

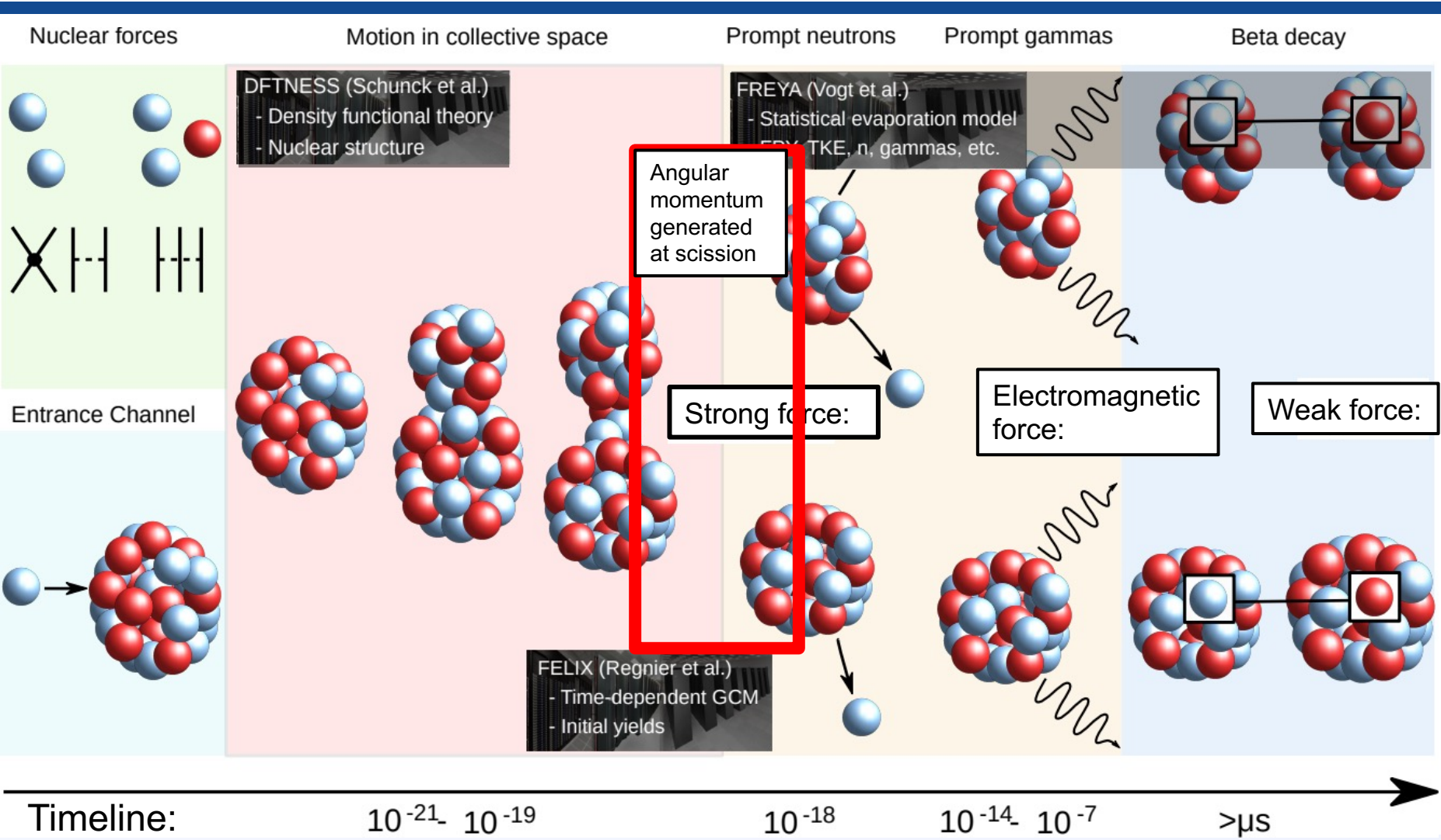
# Observational Consequences of Angular Momentum in Fission



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**Jorgen Randrup (LBNL)**



# Fission is a complicated process involving multiple scales



# Specialized physics models required to study fission: phenomenology required to model complete events

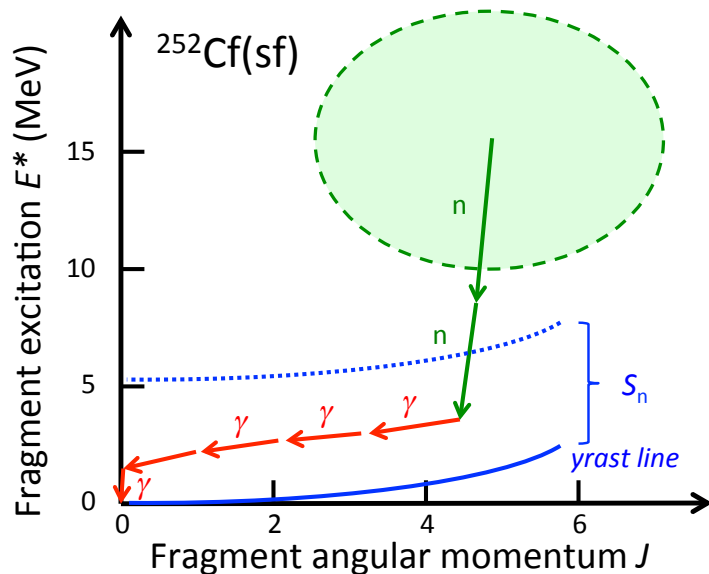
We have been developing **FREYA** (Fission Reaction Event Yield Algorithm) since 2009 to study fission event-by-event

- First such fission model ever published and made generally available
- Showed the importance of energy and momentum conservation for understanding fission data
- **FREYA** is fast enough for users to study fission of different isotopes and energies in real time with a laptop – a big advantage (focus on **FREYA** here for simplicity)

Other codes similar in concept to **FREYA** have also appeared, differ in details:

- **CGMF**; **FIFRELIN**; **GEF**

Older codes deterministic, based on average events, tuned to subsets of average data



# Event-by-event modeling is efficient framework for studying fission

Event-by-event (Monte Carlo) modeling has been used in high energy nuclear and particle physics when there are multiple outcomes – useful for studying detector response and predicting outcomes of experiment

Calculational framework easily adoptable for studying fission

**Goal(s): *Fast* generation of (large) samples of complete fission events**

***Complete* fission event: Full kinematic information on all final particles**

Two product nuclei:  $Z_H, A_H, \mathbf{P}_H$  and  $Z_L, A_L, \mathbf{P}_L$

$\nu$  neutrons:  $\{ \mathbf{p}_n \}, n = 1, \dots, \nu$

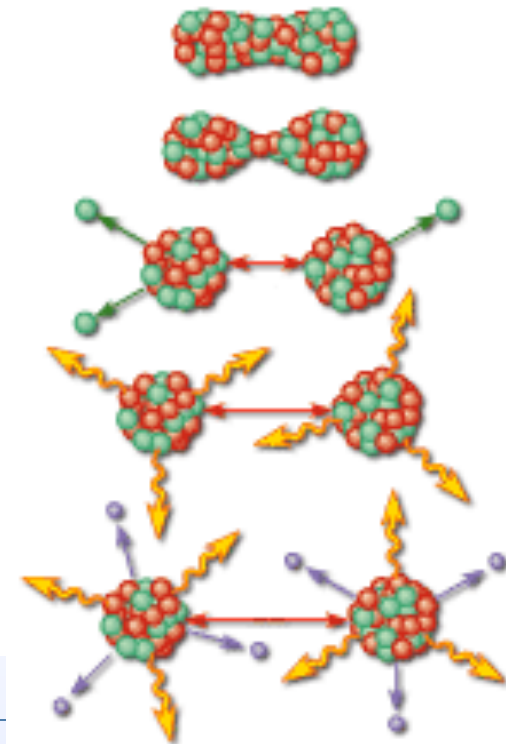
$N_\gamma$  photons:  $\{ \mathbf{p}_m \}, m = 1, \dots, N_\gamma$

**Advantage of having *samples* of complete events:**

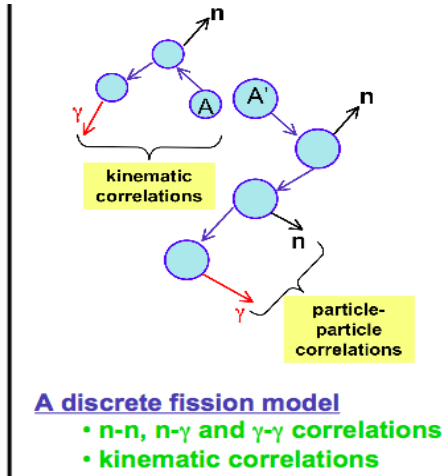
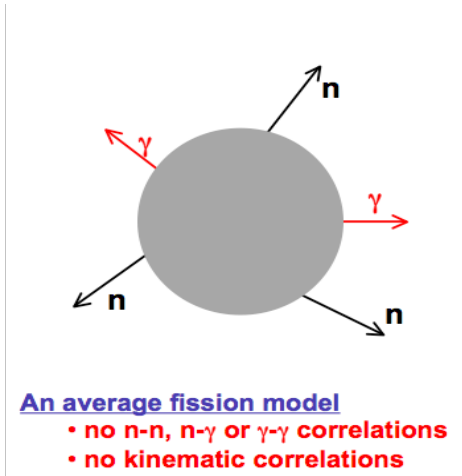
Straightforward to extract *any* observable, including fluctuations and correlations, and to take account of cuts & acceptances

**Advantage of *fast* event generation:**

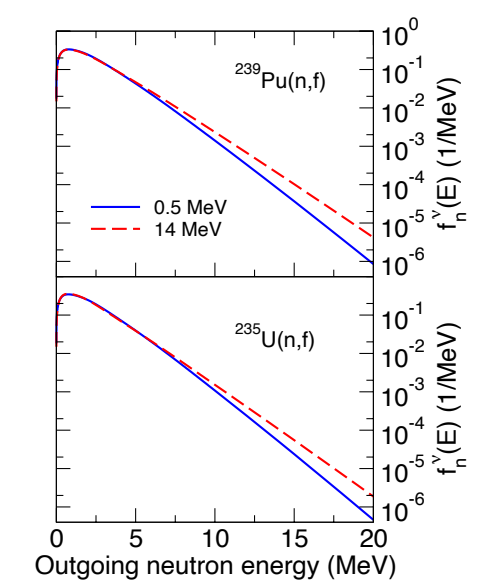
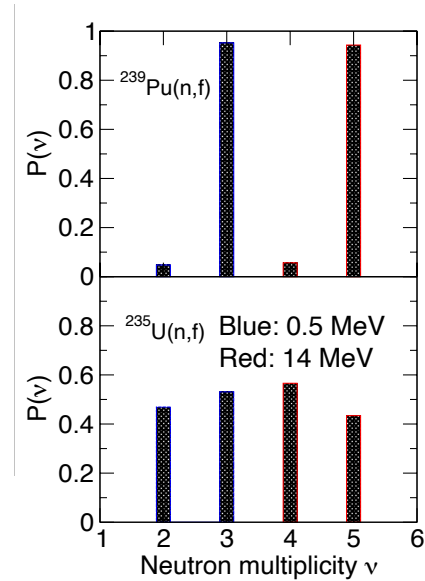
Can be incorporated into transport codes



# Every part of a fission event is correlated



- In ‘average’ models, fission is a black box, neutron and gamma energies sampled from same average distribution, regardless of multiplicity and energy carried away by each emitted particle; **fluctuations and correlations cannot be addressed**
- Detailed models generate complete fission events: energy & momentum of neutrons, photons, and products in each individual fission event; **correlations are automatically included**



- Traditionally, neutron multiplicity sampled between nearest values to get correct average value
- All neutrons sampled from same spectral shape, independent of multiplicity – **no conservation of energy or momentum!**



# Brief synopsis of how FREYA works

- For a given  $Z$ ,  $A$  and energy ( $E_n = 0$  for spontaneous fission), FREYA selects mass and charge of fragment from either data or a model (5 gaussian) parameterization
- Second fragment mass and charge obtained assuming binary fission, mass and charge conservation
- From fragment identities, fission  $Q$  value is obtained
- $TKE(A_H)$  sampled from distribution; TXE obtained by energy conservation
- **'Spin temperature' sets level of rotational energy, remaining TXE given to intrinsic excitation energy**
- Intrinsic excitation divided between fragments, based on level densities, then thermal fluctuations introduced to obtain final excitation energy sharing
- Thermal fluctuations remove energy from TKE to maintain energy conservation, equivalent to width of TKE distribution
- **Spin fluctuations (conserving angular momentum), introduced for wriggling and bending modes**
- Pre-equilibrium emission and n-th chance fission included for  $E_n \leq 20$  MeV
- After scission, fragments are de-excited first by emitting neutrons (Weisskopf-Ewing spectra) until the remaining energy is less than the neutron separation energy
- Photon emission follows until fragment no longer excited (statistical, then discrete emission)



# FREYA has five physics-based parameters

- The fissioning nucleus, with  $A_0$  nucleons, has an initial excitation energy  $E_{sc}$  including statistical and rotational excitation of the fragments
- The level density parameter,  $a \sim A_0/e_0$ , relates the temperature to the excitation energy, as in  $E_{sc} = (A_0/e_0) T_{sc}^2 - e_0$  is the first parameter
- **The fragment ‘spin temperature’ fluctuates around the scission temperature  $T_{sc}$  according to second parameter  $c_s$ ,  $T_s \sim c_s T_{sc}$ , affecting rotational energy  $E_{rot}$  and photon observables**
- Total excitation energy,  $E_{sc} = E_{rot} + E_{stat}$ ,  $E_{stat}$  is dissipated through neutron emission
- Statistical energy is partitioned between light and heavy fragments according to level density parameters,  $E_{stat} = E^*_L + E^*_H$
- The light fragment energy is enhanced by third parameter,  $x > 1$ , by  $E^{**}_L = x E^*_L$  so that  $E^{**}_H = E_{stat} - E^{**}_L$ , affecting neutron multiplicity vs fragment mass
- Fragments get thermal variance, fourth parameter,  $c$ , controlling maximum available excitation and affecting neutron multiplicity distribution and moments
- Fifth parameter **dTKE** adjusts average TKE to fix average neutron multiplicity

# FREYA parameters have been optimized for spontaneous fission

Work by J. Van Dyke, L. Bernstein, and RV, Nucl. Instrum. Meth. A 922 (2019) 36-46, arXiv:1809.05587.

Parameters  $c$ ,  $x$ , and  $c_s$  have direct effects on  $P(\nu)$ ;  $\nu(A)$ ; and photon observables: no other parameters significantly affect these observables

Shape of neutron spectra affected by all parameters



# Optimized Parameters for all Spontaneous Fission in FREYA

|                   |            | $e_0$<br>/MeV | $x$         | $c$         | $c_S$       | dTKE<br>(MeV) | #Data<br>sets | #Evals |
|-------------------|------------|---------------|-------------|-------------|-------------|---------------|---------------|--------|
| $^{238}\text{U}$  | $\vec{x}$  | 10.391        | 1.220       | 0.939       | 0.899       | -1.375        | 0             | 1      |
|                   | $\sigma_x$ | $\pm 0.352$   | $\pm 0.071$ | $\pm 0.238$ | $\pm 0.280$ | $\pm 0.727$   | -             | -      |
| $^{238}\text{Pu}$ | $\vec{x}$  | 10.521        | 1.232       | 1.968       | 0.893       | -1.408        | 0             | 1      |
|                   | $\sigma_x$ | $\pm 0.581$   | $\pm 0.221$ | $\pm 0.071$ | $\pm 0.071$ | $\pm 3.424$   | -             | -      |
| $^{240}\text{Pu}$ | $\vec{x}$  | 10.750        | 1.307       | 3.176       | 0.908       | -3.219        | 1             | 1      |
|                   | $\sigma_x$ | $\pm 0.138$   | $\pm 0.071$ | $\pm 0.355$ | $\pm 0.023$ | $\pm 0.112$   | -             | -      |
| $^{242}\text{Pu}$ | $\vec{x}$  | 10.018        | 1.144       | 3.422       | 0.911       | -1.662        | 1             | 1      |
|                   | $\sigma_x$ | $\pm 1.768$   | $\pm 0.152$ | $\pm 0.341$ | $\pm 0.257$ | $\pm 0.118$   | -             | -      |
| $^{244}\text{Cm}$ | $\vec{x}$  | 10.488        | 1.239       | 1.391       | 0.906       | -4.494        | 2             | 1      |
|                   | $\sigma_x$ | $\pm 1.519$   | $\pm 0.187$ | $\pm 0.582$ | $\pm 0.322$ | $\pm 0.167$   | -             | -      |
| $^{252}\text{Cf}$ | $\vec{x}$  | 10.429        | 1.274       | 1.191       | 0.875       | 0.525         | 4             | 2      |
|                   | $\sigma_x$ | $\pm 1.090$   | $\pm 0.187$ | $\pm 0.362$ | $\pm 0.020$ | $\pm 0.078$   | -             | -      |



# Trends can be seen in the parameter values

The parameter  $e_0$  is around 10.5/MeV on average for all cases

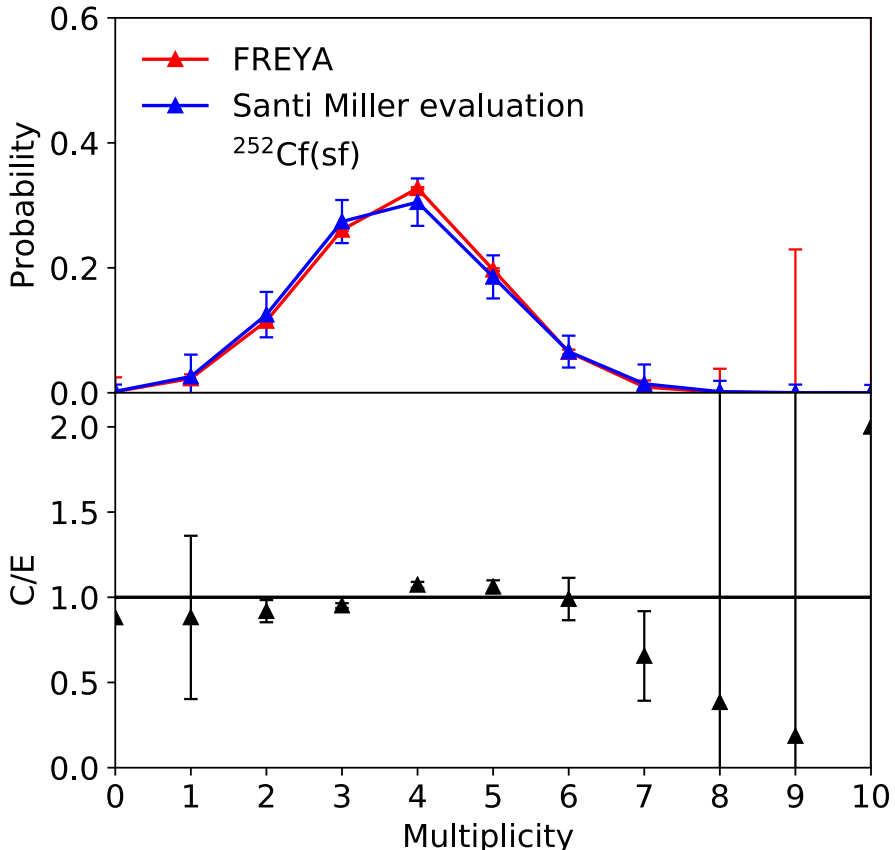
The parameter  $x$  is around 1.2 while  $c_s$  is around 0.9 for all

The parameter  $c$  should be around 1; it is 2-3 for the Pu(sf) isotopes, suggesting they are broader than should be (compare to  $^{238}\text{U}(sf)$  where  $c < 1$ ; both have average neutron multiplicities of around 2-2.1 per fission)

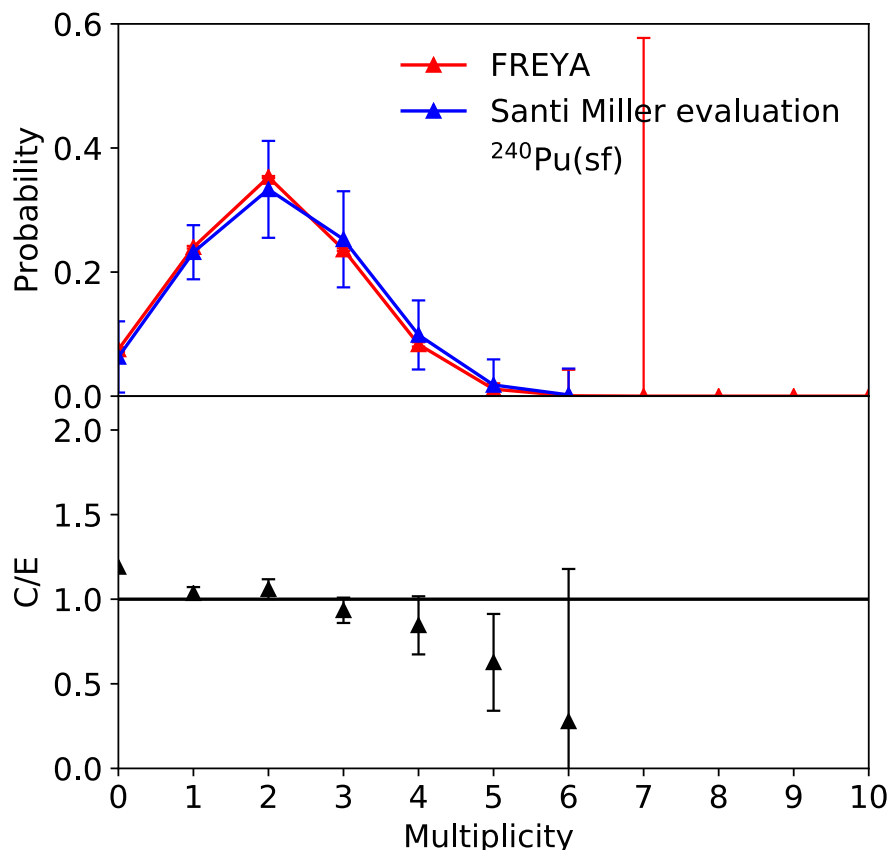
# Neutron multiplicity distributions

Evaluations by Santi & Miller (NSE **160** (2008) 190) have very small uncertainties and an unrealistic number of significant figures.

## $^{252}\text{Cf}(sf)$



## $^{240}\text{Pu}(sf)$



# Revisiting $^{238,240,242}\text{Pu}$ optimization with new data

New Prompt Fission Neutron Spectral data for  $^{240}\text{Pu}(\text{sf})$  in PRC 109, 064611 (2024)

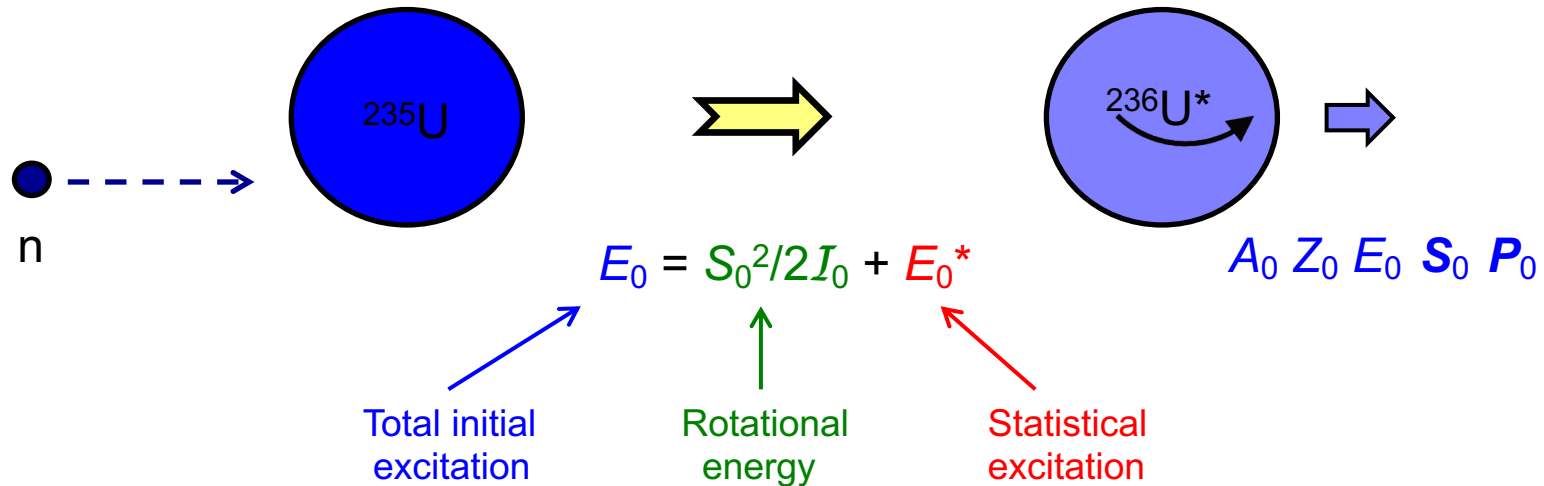
More data to come from LANL-CEA collaboration, working with Alex Tuckey (UM) to optimize the FREYA parameters for all Pu(sf) isotopes with these new data

Once the parameters have been fit to some data for a given isotope, other observables can be predicted, such as those related to angular momentum

# Angular Momentum Generation in FREYA



# Start from the rotating compound nucleus generated by the incoming neutron



The plane of rotation is determined by the impact parameter of incident neutron; the plane may change due to *pre-fission neutron evaporation* (which is treated the same as the post-fission neutron evaporation from the rotating fragments)

Introduced (half) integer spin and can now specify fragments more precisely

RV & JR, PRC **103** (2021) 014610

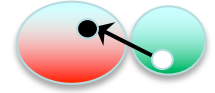
JR & RV, PRL **127** (2021) 062502

JR, T. Dossing & RV, PRC **106** (2022) 014609

JR, PRC **106** (2022) L051601

# FREYA mechanism of fragment spin generation: nucleon exchange

Relevant theory of nucleon exchange



*Damped heavy-ion collisions*, W.U. Schröder and J.R. Huizenga, *Ann. Rev. Nucl. Sci.* (1977) 465

Intimate relationship between nucleon exchange and energy dissipation

*Theory of transfer-induced transport in nuclear collisions*, J. Randrup, *Nucl. Phys.* **A327** (1979) 490:

Each transfer changes the nucleon numbers and the excitation energies of the fragments,  
as well as their linear & angular momenta

*Transport of angular momentum in damped nuclear reactions*, J. Randrup, *Nucl. Phys.* **A383** (1982) 468:

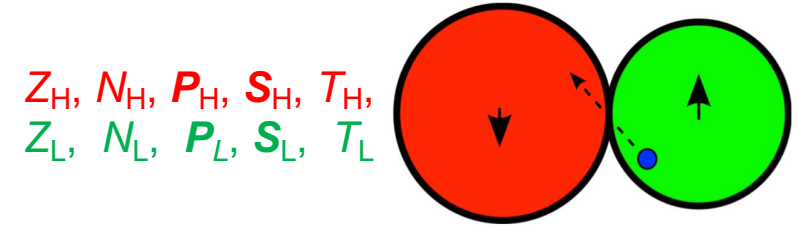
Mobility (friction) tensor: anisotropic

*Dynamical evolution of angular momentum in damped nuclear reactions*, T. Døssing and J. Randrup, *Nucl. Phys.* **A433** (1985) 215:

Relaxation times

$t_{\text{wriggling}} \ll t_{\text{bending}} \ \& \ t_{\text{twisting}} \ \ll\ll t_{\text{tilting}}$   
fast  slow

# Expectations based on the *Nucleon Exchange Transport* model



J. Randrup, Nucl. Phys. A **327**, 490 (1979)  
 J. Randrup, Nucl. Phys. A **383**, 468 (1983)  
 T. Døssing & J. Randrup, NPA **433**, 215 (1985)

Multiple nucleon transfers produce a dissipative force that affects the linear and angular momenta of the binary partners

Time scale:

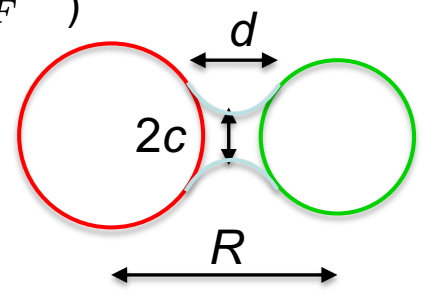
The mobility coefficients for the rotational modes:

$$\left\{ \begin{array}{l} M_{\text{wrig}} = m\mathcal{N}R^2 \\ M_{\text{bend}} = m\mathcal{N} \left[ \left( \frac{\mathcal{I}_H R_L - \mathcal{I}_L R_H}{\mathcal{I}_L + \mathcal{I}_H} \right)^2 + c_{\text{ave}}^2 \right] \\ M_{\text{twst}} = m\mathcal{N}c_{\text{ave}}^2 \end{array} \right.$$

One-way nucleon current:  $\mathcal{N} \approx \frac{1}{4}\rho\bar{v}\pi c^2 \quad (\bar{v} = \frac{3}{4}v_F)$

Center separation:  $R = R_L + R_H + d$

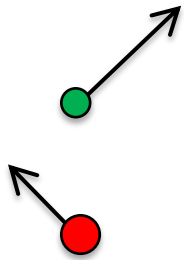
Neck radius:  $c \quad (c^2 \ll R^2)$





# Basic Kinematic Setup

## Two moving particles

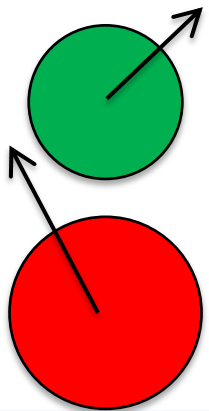


$$\mathbf{p}_1 = m_1 \mathbf{v}_1 \quad \mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 = M\mathbf{V} = \mathbf{p}_+ \quad m_+ = M = m_1 + m_2 \quad \text{TOTAL}$$

$$\mathbf{p}_2 = m_2 \mathbf{v}_2 \quad \mathbf{p} = \mu(\mathbf{v}_1 - \mathbf{v}_2) = \mu\mathbf{v} = \mathbf{p}_- \quad \frac{1}{m_-} = \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \quad \text{RELATIVE}$$

$$E_{\text{kin}} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{P^2}{2M} + \frac{p^2}{2\mu} = \frac{p_+^2}{2m_+} + \frac{p_-^2}{2m_-}$$

## Two rotating spheres

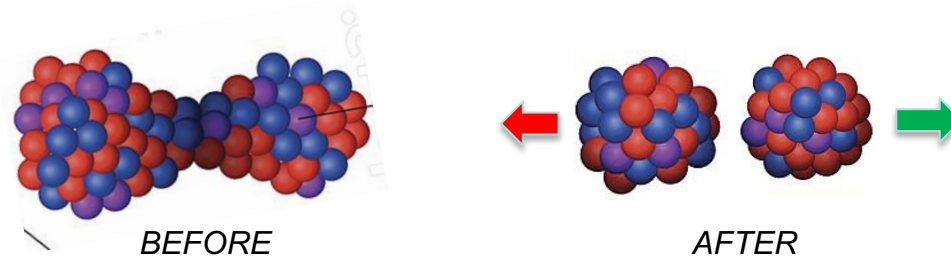


$$\mathbf{S}_1 = I_1 \boldsymbol{\omega}_1 \quad \mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = \mathbf{s}_+ \quad I_+ = I_{\text{tot}} = I_1 + I_2 \quad \text{TOTAL}$$

$$\mathbf{S}_2 = I_2 \boldsymbol{\omega}_2 \quad \mathbf{s} = I_{\text{rel}}(\boldsymbol{\omega}_1 - \boldsymbol{\omega}_2) = \mathbf{s}_- \quad \frac{1}{I_-} = \frac{1}{I_{\text{rel}}} = \frac{1}{I_1} + \frac{1}{I_2} \quad \text{RELATIVE}$$

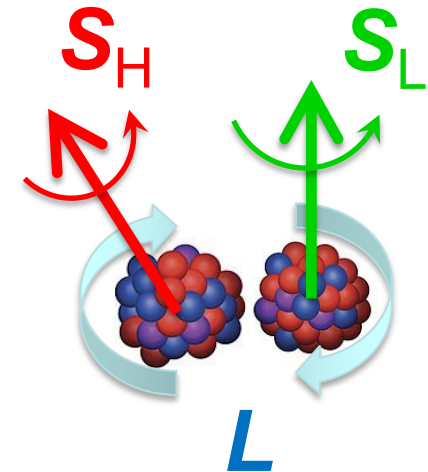
$$E_{\text{rot}} = \frac{S_1^2}{2I_1} + \frac{S_2^2}{2I_2} = \frac{s_+^2}{2I_+} + \frac{s_-^2}{2I_-}$$

# Angular momentum after scission



Three coupled  
angular momenta:

- Spin of the light fragment  $S_L$
- Spin of the heavy fragment  $S_H$
- Orbital angular momentum  $L$




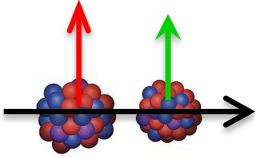
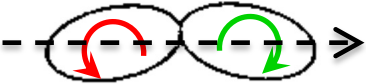
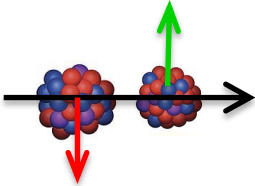
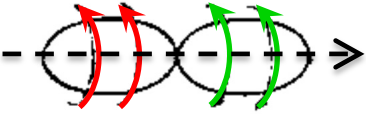
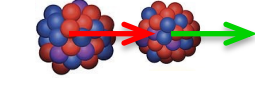
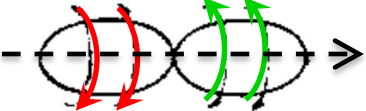
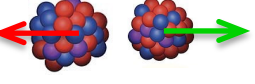
The total angular momentum is *conserved*:

$$S_L + S_H + L = S_0 \quad (\approx 0 \text{ for spontaneous fission})$$

=> Six independent internal rotational modes

# Rotational modes of the two-fragment system

J.R. Nix & W.J. Swiatecki,  
Nucl. Phys. 71, 1 (1963):

|     |   |   |                   |   |   |  |
|-----|---|---|-------------------|---|---|--|
| + + | # | 2 | <i>Wriggling:</i> |    |    | mutually parallel,<br>perpendicular to axis      |
| - - | # | 2 | <i>Bending:</i>   |    |    | mutually anti-parallel,<br>perpendicular to axis |
| +   | # | 1 | <i>Tilting:</i>   |    |    | mutually parallel,<br>parallel to axis           |
| -   | # | 1 | <i>Twisting:</i>  |  |  | mutually anti-parallel,<br>parallel to axis      |

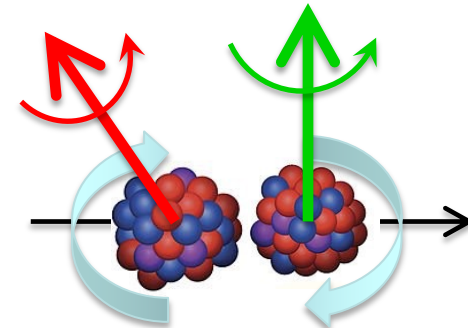
Added to FREYA in 2014

# Rotational modes in dinuclear complex: damped nuclear reactions

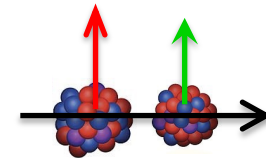
$$E_0^{\text{rot}} = \frac{S_L^2}{2\mathcal{I}_L} + \frac{S_H^2}{2\mathcal{I}_H} + \frac{(S_0 - S_L - S_H)^2}{2\mathcal{I}_R}$$

$$= \frac{S_0^2}{2\mathcal{I}_0} + \frac{s_{\text{wrig}}^2}{2\mathcal{I}_{\text{wrig}}} + \frac{s_{\text{bend}}^2}{2\mathcal{I}_{\text{bend}}} + \frac{s_{\text{twst}}^2}{2\mathcal{I}_{\text{twst}}} + \frac{s_{\text{tilt}}^2}{2\mathcal{I}_{\text{tilt}}}$$

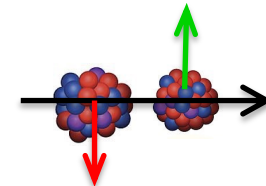
$S_0 = S_L + S_H + L$



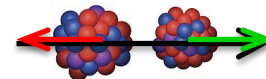
2 *Wriggling:*  $\Rightarrow \delta S_{L,H}^{\text{wrig}} = \frac{\mathcal{I}_{L,H}}{\mathcal{I}_L + \mathcal{I}_H} s_{\text{wrig}}, \quad \delta L^{\text{wrig}} = -s_{\text{wrig}}$



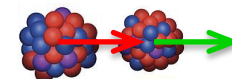
2 *Bending:*  $\Rightarrow \delta S_L^{\text{bend}} = s_{\text{bend}}, \quad \delta S_H^{\text{bend}} = -s_{\text{bend}}, \quad \delta L^{\text{bend}} = 0$



1 *Twisting:*  $\Rightarrow \delta S_L^{\text{twst}} = s_{\text{twst}}, \quad \delta S_H^{\text{twst}} = -s_{\text{twst}}, \quad \delta L^{\text{twst}} = 0$



1 *Tilting:*  $\Rightarrow$  is not directly agitated in because  $L \cdot R = 0$



Total angular momentum still conserved

# FREYA can be used to explore different scenarios:

In order to explore a variety of rotational scenarios, we introduce the *mode temperatures*  $T_m = c_m T_{sc}$

The mode amplitudes  $\{s_m\}$  are thus sampled from

*Mode spins:*

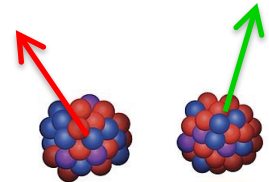
$$P_{\text{wrig}}(s_{\text{wrig}}) \sim \exp(-s_{\text{wrig}}^2 / 2\mathcal{I}_{\text{wrig}}T_{\text{wrig}}), \quad T_{\text{wrig}} = c_{\text{wrig}}T_{sc}$$

$$P_{\text{bend}}(s_{\text{bend}}) \sim \exp(-s_{\text{bend}}^2 / 2\mathcal{I}_{\text{bend}}T_{\text{bend}}), \quad T_{\text{bend}} = c_{\text{bend}}T_{sc}$$

$$P_{\text{twst}}(s_{\text{twst}}) \sim \exp(-s_{\text{twst}}^2 / 2\mathcal{I}_{\text{twst}}T_{\text{twst}}), \quad T_{\text{twst}} = c_{\text{twst}}T_{sc}$$

*Fragment spins:*

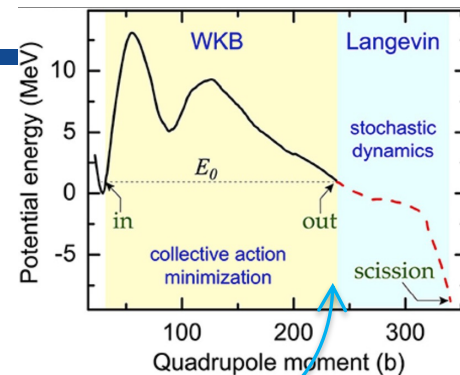
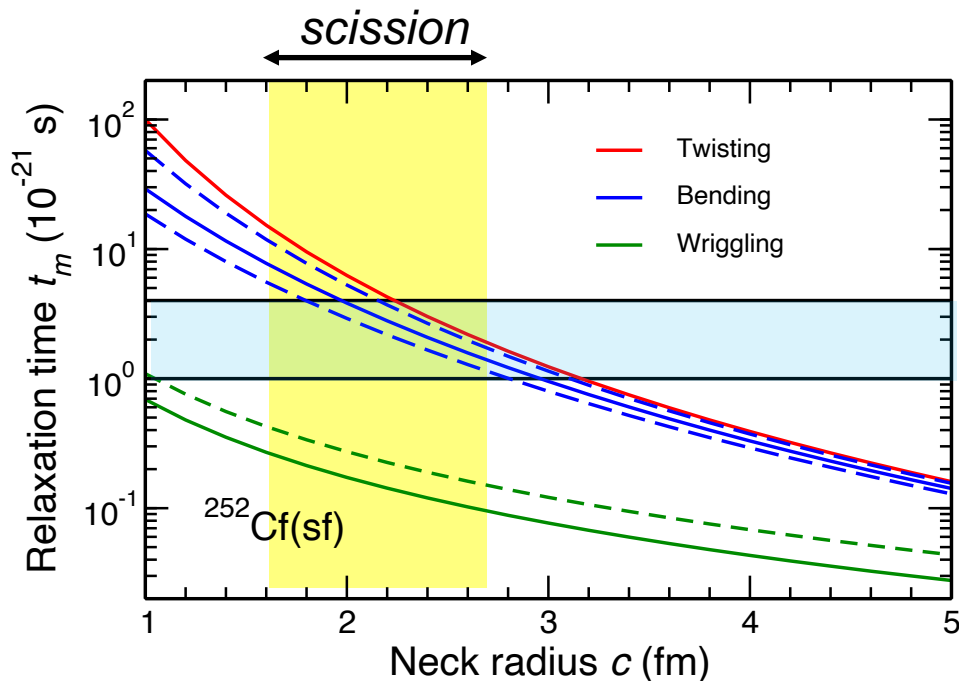
$$\Rightarrow \begin{cases} \mathbf{S}_L = (\mathcal{I}_L/\mathcal{I}_+) \mathbf{s}_{\text{wrig}} + \mathbf{s}_{\text{bend}} + \mathbf{s}_{\text{twst}} \\ \mathbf{S}_H = (\mathcal{I}_H/\mathcal{I}_+) \mathbf{s}_{\text{wrig}} - \mathbf{s}_{\text{bend}} - \mathbf{s}_{\text{twst}} \end{cases}$$



The relative presence of the different modes  $m$  can then be tuned by the coefficients ( $c_{\text{wrig}}$ ,  $c_{\text{bend}}$ ,  $c_{\text{twst}}$ )

*Example:* Standard FREYA  $\rightarrow (1,1,0)$ :  
full wriggling & bending, no twisting

# Relaxation times of dinuclear rotational modes



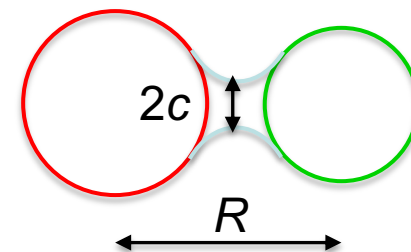
$t_{\text{fiss}} : \text{fission time} \approx 1 - 4 \cdot 10^{-21}$  s

Expectations from nucleon exchange:

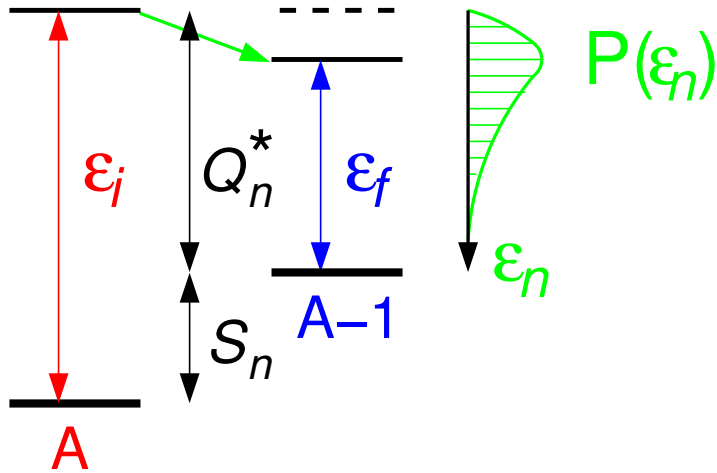
Wriggling is probably fully agitated

Twisting is unlikely to play a major role; it grows more prominent with excitation

Bending probably has some presence; it increases with the mass asymmetry



# Neutron evaporation from rotating fragments



$$M_i^* = M_i^{\text{gs}} + \epsilon_i \quad M_f^* = M_f^{\text{gs}} + \epsilon_f \quad M_i^* = M_f^* + m_n + \epsilon$$

$$Q_n \equiv Q_n^*(\epsilon_i=0) = M_i^{\text{gs}} - M_f^{\text{gs}} - m_n = -S_n$$

$$Q_n^* = \epsilon_i + Q_n = \epsilon_i - S_n$$

$$\epsilon + \epsilon_f = M_i^* - M_f^{\text{gs}} - m_n = Q_n^* = \begin{cases} \epsilon_f^{\text{max}} \\ \epsilon^{\text{max}} \end{cases}$$

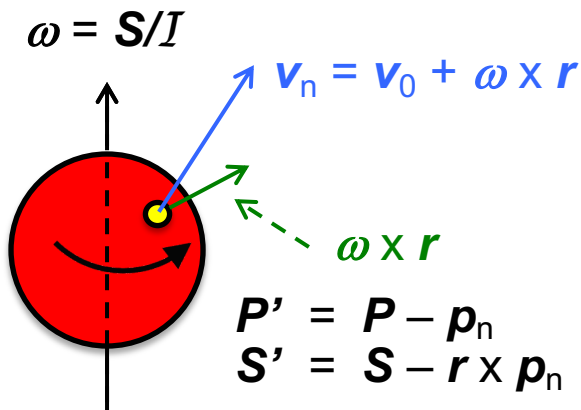
$$T_f^{\text{max}} = \sqrt{\epsilon_f^{\text{max}}/a_f} = \sqrt{Q_n^*/a_f}$$

Weisskopf-Ewing neutron energy spectrum:  $d^3\mathbf{p} \sim \sqrt{\epsilon} d\epsilon d\Omega$  (non-relativistic)

$$\frac{d^3 N}{d^3 \mathbf{p}} d^3 \mathbf{p} \sim \sqrt{\epsilon} e^{-\epsilon/T_f^{\text{max}}} \sqrt{\epsilon} d\epsilon d\Omega = e^{-\epsilon/T_f^{\text{max}}} \epsilon d\epsilon d\Omega$$

When fragment is rotating, emission from moving surface,  $\mathbf{v}_0$ , is boosted by local rotational velocity  $\boldsymbol{\omega} \times \mathbf{r}$  and daughter nucleus absorbs recoil linear and angular momentum

Neutron and daughter nucleus Lorentz boosted from emitter frame to laboratory frame



Neutron emission conserves energy and linear & angular momentum

# Photon emission follows neutron emission

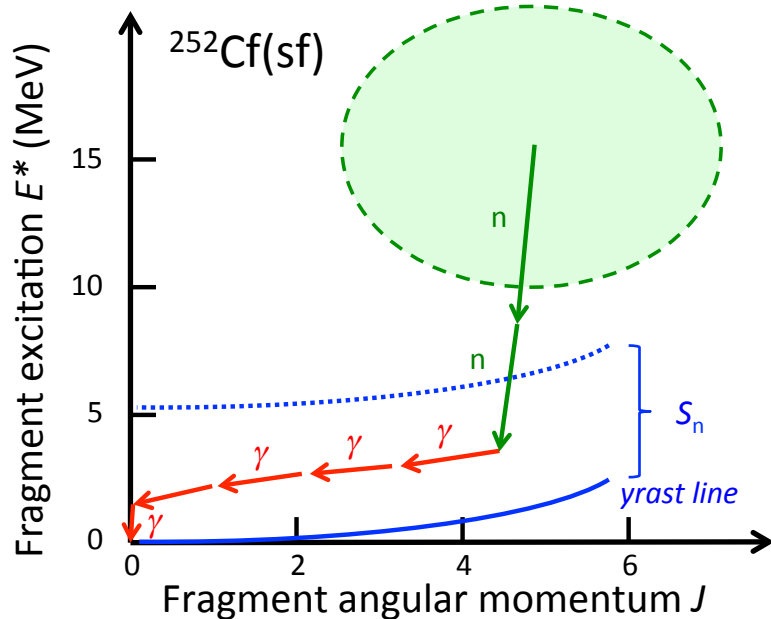
Neutron evaporation ceases when  $E^* < S_n$  (neutron separation energy);  
the remaining excitation energy is disposed of by sequential photon emission ...

... first by statistical photon cascades down to the yrast line ...

$$\frac{d^3 N}{d^3 \mathbf{p}_\gamma} d^3 \mathbf{p}_\gamma \sim \left[ \frac{\Gamma_{\text{GDR}}^2 \epsilon^2}{(\epsilon^2 - \epsilon_{\text{GDR}}^2)^2 + \Gamma_{\text{GDR}}^2 \epsilon^2} \right] \epsilon^2 e^{-\epsilon/T_i} \quad \Leftarrow \quad d^3 \mathbf{p}_\gamma \sim \epsilon^2 d\epsilon d\Omega$$

*(ultrarelativistic)*

$$S_f = S_i - 1 \quad E_f^* = E_i^* - \epsilon_\gamma \quad \epsilon_{\text{GDR}} = \left( 31.2A^{-1/3} + 20.6A^{-1/6} \right) \text{ MeV} \quad \Gamma_{\text{GDR}} = 5 \text{ MeV}$$



.. then by stretched E2 photons along the yrast line ...

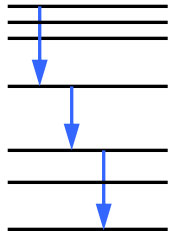
$$S_f = S_i - 2$$

$$\epsilon_\gamma = S_i^2 / 2\mathcal{I}_A - S_f^2 / 2\mathcal{I}_A$$

$$\mathcal{I}_A = 0.5 \times \frac{2}{5} A m_N R_A^2$$

... whenever possible, the RIPL decay tables are used instead...

Each photon is Lorentz boosted from the emitter to the laboratory frame



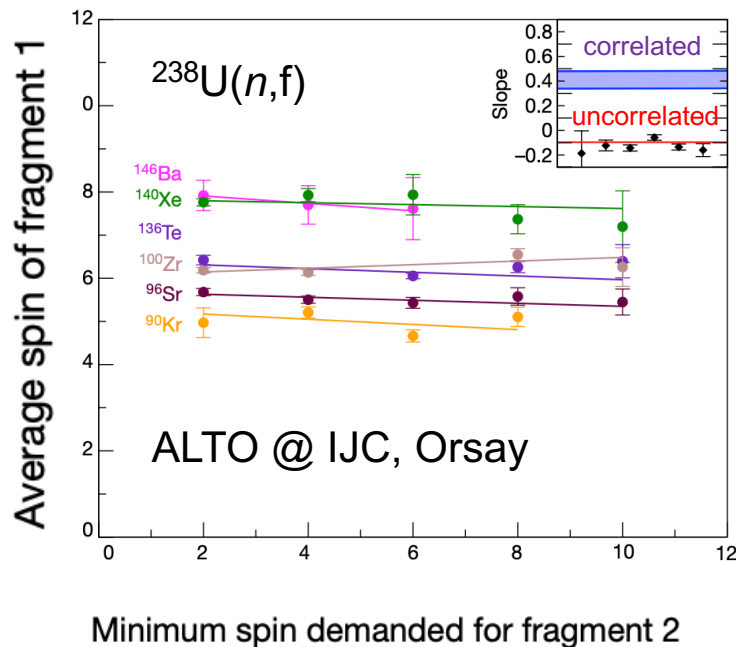


# How can we learn about angular momentum based on measurements?



# Recent experimental information on spin correlations

Angular momentum generation in nuclear fission,  
J. N. Wilson *et al.*, Nature **590** (2021) 566



OBSERVATION:

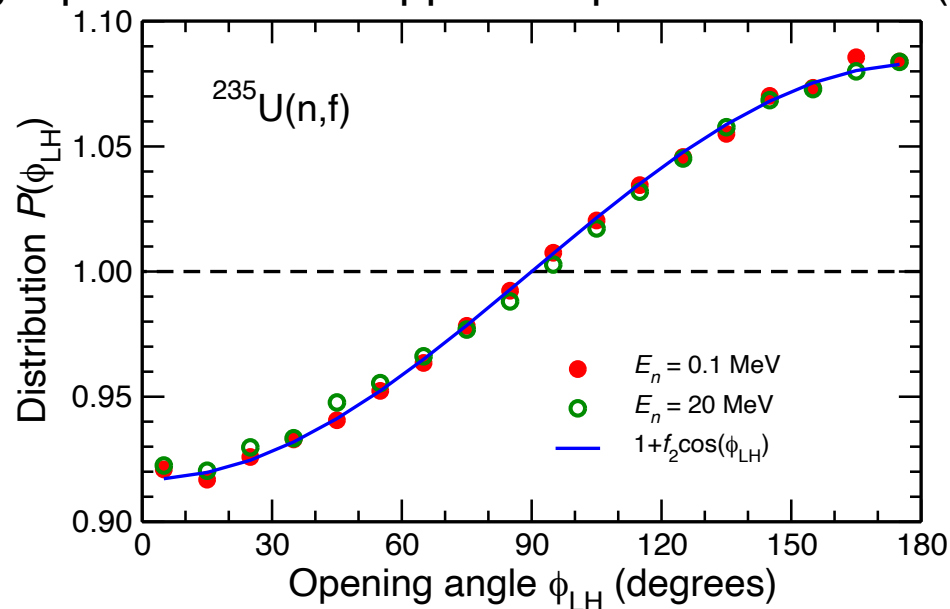
*“There is no significant correlation between the spins of the fragments”*

INTERPRETATION:

*Therefore “the fragment spins are generated after the nucleus splits”, i.e. “after the fragments have become two separate, independent systems”*

# Dominance of fluctuations results in very weak fragment spin correlation

- The fragment spins  $\mathbf{S}_L$  &  $\mathbf{S}_H$  are dominated by wriggling & bending fluctuations and are only very weakly correlated (both mutually and w.r.t  $\mathbf{S}_0$ )
- Recoil from wriggling creates some orbital motion and the subsequent Coulomb trajectory reorients the direction of the relative fragment motion by about  $2^\circ$
- The remaining weak directional correlation is effectively independent of the initial energy, the compound nuclear spin, and the fragment mass division
- There is a slight preference for opposite spin directions:  $P(180^\circ)/P(0^\circ) = 1.18$



R.V. and J. Randrup,  
PRC **103** (2021) 014610

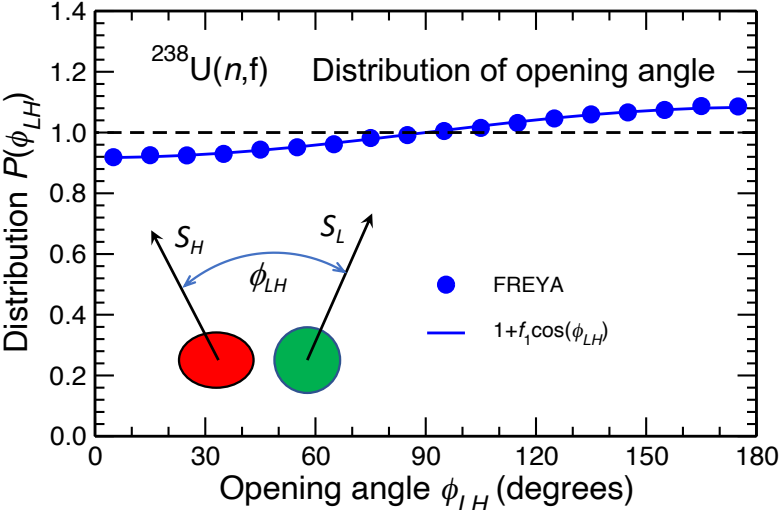
# Moment of inertia for orbital motion is large: $I_R \gg I_H, I_L \rightarrow$ very weak fragment spin correlations

We can calculate the direction and magnitude of spin correlations

spin-spin  
 correlation  
 coefficient:

$$c(S_L, S_H) \equiv \frac{\langle \delta S_L \cdot \delta S_H \rangle}{[\langle \delta S_L^2 \rangle \langle \delta S_H^2 \rangle]^{1/2}} = - \left[ \frac{I_L I_H}{(I_R + I_L)(I_R + I_H)} \right]^{\frac{1}{2}} \ll 1$$

Correlation between the spin *directions*:



Correlation between the spin *magnitudes*:

| Case:                             | <sup>235</sup> U(n, f) | <sup>238</sup> U(n, f) | <sup>239</sup> Pu(n, f) | <sup>252</sup> Cf(sf) |
|-----------------------------------|------------------------|------------------------|-------------------------|-----------------------|
| $\bar{S}_L = \langle S_L \rangle$ | 4.27                   | 4.43                   | 4.58                    | 5.08                  |
| $\bar{S}_H = \langle S_H \rangle$ | 5.66                   | 5.80                   | 5.93                    | 6.33                  |
| $c(S_L, S_H)$ (%)                 | 0.2                    | 0.2                    | 0.1                     | 0.1                   |
| $f_1$ (%)                         | -8.2                   | -8.3                   | -8.3                    | -8.4                  |

magnitude of correlation coefficient:

$$c(S_L, S_H) \equiv \frac{\langle \delta S_L \delta S_H \rangle}{[\langle \delta S_L^2 \rangle \langle \delta S_H^2 \rangle]^{1/2}}$$

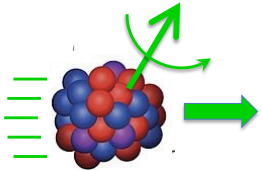
JR & RV, PRL **127** (2021) 062502, RV & JR, PRC **103** (2021) 014610



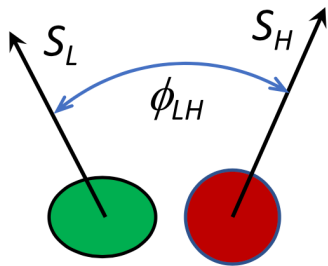
# How can we differentiate between different levels of spin fluctuations?



# Probing fragment spin directions and thus spin modes using photon measurements



Orientation of the fragment spins relative to the fragment motion?



Relative orientation of the fragment spins?

JR, T Dossing & RV, PRC **106** (2022) 014609

# Angular distribution relative to fragment direction $dN/d\cos(\theta_{\gamma,f})$

Look only at E2 emissions in even-even product nuclei

*Pioneering experiments:* J.B. Wilhelmy *et al.*, Phys. Rev. C **5**, 2041 (1972)  
 A. Wolf & E. Cheifetz, Phys. Rev. C **13**, 1952 (1976)

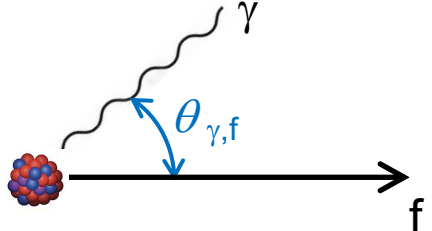
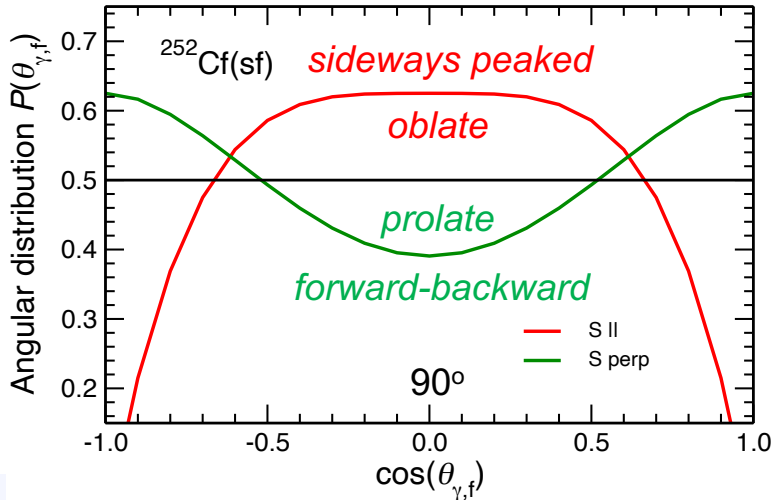
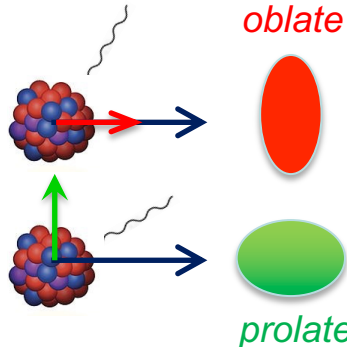
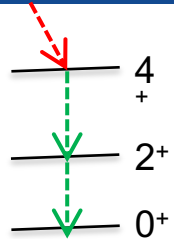
## Two reference scenarios:

The fragment spin is *parallel* to the direction of motion:

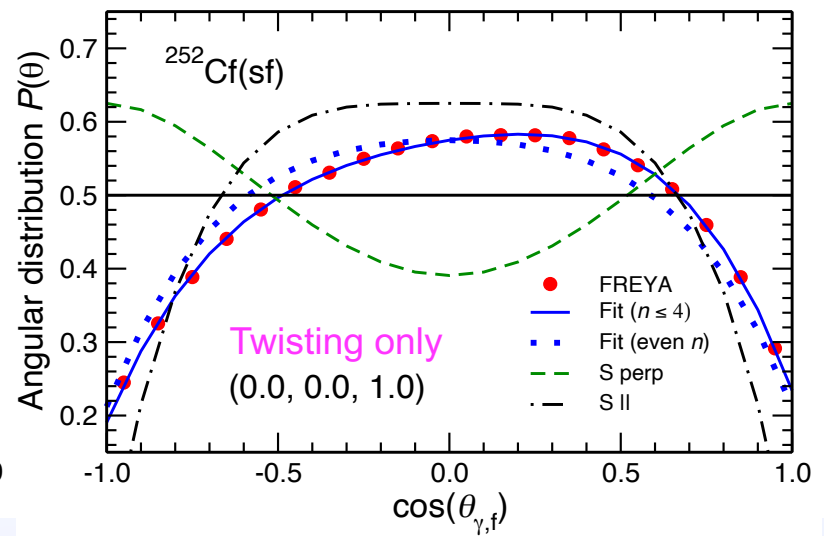
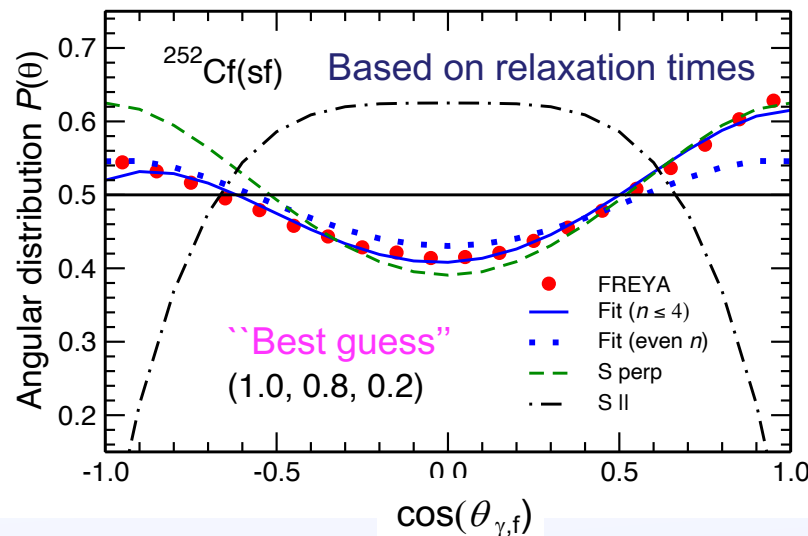
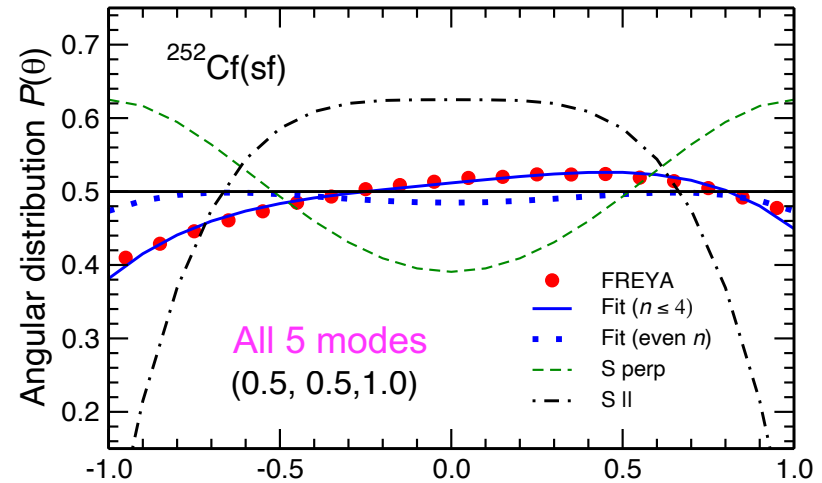
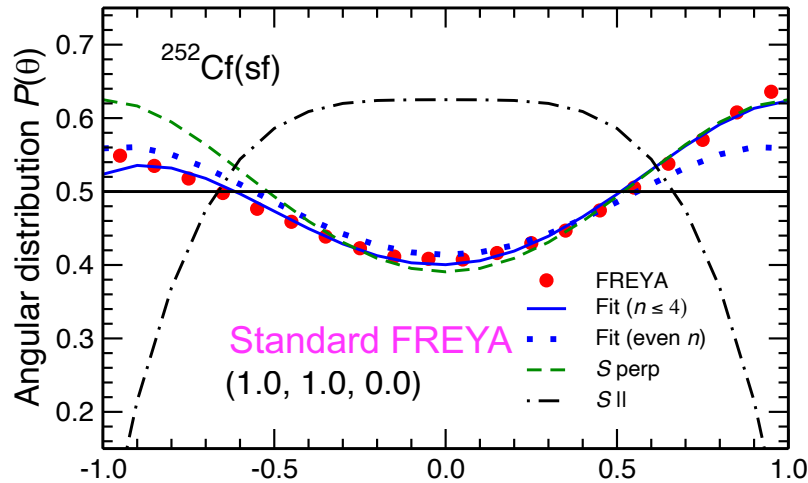
$$W_{\parallel}(\theta_{\gamma f}) \sim 1 - \frac{5}{7}P_2(\cos \theta_{\gamma f}) - \frac{2}{7}P_4(\cos \theta_{\gamma f})$$

Fragment spin is *perpendicular* to the direction of motion:

$$W_{\perp}(\theta_{\gamma f}) \sim 1 + \frac{5}{14}P_2(\cos \theta_{\gamma f}) - \frac{3}{28}P_4(\cos \theta_{\gamma f})$$

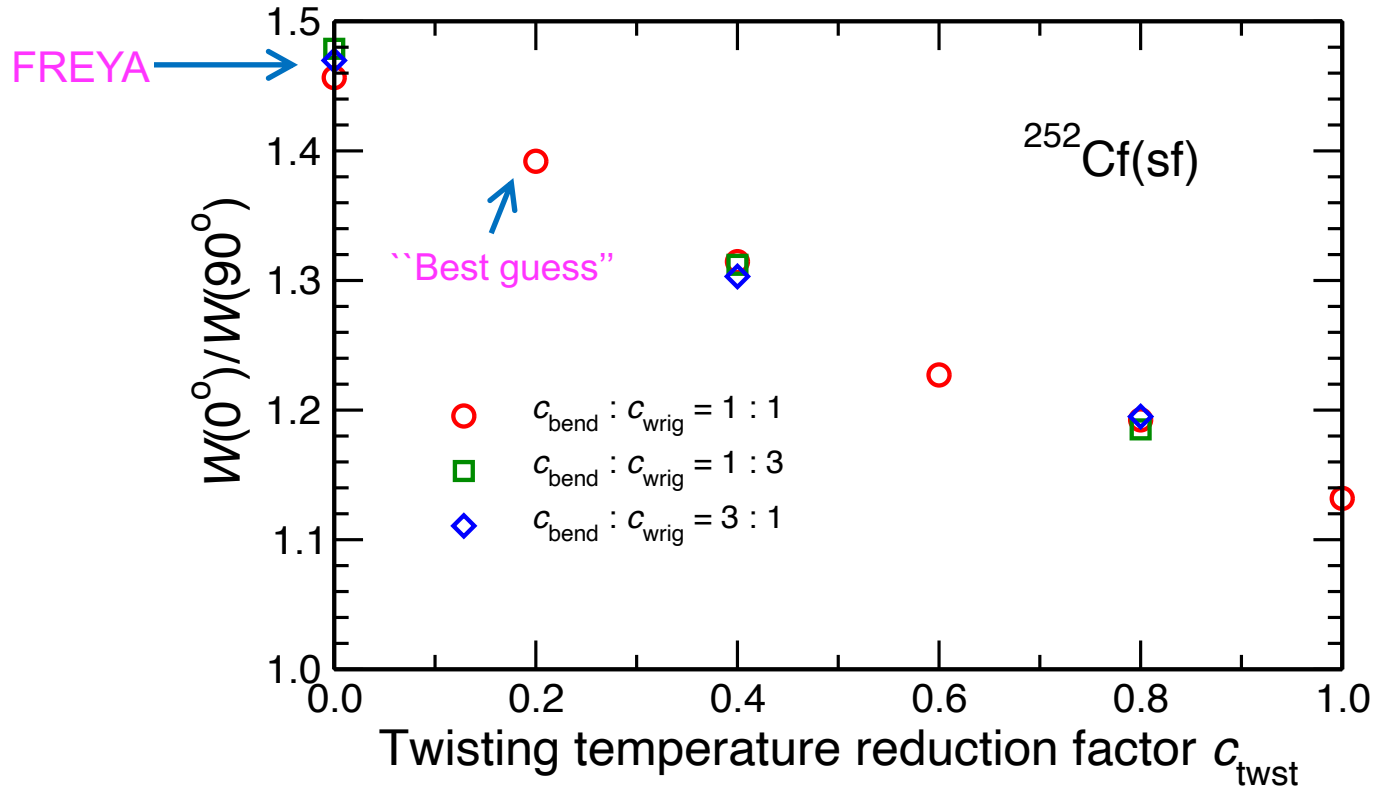


# Testing different contributions from twisting mode





# Yields at 0° relative to yield at 90° adding more twisting



Repeat the experiment by *Wilhelmy et al.*:  
Modern equipment => more accurate data  
=> measure the degree of twisting

# Summary

- Angular momentum has been a hot topic in fission for more than 60 years
- Experiment suggests that fission fragments typically carry  $S = 5-7\hbar$ , approximately directed perpendicular to the fission axis
- Because FREYA conserves energy, linear & angular momentum, it can straightforwardly elucidate the influence of angular momentum
- We have studied the influence of the overall angular momentum and showed that the two fragment spins are nearly uncorrelated even though they are built up in highly correlated increments (by nucleon exchange)
- Fragment rotation has numerous consequences, it:
  - causes neutron emission to be anisotropic;
  - influences photon emission;
  - affects searches for novel effects like scission neutrons
- Spin-spin correlations can provide information on the scission geometry

**FREYA is ideal for studying spin effects in fission**

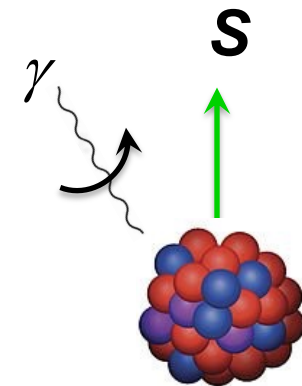
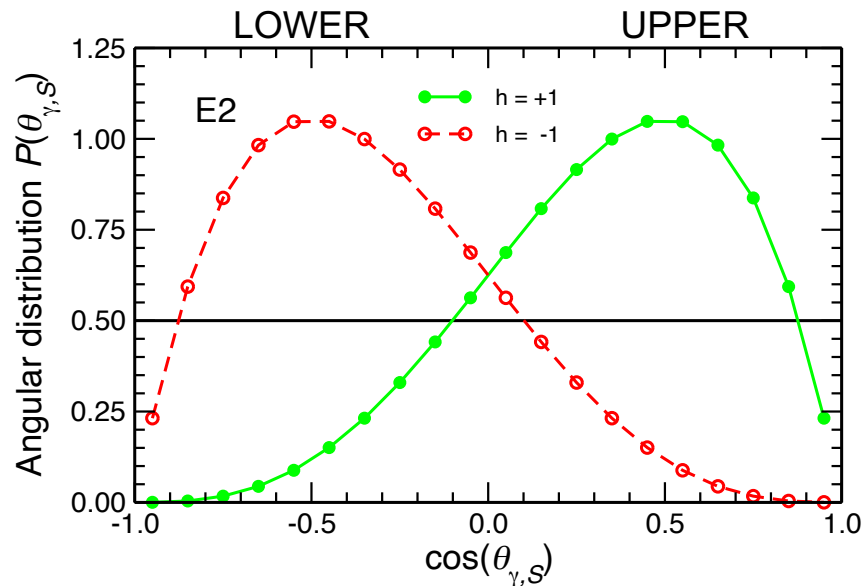
# FREYA references

- **FREYA** developed in collaboration with J. Randrup (LBNL); neutron-transport code integration by J. Verbeke (LLNL); available in MCNP6.2
- **FREYA** journal publications: Phys. Rev. C **80** (2009) 024601, 044611; **84** (2011) 044621; **85** (2012) 024608; **87** (2013) 044602; **89** (2014) 044601; **90** (2014) 064623; **96** (2017) 064620; **99** (2019) 054619; **103** (2021) 014610; Phys. Rev. Lett. **127** (2021) 062502;
- Parameter optimization for spontaneous fission: NIM A **922** (2019) 36
- **FREYA** published in Comp. Phys. Comm. **191** (2015) 178; **222** (2018) 263.
- “Nuclear Fission”, Chapter 5 of ‘100 Years of Subatomic Physics’, World Scientific, 2013
- Review in Eur. Phys. J. A **54** (2018) 9
- Papers with experimentalists: neutron polarization in photofission: Mueller *et al.*, Phys. Rev. C **89** (2014) 034615; photon production: Gjerstvang *et al.*, Phys. Rev. C **103** (2021) 034609; neutron-gamma correlations: Wang *et al.*, Phys. Rev. C **93** (2016) 014606, Marcath *et al.*, Phys. Rev. C **97** (2018) 044622, Marin *et al.*, NIM A **968** (2020) 163907, PRC **104** (2021) 024602; neutron-neutron correlations, Schuster *et al.*, Phys. Rev. C **100** (2019) 014605; Verbeke *et al.*, Phys. Rev. C **97** (2018) 044601; Pozzi *et al.*, Nucl. Sci. Eng. **178** (2014) 250.
- Fission in Astrophysics: Vassh *et al.*, J. Phys. G **46** (2019) 065202; Wang *et al.*, Ap. J. Lett. **903** (2020) L3
- Isotopes currently included: spontaneous fission of  $^{252}\text{Cf}$ ,  $^{244}\text{Cm}$ ,  $^{238,240,242}\text{Pu}$ ,  $^{238}\text{U}$  and neutron-induced fission of  $^{233,235,238}\text{U}(n,f)$ ,  $^{239,241}\text{Pu}(n,f)$  for  $E_n \leq 20$  MeV

# Correlation between the two fragment spin *directions*

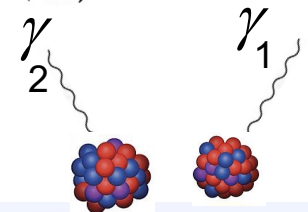
A photon having *positive* helicity tends to emerge in the *upper* hemisphere

$$P_{2,h}^{E2}(\theta_{\gamma S}) \sim \frac{1}{4}(1 + h \cos \theta_{\gamma S})^2(1 - \cos^2 \theta_{\gamma S})$$

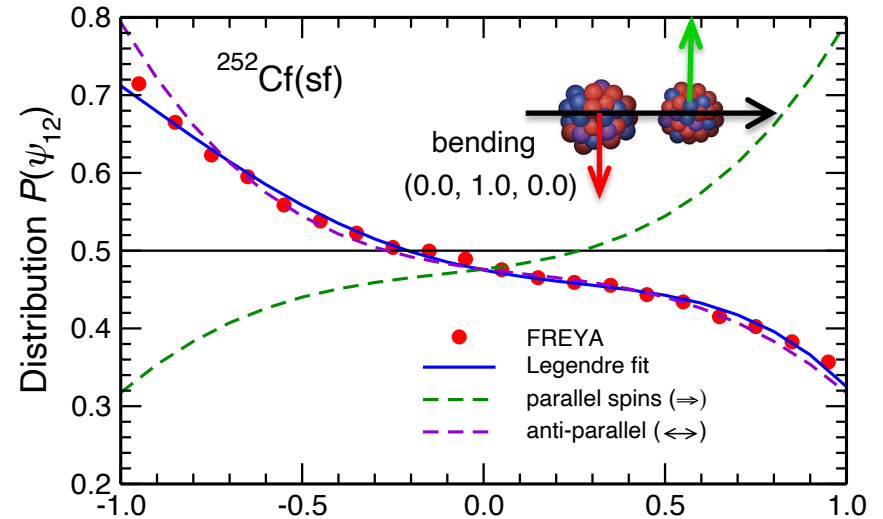
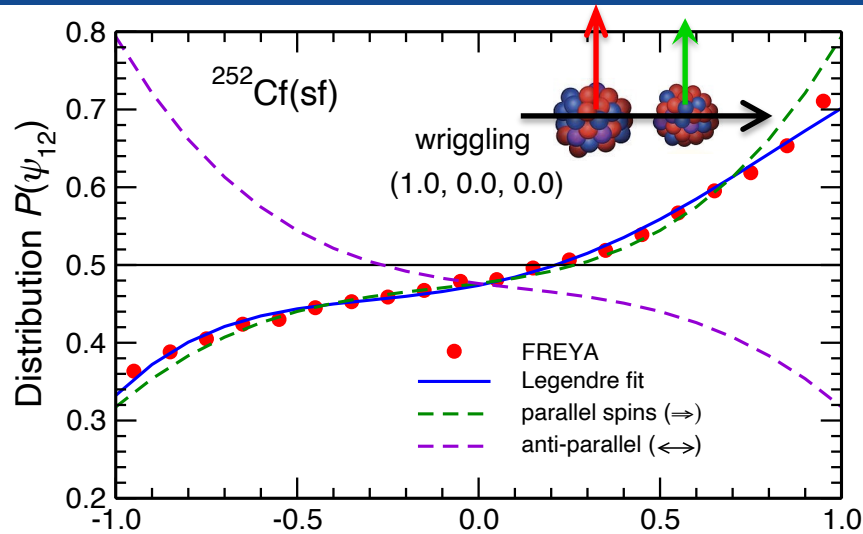


$$dN_i/d \cos \theta_i = \sum_{n \geq 0} \alpha_n^{(i)} P_n(\cos \theta_i) \quad \Rightarrow \quad P(\psi_{12}) = 2 \sum_{n \geq 0} \frac{\alpha_n^{(1)} \alpha_n^{(2)}}{2n + 1} P_n(\cos \psi_{12})$$

Measure the distribution of the opening angle  $\psi_{12}$  between two E2 photons with identified *helicities* originating from partner fragments



# Distribution of the opening angle between two E2 photons having the same helicity



Wriggling: same hemisphere, small angle

Bending: opposite hemisphere, large angle

