Testing the quality of Nuclear Level Density models on Oslo data

S. Goriely, A.C. Larsen, D. Muecher

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Comprehensive test of nuclear level density models

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About the Constant-T behaviour of the Oslo NLD

“All the level density distributions follow the constant-temperature description” (Giacoppo et al. 2014)

“The level-density function follows closely the constant-temperature level-density formula” (Tornyi et al. 2014)

“It was found that the Gilbert and Cameron level density model is best to reproduce experimental data.” (Voinov et al. 2013)

“The level densities, which were extracted using the Oslo method, show a constant temperature behavior” (Guttormsen et al. 2013)

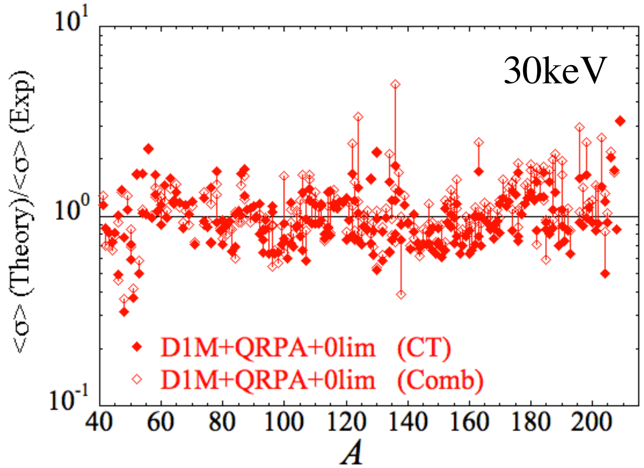
Clear message:

“NLD follow a Constant-T behaviour”

or

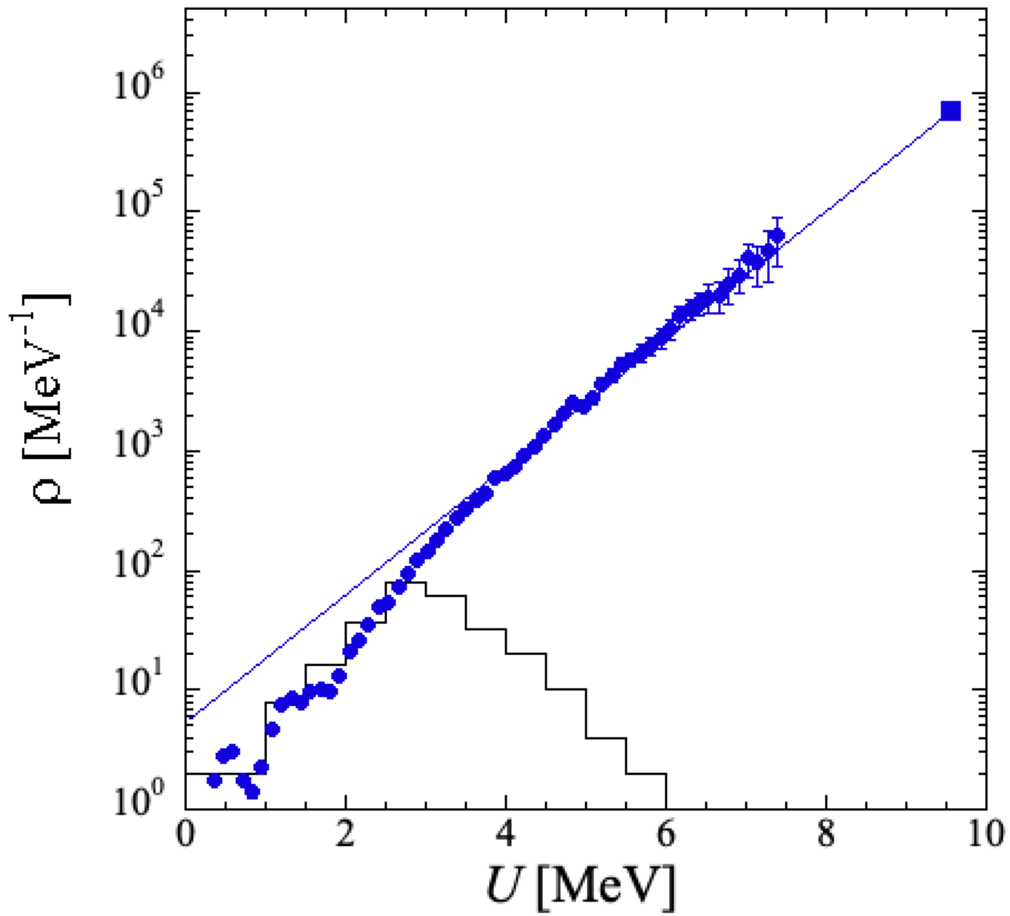
“NLD follow a Constant-T behaviour above the pair-breaking energies”

Impact on MACS



The ^{106}Pd Nuclear Level Densities

"Above the pair-breaking energies the characteristics of the level densities can be described by the constant temperature formula" (Eriksen et al. 2013)



D_0=10.9±0.5 eV

$\rho(S_n)$=7.1 10^5 MeV^{-1}

CT formula

$$D = \frac{1}{\rho(S_n, J_0+1/2, \pi_0) + \rho(S_n, J_0-1/2, \pi_0)} \quad J_0>0$$

$$= \frac{1}{\rho(S_n, J_0+1/2, \pi_0)} \quad J_0=0$$

↔ Model-dependent transformation

$$\rho_{obs}(U) = \sum_{J,\pi} \rho(U, J, \pi)$$

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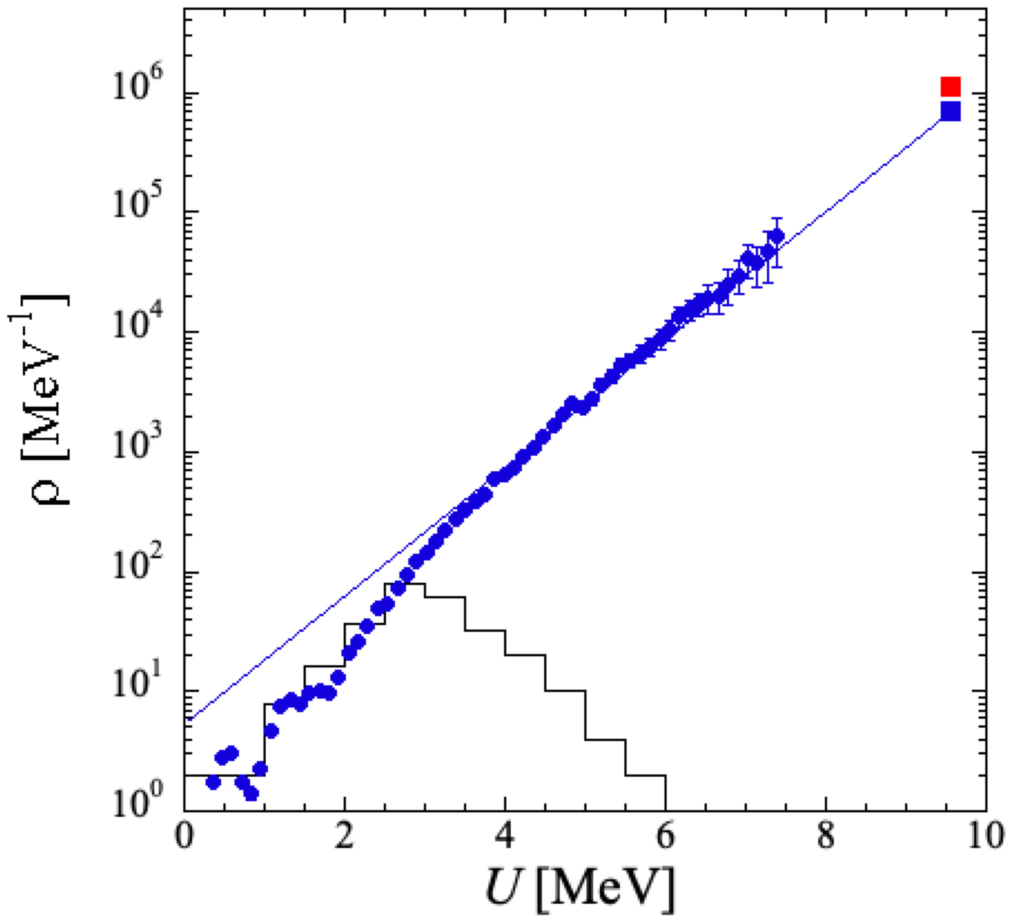

D_0=10.9±0.5 eV

$\rho(S_n)$=7.1 10^5 MeV^{-1}

CT formula

$\rho(S_n)$=1.2 10^6 MeV^{-1}

HFB+Comb model

(SG & Hilaire, 2008)

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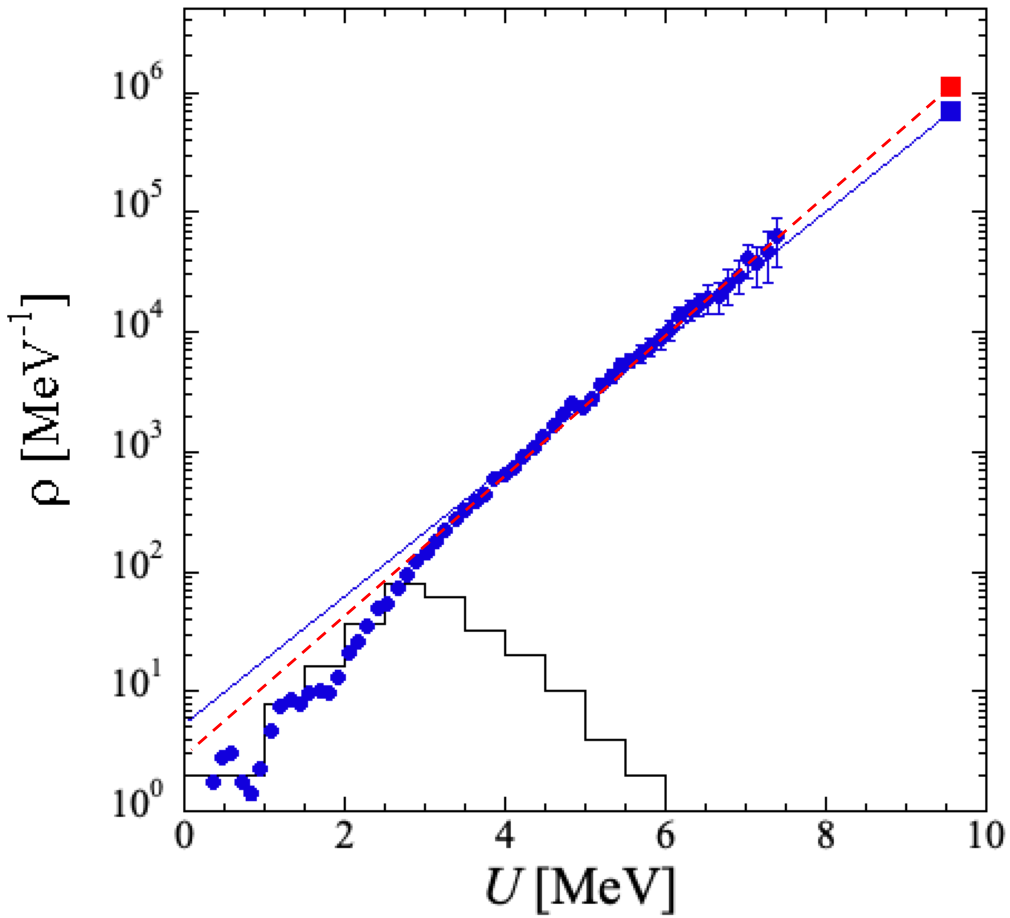

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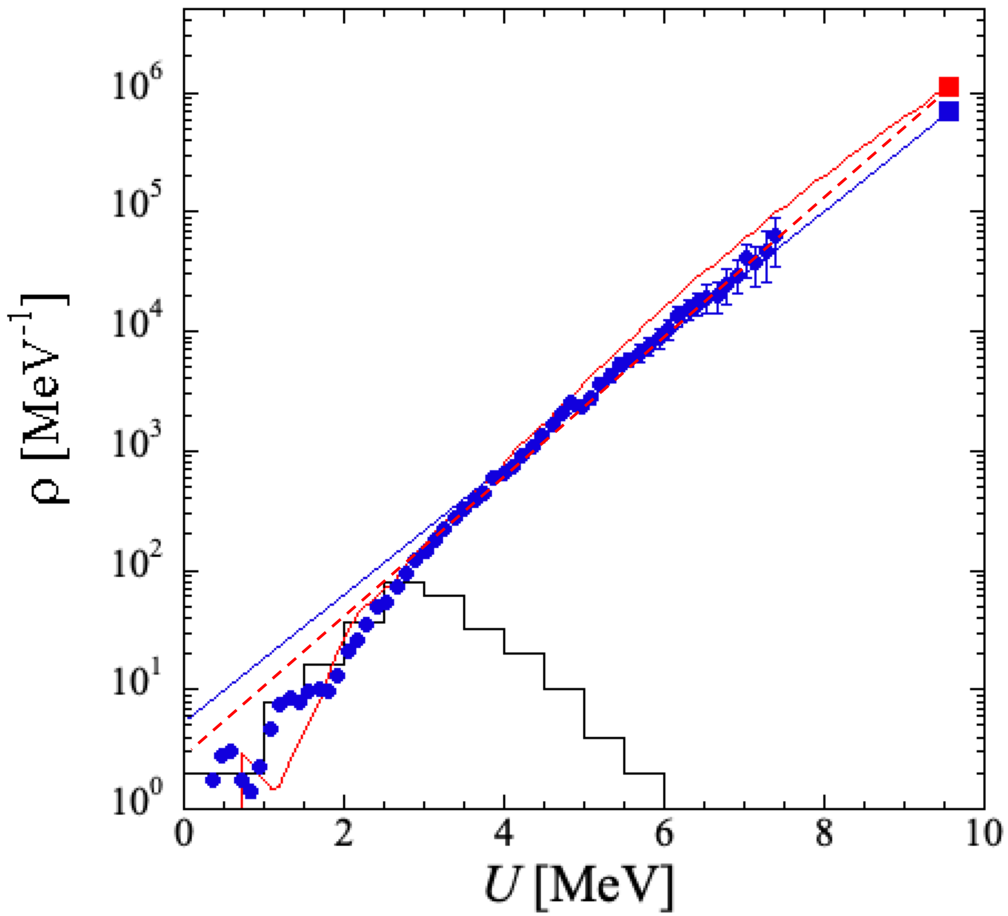

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Does it fail ??

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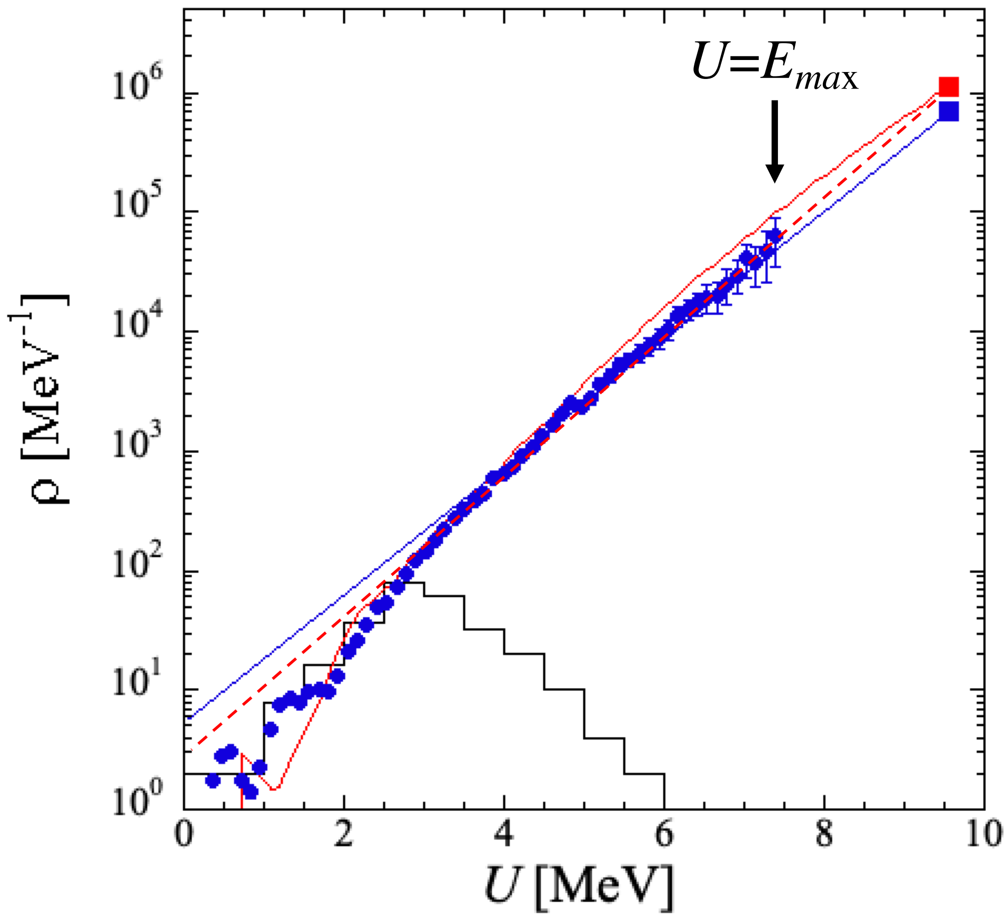

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Normalisation at $\rho(U=E_{max})$

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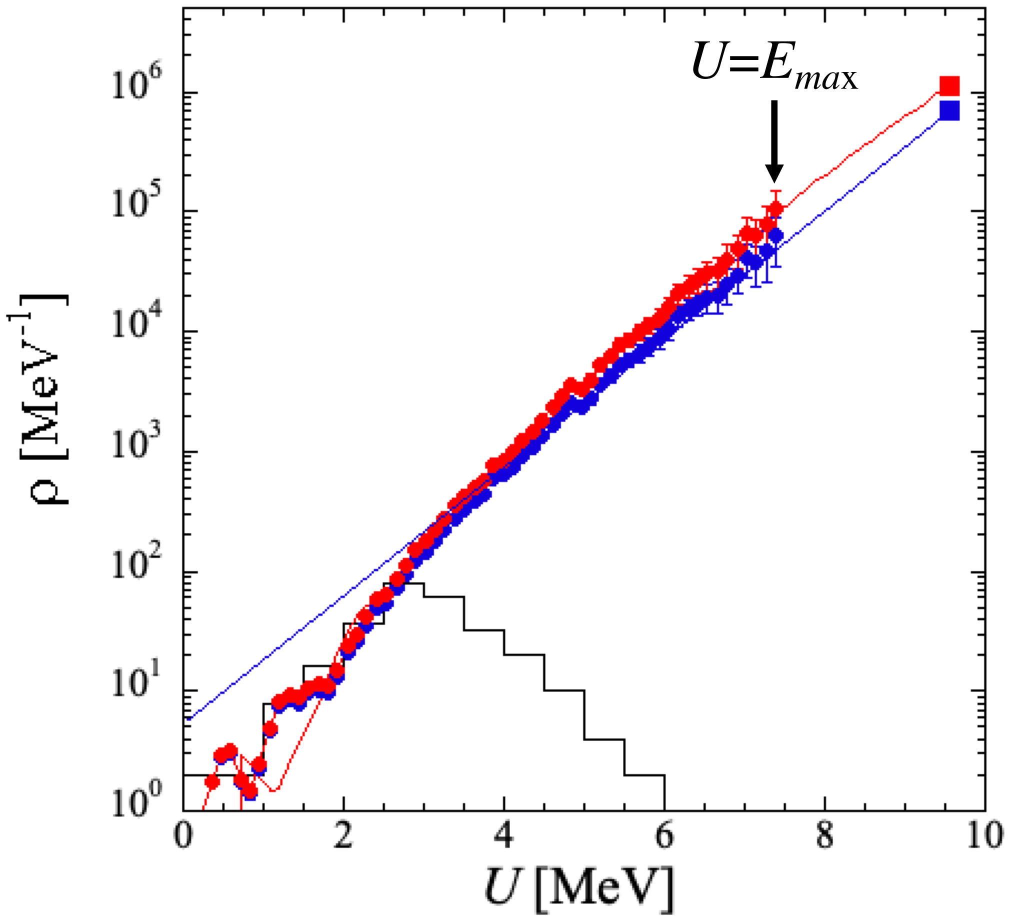

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Normalisation at $\rho(U=E_{max})$

$$\tilde{\rho}(E_i - E_\gamma) = A \exp[\alpha\,(E_i - E_\gamma)]\,\rho(E_i - E_\gamma)$$

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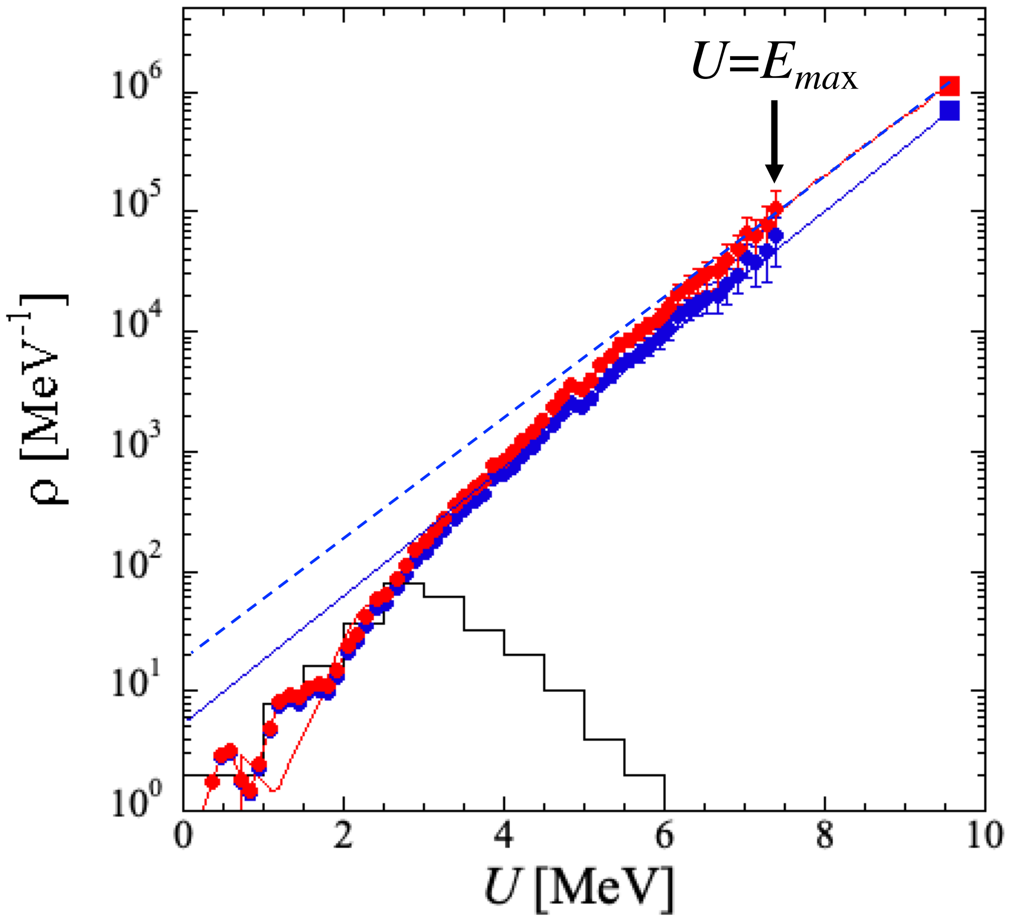

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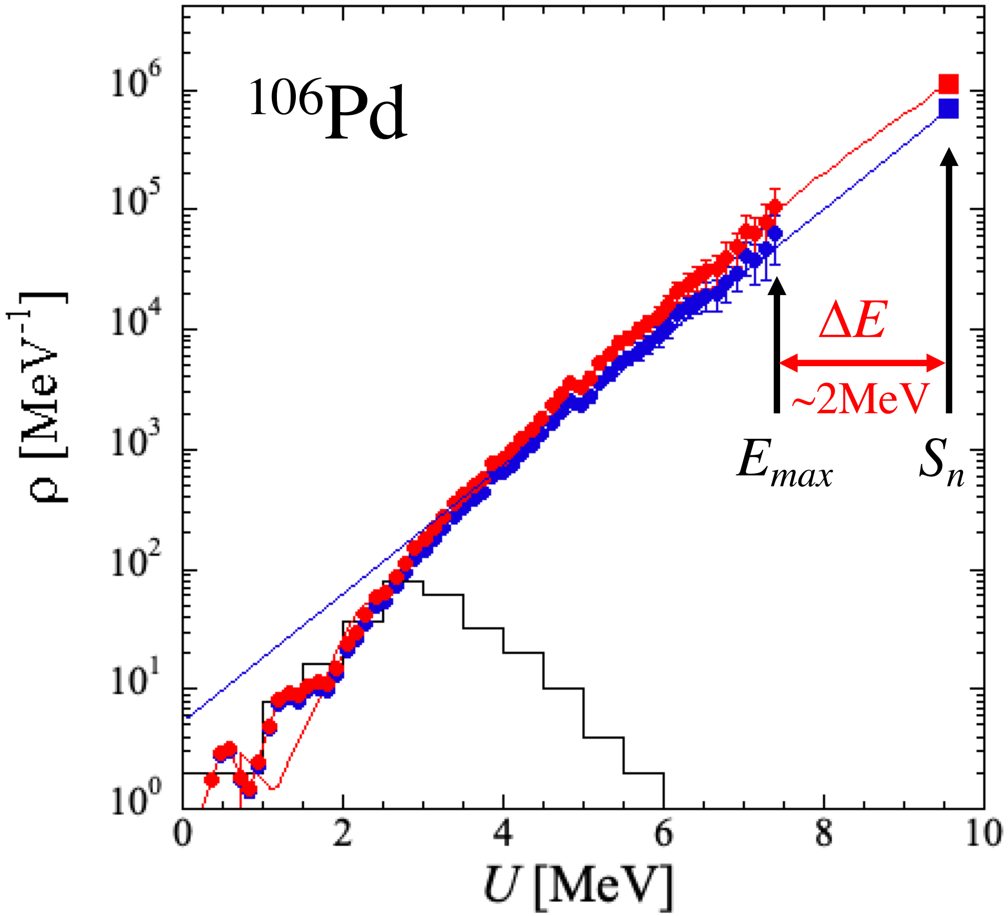
Normalisation at $\rho(U=E_{max})$

CT fails ?

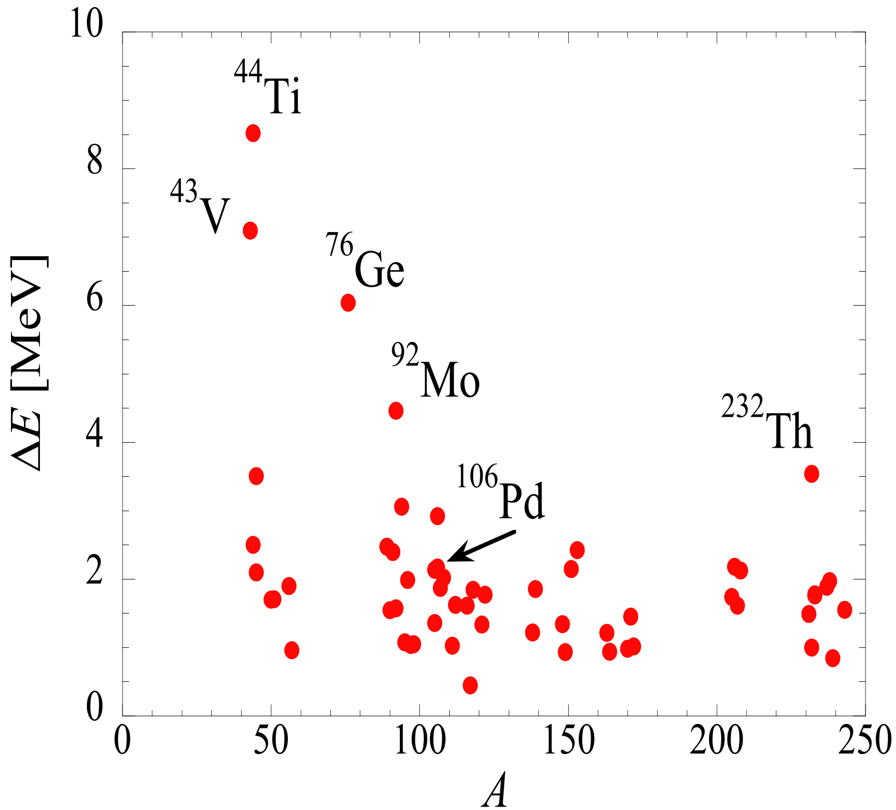
$$\tilde{\rho}(E_i - E_\gamma) = A \exp[\alpha\,(E_i - E_\gamma)]\,\rho(E_i - E_\gamma)$$

The NLD renormalization needed in the Oslo method significantly depends on the interpolation between E_{max} and S_n

$$\tilde{\rho}(E_i - E_\gamma) = A \exp[\alpha\,(E_i - E_\gamma)]\,\rho(E_i - E_\gamma)$$

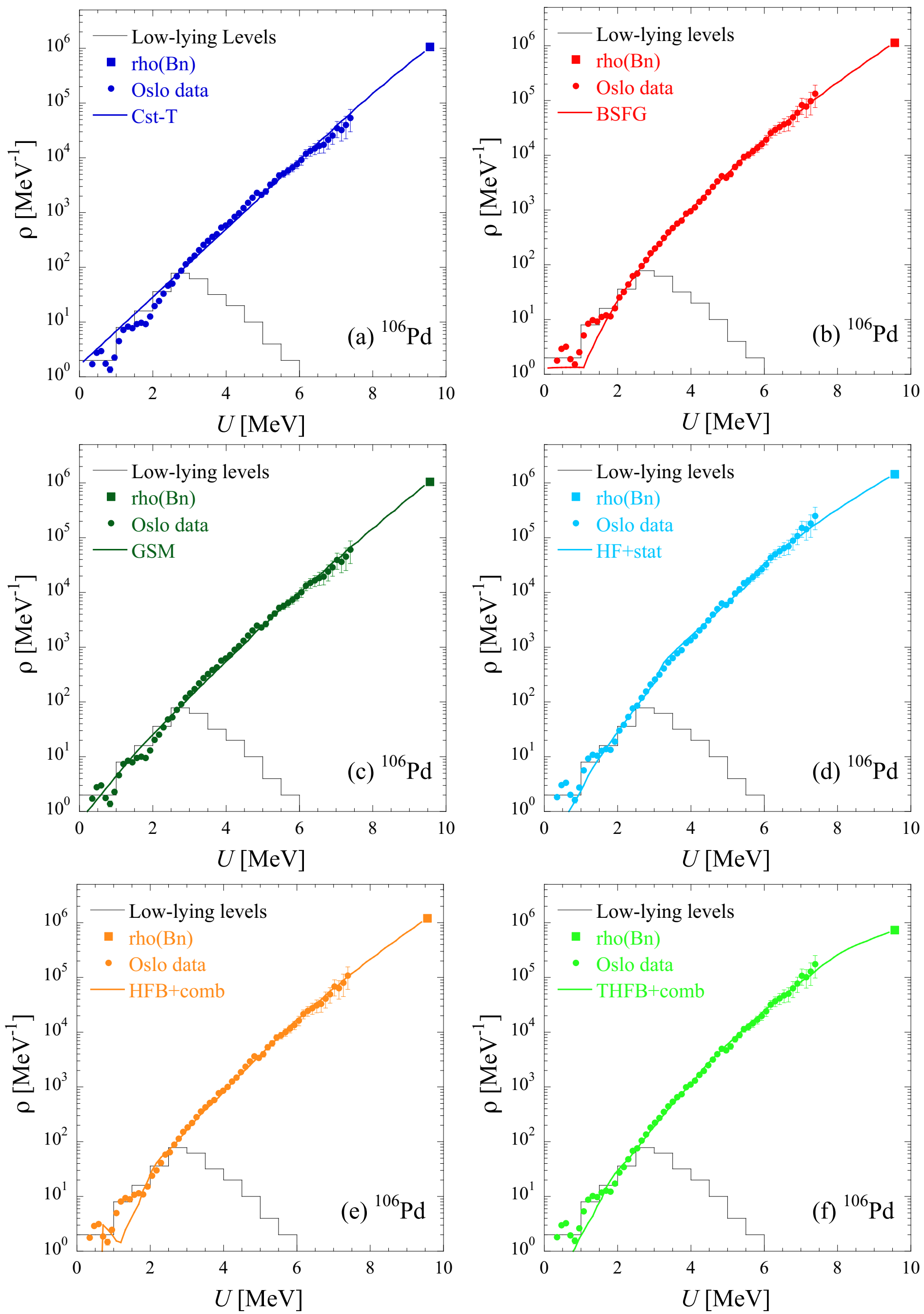


$\Delta E = S_n - E_{max}$

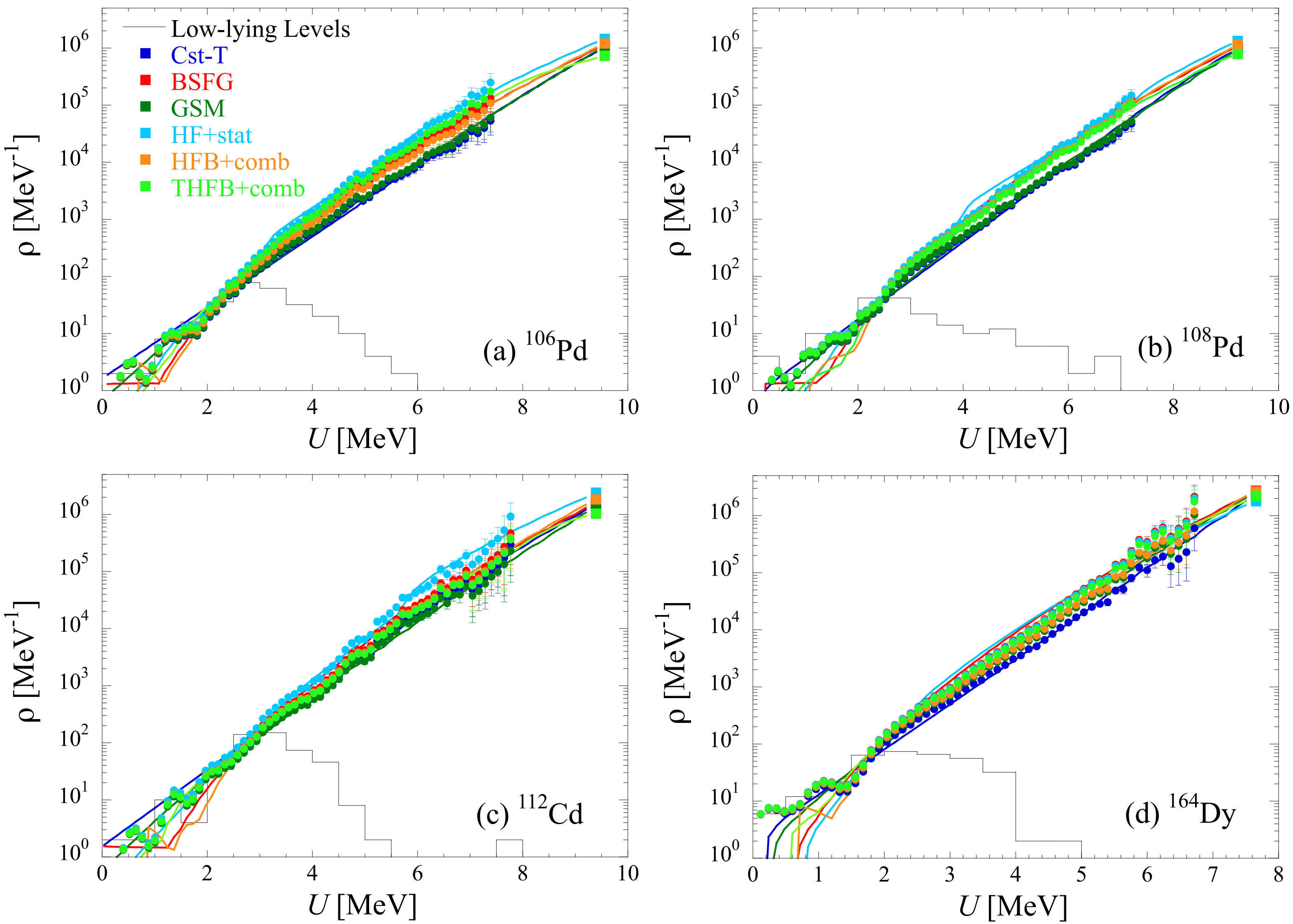


57 Oslo experimental NLD

The ^{106}Pd Nuclear Level Densities

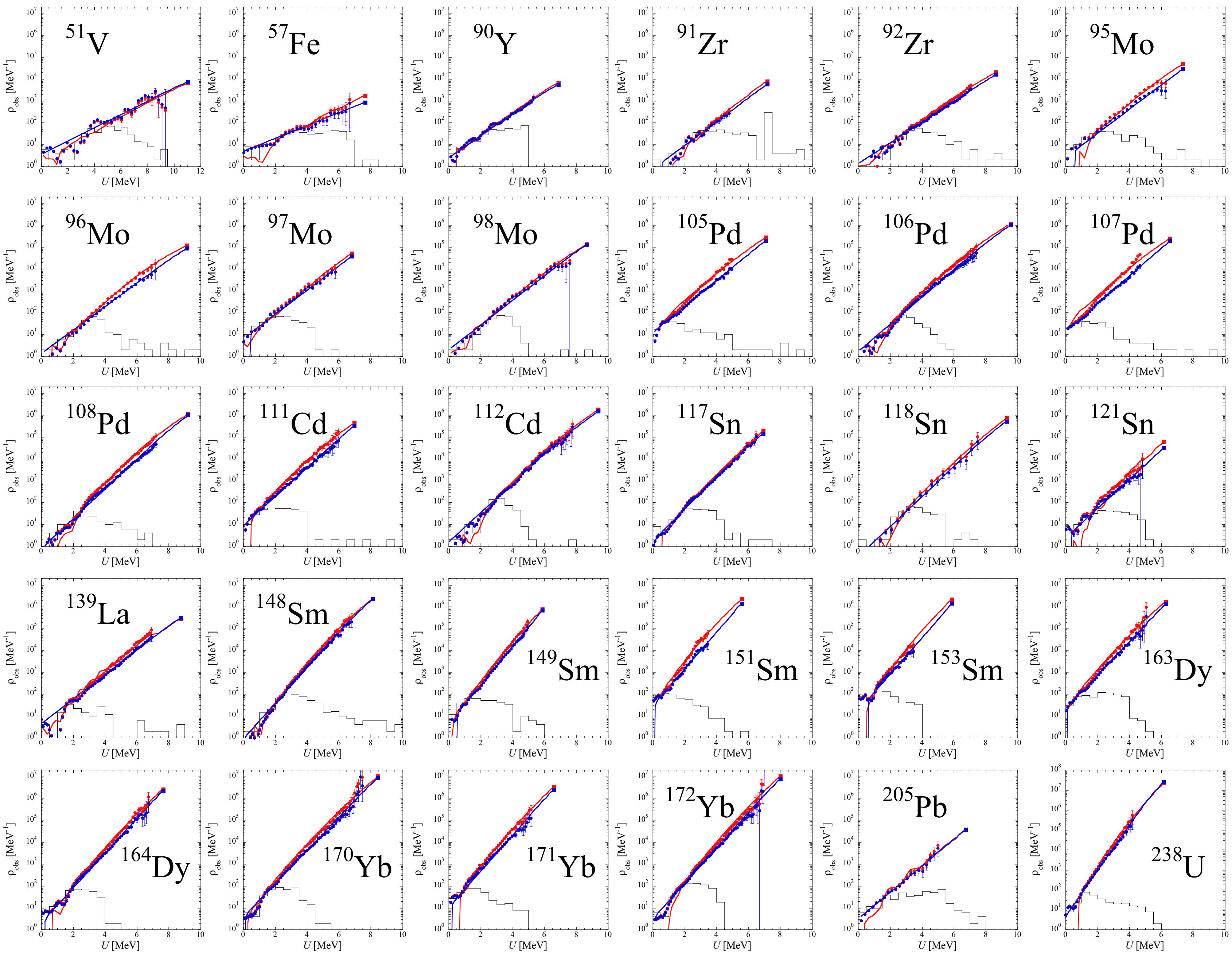


Testing NLD models on Oslo data



HFB plus Combinatorial model versus CT + Fermi Gas

30 (out of 39) nuclei for which Oslo NLD & experimental D_0 are available



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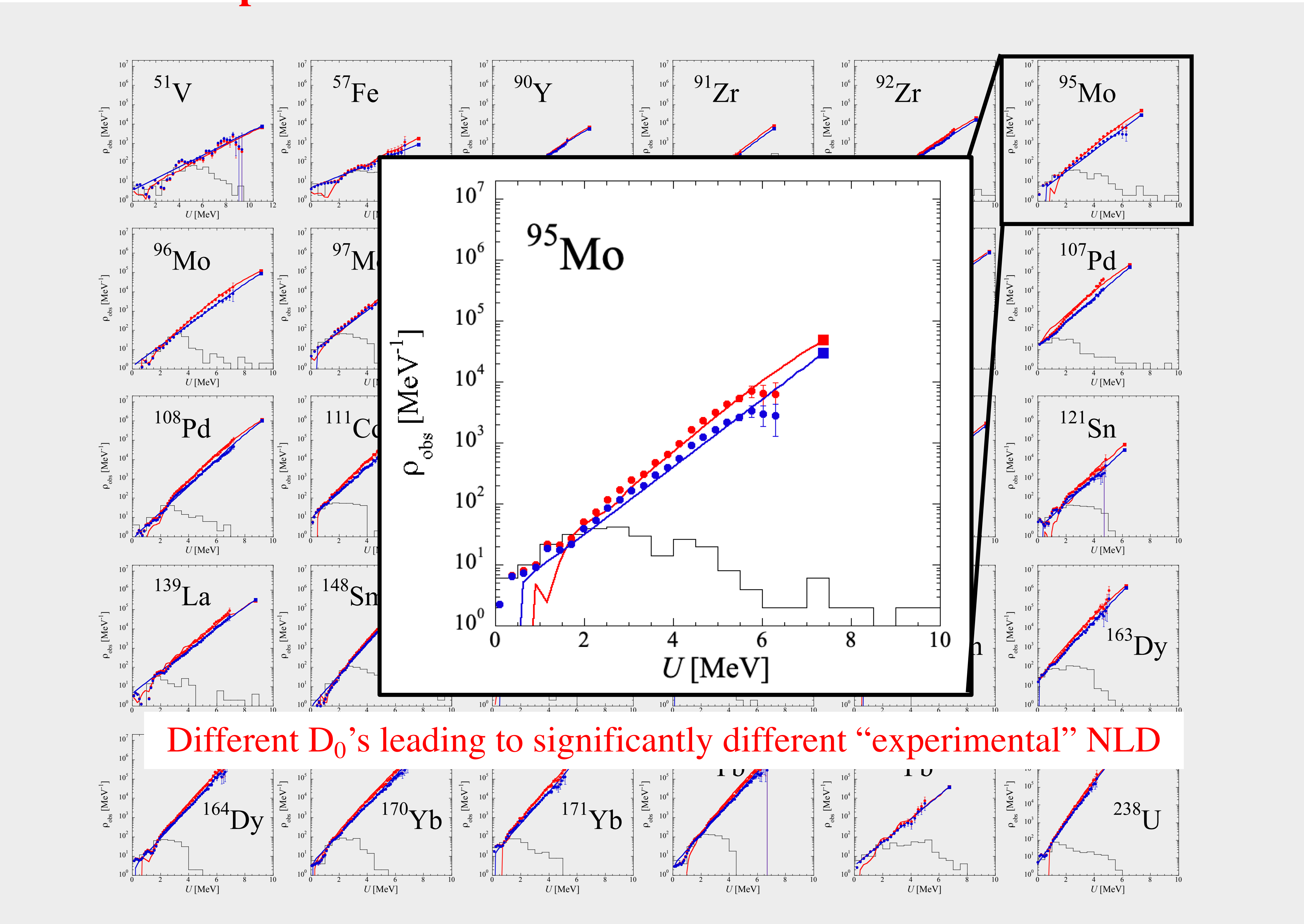
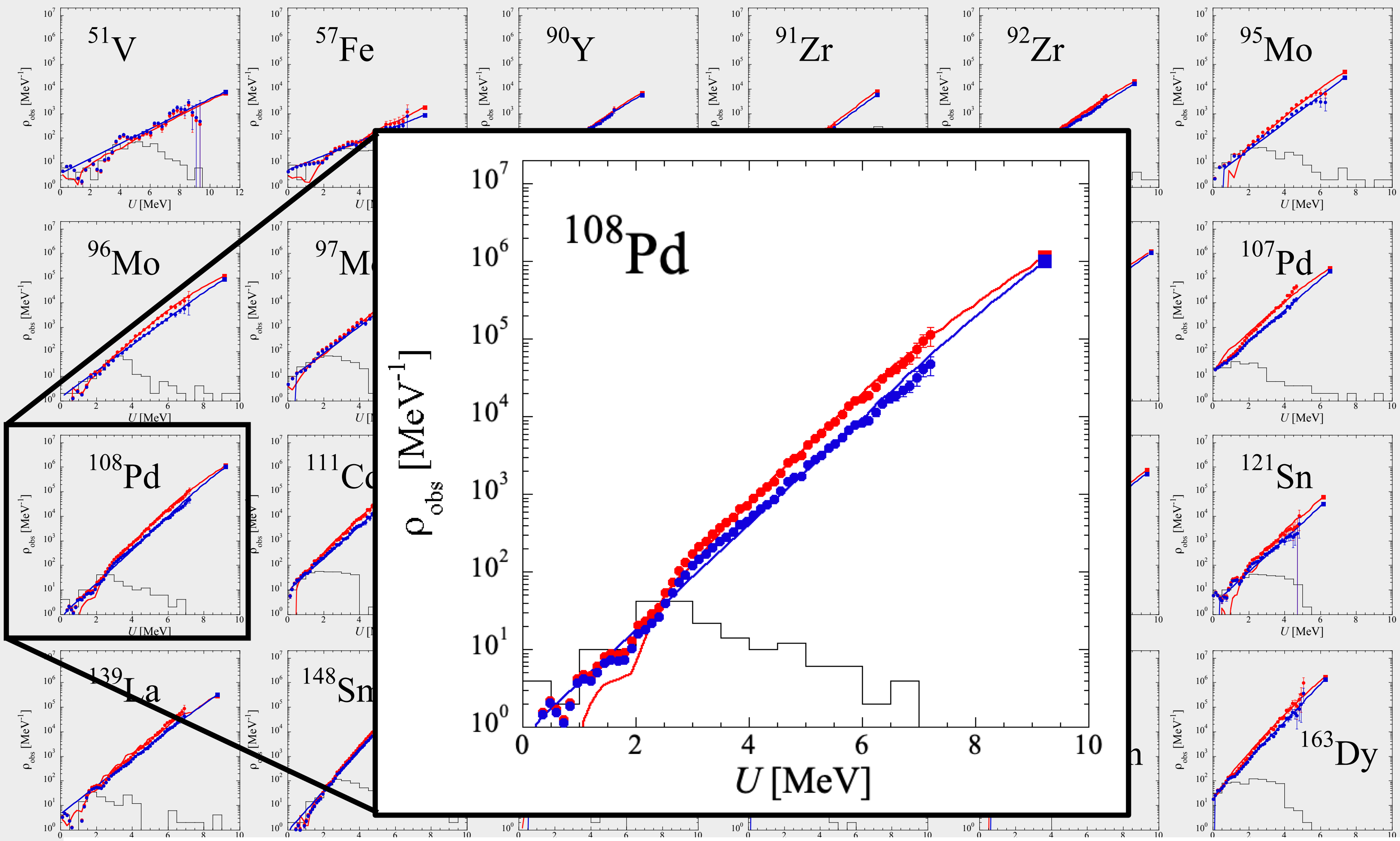
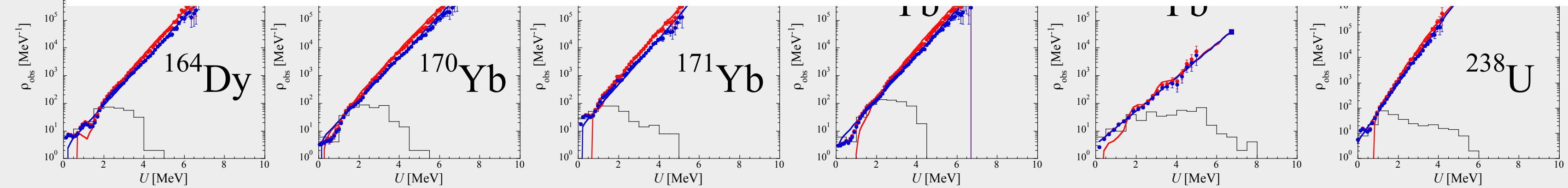


Different D_0's leading to significantly different "experimental" NLD

HFB plus Combinatorial model versus CT + Fermi Gas



Relatively similar D_0, still significantly different "experimental" NLD



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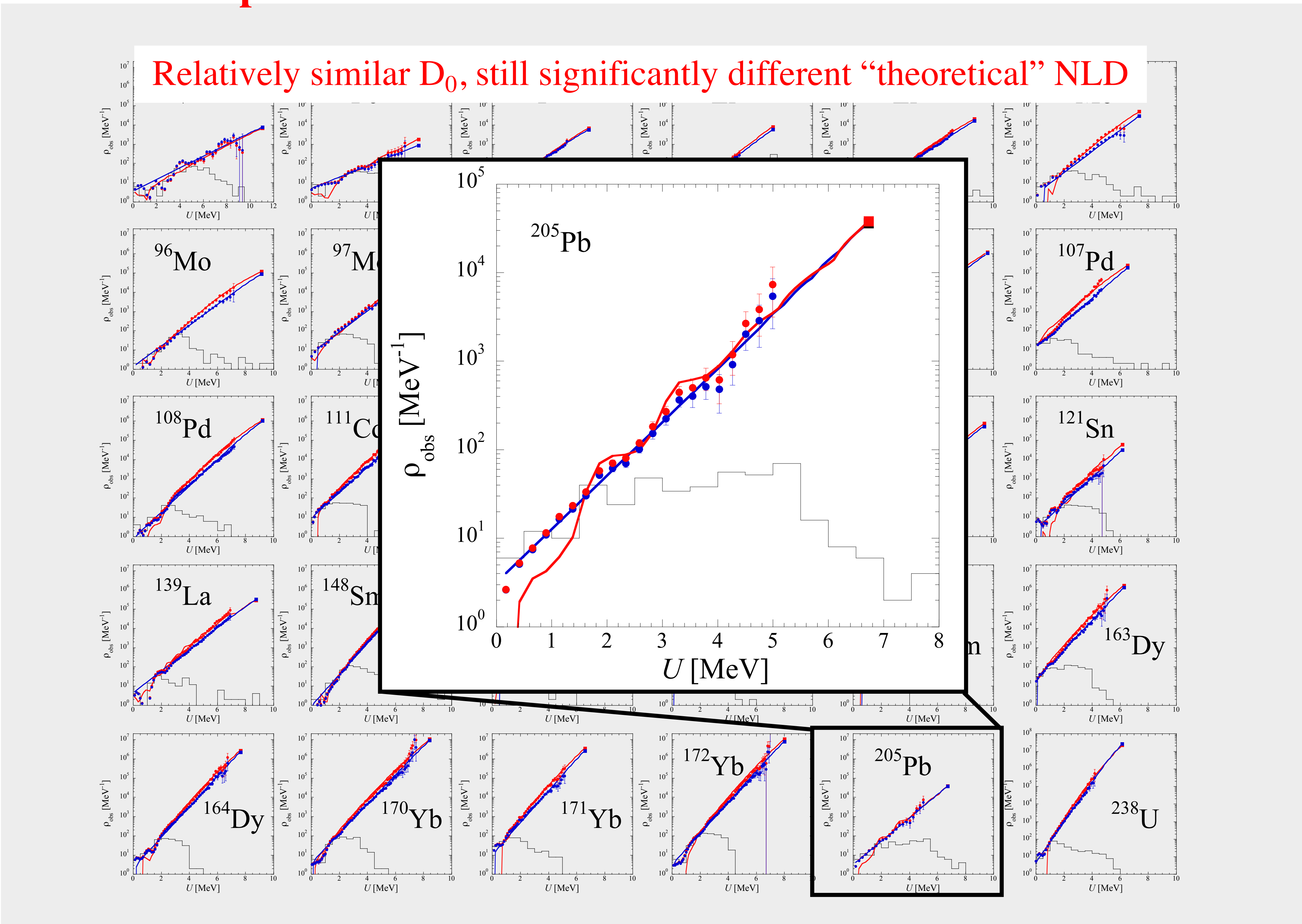
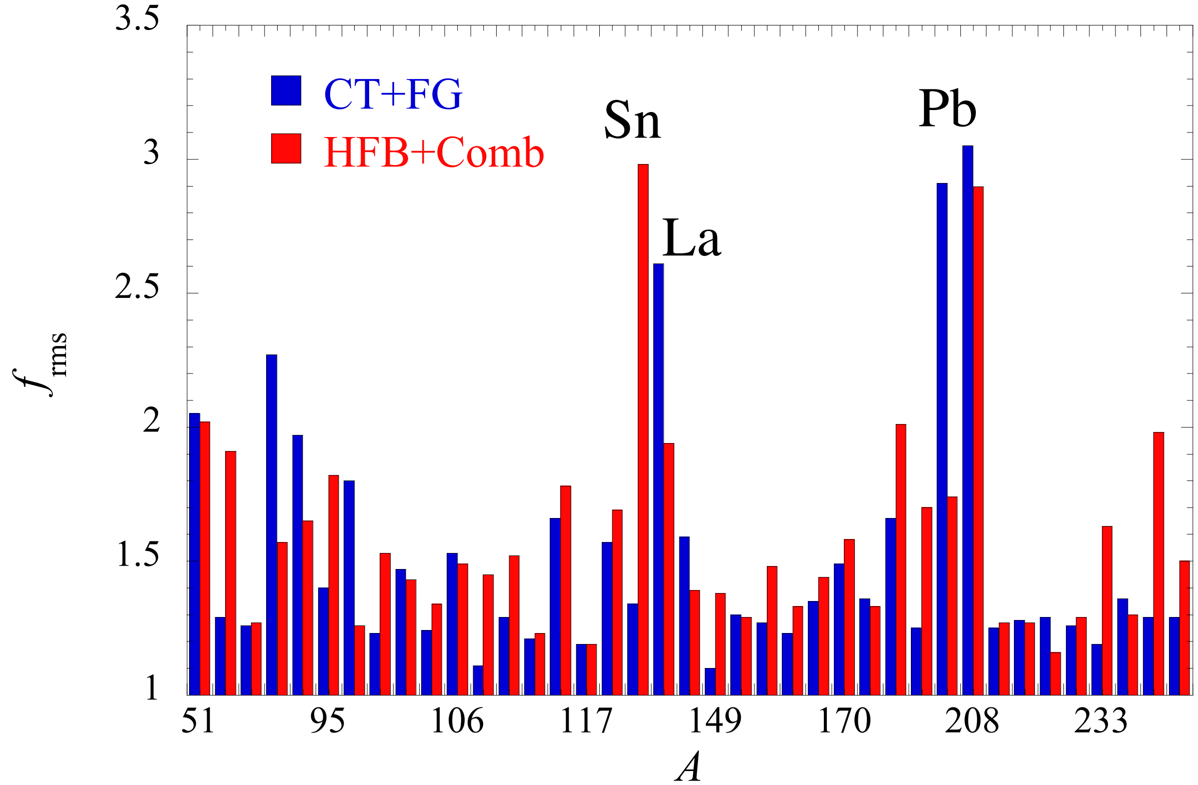


A statistical f_{rms} test of the "quality" of the NLD model versus Oslo data

$$f_{\rm rms} = \exp\left[\frac{1}{N_e}\sum_{i=1}^{N_e}\ln^2 r_i\right]^{1/2}$$



<f_{rms}> = 1.52 for CT+FG model
<f_{rms}> = 1.59 for HFB+Comb model

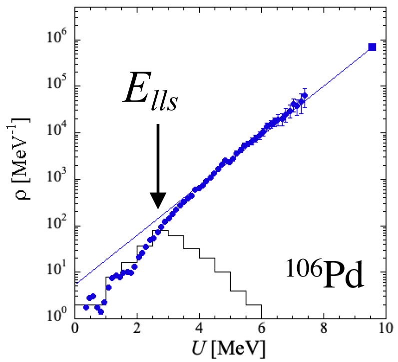
→ Virtually, same accuracy on the 39 nuclei with known D_0

Not possible to rationally favour one of these two models !

P.S. <f_{rms}> = 1.8 for BSFG model, <f_{rms}> = 1.9 for GSM

A statistical f_{rms} test of the "quality" of the NLD model versus Oslo data

Mean ε and rms σ deviations for all the $N_n = 42$ nuclei corresponding, for a given NLD model, to the differences between the NLD predictions and the newly renormalized Oslo data. E_{lls} is the energy at which the level scheme is assumed to be complete from a comparison with Oslo data.



NLD model	ε(all)	σ(all)	$\varepsilon(E > E_{lls})$	$\sigma(E > E_{lls})$
Cst-T	1.02	1.45	0.97	1.21
BSFG	0.92	1.68	1.01	1.25
GSM	0.97	1.69	1.00	1.34
HF+stat	0.94	1.53	1.02	1.27
HFB+comb	0.94	1.47	0.99	1.25
THFB+comb	0.95	1.64	1.02	1.30

$$f_{\rm rms} = \exp\left[\frac{1}{N_e}\sum_{i=1}^{N_e}\ln^2 r_i\right]^{1/2}$$ where

$$\begin{aligned} r &= \frac{\rho_{th}}{\rho_{\exp} - \delta\rho_{\exp}} \quad \text{if} \quad \rho_{th} < \rho_{\exp} - \delta\rho_{\exp} \\ &= \frac{\rho_{th}}{\rho_{\exp} + \delta\rho_{\exp}} \quad \text{if} \quad \rho_{th} > \rho_{\exp} + \delta\rho_{\exp} \\ &= 1 \quad \text{otherwise.} \end{aligned}$$

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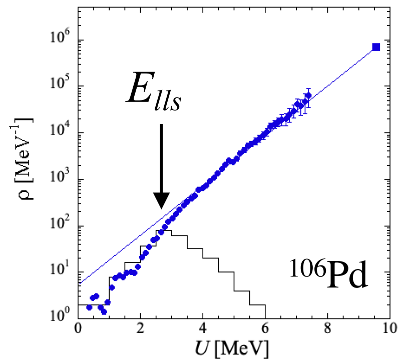

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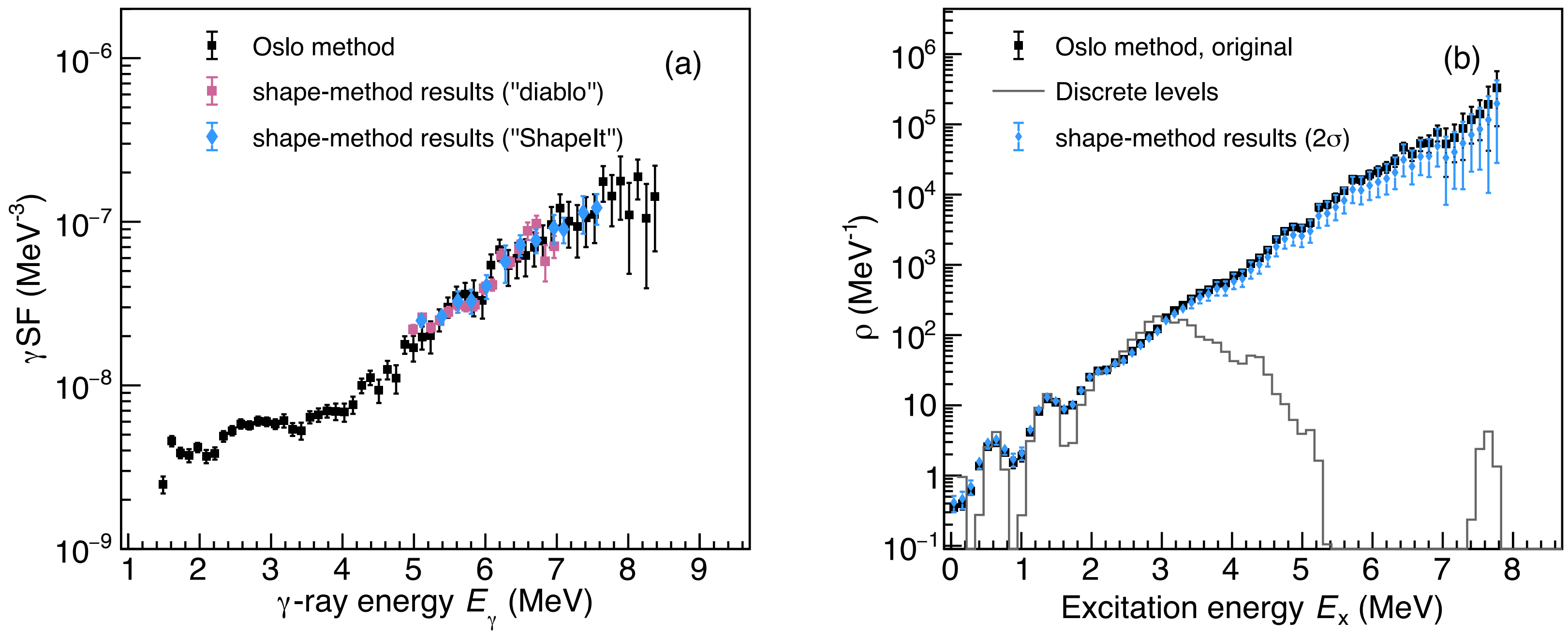
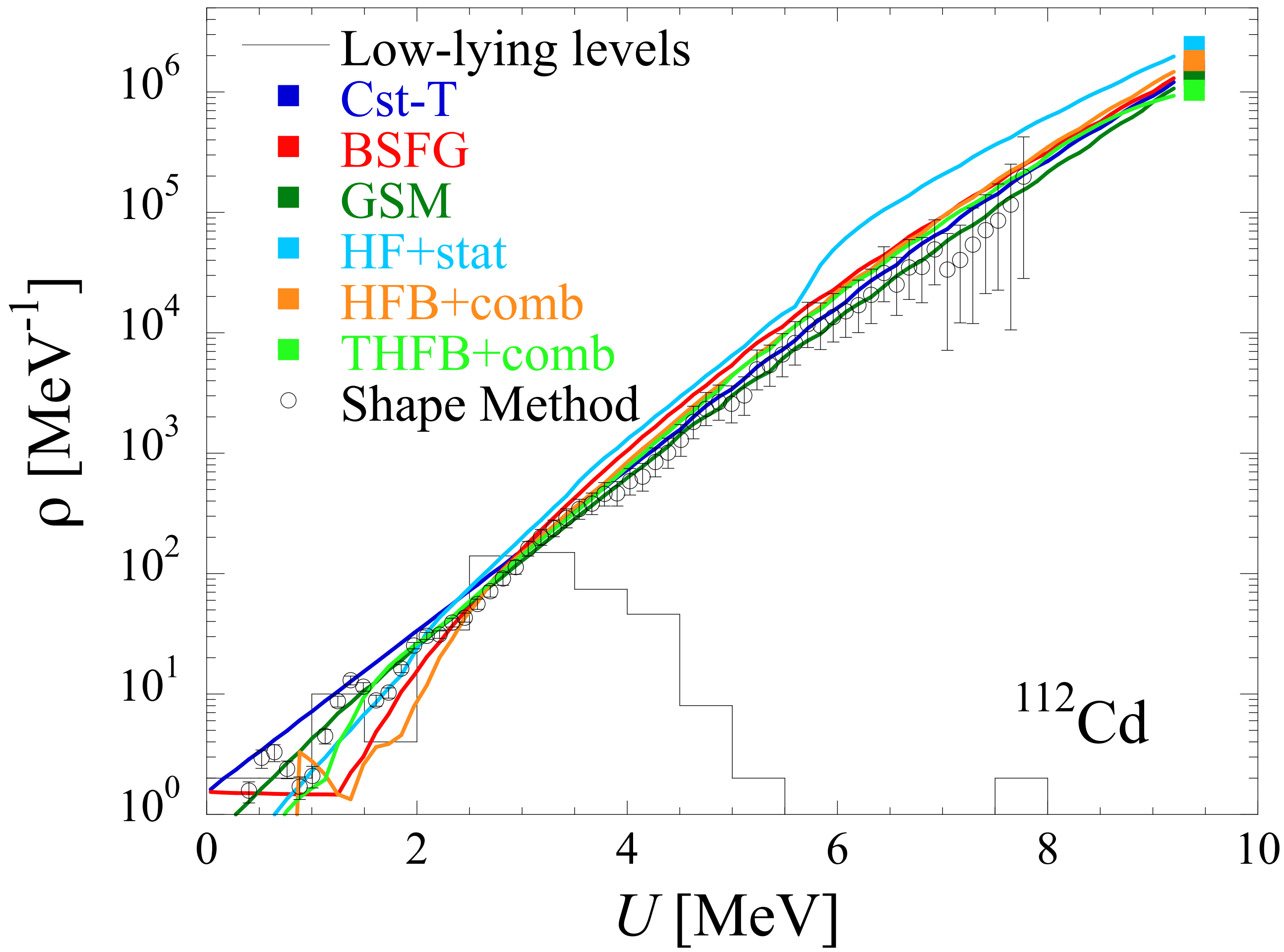
→ But the Shape Method should be able to reduce the uncertainties related to the renormalisation

NLD and the Shape Method

^{112}Cd



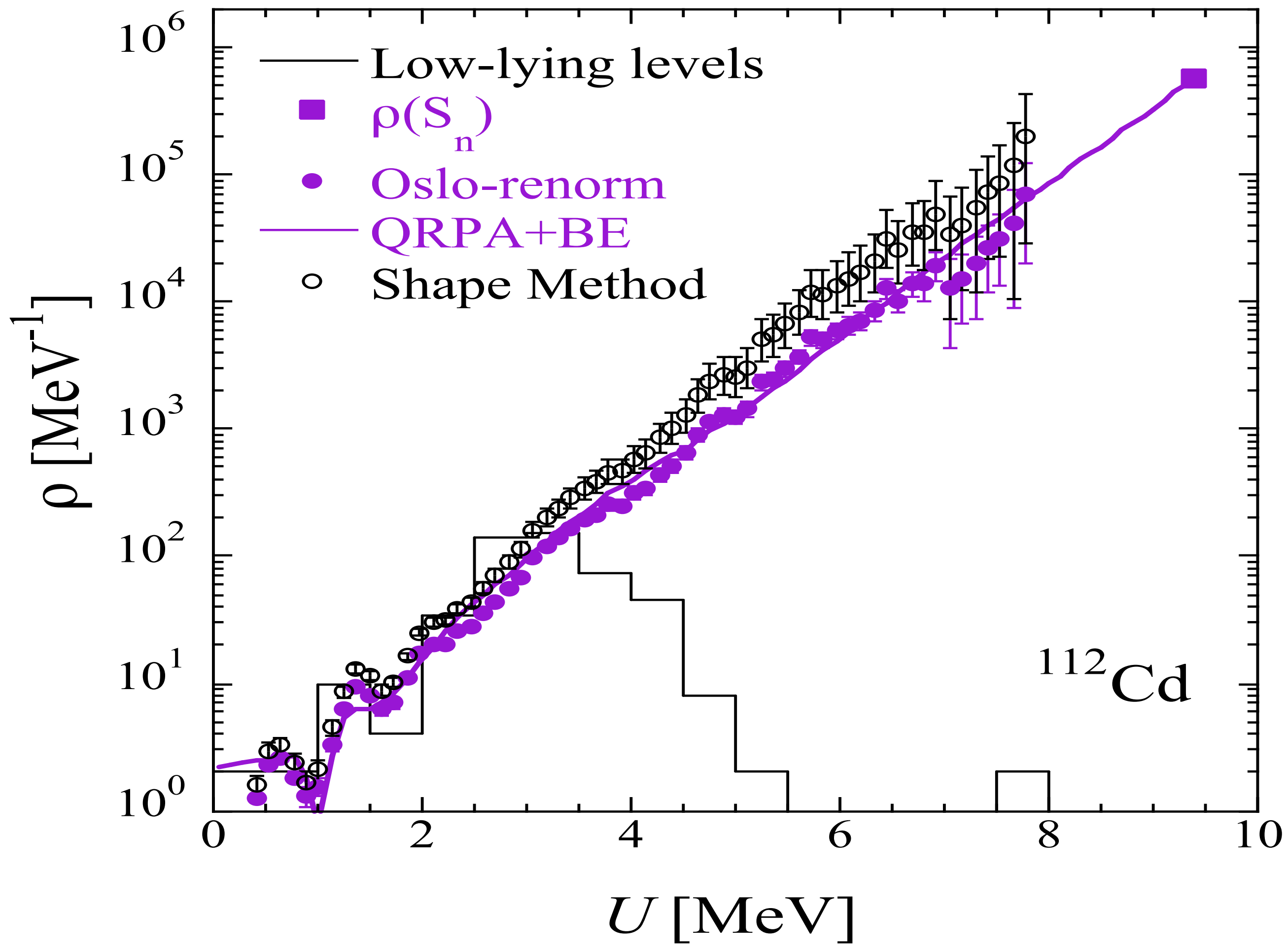
NLD and the Shape Method



→ **Absolute determination of the NLD**

→ **Possibility to test the quality of the NLD models, but not on 1 nucleus !**

NLD and the Shape Method



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THANK YOU
FOR
YOUR ATTENTION