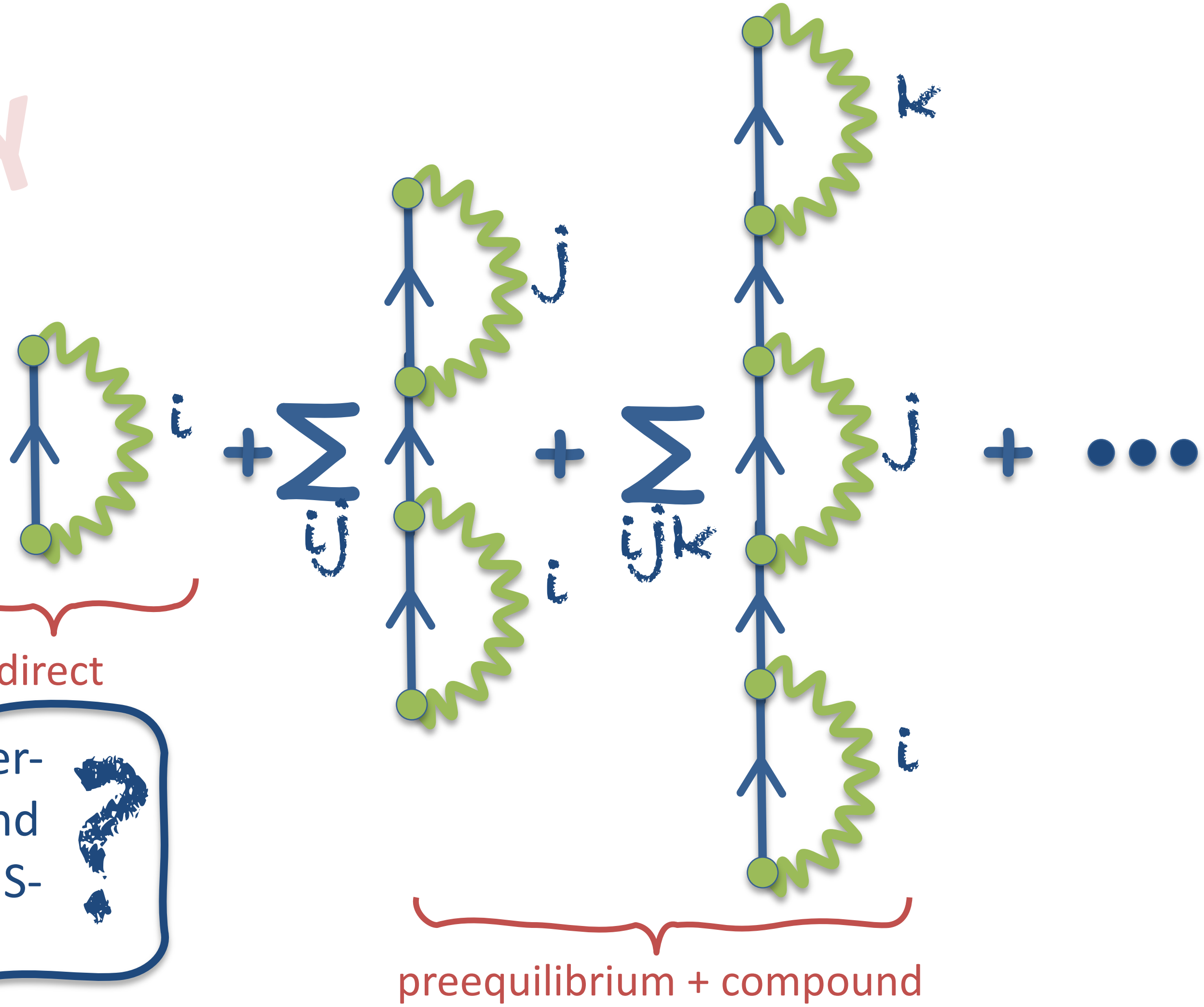


How does this compare with the Hauser-Feshbach distinction between direct and compound, based on energy-averaged S-matrix ?

An equivalent expansion in powers of the couplings can shed light on direct, preequilibrium, and compound processes



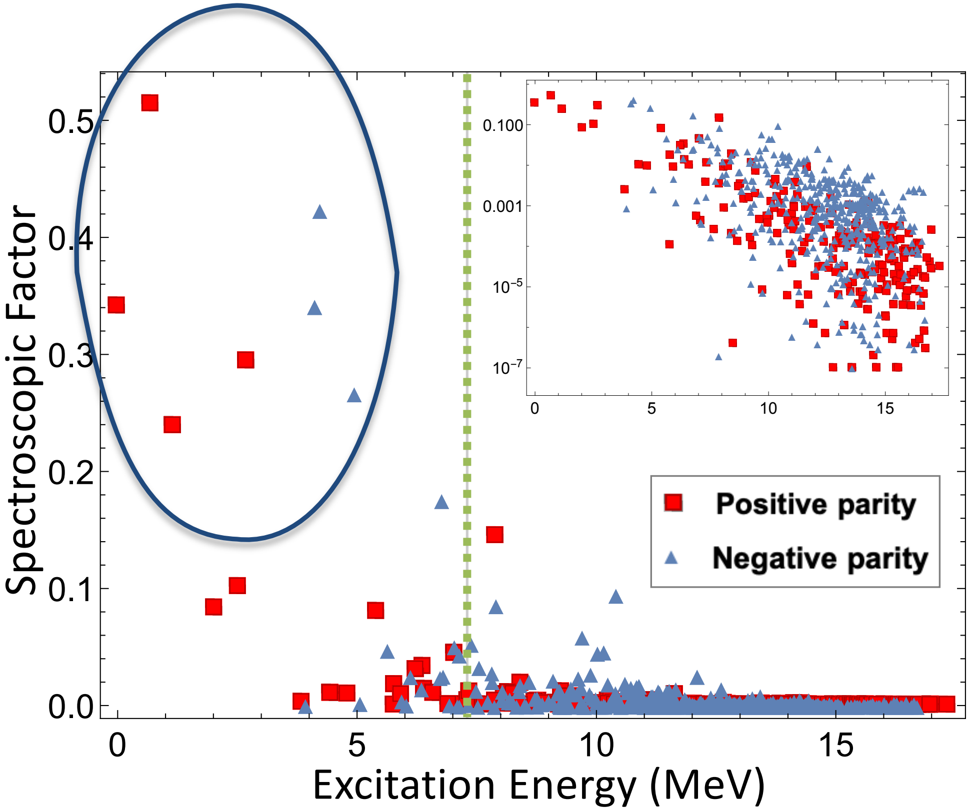
shape elastic



How does this compare with the Hauser-Feshbach distinction between direct and compound, based on energy-averaged S-matrix ?

Some strong single-particle states are also strong γ absorbers/emitters

some states couple strongly
to the neutron channel



ANNALS OF PHYSICS: **63**, 171–218 (1971)

Partial Width Correlations and Common Doorway States

A. M. LANE

PHYSICAL REVIEW LETTERS **125**, 102503 (2020)

Accessing the Single-Particle Structure of the Pygmy Dipole Resonance in ^{208}Pb

M. Spieker,[1,*] A. Heusler,[2] B. A. Brown,[3,4] T. Faestermann,[5] R. Hertenberger,[6] G. Potel,[7] M. Scheck,[8,9] N. Tsoneva,[10] M. Weinert,[11] H.-F. Wirth,[6] and A. Zilges[11]

PHYSICAL REVIEW LETTERS **127**, 242501 (2021)

Microscopic Structure of the Low-Energy Electric Dipole Response of ^{120}Sn

M. Weinert,[1,*] M. Spieker,[2] G. Potel,[3] N. Tsoneva,[4] M. Müscher,[1] J. Wilhelmy,[1] and A. Zilges[1]