Bifurcation-driven vertical plasma displacement

Dmitrii I. Kiramov and Boris N. Breizman

Institute for Fusion Studies, The University of Texas at Austin

Dmitrii.Kiramov@austin.utexas.edu

TEXAS

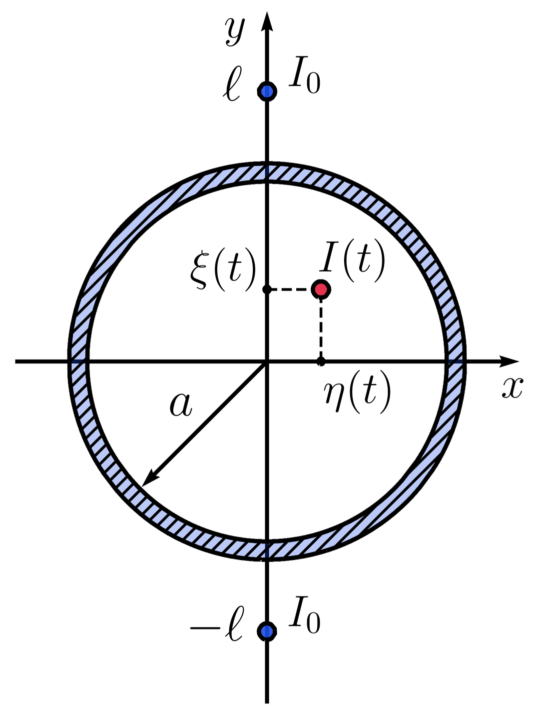
The University of Texas at Austin

1. Motivation

- As commonly observed in experiments, the disruptive plasma tends to move vertically, and the **time scale** of this motion is rather **resistive** than Alfvenic.
- These observations allow one to view the VDE as an **adiabatic equilibrium change** rather than a rapid MHD instability.
- In the case of **cold VDEs** (the wall resistive time is much longer than the plasma resistive time), the wall acts as an ideal conductor, confining the magnetic flux.

2. Main assumptions model

- We consider a plasma column confined within a **perfectly conducting** cylindrical wall.
- We assume that the wall is circular, but an **external magnetic field** breaks the cylindrical symmetry. This field represents the shaping coils in a tokamak.
- We model the plasma as a **rigid wire** that retains its cross-section. This wire can move within the cylindrical chamber in a force-free way.



3. Magnetic field

In the **ideal wall limit**, the magnetic flux outside the vacuum vessel must be stationary:

$$\psi(r>a, t>0) = \psi(r>a, t=0)$$

The magnetic flux inside and outside the vacuum vessel obeys the **time-dependant Poisson equation**:

$$\Delta\psi = \frac{4\pi}{c} j\,(x, y, t)$$

The magnetic flux is continuous:

$$\psi(a-0, \phi, t) = \psi(a+0, \phi, t)$$

The discontinuity of the tangential component of the magnetic field determines the wall surface current $\sigma\,(\phi, t)$:

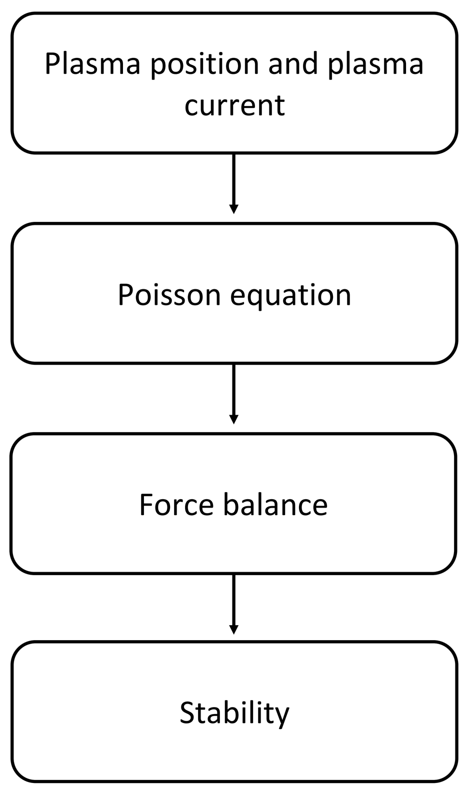
$$\frac{\partial\psi}{\partial r}(a+0, \phi, t) - \frac{\partial\psi}{\partial r}(a-0, \phi, t) = \frac{4\pi}{c}\sigma\,(\phi, t)$$

4. Force-free constraint

We assume that the plasma column stays in a **force-free state** as the evolution occurs on the plasma current decay time scale. This implies that the force $\mathbf{F}$ exerted on the plasma wire vanishes for all $t \geq 0$:

$$\mathbf{F} = \mathbf{F}_{wp} + \mathbf{F}_{ep} = 0$$

where $\mathbf{F}_{ep}$ and $\mathbf{F}_{wp}$ is the force due to the external and wall currents, respectively.



5. Effective potential

In our limiting case of slow motion, the force acting on the plasma wire renders the following expression for the **effective potential**:

$$U_{eff}\,(x, y) = -\frac{I^2}{c^2}\ln\left(a^2 - x^2 - y^2\right) + \frac{II_0}{c^2}\ln\left[\left(x^2 + (\ell - y)^2\right)\left(x^2 + (\ell + y)^2\right)\right]$$

Since the plasma wire is initially stable and located at the origin, the following inequality must be satisfied:

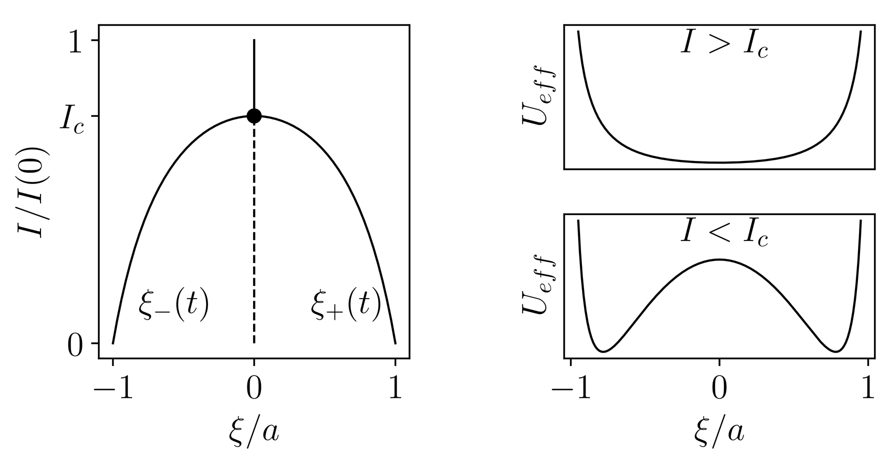
$$I\,(t=0) > 2I_0\frac{a^2}{\ell^2} \equiv I_c$$

The plasma current must stay at the origin while $I(t) < I_c$.

6. Bifurcation

The **initial equilibrium bifurcates** once the plasma current passes the critical value I_c, creating new equilibria. However, the one at the origin is **unstable**. Hence, the **plasma must move to remain force-free and stable simultaneously**.

$$\xi_{\pm} = \pm a\left(1 - I\,(t)\,I_c^{-1}\right)^{\frac{1}{2}}\left(1 - I\,(t)\,I_c^{-1}a^2\ell^{-2}\right)^{-\frac{1}{2}}$$



7. Poloidal magnetic flux

For $I(t) \geq I_c$, **the plasma vertical position is a function of the plasma current**. Because of the **up-down symmetry** of the model, a slight **jitter** at the bifurcation point determines the direction of the plasma drift. Without loss of generality, we can pick one of the branches of $\xi\,(t)$ and find the **total magnetic flux** inside

$$\psi\,(r<a, \phi) = \frac{I_0}{c}\ln\left(\left(r^2+\ell^2\right)^2 - 4r^2\ell^2\sin^2(\phi)\right) + \frac{I\,(t)}{c}\ln\left(\frac{r^2 - 2r\xi_{\pm}\,(t)\sin(\phi) + \xi_{\pm}^2\,(t)}{a^2 - 2r\xi_{\pm}\,(t)\sin(\phi) + \frac{r^2}{a^2}\xi_{\pm}^2\,(t)}\right)$$

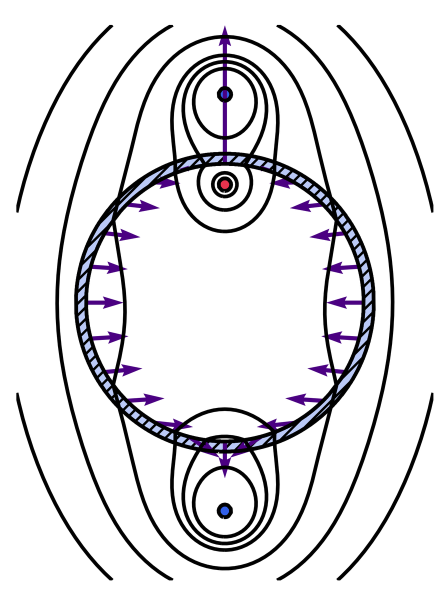
and outside the vacuum vessel

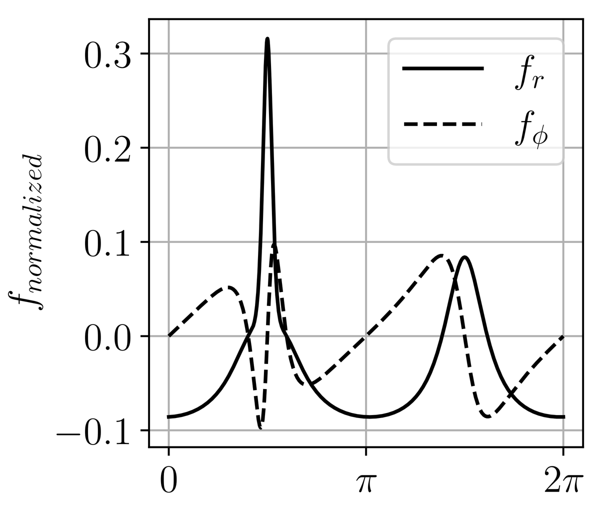
$$\psi\,(r>a, \phi) = \frac{I\,(0)}{c}\ln\left(\frac{r^2}{a^2}\right) + \frac{I_0}{c}\ln\left(\left(r^2+\ell^2\right)^2 - 4r^2\ell^2\sin^2(\phi)\right)$$

8. Wall force

The surface density of the electromagnetic force acting on the vacuum vessel:

$$\mathbf{f}\,(\phi) = -\frac{\mathbf{e}_r}{8\pi}\left[\left(\frac{\partial\psi}{\partial r}\right)^2\right]_{a-0}^{a+0} - \frac{\mathbf{e}_\phi}{4\pi a}\frac{\partial\psi}{\partial\phi}\left[\frac{\partial\psi}{\partial r}\right]_{a-0}^{a+0}$$





9. Conclusions

- In the ideal wall limit, the plasma with a subcritical current must move vertically to remain force-free. We attribute this motion to the asymmetry of the magnetic field created by the external shaping conductors.
- The plasma remains stable en route to the first wall, and the time scale of this motion is roughly the plasma current decay time. This adiabatic adjustment of the equilibrium position is conceptually different from the instability driven picture.
- The initial steady-state equilibrium remains stable if the decaying plasma current is higher than the threshold value. The critical current that triggers the bifurcation is determined by the external conductors or the shape of the wall. It is desirable to optimize the critical current to minimize mechanical and thermal loads on the wall.

*This work was supported by the U.S. Department of Energy Contract Nos. DEFG02-04ER54742 and DESC0016283.