

# ALTERNATING PHASE FOCUSING BEAM DYNAMICS FOR DRIFT TUBE LINACS

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## Abstract

In contrast to conventional E-mode resonance accelerators, H-mode DTLs provide for compact linac sections and have been established as highly efficient resonators during the last decades. Thus, H-mode structures are widely applied for heavy-ion acceleration with medium beam energies because of the outstanding capability to provide for high acceleration gradients with relatively low energy consumption. To build upon those advantages, an Alternating Phase Focusing beam dynamics layout has been applied to provide for a resonance accelerator design without internal lenses, which allows for eased commissioning, routine operation, maintenance, and potential future upgrades. The features of such channel are going to be demonstrated on the example of two Interdigital H-mode cavities, separated by an external quadrupole triplet. This setup provides for heavy ion (mass-to-charge  $\leq 6$ ) acceleration from 300 to 1400 keV/u and is used as injector part of the superconducting continuous wave accelerator HELIAC. Hence, this promising approach generally enables effective and compact routine operation for various applications as super heavy ion research, material science and radiobiological applications as heavy-ion tumor therapy.

## 1. INTRODUCTION

GSI Helmholtzzentrum für Schwerionenforschung (GSI, Germany) is one of the leading institutions in the research field of superheavy elements. Using heavy ion beams, delivered by the Universal Linear Accelerator (UNILAC) (Barth, et al., 2020), six new heavy elements of the Mendeleyev periodic table have been discovered during the past decades, namely element 107 – 112. Recently, the discovery of new superheavies has become more difficult with increased targeted mass of the new elements, as the probability of successful fusion to obtain new elements decreases. The low fusion probability requires weeks and months longer duration of the experiments (Khuyagbaatar, et al., 2020) with use of the same linac. An increased average beam current will resolve this problem. For this purpose, either the current per bunch or the duty cycle of the delivering machine has to be increased. Since a high peak beam current is impractical to be delivered to the beam targets due to possible damage from heat, a high duty cycle is preferable. For continuous wave and high duty cycle applications of heavy ion beams, superconducting machines have been proven to be more economic than their normal conducting counterparts (Podlech, 2013).

Currently, the GSI main linac for heavy ion research UNILAC is upgraded (Barth, et al., 2007; Barth, et al., 2022) for beam delivery to the FAIR SIS100 (Spiller, et al., 2020) synchrotron and its various experimental areas, as APPA (Stöhlker, et al., 2015), CBM (Herrmann, 2022), HADES (Lapidus, Gumberidze, Hennino, Rosier, & Ramstein, 2012), NUSTAR (Nilsson, 2015) and PANDA (Schmidt, 2019) among others. The new requirements to the beam are drastically different from the former demands, as the UNILAC will need to deliver high peak-current beam at a low duty cycle. The new objectives of UNILAC operation are different from the beforementioned requirements for superheavy element research. Therefore, a new linear accelerator has been proposed to provide for energy variable, continuous wave heavy ion beam, dedicated to the discovery of new superheavy elements (Minaev, Ratzinger, Podlech, Busch, & Barth, 2009; Schwarz, et al., 2019; Barth, et al., 2017). The Helmholtz Linear Accelerator is going to deliver 1 mA average beam current of different ions from protons to uranium (see TABLE 1). The high average beam current will improve the timeframe for measurement campaigns.

TABLE 1. Helmholtz Linear Accelerator specifications

Quantity	Value	Unit
Frequency	108.408	MHz
Mass-to-charge ratio	1 to 6	
Repetition rate	100	%
Average beam current	1	mA
Beam energy	3.5 to 7.3	MeV/u
Cryomodules	4	
No. SC cavities	12	

The Helmholtz Linear Accelerator was previously planned as superconducting extension (Schwarz, et al., 2019) to the already existing GSI High Charge State Injector (Hochladungsinjektor, HLI) (Klabunde, 1992). Due to new planning directives in conjunction with the upgrade of the UNILAC, it has been decided to provide for a new dedicated injector, employing the HELIAC as an independent accelerator, nevertheless integrated into GSI complex. Thus, a new design of the dedicated HELIAC injector has to be delivered. Following the bunch formation and pre-acceleration in the RFQ, a normal conduction linac section is going to supply beam to the superconducting main linac part. Two normal conduction Interdigital H-mode (IH) cavities will provide for beam acceleration from 300 to 1400 keV/u beam energy (Lauber & others).

It has been decided to adopt IH cavities, as Crossbar H-mode cavities would be too compact for manufacturing, whereas Alvarez-type DTLs lack of energy efficiency. Three approaches for design of the NC injector linac have been previously investigated: designs with one, two, and three separate DTL cavities for heavy ion acceleration from 300 to 1400 keV/u. The layout with two cavities and an intermediate tank is preferred.

A draft with three separately powered IH cavities was dismissed, as the two intertank sections in between the three cavities would have bloated the accelerator length. Furthermore, the operation of three Radio Frequency (RF) amplifiers and many quadrupole lenses could have aggravated operation of such linac due to a high number of control parameters. The operation of one single IH cavity for the design specifications (see TABLE 1) is generally possible. The already existing HLI injector IH cavity employs a single resonator with

embedded quadrupole lenses and is based on the Combined Zero Degree Structure (Kombinierte Null Grad Struktur, KONUS) beam dynamics concept, offering a space-efficient linac (Ratzinger, Hähnel, Tiede, Kaiser, & Almomani, 2019). However, such compact design results in the structure being sensitive to fluctuations of the control parameters during operation and does not feature the desirable beam diagnostics for eased operation.

Another approach to design an efficient single DTL cavity is the application of Alternating Phase Focusing (APF, see SECTION 1.1) beam dynamics. In this concept, internal magnetic lenses for transverse beam focusing are omitted inside the DTL cavity. Instead, positive synchronous phases are used to provide for the required transverse beam focusing. The mandatory longitudinal focusing is achieved with negative synchronous phases, traditionally used in the layout of linacs. In order to achieve beam focusing in all three room directions, positive and negative synchronous phases are used successively in an alternating sequence.

At the Heavy Ion Medical Accelerator facility (HIMAC, Japan), a single-cavity APF-linac is employed (Iwata, et al., Alternating-phase-focused IH-DTL for an injector of heavy-ion medical accelerators, 2006), partially comparable to our design specifications (cf. TABLE 1): a mass to charge ration  $A/Z = 3$ , an injection energy of 400 keV/u and an output energy of 4 MeV/u of  $^{12}\text{C}^{4+}$  carbon ions. The HIMAC APF structures length is 3.4 m at a resonance frequency of 200 MHz and a duty cycle of 0.4 %.

The HIMAC APF linac proves the advantages of APF beam dynamics: the linac DTL is uncomPLICATE for operation, as the only control parameters are the cavity phase and voltage. Therefore, it is highly suited as a medical injector due to rapid recommissioning periods. In general, it has been reported that operation of APF linacs in general can reduce construction and operation costs by about 30 % (Minaev, Ratzinger, & Schlitt, APF or KONUS drift tube structures for medical synchrotron injectors-a comparison, 1999). In comparison to HIMAC, for HELIAC the transported mass-to-charge ratio is twice as high (Lauber & others). Preliminary investigations of a single-cavity APF acceleration system (see SECTION 3.1) for the HELIAC injector indicated high quality beam transport with up to 90% of the design emittance, but no satisfying solution has been found for the 10 % higher design emittance. Also obtaining high beam quality with such high beam emittance requires strict fabrication tolerances.

Thus, for the HELIAC injector it has been decided to adopt a linac design using two APF cavities (see SECTION 3.2), separated by an intertank equipped with a quadrupole triplet for extra transverse beam focusing. The hybrid approach combines the advantages of highly adjustable quadrupole focusing with the low number of control parameters from the APF-concept and reduced construction costs. The intertank also allows the installation of transport and diagnostic equipment, that could not have been installed in a single-cavity machine. In particular, the additional quadrupole triplet is mandatory to cope with varying beam parameters, which could be employed operation of the ECR ion source with very different ion species, as required for material and superheavy ion research.

### 1.1. Alternating Phase Focusing

The principle of Alternating Phase Focusing has been proposed in 1953 by J. Adlam (Adlam, 1953) and M. Good (Good, 1953) and independently by I. Fainberg in 1956 (Fainberg, 1956). The theoretic background has been developed further in the following years by I. M. Kapchinsky (Kapchinskiy, 1985). But the actual operation of an APF linac has been reported by Y. Iwata et al. (Iwata, et al., Performance of a compact injector for heavy-ion medical accelerators, 2007) in 2007. Apparently, APF was not widely used due to a lack of computer power for design.

APF cavities are highlighted by the absence of magnetic lenses inside the cavity. In order to omit magnetic focusing elements within the cavity, the action of the electric field of the RF gaps is used for beam acceleration and additionally for focusing. But Gauss's law, one of the fundamental Maxwell equations, does not allow simultaneously focusing along all directions in charge free space,  $\nabla \cdot \vec{E} = 0$ . Thus, subsequential longitudinal and transverse electric focusing is necessary to provide for overall beam focusing. Positive and negative synchronous phases (i.e., the RF phase when the accelerated particle beam passes the RF gap) are applied alternately to provide for the transversal and longitudinal focusing. Negative phases are routinely applied for acceleration and longitudinal focusing, whereas positive phases for transverse focusing have found wider application during recent decades, although proposed already in 1953 and refined in following years [1-3]. Since then, computational power has increased by several orders as predicted by Moore's Law (Moore, 1965). Recently, it is possible to provide for a design and detailed analysis of the complex beam transport in Alternating Phase Focusing accelerators.

From a beam dynamics point of view, the core task in APF cavity design is selecting the synchronous phases  $\phi_i$  for each gap to obtain the preferred accelerating/focusing properties. The gradual change from negative to positive synchronous phases is realized by altering the  $\beta\lambda/2$  resonance acceleration geometry of a cavity. The introduced synchronous phase change  $\Delta\phi$  in between two neighboring RF gaps leads to a change of the resonator geometry: the lengths of the tubes inside the DTL cavity are decreased/increased:

$$L_{\text{cell}} = \frac{\beta\lambda}{2} + \beta\lambda \frac{\Delta\phi}{360^\circ} \quad (1)$$

The changed cell length affects the time a particle bunch needs to travel from one RF gap to another. The altered arrival timing of the bunch in the next gap thus leads to a changed synchronous phase.

## 2. METHODS

In general, the energy gain of a particle with charge  $q$  depends on the voltage  $U_0$  in a RF gap, the transit-time factor  $T_{\text{TF}}$  and the synchronous phase  $\phi$  (Reiser, 2008).

$$\Delta W = qU_0 T_{\text{TF}} \cos(\phi) \quad (2)$$

Furthermore, the transverse focusing strength  $k_{x,y}$  depends on the mass  $m_0$ , velocity  $v$ , the Lorentz factor  $\gamma$  and the RF wavelength  $\lambda$  (Reiser, 2008)

$$k_{x,y} = -1 \frac{\pi q U_0 T_{\text{TF}}}{m_0 v^2 \gamma^2 \lambda} \sin(\phi) \quad (3)$$

The longitudinal focusing strength  $k_z$ , is twice as strong (Reiser, 2008):

$$k_z = 2 \frac{\pi q U_0 T_{\text{TF}}}{m_0 c^2 \beta^2 \lambda} \sin(\phi) \quad (4)$$

Thus, the focusing properties in all three room-dimensions  $u \in \{x, y, z\}$  could be calculated by means of a matrix multiplication of the particle coordinates  $x$  in mm and relative velocities  $x'$  in mrad with the transport matrix  $M$  (Reiser, 2008).

$$M_u \vec{x}_u = \begin{pmatrix} 1 & 0 \\ k_u/(\beta\gamma)_f & (\beta\gamma)_i/(\beta\gamma)_f \end{pmatrix} \cdot \begin{pmatrix} x_u \\ x'_u \end{pmatrix} \quad (5)$$

For accurate calculation of the beam transport, the volumetric transit-time factor could be used, considering the radial position of the particle  $r$ , the aperture radius  $a$ , and the gap length  $g$  (Reiser, 2008).

$$T_{\text{TF}}(r) = I_0(Kr) \frac{J_0(2\pi a/\lambda) \sin(\pi g/(\beta\lambda))}{I_0(Ka) \pi g/(\beta\lambda)} \quad (6)$$

The constant  $K$  scales reciprocal with the particle velocity  $K = 2\pi/(\gamma\beta\lambda)$ . The Bessel and modified Bessel functions are denoted as  $I_0(x)$  and  $J_0(x)$ .

EQ. (5) is routinely used for efficient calculations of beam dynamics transport because of its vectorized format, by assigning the average phase  $\phi_{\text{ref}}$  and a common transit-time factor to all particles. But the mathematical averaging to achieve maximum software performance is not expedient for calculation of the beam dynamics in an APF channel. To cover the features of the overall non-linear beam transport, the tracking must be accurately conducted for each individual particle to account for the coupling of particle phase to transverse focusing. Either, above equations are implemented for tracking of individual particles separately, or already existing modern particle tracking software could be employed.

Nevertheless, the particle tracking from one RF gap to the next could be implemented efficiently by using the drift matrix  $D$  and the cell length according to EQ. (1) (Reiser, 2008).

$$D_u \vec{x}_u = \begin{pmatrix} 1 & L_{\text{cell}}/K_u \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_u \\ x'_u \end{pmatrix} \quad (7)$$

The constant  $K$  equals to  $K_{x,y} = 1$  transversely and to  $K_z = \gamma^2$  longitudinally.

The transport through the APF linac is calculated iteratively by updating the particle coordinates  $x_{u,i}$  and relative velocities  $x'_{u,i}$ , as well as the beam energy  $E_{\text{kin},i}$  for each gap  $i$  directly by applying EQ. (2), (5) and (7).

Also, an input particle distribution must be selected as a starting point of the particle tracking. To analyze the beam dynamics with the lowest number of particles, it is proposed to solely cover the border of the 6D phase space, whilst the inner positions within the hypersphere could be transported with even higher beam quality. To obtain a 6D hypersphere, a multivariate normal distribution must be rescaled according to the 6D Twiss equation (Shor, Feinberg, Halfon, & Berkovits, 2004)

$$\frac{\widehat{\gamma}_x x^2 + 2\widehat{\alpha}_x x x' + \widehat{\beta}_x x'^2}{\widehat{\epsilon}_x} + \frac{\widehat{\gamma}_y y^2 + 2\widehat{\alpha}_y y y' + \widehat{\beta}_y y'^2}{\widehat{\epsilon}_y} + \frac{\widehat{\gamma}_z z^2 + 2\widehat{\alpha}_z z z' + \widehat{\beta}_z z'^2}{\widehat{\epsilon}_z} = 1, \quad (8)$$

using the Twiss parameters  $\widehat{\alpha}_u$ ,  $\widehat{\beta}_u$ ,  $\widehat{\gamma}_u$  and  $\widehat{\epsilon}_u$ . The presented distribution has a total- to RMS-emittance ratio of 6 and therefore corresponds to a Waterbag distribution.

The beam focusing and acceleration within the cavity should be designed to obtain maximum acceleration efficiency with minimum emittance growth. To obtain an appropriate solution, the input Twiss parameters  $\widehat{\alpha}_u$ ,  $\widehat{\beta}_u$ ,  $\widehat{\gamma}_u$  and  $\widehat{\epsilon}_u$ , as well as the synchronous phase  $\phi_i$  in each gap must be selected correspondingly. For identification of the optimum variables, many different numeric global and local optimization strategies are available (Virtanen, et al., 2020). A key aspect for optimization of the variables is the adoption of an objective function, translating the designer's requirements to a formal measure. The implemented objective function is detailed in (Lauber & others) and targets minimum emittance growth  $\xi_u$ , as well as a high output energy  $W_{\text{out}}$ .

$$f = \left( \frac{\xi_{x,y} - 1}{t_{x,y}} \right)^2 + \left( \frac{\xi_z - 1}{t_z} \right)^2 + \frac{W_{\text{target}} - W_{\text{out}}}{t_E} \quad (9)$$

The terms of the objective functions are designed to yield a value in between 0 and 1 if the variables are below their corresponding target tolerance  $t$ , otherwise the result is a value greater 1. The target energy is intentionally left without exponent to allow for an even higher output energy than targeted, provided that the emittance growth does not increase dramatically.

The variables of the optimization, i.e., the input Twiss parameters  $\widehat{\alpha}_u$ ,  $\widehat{\beta}_u$ ,  $\widehat{\gamma}_u$  and  $\widehat{\epsilon}_u$ , and the synchronous phase  $\phi_i$  in each gap, are constrained. Extreme combinations of Twiss parameters are not desired, as the actual transport systems might not be able to deliver them. The phases in all gaps are at least constrained by the physical length of the drift tubes, as too short cells from rapid changes in the synchronous phase could cause too narrow cell lengths. From an RF point of view, also too long tubes could be impractical due to heat overload. Therefore, in addition to a  $+90^\circ$  to  $-90^\circ$  boundary of the synchronous phases, multiple constraints must be considered in advance during optimization.

### 3. RESULTS

In this section, the results of a preliminary single cavity design are discussed (SEC. 3.1), as well as the final design of the HELIAC APF injector linac with two separate cavities (SEC. 3.2). Both designs were obtained using a specifically previously developed optimization framework, wrapping the multi-particle tracking code DYNAMION. In order to put the results in perspective, TABLE 2 outlines the results of the presented and similar APF channels from other authors.

#### 3.1. Single Cavity with Alternating Phase Focusing

A feasibility study was conducted in order to determine, if a single cavity with APF beam dynamics could be realized with the required input emittance. The cavity was optimized for beam transport with a transverse normalized input emittance of 0.8 mm mrad and 64 deg keV/u longitudinally (foreseen by to be delivered by the

preceding RFQ), and a field gradient of 3 MV/m for acceleration from 300 keV/u to 1400 keV/u. The transverse envelopes are limited by the aperture of 10 mm.

The resulting trajectories after optimization are depicted in FIG. 1. No particle loss occurs within the 3.4 m long structure, whereas the 100%-transverse envelope is very close to the aperture and the 90 % envelope size is smaller than half of the aperture along the DTL cavity. The longitudinal 100%-envelope is asymmetric, whereas the 90 % envelope is almost symmetrical. This observation is also reflected by the emittance growth metric. The 90 %-effective emittance growth is about 25 % transversely and only 4 % longitudinally. Those figures of merit are superb, but the total emittance growth and consequently the beam size and potential losses render this result unpreferable for application in a continuous wave linear accelerator, as even few percent particle loss along decades of actual operation could impose degradation of the machine performance.

The model of this single cavity with Alternating Phase Focusing was obtained in about a week of work and does not reflect a final optimum solution, but rather an intermediate one, as it was decided early to design an APF channel using two separate cavities to allow a highly flexible robust routine operation, also necessary to compensate varying beam conditions from operation of the ion source with very different ion species

TABLE 2. Overview on APF linacs worldwide

	HIMAC Medical Synchrotron injector [24]	Compact IH [29]	HELIAC injector - two cavity design [16]	J-PARC Muon Linac [30]	HELIAC injector – one cavity	108MHz Medical Synchrotron injector [19]
Built/Operated	yes	yes	planned	planned	no	no
Year	2007	2000	-	-	-	-
A/Z	3	1	6	0.1	6	3
Input energy (keV/u)	608	40	300	3000	300	300
Output energy (keV/u)	4000	2000	1400	40000	1400	7000
Max gap voltage (kV)	350 [2]	180	260	NA	180	450
Length (m)	3.4374	1.5	4.5 (3) <sup>1</sup>	1.3	3.38	4.3
Frequency (MHz)	200	100	108.4	324	108.4	216.8
Duty factor (%)	0.4	NA	100	0.1 [31]	NA	NA
Number of cells	72	22	56	16	60	78
Aperture (mm)	7	NA (6-8)	9	NA	10	12 and 16
Kilpatrick	1.6	NA	2.5	1.8	NA	NA
Transmission (%)	99.6	80	100	98	100	100
Energy gain (MeV)	10.176	1.96	6.6	3.7	6.6	20.1
Effective gradient (MV/m)	3	1.3	1.5	2.8	2	4.7
Transv. 90% input emit. (norm. mm mrad)	0.68 [18]	NA (1 acceptance)	0.4	NA (0.3 RMS)	0.4	0.32
Transv. emit. growth (%)	0.68 [18]	NA	5	NA (6 RMS)	25	70
Long. 90% input emit. (ns keV/u)	0.68 [18]	NA (0.077 acceptance)	1.64	NA (1.23 RMS)	1.64	0.88
Long. emit. growth (%)	0.68 [18]	NA	3	NA (24 RMS)	4	11

<sup>1</sup> 3 m without the intertank (containing a quadrupole triplet)

### 3.2. Two Separate Cavities with Alternating Phase Focusing

The second variant to design an APF channel for continuous wave application was realized by employing two separate IH cavities (Cavity-1 & 2) with a quadrupole triplet in between them (Intertank). A detailed report on the design of the channel is published in (Lauber & others). As already mentioned, the triplet is used to compensate beam parameters different from the reference design. Also beam diagnostics as phase probe sensors and beam position monitors will be installed to the channel in the intertank region. The additional beam diagnostics will provide for easy commissioning, as well as routine operation.

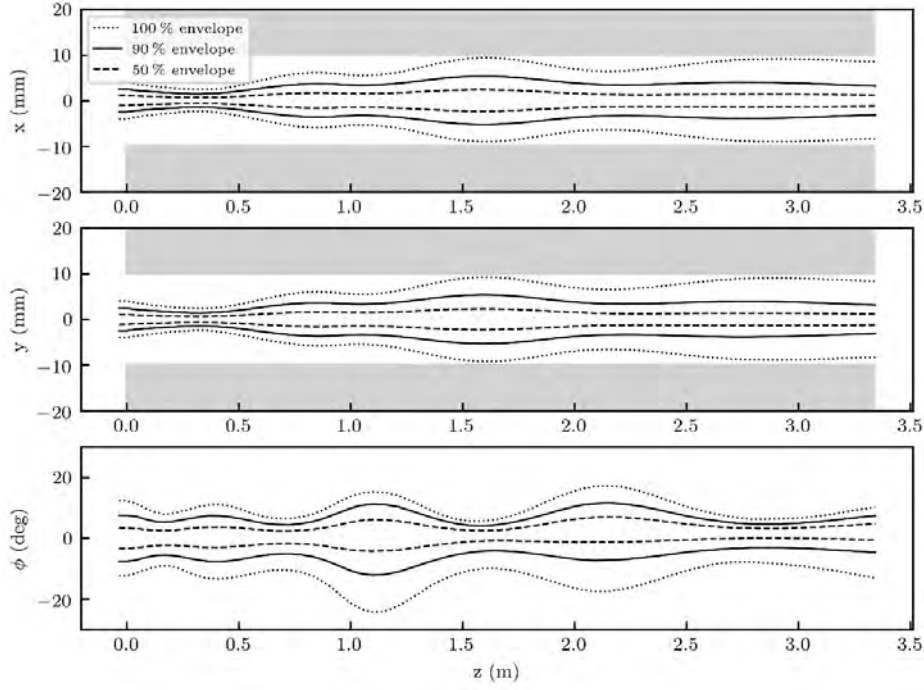


FIG. 1. Beam dynamics draft design of an Alternating Phase Focusing single cavity for acceleration from 300 to 1400 keV/u. An aperture of 10 mm is employed.

The channel was designed employing a field gradient of 3 MV/m and an aperture radius of 9 mm for acceleration of a beam with a transverse normalized emittance of 0.8 mm mrad and 64 deg keV/u longitudinally (the same as for option 1).

For integration into the HELIAC injector, the corresponding beam transport lines have been designed as well and are (additionally to the two designed cavities) depicted in FIG. 2. The Medium Energy Beam Transport system (until 1.9 m), equipped with a quadrupole duplet and a triplet, as well as a rebuncher for longitudinal beam matching to the acceptance of the first cavity. Cavity-1 (1.7 m to 3 m) accelerates the beam to an intermediate energy of 700 keV/u. The quadrupole triplet in the intertank (3 m to 4.5 m) refocuses the transversely divergent beam. A rebuncher is not installed to the intertank, as Cavity-1 provides for dedicated output parameters to match the beam longitudinally to Cavity-2. The beam is accelerated to the final energy of 1400 keV/u along Cavity-2 (4.2 m to 6.1 m). Due to the preceding transverse beam focusing, the synchronous phase pattern in Cavity-2 is oriented rather to beam acceleration than to transverse focusing. The final matching section ( $> 6.1$  m) is equipped with two quadrupole doublets and two rebuncher cavities for full 6D matching of the beam to the acceptance of the superconducting HELIAC. The 90 %-effective emittance growth of the channel is about 5 % transversely and 3 % longitudinally, and thus suited to supply high quality beam to the superconducting HELIAC and subsequent experiments.

#### 4. CONCLUSION

Alternating Phase Focusing cavities are a highly attractive option to extend the length of DTL cavities without employing embedded magnetic lenses within the cavity whilst retaining compactness and effective acceleration. The thereby reduced number of control parameters eases fabrication and facilitates rapid commissioning and stable operation. Two implementations of the APF beam dynamics scheme for  $\beta\lambda/2$  drift tube linacs have been elaborated for acceleration of heavy ions from 300 to 1400 keV/u: a single DTL without any magnetic lens, and a channel with two APF cavities separated by an intertank, equipped with an external quadrupole triplet. The first option provides for a 90 %-effective emittance growth of about only 4 % longitudinally, but 25 % transversely. Potential losses due to the total beam size make this option unsuitable for continuous wave operation with 1 mA beam current. This preliminary design could be improved and therefore could be of high interest for applications with a decreased average beam current due to the compactness,

effectiveness and low number of control parameters, i.e., tank phase and voltage. The well-developed second option, with two separate APF cavities and external quadrupole focusing, provides for high beam quality (90 %-emittance growth of about 5 % transversely, 3 % longitudinally) and is adopted as main linac part for the injector of the superconducting Helmholtz Linear Accelerator (HELIAC).

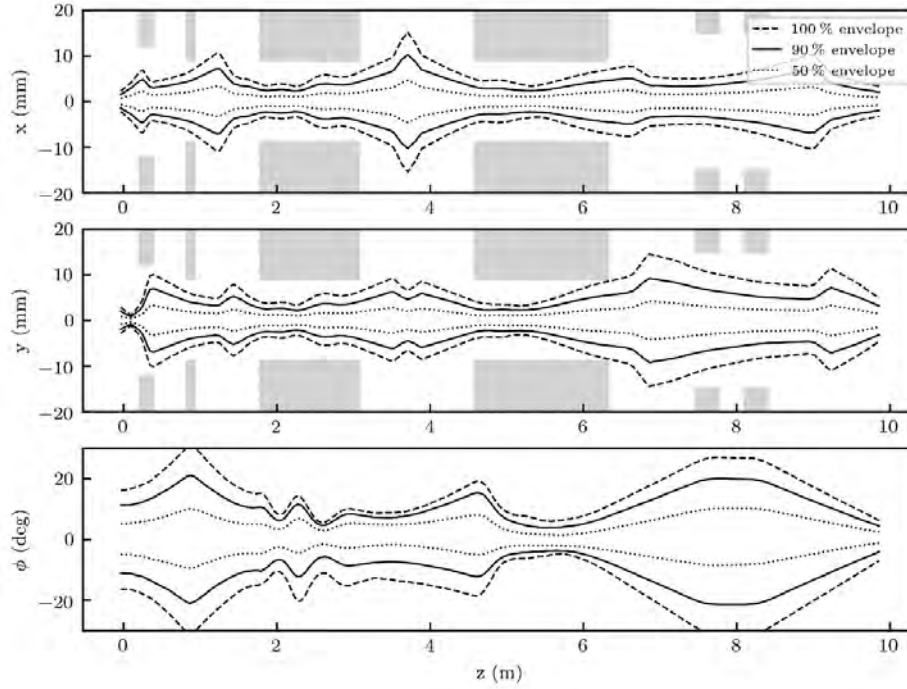


FIG. 2. Design beam envelopes along HELIAC injector linac from RFQ output to SC HELIAC input, employing two APF cavities. The gray blocks indicate the apertures of the quadrupoles, IH cavities and rebunchers.

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