Isotopic Fuel Tailoring as Actuator for Burn Control in Tokamak Reactors: Tritium Concentration Effects on Operations & Control

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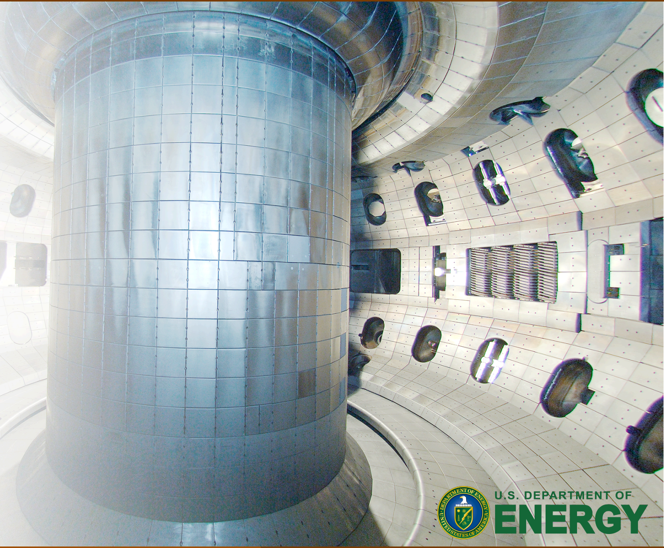


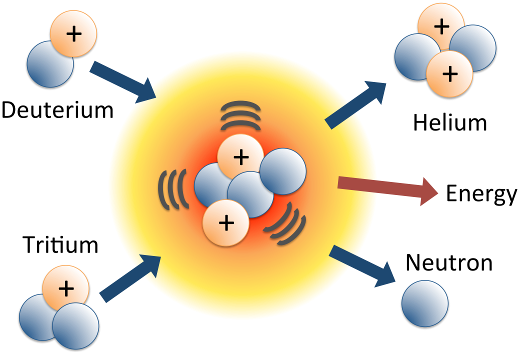
U.S. DEPARTMENT OF ENERGY

Isotopic Fuel Tailoring as Actuator for Burn Control

1. **Kinetic (Burn) Control**
   - What Type of Model Do We Need for Burn-Control Design?
   - Why a Nonlinear Control Solution is Needed?
   - Synthesis of Nonlinear Controller for the Regulation the Burn Condition
   - How Do We Deal With Model Uncertainties and Unmodeled Dynamics?
   - How Do We Select the Correct Reference for the Controller?
   - How Do We Close the Loop if State Is Not Fully Measurable?
   - How Do We Handle Actuator Dynamics and Constraints?
2. **Robust Burn Control Against Drift/Biases in Fueling-line Tritium Fractions**
3. **Impact of Fueling-line Tritium Fractions on ITER's Operational Space**
4. **Summary of the Presentation**

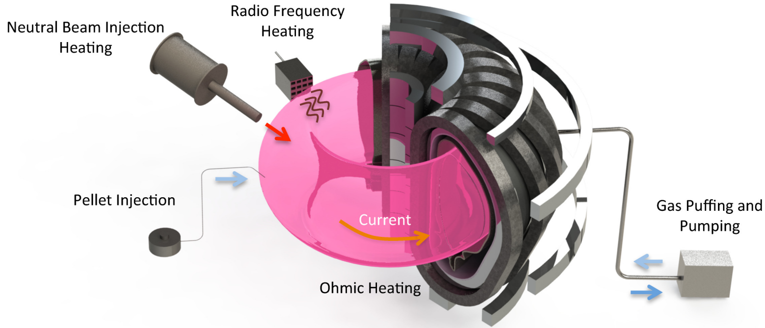
Burn Control $\equiv$ Density and Temperature Control of All Species

$$
{}^2_1D + {}^3_1T \rightarrow \underbrace{{}^4_2He}_{3.5MeV} + \underbrace{{}^1_0n}_{14.1MeV}, \qquad Q_{DT} = \Delta mc^2 = 17.6MeV
$$



- **Fusion power regulation in DT plasma $\leftrightarrow$ species density/temperature control**
- **Dimensionality and nonlinearity of the burning plasma $\rightarrow$ model-based control**

Actuators Used to Control the Plasma Kinetic Condition



- **Plasma current** contributes to heating through *Ohmic heating* (small in reactor)
- **Magnetic configuration** affects burn condition through *confinement time*
- **Neutral beam injectors** and **radio frequency waves** heat the plasma
- Refueling at the plasma boundary is achieved through **gas puffing**
- Refueling in the plasma core is achieved through **pellet injection**
- **Impurity injection** dilutes the fuel content and increases *radiation losses*
- **Gas pumping** removes exhausted fuel, alpha particles, and impurities

Isotopic Fuel Tailoring as Actuator for Burn Control

Burning Plasma Model (Dynamics): 1D Conservation Equations

Alpha Particle Density

$$\frac{\partial}{\partial t} n_\alpha = -\nabla \cdot \vec{\Gamma}_\alpha + S_\alpha$$

Deuterium Density:

$$\frac{\partial}{\partial t} n_D = -\nabla \cdot \vec{\Gamma}_D - S_\alpha + S_D^R + S_D$$

Tritium Density:

$$\frac{\partial}{\partial t} n_T = -\nabla \cdot \vec{\Gamma}_T - S_\alpha + S_T^R + S_T$$

Impurity Density:

$$\frac{\partial}{\partial t} n_I = -\nabla \cdot \vec{\Gamma}_I - S_I$$

Energy Density:

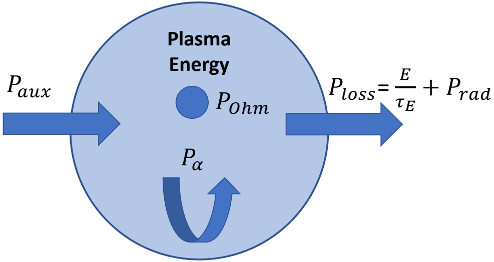
$$\frac{\partial}{\partial t}\left[\frac{3}{2} nT\right] = -\nabla \cdot \vec{\Gamma}_E + Q_\alpha S_\alpha + P_{ohm} - P_{brem} + P_{aux}$$

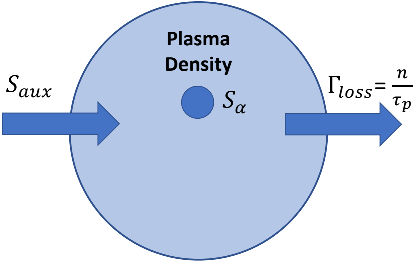
- T: Plasma Temperature ($T \triangleq T_i = T_e$)
- n: Plasma Density
- P_{aux}: Auxiliary Power
- P_{ohm}: Ohmic Power
- P_{rad}: Radiation Power
- S_D, S_T: Deuterium, Tritium Refueling
- S_D^R, S_T^R: Deuterium, Tritium Recycling
- Fusion Reaction Rate: $S_\alpha = n_D n_T \langle \sigma \nu \rangle$
- Reactivity: $\langle \sigma \nu \rangle$
- $\vec{\Gamma}_\alpha$, $\vec{\Gamma}_D$, $\vec{\Gamma}_T$, $\vec{\Gamma}_I$, $\vec{\Gamma}_E$: Transport Fluxes

Particle Densities and Temperature are functions of time (t) and radial position ($\hat{\rho}$)

Burning Plasma Model (Controls): 0D Balance Equations

- 1D burning plasma dynamic model may be intractable for control synthesis
  - Transport models are too complex and still under development
- Control goal is 0D $\Rightarrow$ 0D response model is what is needed for control synthesis
- Control synthesis: 0D model $\rightarrow$ Performance assessment (simulations): 1D model





$$\frac{dE}{dt} = -P_{loss} + P_\alpha + P_{Ohm} + P_{aux}$$

$$\frac{dn}{dt} = -\Gamma_{loss} \pm B + S$$

- 0D response model is based on energy/particle balance equations
- Particle balance equations are needed for all species

Nonlinear Plasma Response Model for Burn Control Synthesis

Energy: $$\frac{dE}{dt} = -\frac{E}{\tau_E} + \underbrace{\overbrace{Q_\alpha S_\alpha}^{P_\alpha} - P_{rad} + P_{Ohm} + P_{aux} + P_{aux}^{burn}}_{P}$$

Alpha particles: $$\frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + S_\alpha$$

Deuterium: $$\frac{dn_D}{dt} = -\frac{n_D}{\tau_D} - S_\alpha + S_D^{rec} + S_D + S_D^{inj}$$

Tritium: $$\frac{dn_T}{dt} = -\frac{n_T}{\tau_T} - S_\alpha + S_T^{rec} + S_T^{inj}$$

Impurities: $$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I^{sp} + S_I^{inj}$$

(actuators/disturbances in red/blue)

Quasi-neutrality: $$n_e = n_D + n_T + 2n_\alpha + Z_I n_I$$

Density: $$n = \overbrace{n_\alpha + n_D + n_T + n_I}^{n_i} + n_e = 2n_D + 2n_T + 3n_\alpha + (Z_I + 1)\, n_I$$

Temperature: $$T = \frac{2}{3}\frac{E}{n} = \frac{2}{3}\frac{E}{2n_D + 2n_T + 3n_\alpha + (Z_I+1)n_I}$$ ($T \triangleq T_i = T_e$ (can be relaxed))

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(actuators/disturbances in red/blue)

- Reaction rate: $S_\alpha = n_D n_T \langle \sigma \nu \rangle \rightarrow S_\alpha = \gamma\,(1-\gamma)\, n_{DT}^2 \langle \sigma \nu \rangle$. Tritium fraction: $\gamma \triangleq n_T / n_{DT}$, $n_{DT} = n_T + n_D$.
- The DT reactivity $\langle \sigma \nu \rangle$ is a highly nonlinear function of the plasma temperature calculated as

$$\langle \sigma \nu \rangle = exp\left(\frac{a}{T^r} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4\right)$$

Nonlinear Plasma Response Model for Burn Control Synthesis

$$\text{Energy:} \quad \frac{dE}{dt} = -\frac{E}{\tau_E} + \underbrace{\overbrace{Q_\alpha S_\alpha}^{P_\alpha} - P_{rad} + P_{Ohm} + P_{aux} + P_{aux}^{burn}}_{P}$$

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- Confinement scaling (IPB98(y,2)): $\tau_E = 0.0562 H_H (I_{coils}^{non-axi}) I_p^{0.93} B_T^{0.15} P^{-0.69} n_{e19}^{0.41} M^{0.19} R^{1.97} \epsilon^{0.58} \kappa_{95}^{0.78}$.
- Particle confinement assumed proportional to τ_E, i.e. $\tau_\alpha = k_\alpha \tau_E$, $\tau_D = k_D \tau_E$, $\tau_T = k_T \tau_E$, $\tau_I = k_I \tau_E$.
- Impurity sputtering source: $S_I^{sp} = f_I^{sp} \left(\frac{n}{\tau_I} + \dot{n} \right) \quad (0 \leq f_I^{sp} << 1) \Rightarrow n_I^{sp} = f_I^{sp} n \quad (n_I = n_I^{inj} + n_I^{sp})$.

Nonlinear Plasma Response Model for Burn Control Synthesis

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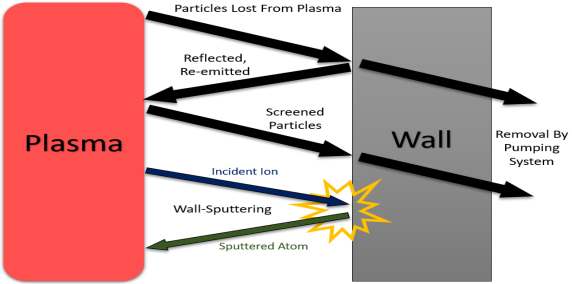
(actuators/disturbances in red/blue)

- Fuel recycling is included in the model through nonlinear functions S_D^{rec} and S_T^{rec} of the states.
- P_{rad}, P_{Ohm} are also highly nonlinear functions of the states (and external variables like I).
- P_{aux}, S_D represent effect of actuators (NBI, RF H&CD) under other competing controllers.
- P_{aux}^{burn}, S_D^{inj}, S_T^{inj}, S_I^{inj} are the actuators available for burn control.

Nonlinear Plasma Response Model for Burn Control Synthesis

$$S_D^R = \frac{f_{eff}}{1-f_{ref}\left(1-f_{eff}\right)}\left\{f_{ref}\frac{n_D}{\tau_D} + \left(1-\gamma^{PFC}\right)\times\left[\frac{\left(1-f_{ref}\left(1-f_{eff}\right)\right)R^{eff}}{1-R^{eff}\left(1-f_{eff}\right)} - f_{ref}\right]\left(\frac{n_D}{\tau_D}+\frac{n_T}{\tau_T}\right)\right\}$$

$$S_T^R = \frac{f_{eff}}{1-f_{ref}\left(1-f_{eff}\right)}\left\{f_{ref}\frac{n_T}{\tau_T} + \gamma^{PFC}\times\left[\frac{\left(1-f_{ref}\left(1-f_{eff}\right)\right)R^{eff}}{1-R^{eff}\left(1-f_{eff}\right)} - f_{ref}\right]\left(\frac{n_D}{\tau_D}+\frac{n_T}{\tau_T}\right)\right\}$$



Recycled fluxes S_D^R, S_T^R are functions of[†,1]:

- f_{ref} : Reflection Fraction
- f_{eff} : Recycling Efficiency
- R^{eff} : Recycling Coefficient
- γ^{PFC} : Plasma-Facing-Component Tritium Fraction

$$P_{rad} = \underbrace{A_{brem}\left(n_D+n_T+4n_\alpha+Z_I^2 n_I\right)n_e\sqrt{T(keV)}}_{\text{Bremmstrahlung}},$$

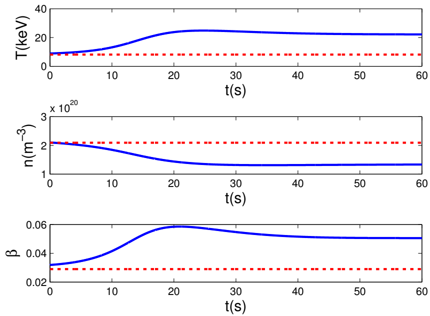
$$P_{Ohm} = 2.8\times 10^{-9}\frac{Z_{eff}I^2}{a^4T^{3/2}}$$

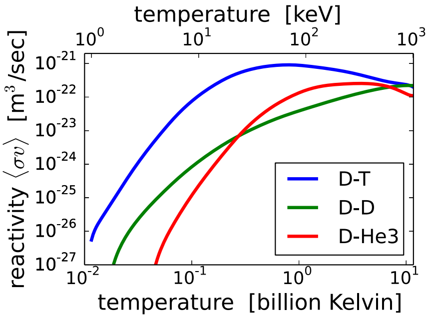
† Ehrenberg J. 1996 Physical Processes of the Interaction of Fusion Plasmas with Solids (New York: Academic)

[1] M.D. Boyer and E. Schuster, Nuclear Fusion 55 (2015) 083021 (24pp).

Isotopic Fuel Tailoring as Actuator for Burn Control

Burn Control Challenges





- Potential for **thermal instability** $\rightarrow$ **excursions and quenching**:

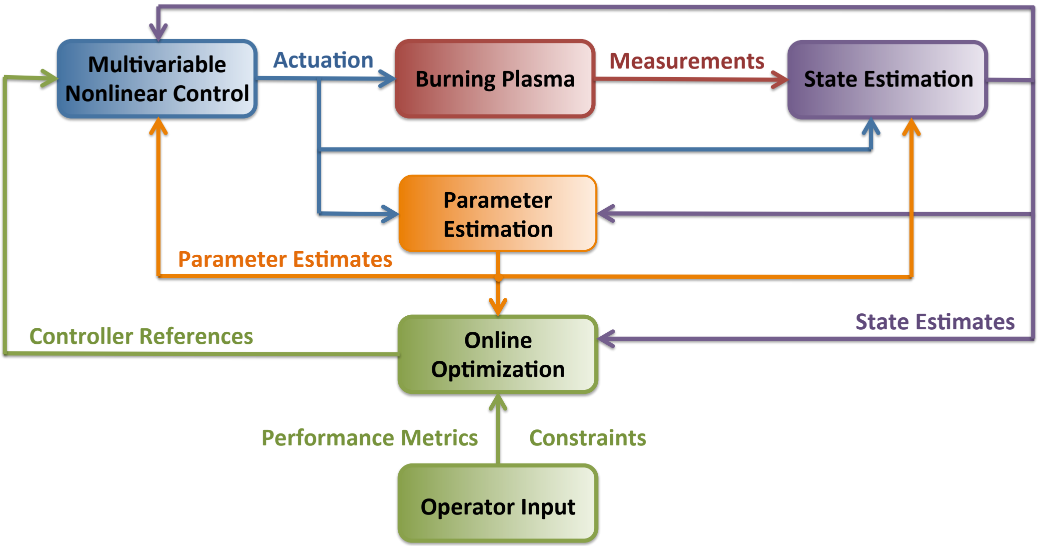
$$P_f = n_D n_T < \sigma v > Q_{DT} \propto \beta^2 B^4, \quad \beta \propto \frac{nkT}{\frac{B^2}{2\mu_o}}$$

- Even when operating at stable equilibria, system **performance during transients and plasma response to disturbances** could be **undesirable without active control.**

Burn Control Needs and Objectives

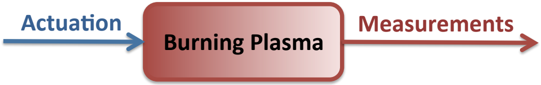
- Burn condition demands effective feedback control scheme to avoid:
  - **Undesirable transient performance** due to **nonlinear/coupled dynamics**
  - **Perturbations** due to **plasma changes (confinement, impurity content)**
  - **Potentially disruptive plasma conditions** due to **thermal instabilities**
- Nonlinear coupling with other control problems and objectives is severe
  - Confinement: PF coils (shape, current), Non-axisymmetric coils (RWM/ELM)
  - Heating/Density: Non-inductive heating & current drive (q-profile, NTM)
- Wall heat/particle load tolerance may impose constraints on core burn regulation
  - Requires controller that can effectively change operating conditions (Q, P_f, etc.)
- Capability of controller designed based on **linearized model:**
  - ✓ **Regulation around a desired burning equilibrium point**
  - × **Drive plasma from one operating point to another (Modify Q or P_f)**
  - × **Access to and exit from the burning plasma mode**
  - × **Handle nonlinear coupling with other competing controllers**
- Reactor-specific additional challenges for effective burn control:
  - $P_\alpha >> P_{aux}$**: control by heating may not be effective**
  - **Wall recycling effects may also make control by fueling not effective**
  - **Limited and noisy set of diagnostics**

Burn Control Solution: Overview of Proposed Approach



Isotopic Fuel Tailoring as Actuator for Burn Control

Stage 1: Synthesis of Nonlinear Feedback Controller



- Nonlinear dynamics
- Multiple inputs and outpus

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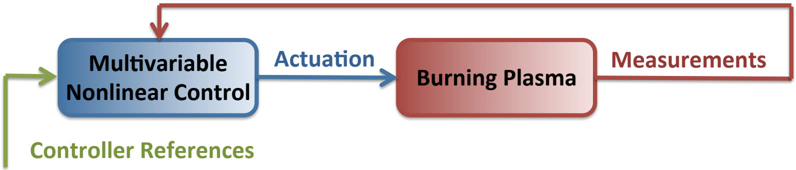

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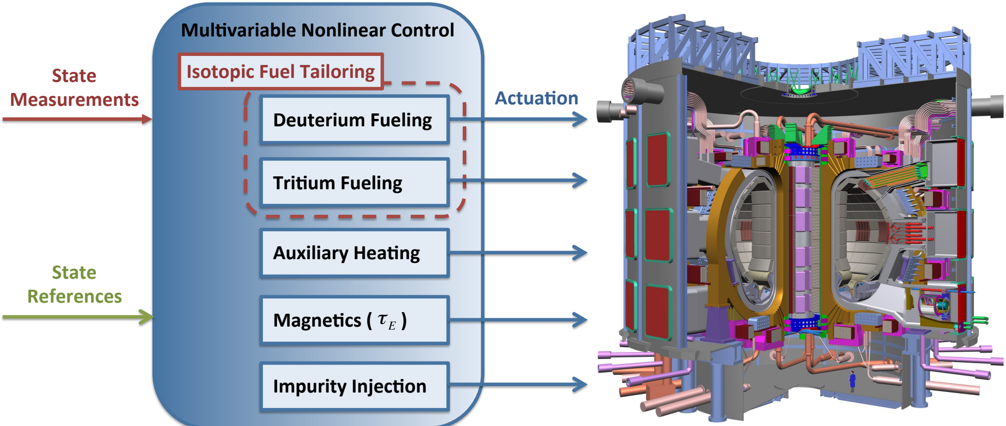


- The approach embeds whole nonlinear dynamics of burning plasma in controller by **avoiding linearization of the model** around operating point.
  - Preserving nonlinear dynamics is key to achieve controller's goals.
- The approach uses combination of actuators **(SISO $\rightarrow$ MIMO)**.

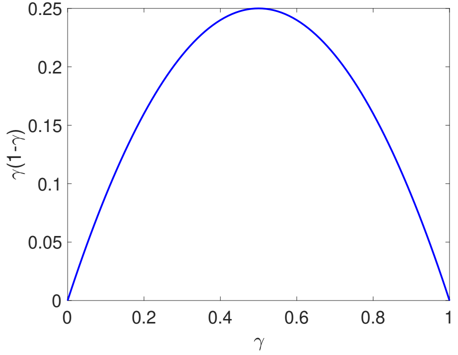
Stage 1: Synthesis of Nonlinear Feedback Controller

Isotopic Fuel Tailoring

- Alpha power depends on tritium fraction

$$P_\alpha = n_D n_T \langle \sigma \nu \rangle Q_\alpha = \gamma\,(1-\gamma) n_{DT}^2 \langle \sigma \nu \rangle Q_\alpha$$

  where $n_{DT} \triangleq n_T + n_D$ and $\gamma \triangleq n_T / n_{DT}$.
- The function $\gamma\,(1-\gamma)$ achieves its maximum of 0.25 at $\gamma = 0.5$ and decreases steeply for smaller/larger γ's.
- Fueling system in ITER (gass puffing (D) and pellet injection (D + DT)) will allow fuel mix regulation.
- Diagnostics for measuring the tritium ratio in both the edge and core plasma should be available in real time.
- Feedback control of the tritium ratio in ITER plasmas through isotopic fuel tailoring should be feasible.
- Effectiveness may be limited due to particle recycling.



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No unique approach (depends on actuators and goals) but usually leads to tandem approach:

Step 1: Calculate value of P_{aux}^{burn} that stabilizes E^r subject to saturation.

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Step 1: Calculate value of P_{aux}^{burn} that stabilizes E^r subject to saturation.

Step 2: Calculate stabilizing values of tritium fraction, γ^*, and/or energy confinement, τ_E^*.

- If the auxiliary power was not saturated, then $\gamma^* = \gamma^r$ and $\tau_E^* = \tau_E^{max}$.

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Step 4: Calculate fuel injection rates that stabilize γ^* and n^r subject to saturation.

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Step 5: If $P_{aux}^{burn} = (P_{aux}^{burn})^{max}$, $\tau_E^* = \tau_E^{max}$ and γ^* cannot be tracked $\rightarrow$ thermal quench.

- Change magnetic plasma parameters to improve energy confinement.

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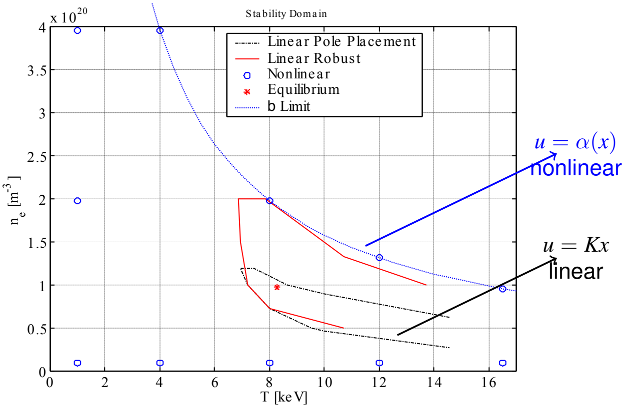
- Change magnetic plasma parameters to improve energy confinement.

Step 6: If $P_{aux}^{burn} = (P_{aux}^{burn})^{min}$, $\tau_E^* = \tau_E^{min}$ and γ^* cannot be tracked $\rightarrow$ thermal excursion.

- Calculate a stabilizing value of impurity density, n_I^*, and the fueling/impurity injection rates needed to track n_I^*, γ^* and n^r.

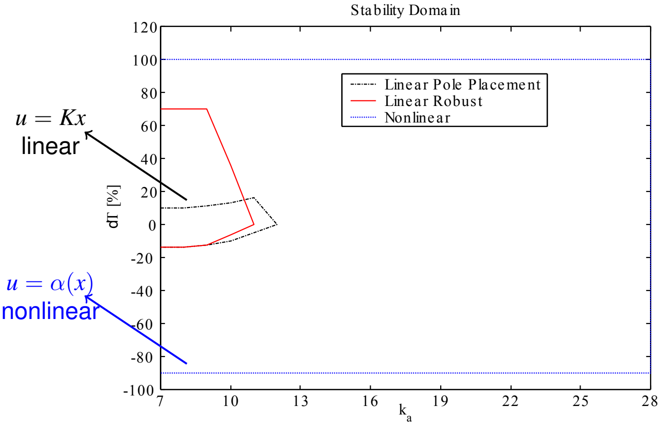
[1] E. Schuster, M. Krstic and G. Tynan, Fusion Engineering and Design, 63-64, pp. 569-575, 2002.
[2] E. Schuster, M. Krstic and G. Tynan, Fusion Science and Technology, vol. 43, no. 1, 2003.
[3] M.D. Boyer and E. Schuster, Nuclear Fusion 55 (2015) 083021 (24pp).
[4] A. Pajares and E. Schuster, Fusion Engineering and Design 123 (2017) 607–611.

Potential of Nonlinear Control: Burn Performance



- Comparative study is carried out generating initial perturbations around the equilibrium for T and n_e keeping $f_\alpha = n_\alpha / n_e$ constant.
- While the boundaries shown for the linear controllers are absolute, for the nonlinear controller they only indicate the test limits.
- **Embedding nonlinear dynamics in control synthesis $\rightarrow$ higher levels of performance!**

Potential of Nonlinear Control: Burn Robustness



- Comparative study is carried out generating initial perturbations around the equilibrium for T.
- While the boundaries shown for the linear controllers are absolute, for the nonlinear controller they only indicate the test limits.
- **Embedding nonlinear dynamics in control synthesis $\rightarrow$ higher levels of robustness!**

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Adding Robustness to Controller by Specific Design Techniques

How Do We Handle Uncertainties and Time-variations In Control-oriented Models?

- One of the main characteristic of feedback is its ability to deal with model uncertainties:
  - Poorly understood and/or unmodeled dynamics in response models
- Moreover, there are specific tools within the body of mathematical theory of control to specifically deal with model uncertainties:
  - **Adaptive Control**
  - **Robust Control**

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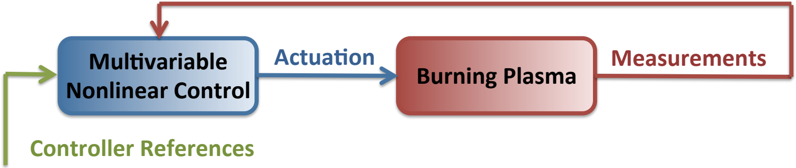
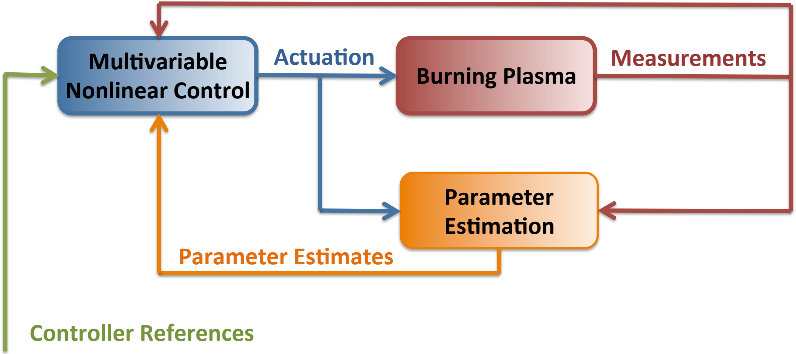
Stage 2: Adaptation by Real-time Estimation of Model Parameters



- Many of the burning plasma model parameters may be uncertain (highlighted in orange).
- The control algorithm must make use of estimated model parameters.
- Adaptive control is proposed to ensure tracking despite uncertainty.

Stage 2: Adaptation by Real-time Estimation of Model Parameters



- Many of the burning plasma model parameters may be uncertain (highlighted in orange).
- The control algorithm must make use of estimated model parameters.
- Adaptive control is proposed to ensure tracking despite uncertainty.

Stage 2: Adaptation by Real-time Estimation of Model Parameters

We define a system observer as

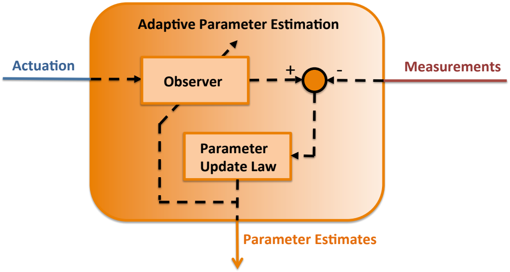
$$\dot{E}^{ob} = -\hat{\theta}_1 \frac{E}{\tau_E} + P_\alpha - P_{rad} + P_{aux} + P_{Ohm} - K_E^{ob}\left(E^{ob} - E\right)$$

$$\dot{n}_\alpha^{ob} = -\hat{\theta}_2 \frac{n_\alpha}{\tau_E} + S_\alpha - K_\alpha^{ob}\left(n_\alpha^{ob} - n_\alpha\right)$$

$$\dot{n}_D^{ob} = -\hat{\theta}_3 \frac{n_D}{\tau_E} + \hat{\theta}_4 \frac{n_T}{\tau_E} - S_\alpha + S_D^{inj} - K_D^{ob}\left(n_D^{ob} - n_D\right)$$

$$\dot{n}_T^{ob} = \hat{\theta}_5 \frac{n_D}{\tau_E} - \hat{\theta}_6 \frac{n_T}{\tau_E} - S_\alpha + S_T^{inj} - K_T^{ob}\left(n_T^{ob} - n_T\right)$$

$$\dot{n}_I^{ob} = -\hat{\theta}_7 \frac{n_I}{\tau_E} + S_I^{inj} + S_I^{sp} - K_I^{ob}\left(n_I^{ob} - n_I\right)$$



The dynamics of the error $\tilde{\theta} = \theta - \hat{\theta}$ can be asymptotically stabilized by taking

$$\dot{\hat{\theta}} = -\frac{1}{\tau_E}\Gamma\left[\begin{array}{ccccccc} \tilde{n}_\alpha^{ob} n_\alpha & \tilde{E}^{ob} E & \tilde{n}_D^{ob} n_D & -\tilde{n}_D^{ob} n_T & -\tilde{n}_T^{ob} n_D & \tilde{n}_T^{ob} n_T & \tilde{n}_I^{ob} n_I \end{array}\right]^T, \Gamma > 0$$

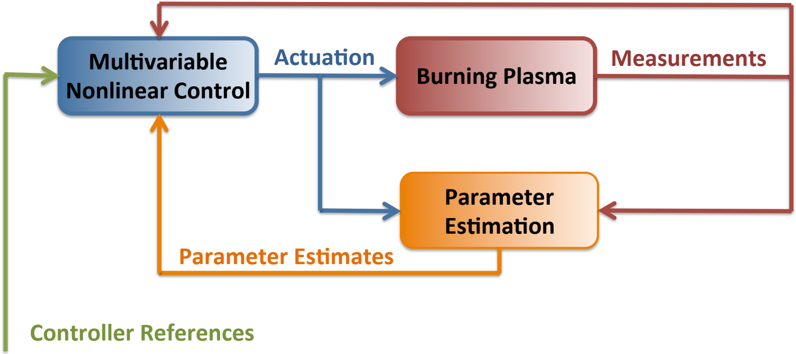
where

$$\tilde{n}_\alpha^{ob} = n_\alpha^{ob} - n_\alpha, \tilde{E}^{ob} = E^{ob} - E, \tilde{n}_I^{ob} = n_I^{ob} - n_I, \tilde{n}_D^{ob} = n_D^{ob} - n_D, \tilde{n}_T^{ob} = n_T^{ob} - n_T.$$

[1] M.D. Boyer and E. Schuster, Plasma Physics and Controlled Fusion 56 104004 (2014).

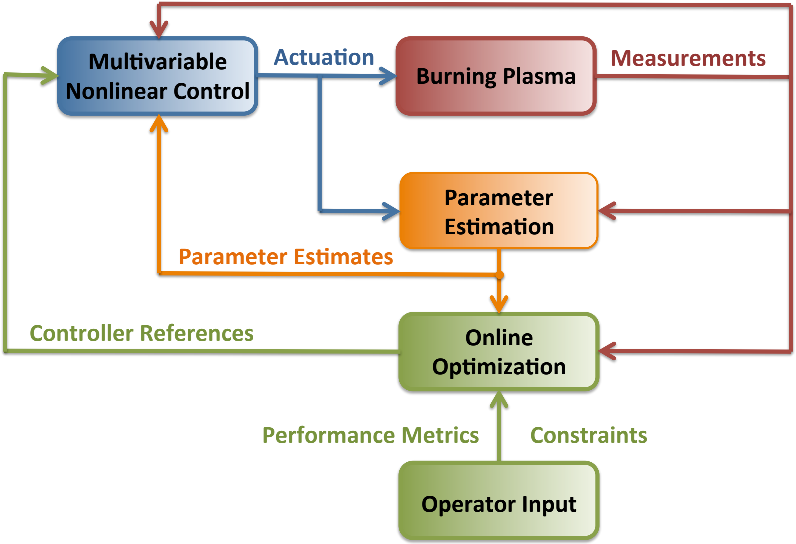
Isotopic Fuel Tailoring as Actuator for Burn Control

Stage 3: Reference Governor by Real-time Optimization



- Part of burn control problem is selection of controller references.
- References must be chosen to optimize figure of merit for performance.
- Convex optimization is proposed to ensure optimal reference selection.

Stage 3: Reference Governor by Real-time Optimization

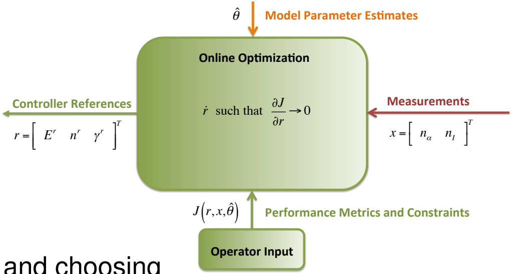


Stage 3: Reference Governor by Real-time Optimization

$$J = \overbrace{\frac{w_T}{2}}^{\text{weight}}\left(T - \underbrace{T^{des}}_{\text{target}}\right)^2 + \overbrace{\frac{w_{P_\alpha}}{2}}^{\text{weight}}\left(P_\alpha - \underbrace{P_\alpha^{des}}_{\text{target}}\right)^2 + \overbrace{\frac{w_\gamma}{2}}^{\text{weight}}\left(\gamma - \underbrace{\gamma^{des}}_{\text{target}}\right)^2 - \overbrace{\frac{1}{\eta_c}\sum_{i=1}^{K}\ln(-g_i)}^{\text{constraints}}.$$

- A **reference for the controlled states** $r = [E^r, n^r, \gamma^r]^T$ determines burn condition.
- T^{des}, P_α^{des}, γ^{des} are desired targets.
- w_T, w_{P_α}, w_γ are tracking weights.
- Constraints by **barrier function**

$$g_i(E, n, \gamma, n_\alpha, n_I) < 0$$



- Optimization is achieved by defining $V_r = \frac{1}{2}\left(\frac{\partial J}{\partial r}\right)^T \frac{\partial J}{\partial r}$ and choosing

$$\dot{r} = -\left(\frac{\partial^2 J}{\partial r^2}\right)^{-1}\left[K_{RTO}\frac{\partial J}{\partial r} + \frac{\partial^2 J}{\partial r \partial x}\dot{x} + \frac{\partial^2 J}{\partial r \partial \hat{\theta}}\dot{\hat{\theta}}\right] \Rightarrow \dot{V}_r \le 0 \Rightarrow \frac{\partial J}{\partial r} \to 0 \Rightarrow r \to r^*$$

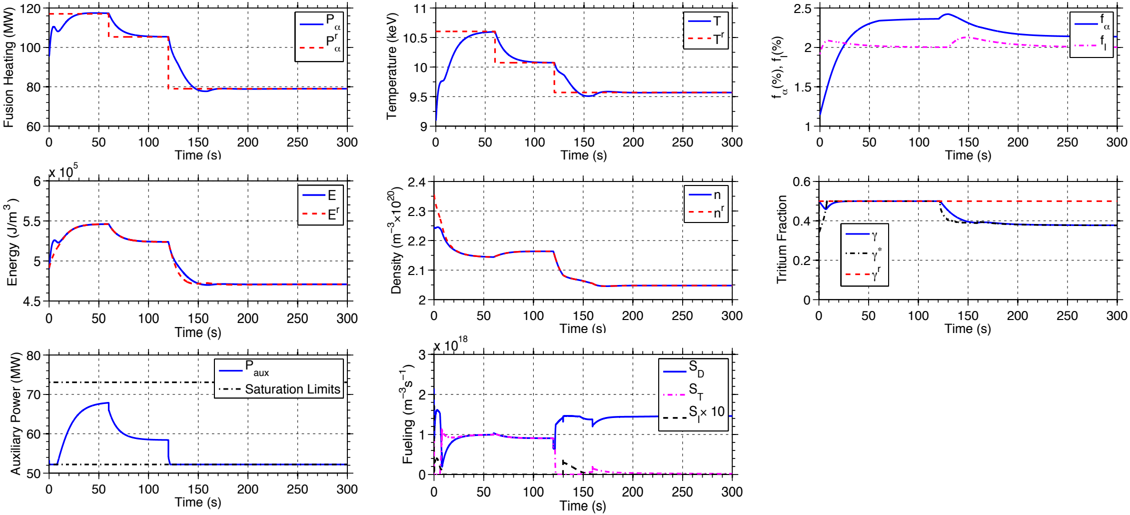
- The cost function is user-defined! More sophisticated optimization problems are possible!

[1] M.D. Boyer and E. Schuster, Plasma Physics and Controlled Fusion 56 104004 (2014).

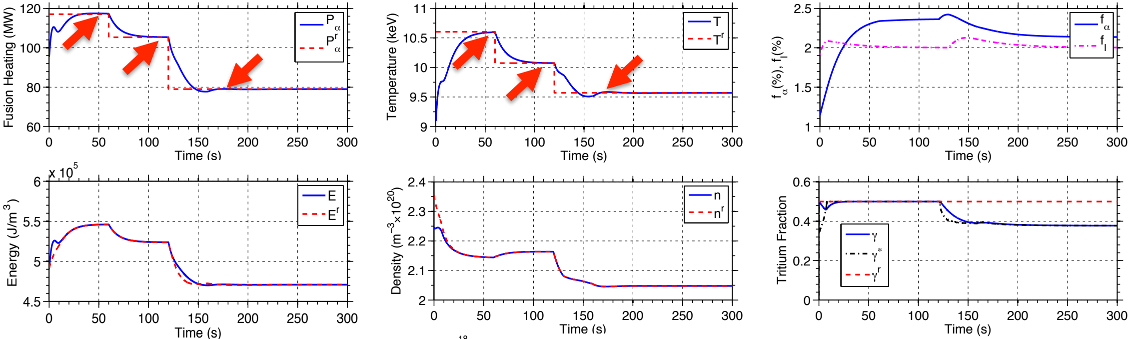
Changing Operating Points via Real-time Optimization

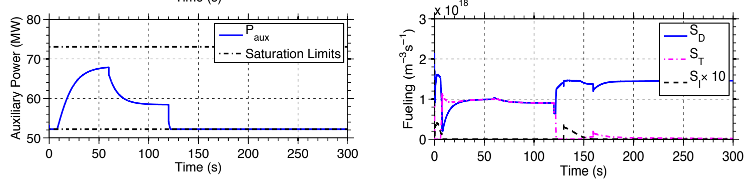
- Conditions: $\gamma^r = 0.5$ is kept constant in this simulation and $w_\gamma \equiv 0$
- Optimization weights: $w_T = 0.1$ and $w_{P_\alpha} = 1$.
- References for fusion heating and temperature modified twice (at $t = 60s$ and $t = 120s$)
- Constraints: 53MW $< P_{aux} <$ 73MW
- Recycling: $\gamma^{PFC} = 0.5, f_{eff} = 0.3, f_{ref} = 0.5, R_{eff} = 0.95 \Rightarrow$ poor γ control
- Simulation conditions chosen to ensure impurity injection is needed

Changing Operating Points via Real-time Optimization



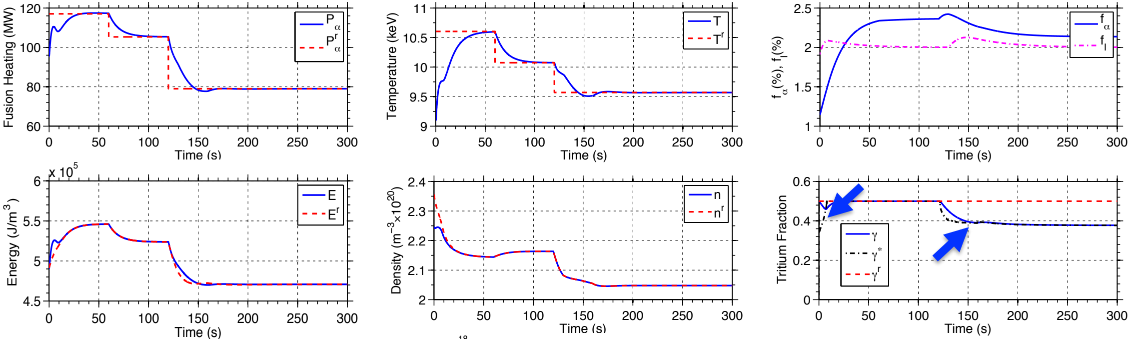
Changing Operating Points via Real-time Optimization

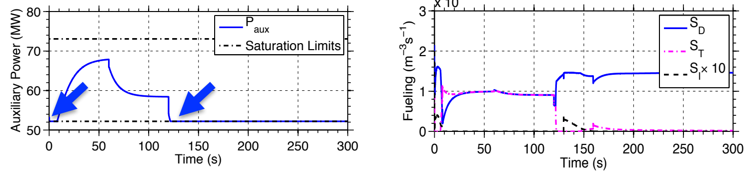




Nonlinear controller and online optimization scheme were able to switch between desired operating points.

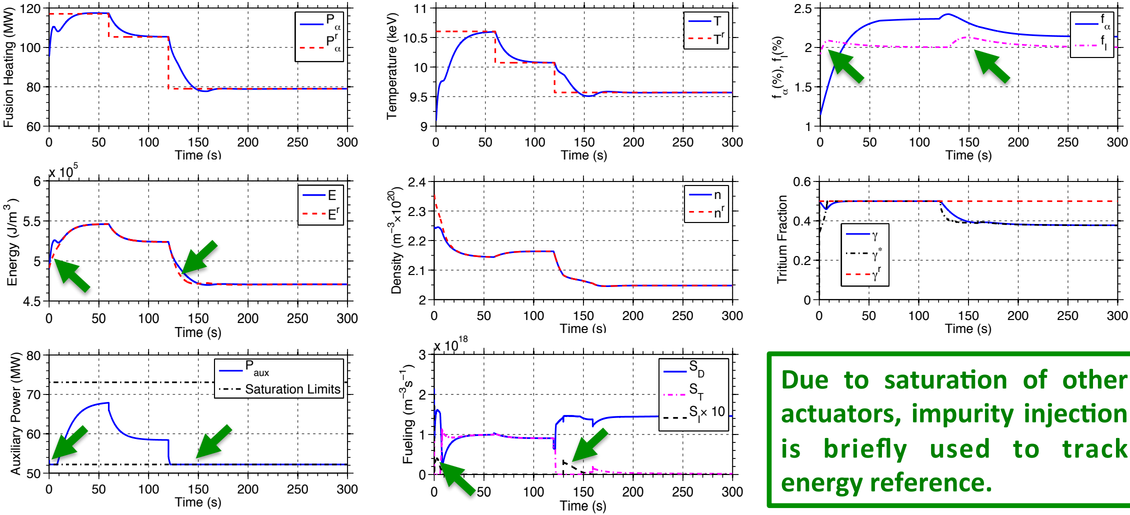
Changing Operating Points via Real-time Optimization





Tritium fraction was reduced in response to saturation of auxiliary power.

Changing Operating Points via Real-time Optimization



Due to saturation of other actuators, impurity injection is briefly used to track energy reference.

Isotopic Fuel Tailoring as Actuator for Burn Control

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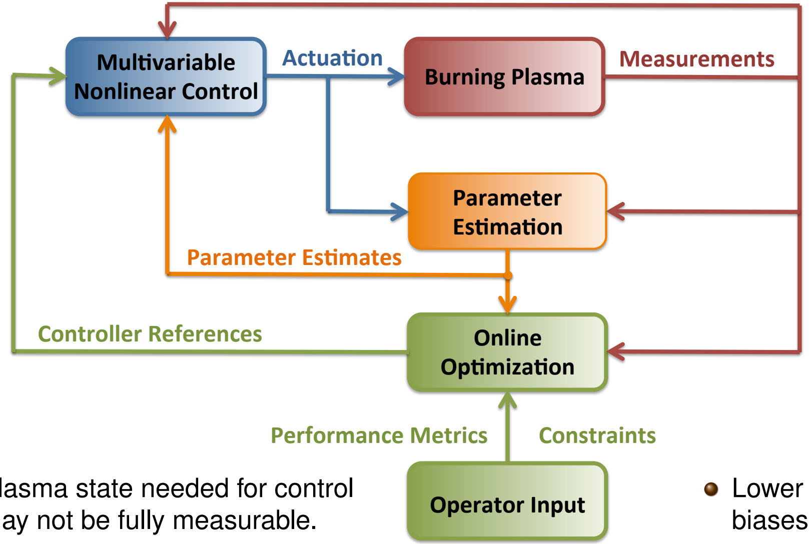
Stage 4: State Estimation by Observer From Noisy/Limited Data



- Plasma state needed for control may not be fully measurable.
- Limited number of diagnostics.
- Lower quality (e.g., noise, drifts, biases, etc.) of diagnostics.
- Critical issue in fusion reactors!

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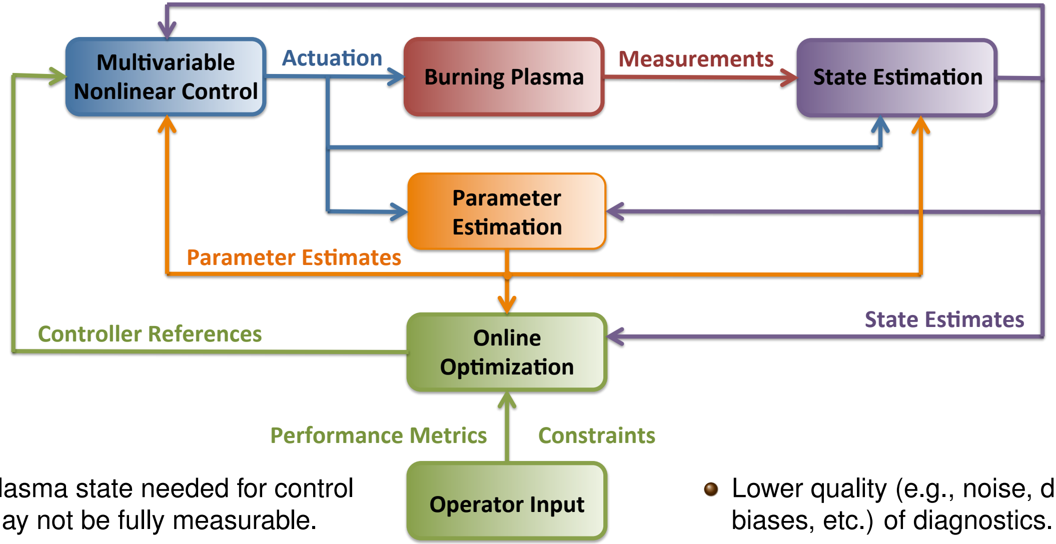

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Stage 4: State Estimation by Observer From Noisy/Limited Data

We define an observer as

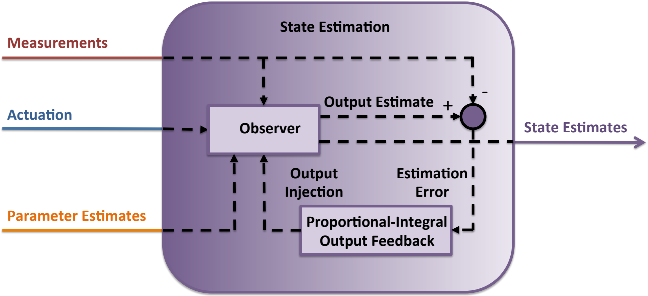
$$\dot{\mathring{E}} = -\hat{\theta}_1 \frac{\mathring{E}}{\tau_E^{sc}} + P_\alpha - P_{rad} + P_{Ohm} + P_{aux} + L_E,$$

$$\dot{\mathring{n}}_\alpha = -\hat{\theta}_2 \frac{\mathring{n}_\alpha}{\tau_E^{sc}} + S_\alpha + L_\alpha,$$

$$\dot{\mathring{n}}_D = -\hat{\theta}_3 \frac{\mathring{n}_D}{\tau_E^{sc}} + \hat{\theta}_4 \frac{\mathring{n}_T}{\tau_E^{sc}} - S_\alpha + S_D^{inj} + L_D,$$

$$\dot{\mathring{n}}_T = \hat{\theta}_5 \frac{\mathring{n}_D}{\tau_E^{sc}} - \hat{\theta}_6 \frac{\mathring{n}_T}{\tau_E^{sc}} - S_\alpha + S_T^{inj} + L_T,$$

$$\dot{\mathring{n}}_I = -\hat{\theta}_7 \frac{\mathring{n}_I}{\tau_E^{sc}} + S_I^{sp} + S_I^{inj} + L_I,$$

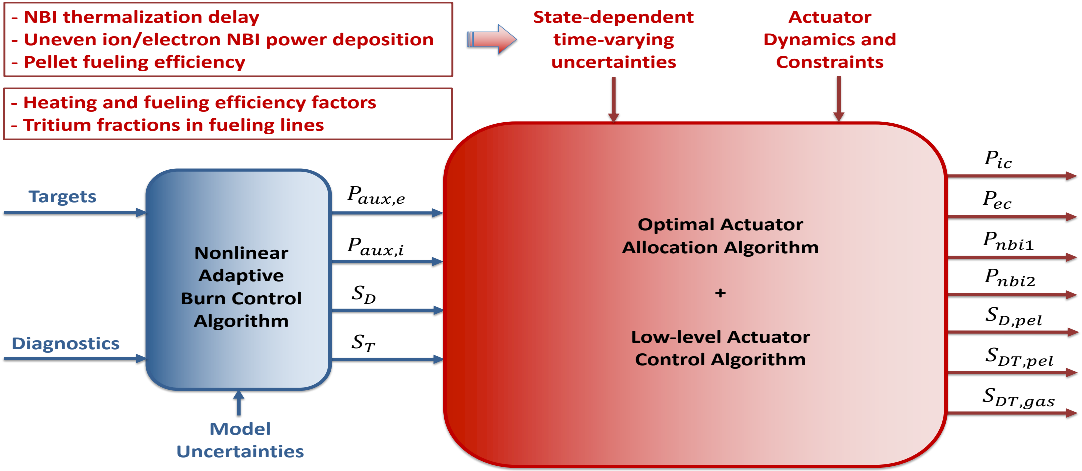


- We consider a general nonlinear output map $y = h(n_\alpha, E, n_I, n_D, n_T)$.
- The system is augmented with an additional state, $\check{z}$, governed by $\dot{\check{z}} = \mathring{y} - y = \check{y}$.
- Lyapunov analysis $\rightarrow$ injection terms L_E, L_α, L_D, L_T, L_I adopt a proportional-integral form.

[1] M. D. Boyer, E. Schuster, International Federation of Automatic Control World Congress (2014).

Stage 5: Optimal Allocation of Actuators with Dynamics

- Two-temperature model ($T_i \neq T_e$). Heating and fueling as actuation.
- Virtual control inputs $\leftrightarrow$ Effector System $\leftrightarrow$ Physical control inputs.



[1] V. Graber and E. Schuster, Nuclear Fusion 62 (2022) 026016 (18pp).

Stage 5: Optimal Allocation of Actuators with Dynamics

The Effector System maps the control efforts v to the actuator efforts u:

$$v = [P_{aux,i}\ P_{aux,e}\ S_D\ S_T]^T \quad \longleftrightarrow \quad u = [P_{ic}\ P_{ec}\ P_{nbi_1}\ P_{nbi_2}\ S_{D_{pel}}\ S_{DT_{pel}}\ S_{DT_{gas}}]^T$$

$$P_{aux,i} = \eta_{ic} P_{ic} + \eta_{nbi_1} \phi_{nbi} P_{nbi_1} + \eta_{nbi_2} \phi_{nbi} P_{nbi_2}$$

$$P_{aux,e} = \eta_{ec} P_{ec} + \eta_{nbi_1} \bar{\phi}_{nbi} P_{nbi_1} + \eta_{nbi_2} \bar{\phi}_{nbi} P_{nbi_2} \qquad (\text{where } \bar{\phi}_{nbi} = 1 - \phi_{nbi})$$

$$S_D = \eta_{nbi_1} \frac{P_{nbi_1}}{\varepsilon_{nbi_0}} + \eta_{nbi_2} \frac{P_{nbi_2}}{\varepsilon_{nbi_0}} + \eta_{pel_1} S_{D_{pel}} + \eta_{pel_2} (1-\gamma_{pel}) S_{DT_{pel}} + \eta_{gas} (1-\gamma_{gas}) S_{DT_{gas}}$$

$$S_T = \eta_{pel_2} \gamma_{pel} S_{DT_{pel}} + \eta_{gas} \gamma_{gas} S_{DT_{gas}}$$

Uncertain Parameters

- Ion cyclotron, electron cyclotron & NBI heating: P_{ic}, P_{ec}, P_{nbi_1}, P_{nbi_2}
- DT pellet & gas injection with Tritium fractions γ_{pel} & γ_{gas}: $S_{D_{pel}}$, $S_{DT_{pel}}$, $S_{DT_{gas}}$
- Efficiency factors: η_{ic}, η_{ec}, η_{nbi_1}, η_{nbi_2}, η_{pel_1}, η_{pel_2}, η_{gas}
- Pellet fueling efficiency decreases with increasing plasma energy: $\eta_{pel_i} = \rho_{pel_i}(1 - E/E_0)$, $i \in \{1, 2\}$
- The NBI ion-heating fraction $\phi_{nbi} = \rho_{nbi} \phi^\star_{nbi}$ contains uncertainty (ρ_{nbi}).
- NBI thermalization delay contains uncertainty: ρ_{th}

$$\tau^{lag}_{nbi} = \rho_{th} \tau^\star_{nbi} = -\rho_{th} \frac{2}{3B} \ln \left[\frac{\left(\frac{\varepsilon_{nbi_{th}}}{\varepsilon_{nbi_0}}\right)^{3/2} + \left(\frac{\varepsilon_c}{\varepsilon_{nbi_0}}\right)^{3/2}}{1 + \left(\frac{\varepsilon_c}{\varepsilon_{nbi_0}}\right)^{3/2}} \right] \qquad (\varepsilon_{nbi_{th}} = T_i)$$

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3. Impact of Fueling-line Tritium Fractions on ITER's Operational Space

4. Summary of the Presentation

Unmeasurable DT Content in Fueling Lines May Vary Over Time

- **Two fueling lines** will be available in ITER:
  1. **D injector ($S_{\mathrm{D}}^{\mathrm{line}}$):** pellets of approximately **100% D**
  2. **DT injector ($S_{\mathrm{DT}}^{\mathrm{line}}$):** pellets of approximately **10% D - 90% T**
- These nominal concentrations in the fueling lines may not be sustained over long discharges
- Moreover, drifts and biases in the tritium fractions would not be measurable in real time
- Can we make the burn controller robust against these unmeasurable drifts and biases?

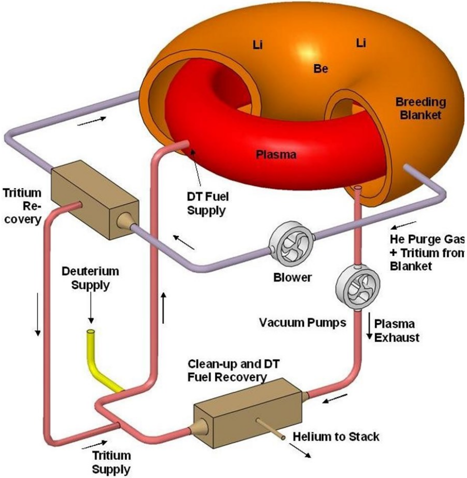

Figure source: ITER webpage, www.iter.org

Adding Robustness to Controller by Specific Design Techniques

How Do We Handle Uncertainties and Time-variations In Control-oriented Models?

- One of the main characteristic of feedback is its ability to deal with model uncertainties:
  - Poorly understood and/or unmodeled dynamics in response models
- Moreover, there are specific tools within the body of mathematical theory of control to specifically deal with model uncertainties:
  - **Adaptive Control**
  - **Robust Control**

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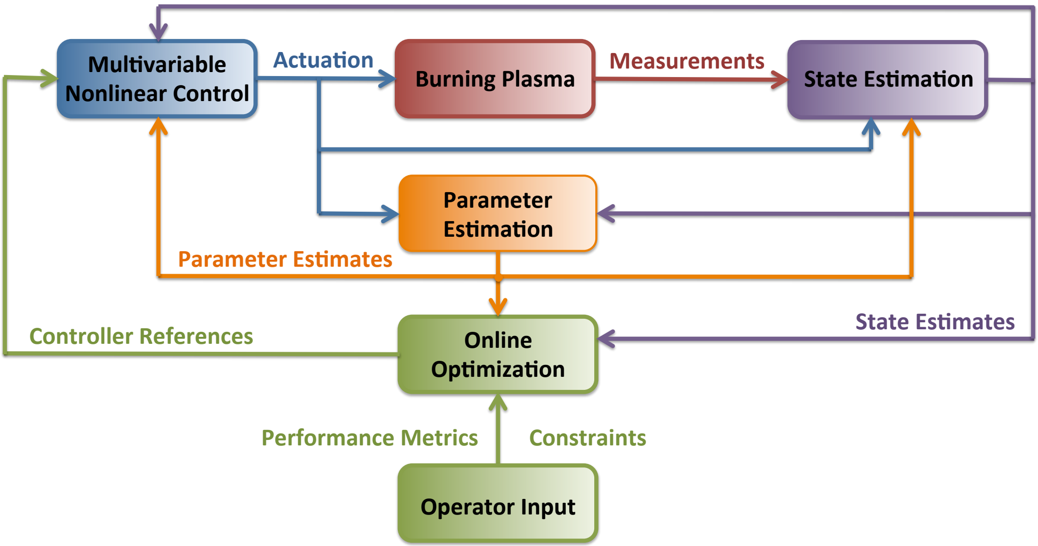
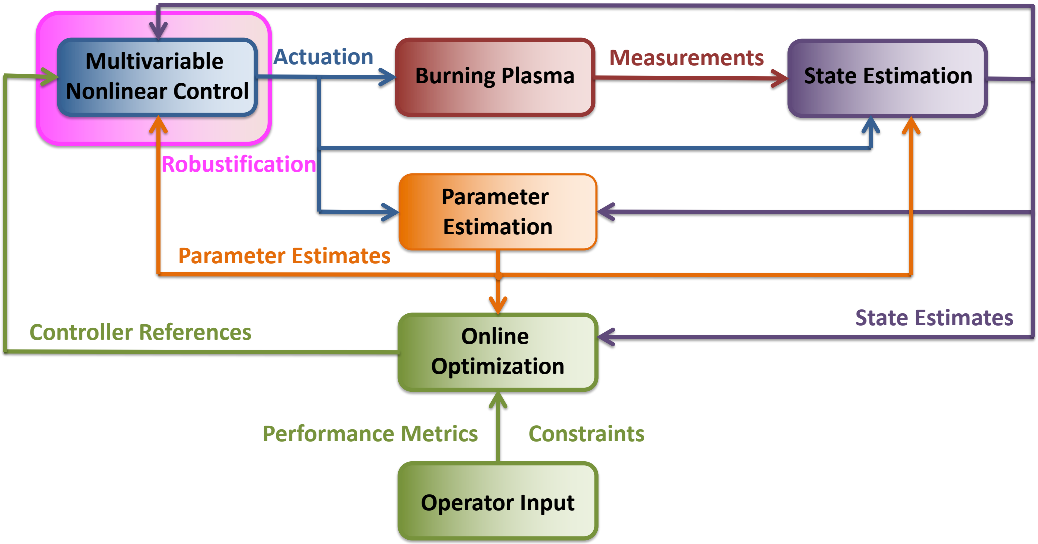
Burn Control Scheme: Overview of Proposed Approach



Burn Control Scheme: Overview of Robust Proposed Approach



Robustness Against Drifts and Biases in Fueling Concentrations

- Injection rates for D and T can be written as

$$S_D^{inj} = (1 - \gamma_{DT}) S_{DT}^{line} + (1 - \gamma_D) S_D^{line}$$
$$S_T^{inj} = \gamma_{DT} S_{DT}^{line} + \gamma_D S_D^{line}$$

  – $\gamma_{DT} \in [0, 1]$, $\gamma_D \in [0, 1]$ characterize the tritium concentration in the DT and D fueling lines.

- In the nominal case, $\gamma_{DT} = \gamma_{DT}^{nom} = 0.9$ and $\gamma_D = \gamma_D^{nom} = 0$.
- Unknown variations over time in the tritium fractions are modeled as

$$\gamma_{DT} = \gamma_{DT}^{nom} + \delta_{DT}, \quad \gamma_D = \gamma_D^{nom} + \delta_D, \tag{1}$$

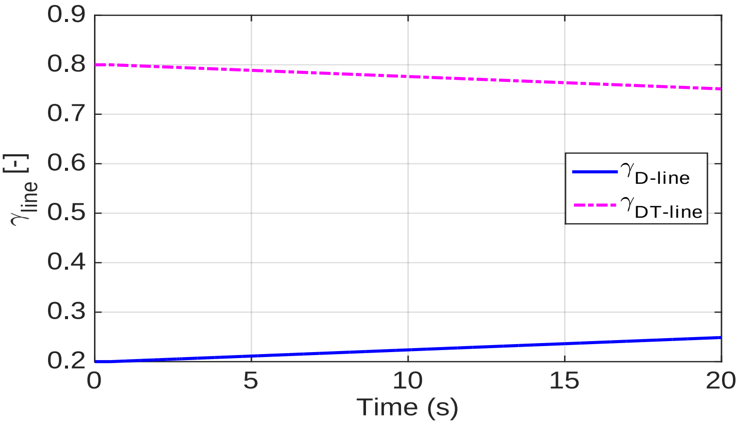
  – δ_{DT} and δ_D are "model uncertainties" in the tritium fractions.

- From definition, $\delta_{DT} \in [-0.9, 0.1]$, $\delta_D \in [0, 1]$ $\Rightarrow$ bounded uncertainties.

[1] A. Pajares, E. Schuster, Nuclear Fusion 59 (2019) 096023 (18pp).

Robustness Against Drifts and Biases in Fueling Concentrations

- Controller tries to regulate the system around a nominal equilibrium point defined by $\bar{T} = 12keV$, $\bar{\gamma} = 0.4$ and $\bar{\beta}_N = 1.5$
- The system starts from a perturbed initial condition of +5% in E.
- Initial biases and time drifts in γ_{DT} and γ_D are simulated (not known by controller).



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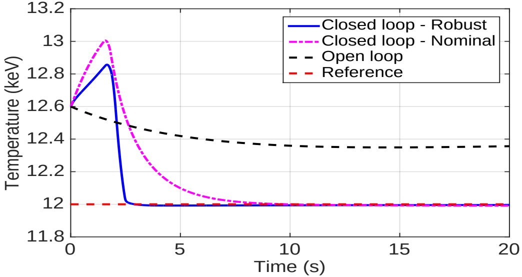

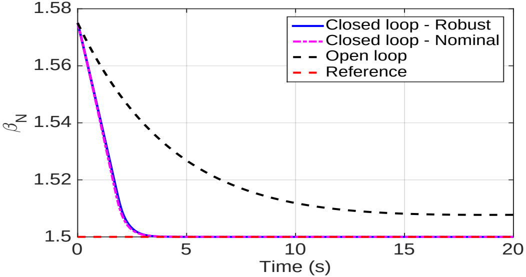

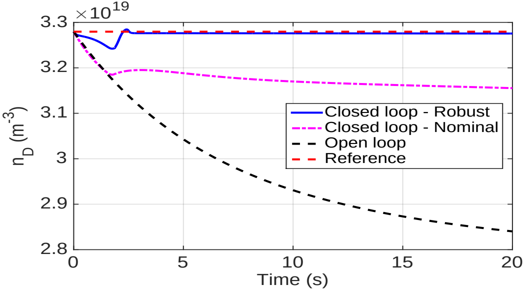

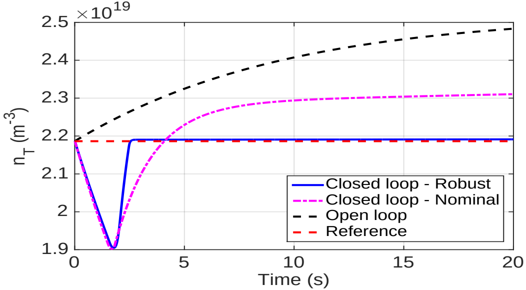

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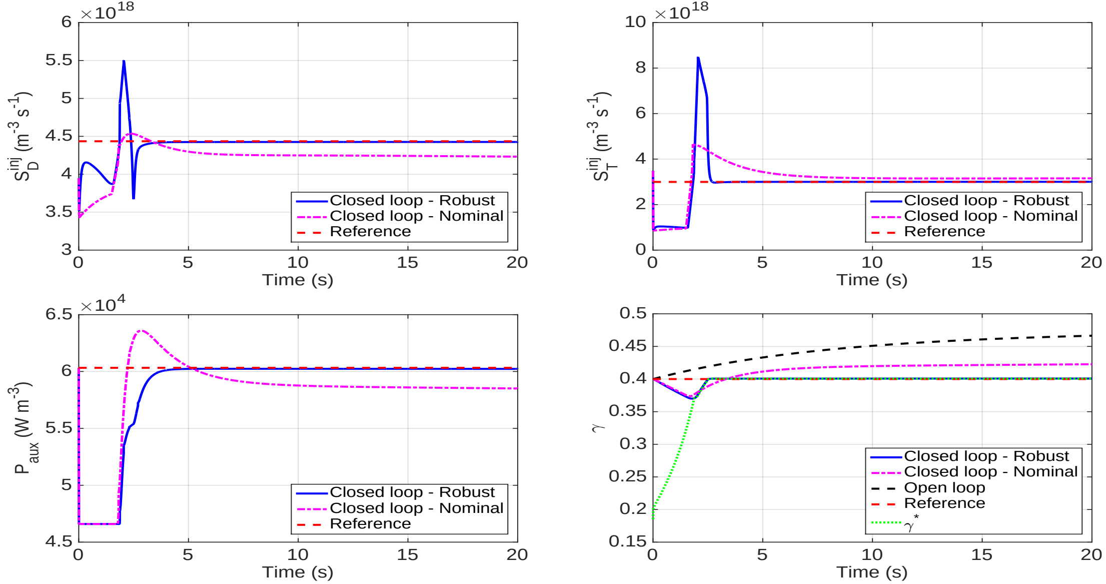

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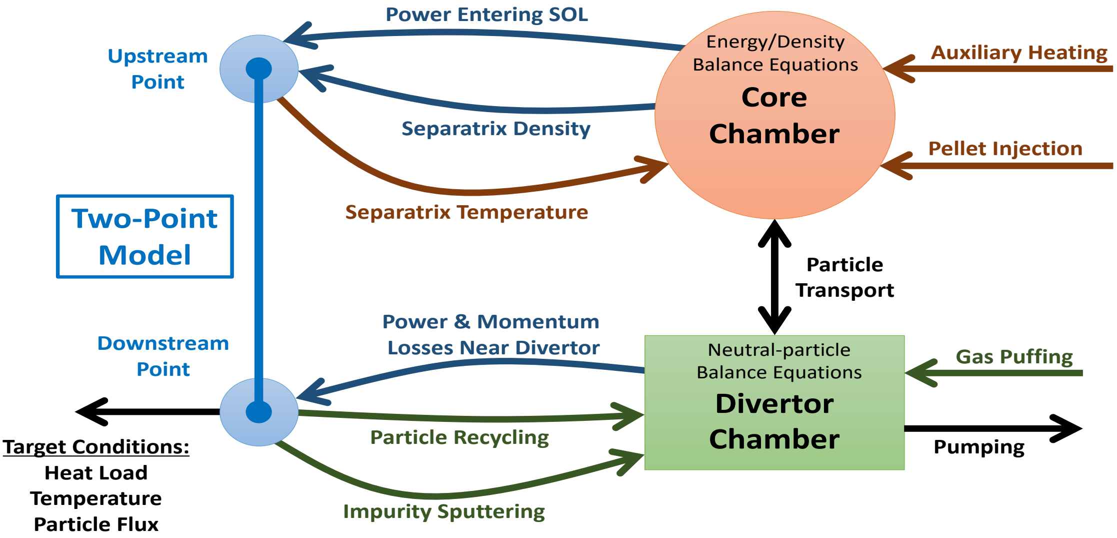
Coupling Two-Chamber Model and Two-Point Model → CSD Model



[1] V. Graber and E. Schuster, Fusion Engineering and Design 171 (2021) 112516.

Core Chamber: Radial Temperature & Density Profiles

Peaked Electron and Ion Temperature Profiles

$$T_e(t,\psi) = (T_{e,0} - T_u)(1 - \psi/\psi_0)^2 + T_u$$

$$T_i(t,\psi) = (T_{i,0} - T_u)(1 - \psi/\psi_0)^2 + T_u$$

Central Ion Temperature: $T_{i,0}$

Central Electron Temperature: $T_{e,0}$

Upstream Separatrix Temperature: T_u

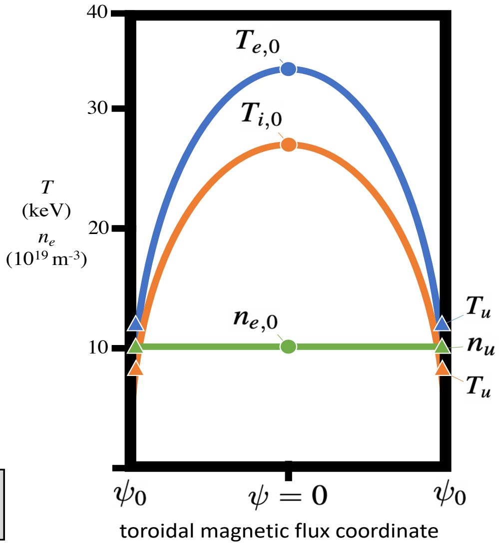
Flat Density Profiles

$$n_e(t,\psi) = n_{e,0} = n_u$$

Central Electron Density: $n_{e,0}$

Upstream Separatrix Density: n_u

Radial profiles couple core conditions (T_0 / n_0) with conditions at the separatrix (T_u / n_u)



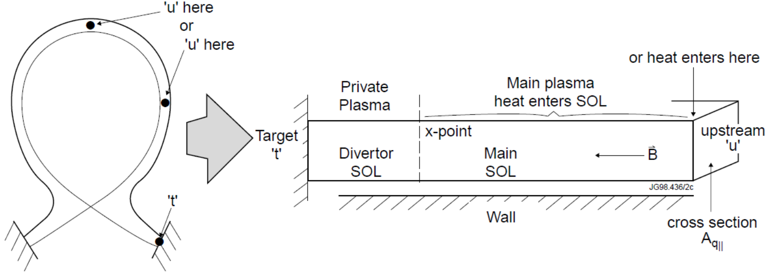
Two-Point Model: Straightening Out the SOL Plasma

- Relates upstream (separatrix) and downstream (target) conditions[†]
- **Upstream**: density n_u, temperature T_u, parallel power flux density $q_{\|}$
- **Downstream**: density n_t, temperature T_t

Two Point Model Equations:

$$2n_tT_t = f_{mom}n_uT_u, \qquad T_u^{\frac{7}{2}} = T_t^{\frac{7}{2}} + \frac{7}{2}\frac{f_{cond}q_{\|}L}{\kappa_0}, \qquad (1-f_{pow})q_{\|} = \gamma_s n_t T_t c_{st}$$

The Two-Point Model can be solved in terms of the **electron density** and **power entering the SOL** which are controllable with core-plasma actuators (pellet injection and auxiliary power).



† P.C. Stangeby, "The Plasma Boundary of Magnetic Fusion Devices," IoP Publishing, 2000.

POPCON (Plasma OPeration CONtour) Analysis

Generating POPCONs:

First, the Core-SOL-Divertor Model is solved in steady-state.

Then, operational constraints and plasma conditions can be plotted in temperature-density space.

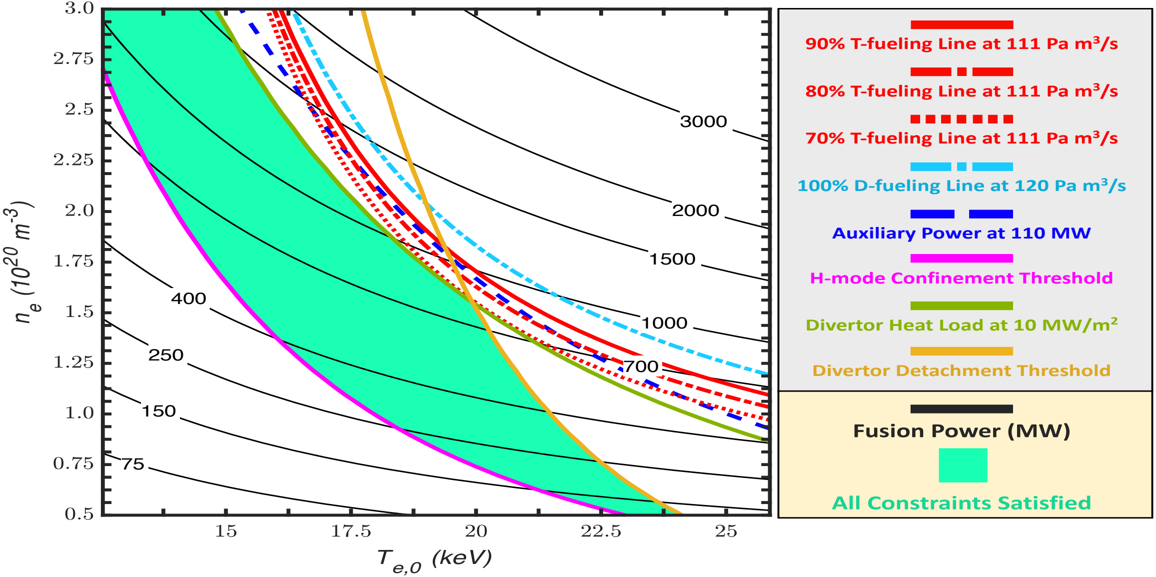
Correction Factors for SOL:

- *Conduction factor* $f_{cond} = 1$ models inclusion of convection.
- *Momentum loss factor* $f_{mom} = 0.4$ models frictional collisions with neutrals.
- *Power loss factor* $f_{pow} = 0.8$ models radiation losses below the X-point.

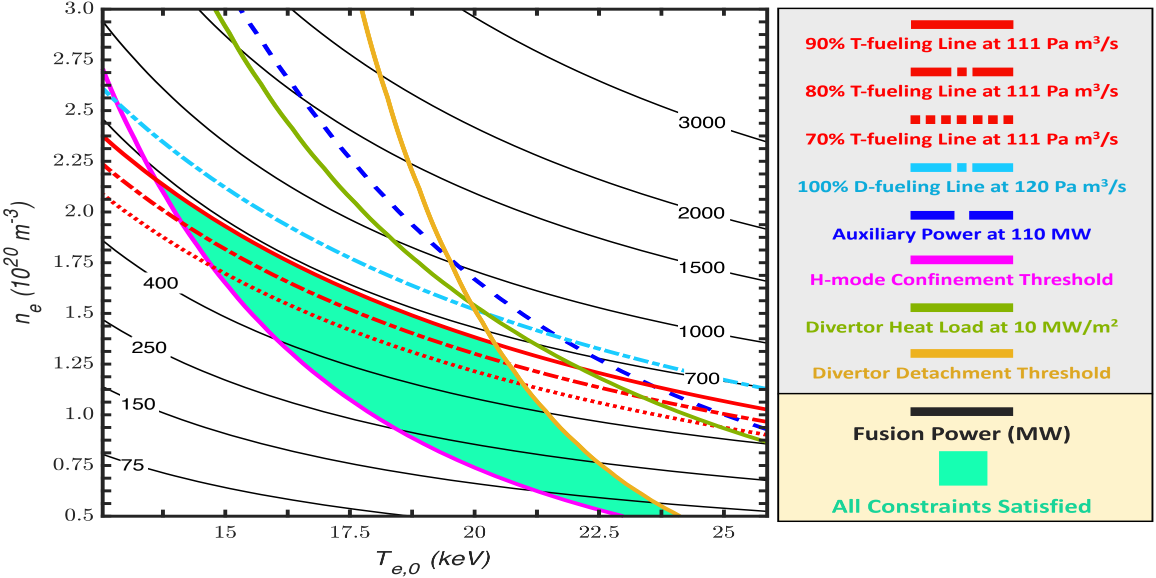
ITER Operational Constraints:

- External Fueling Saturation
  - 100% D Pellet Injector
  - 90%T-10%D Pellet Injector
  - Tritium concentration in fueling lines can fall during long pulses
- Auxiliary Heating Saturation
  - Neutral beam injection
  - Radio frequency heating
- Power Threshold for H-mode
  - Total power must exceed threshold power
- Maximum Divertor Heat Load
  - Maximum: 10 MW/m^2
- Divertor Detachment ($T_t < 7eV$)

POPCON: ITER Plasma With Significant DT Recycling



POPCON: ITER Plasma Without Any DT Recycling



Isotopic Fuel Tailoring as Actuator for Burn Control

Isotopic Fueling Can Play Key Role in Burn-Control Schemes

- **Burn-control solution:** feedback controller, state observer, actuator allocator, reference governor.
  - *Control techniques are available to incorporate nonlinear dynamics into the design $\rightarrow$ Performance $\uparrow$*
  - *Adaptive & robust control techniques are available to deal with model uncertainties $\rightarrow$ Robustness $\uparrow$*
- **Burn control by fueling becomes critical when control by heating becomes ineffective.**
  - Density control by fueling is constrained by efficiency of actuators (gas puffing, pellet injection)
- **Burn control designs can effectively incorporate isotopic fueling as an "actuator."**
- **Control by (isotopic) fueling may lose effectiveness as particle recycling increases.**
- ***Burn controllers can be robustified against tritium-concentration drifts in fueling lines.***
- ***Tritium fraction in fueling lines could impose limits on operational space in low recycling scenarios but drops of tritium fraction below nominal values are expected to be small.***
- **Testing of proposed density$\rightarrow$burn-control algorithms in 1D simulations is needed.**
  - Further work on actuator/diagnostic/transport modeling is needed
  - *Need for multi-zone response model for control synthesis could be determined from simulation results*
- **Testing of proposed density$\rightarrow$burn-control algorithms in present devices is needed.**
  - Emulation of α heating, and even particle recycling, is possible through different mechanisms.
  - Emulation of ITER's actuators and diagnostics is also possible.