4th IAEA Technical Meeting on Fusion Data Processing , Validation and Analysis

An advanced plasma current tomography based on Bayesian inference and neural networks

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CONTENTS

PART 01

Background

Background

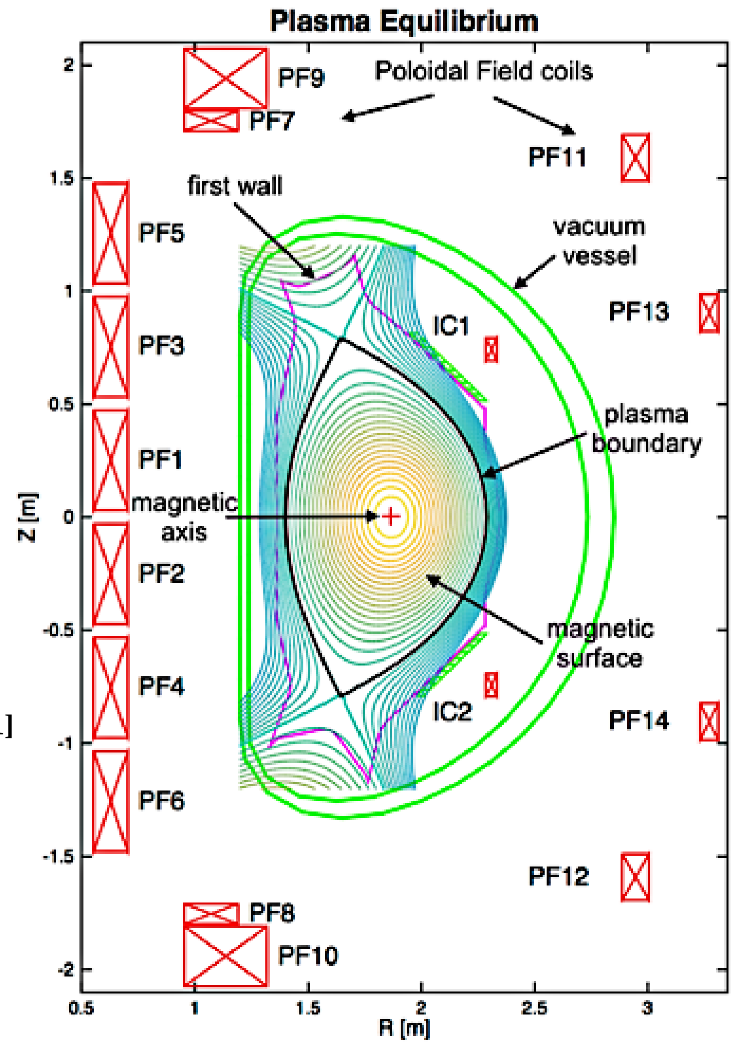
Advantages:

- The Bayesian inference takes the prior information $\boldsymbol{p}(\bar{\boldsymbol{I}})$, the likelihood probability $\boldsymbol{p}(\bar{\boldsymbol{D}}^{\boldsymbol{Mag}}|\,\bar{\boldsymbol{I}})$ into consideration to give the current distribution in a probabilistic manner.
- The error is visually represented by a given uncertainty
- Other diagnostics were easily integrated by joint probabilities

Previous works :

plasma current tomography based on CAR(conditional autoregressive) prior [1]

- The model was severely affected by the diagnostics (damaged)
- The current at the core is always under estimated



[1] Z. J. Liu and et al. Plasma current profile reconstruction for east based on Bayesian inference. Fusion Engineering and Design

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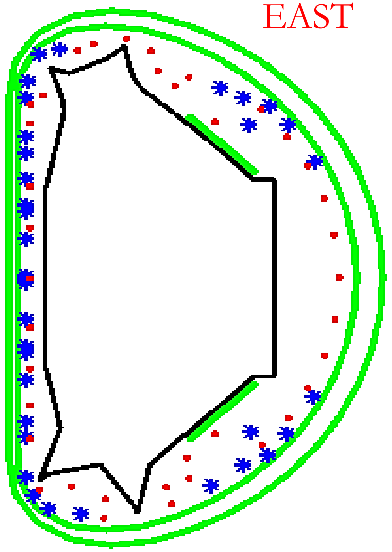
PART 02

Plasma current tomography

Plasma current tomography

$$p(B|A) = p(B)\frac{p(A|B)}{p(A)} \propto p(B)\cdot p(A|B)$$

$$p(\bar{I}|\bar{D}^{Mag}) \propto p(\bar{I})\cdot p(\bar{D}^{Mag}|\bar{I})$$



Likelihood $p(\bar{D}^{Mag}|\bar{I})$

Assume: The fitting error satisfies with the Gaussian distribution

Biot-Savart Law : $\vec{B} = \int d\vec{B} = I\oint \frac{\mu_0}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3} = GI$

$$\bar{D}^{Mag} = \bar{\bar{G}}\bar{I} + \bar{C}$$

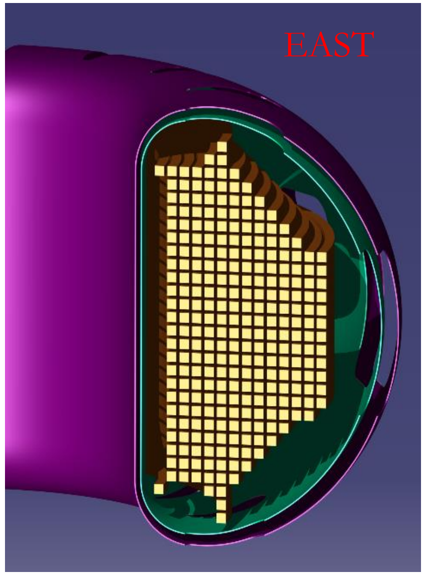
The contribution of the PF coil current to the diagnostic signal

Magnetic diagnostics on EAST

- Pickup coils: 38(red dot)
- Magnetic flux loops: 35 (blue asterisk)
- Rogowski loop: 1

Covariance matrix, determined by the diagnostics

$$p(\bar{D}^{Mag}|\bar{I}) = \frac{1}{(2\pi)^{N_D/2}|\bar{\bar{\Sigma}}_D|^{1/2}} exp\left(-\frac{1}{2}(\bar{\bar{G}}\bar{I} + \bar{C} - \bar{D}^{Mag})^T \bar{\bar{\Sigma}}_D^{-1}(\bar{\bar{G}}\bar{I} + \bar{C} - \bar{D}^{Mag})\right)$$



The yellow grids contain the plasma

ASE (Advanced squared exponential) prior

$p(\bar{I})$

$$\mathcal{K}_{SE}(\bar{x}_i,\bar{x}_j) = \sigma^2 exp(-\frac{(\bar{x}_i-\bar{x}_j)^2}{2\ell^2})$$

$$\bar{\bar{\Sigma}}_I = \begin{pmatrix} \mathcal{K}(\bar{x}_1,\bar{x}_1) & \cdots & \mathcal{K}(\bar{x}_1,\bar{x}_n) \\ \vdots & \ddots & \vdots \\ \mathcal{K}(\bar{x}_n,\bar{x}_1) & \cdots & \mathcal{K}(\bar{x}_n,\bar{x}_n) \end{pmatrix}$$

Construcr σ as a proportional function of current

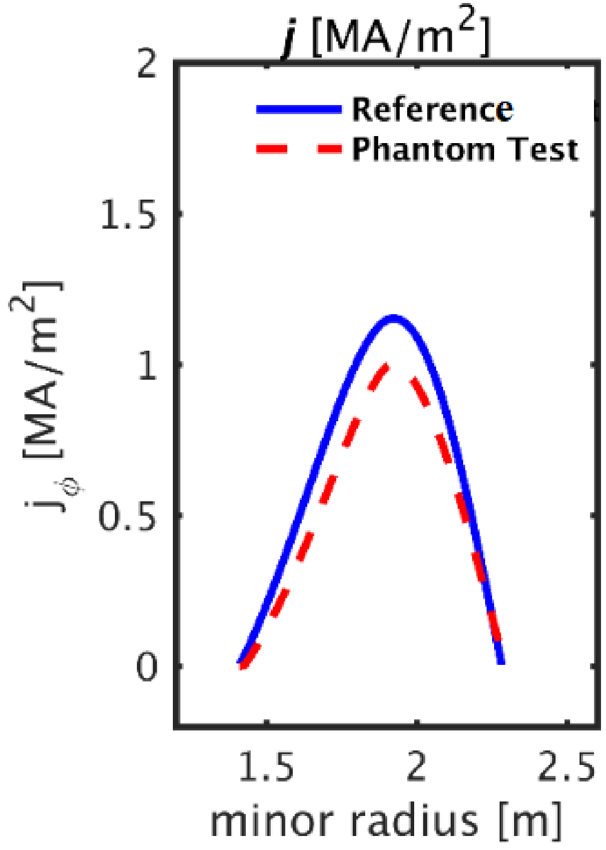
(The current is from reference discharge)

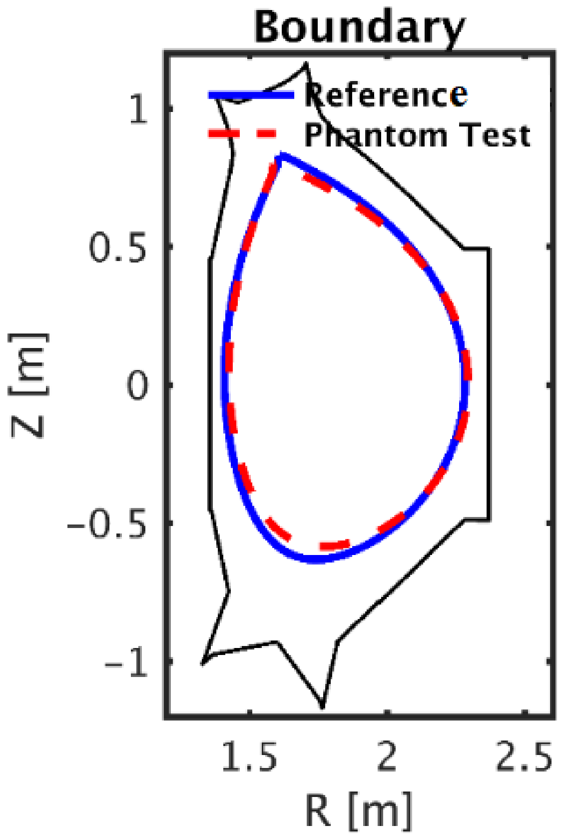
If (i = j) $\sigma = k * \bar{I'}_i$

Else $\sigma = k * \sqrt{\bar{I'}_i * \bar{I'}_j}$

Where k is a constant

$$P(\bar{I}) = \frac{1}{(2\pi)^{N_I/2}|\bar{\bar{\Sigma}}_I|^{1/2}} exp(-\frac{1}{2}\bar{I}^T\bar{\bar{\Sigma}}_I^{-1}\bar{I})$$





Posterior

$$p(\bar{I}|\bar{D}^{Mag})$$

- **Prior probability:**

$$p(\bar{I}) = \frac{1}{(2\pi)^{N_I/2}|\bar{\bar{\Sigma}}_I|^{1/2}} exp[-\frac{1}{2}(\bar{I})^T\bar{\bar{\Sigma}}_I^{-1}(\bar{I})]$$

- **Likelihood probability:**

$$p(\bar{D}^{Mag}|\bar{I}) = \frac{1}{(2\pi)^{N_D/2}|\bar{\bar{\Sigma}}_D|^{1/2}} exp[-\frac{1}{2}(\bar{\bar{G}}\bar{I} + \bar{C} - \bar{D}^{Mag})^T\bar{\bar{\Sigma}}_D^{-1}(\bar{\bar{G}}\bar{I} + \bar{C} - \bar{D}^{Mag})]$$

Posterior probability: $p(\bar{I}|\bar{D}^{Mag}) = \frac{1}{(2\pi)^{N_I/2}|\bar{\bar{\Sigma}}|^{1/2}} exp[-\frac{1}{2}(\bar{I} - \bar{m})^T\bar{\bar{\Sigma}}^{-1}(\bar{I} - \bar{m})]$

Covariance matrix: $\bar{\bar{\Sigma}} = (\bar{\bar{G}}^T\bar{\bar{\Sigma}}_D^{-1}\bar{\bar{G}} + \bar{\bar{\Sigma}}_I^{-1})^{-1}$

Mean: $\bar{m} = (\bar{\bar{G}}^T\bar{\bar{\Sigma}}_D^{-1}\bar{\bar{G}} + \bar{\bar{\Sigma}}_I^{-1})^{-1}\bar{\bar{G}}^T\bar{\bar{\Sigma}}_D^{-1}(\bar{D}^{Mag} - \bar{C} - \bar{\bar{G}}\bar{m}_I)$

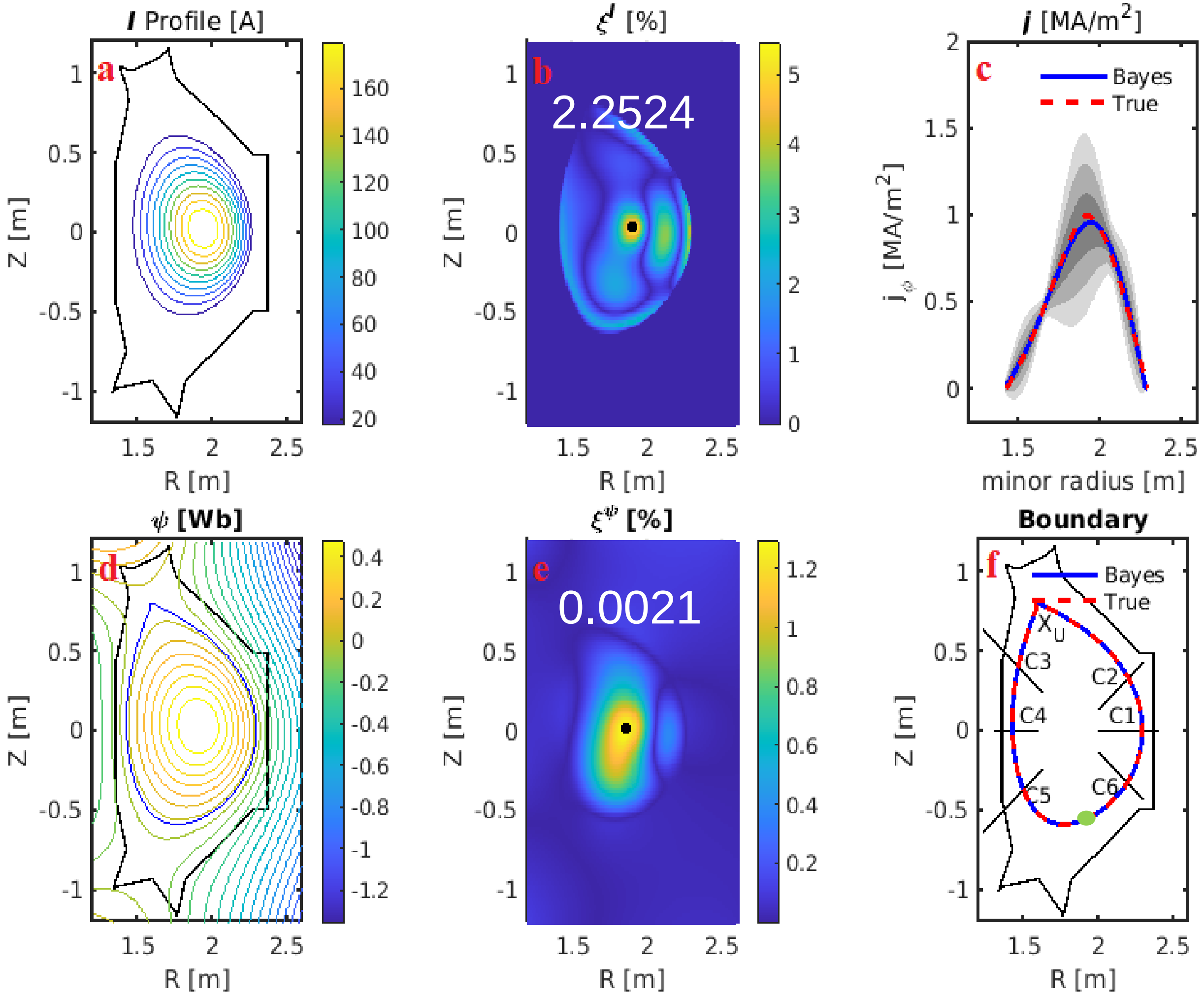
Result from simulation

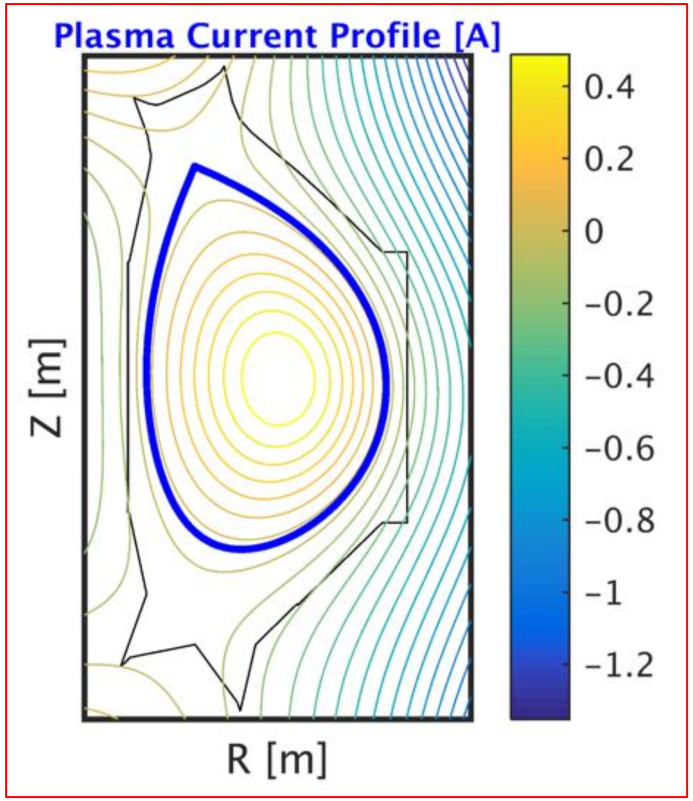
Relative error :

$$\xi_i = \frac{|\bar{I}_i^{rec} - \bar{I}_i|}{max\{\bar{I}\}}$$

Rooat-mean-square deviation :

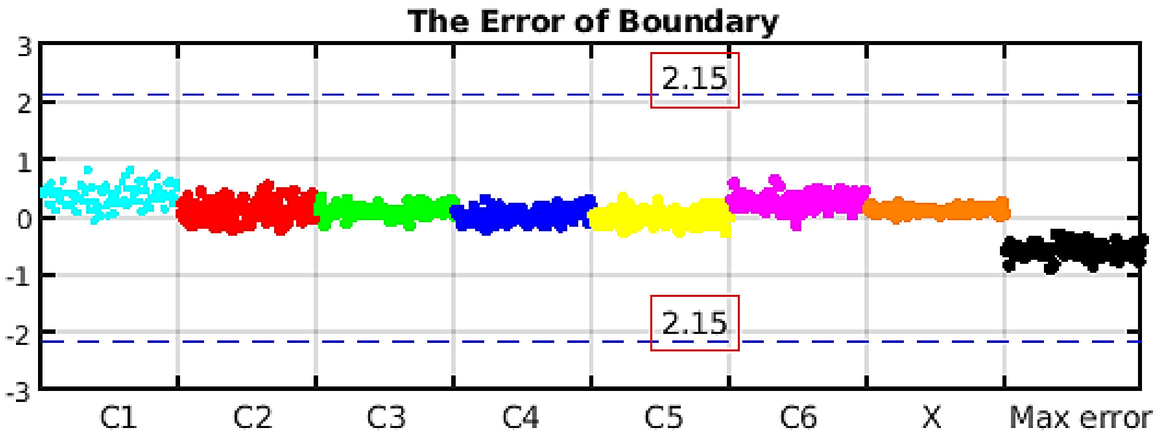
$$RMSD = \sqrt{\frac{\sum_{i=1}^{n}(\bar{I}_i^{rec} - \bar{I}_i)^2}{n}}$$

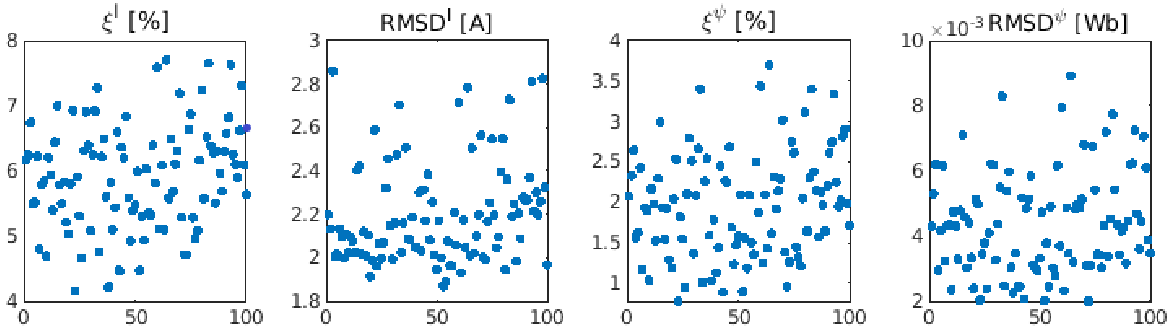




Robustness

Add 3% noise





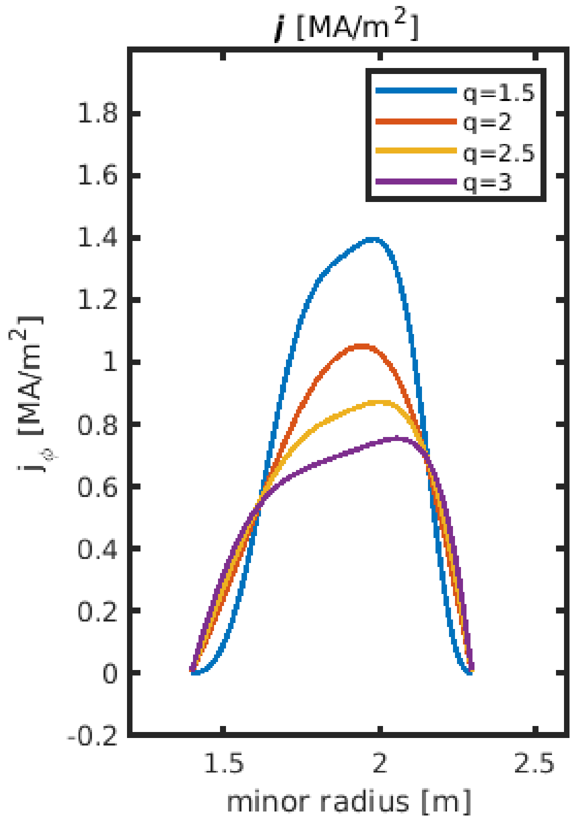
Different configration

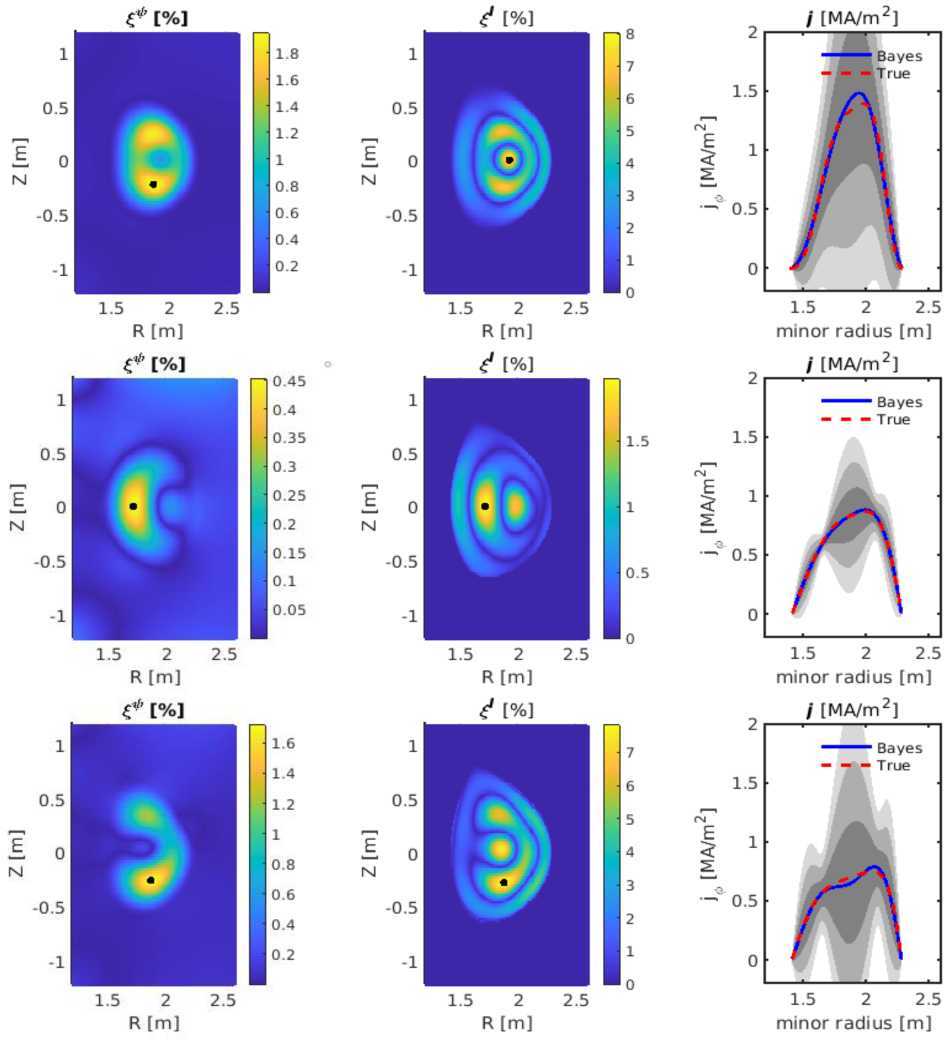
Configuration	USN	LSN	DN
C1 (cm)	0.0545	0.0476	0.0850
C2 (cm)	0.0953	0.0093	0.0372
C3 (cm)	0.1100	0.0474	0.1600
C4 (cm)	0.0107	0.0652	0.0820
C5 (cm)	0.0535	0.0305	0.0539
C6 (cm)	0.0161	0.0589	0.0561
X_U(cm)	0.0668		0.0305
X_L(cm)		0.0289	0.1580
Max Error(cm)	0.4800	0.1700	0.7800
Max ξ Current (%)	5.47%	3.34%	5.95%
RSMD Current (A)	2.2524	1.1884	1.7880
Max ξ Flux (%)	1.30%	0.97%	0.86%
RSMD Flux (Wb)	0.0021	0.0017	0.0018

Robustness

Reference discharge

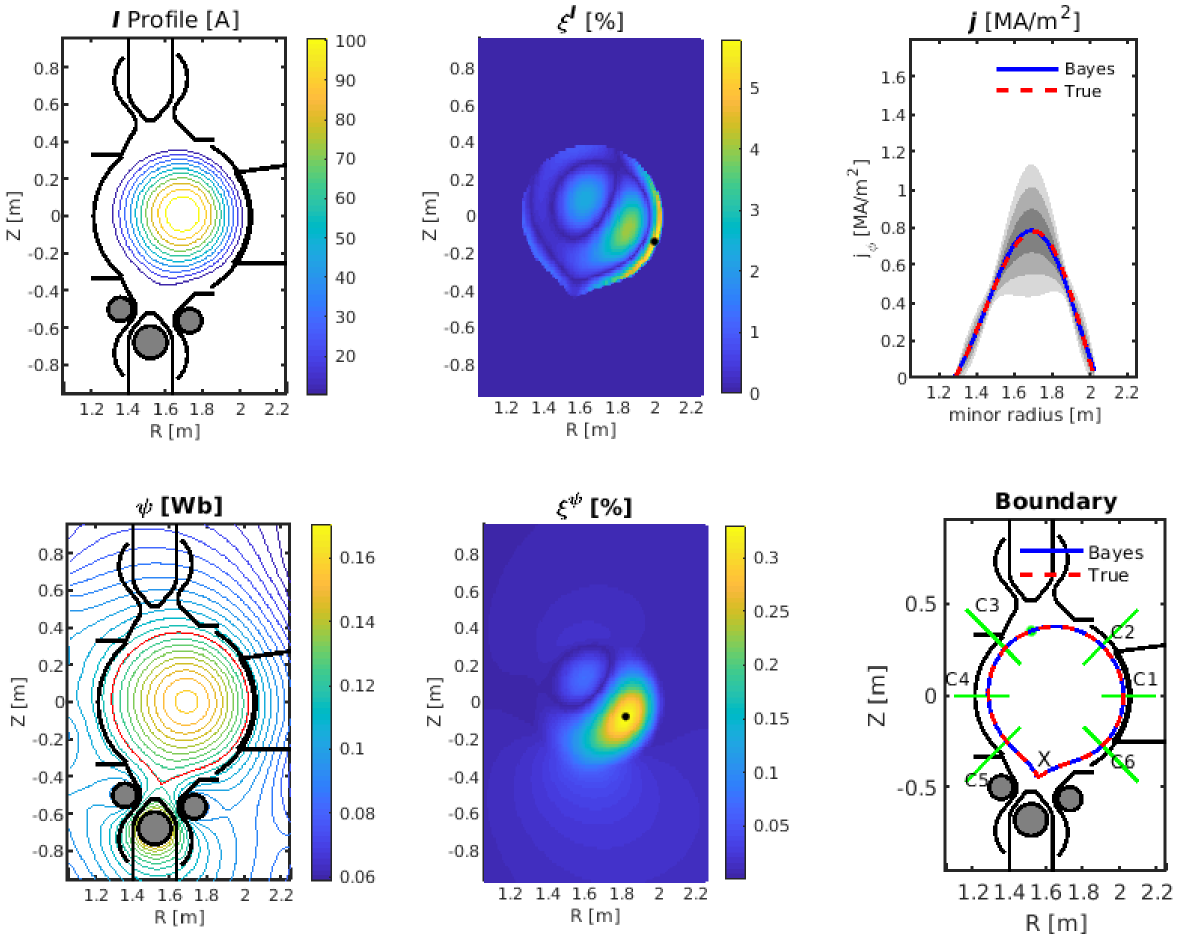
- The different current distributions were obtained by changing the q.
- Select q = 2 as reference discharge and reconstruct plasma current at q = 1.5, 2.5 and 3

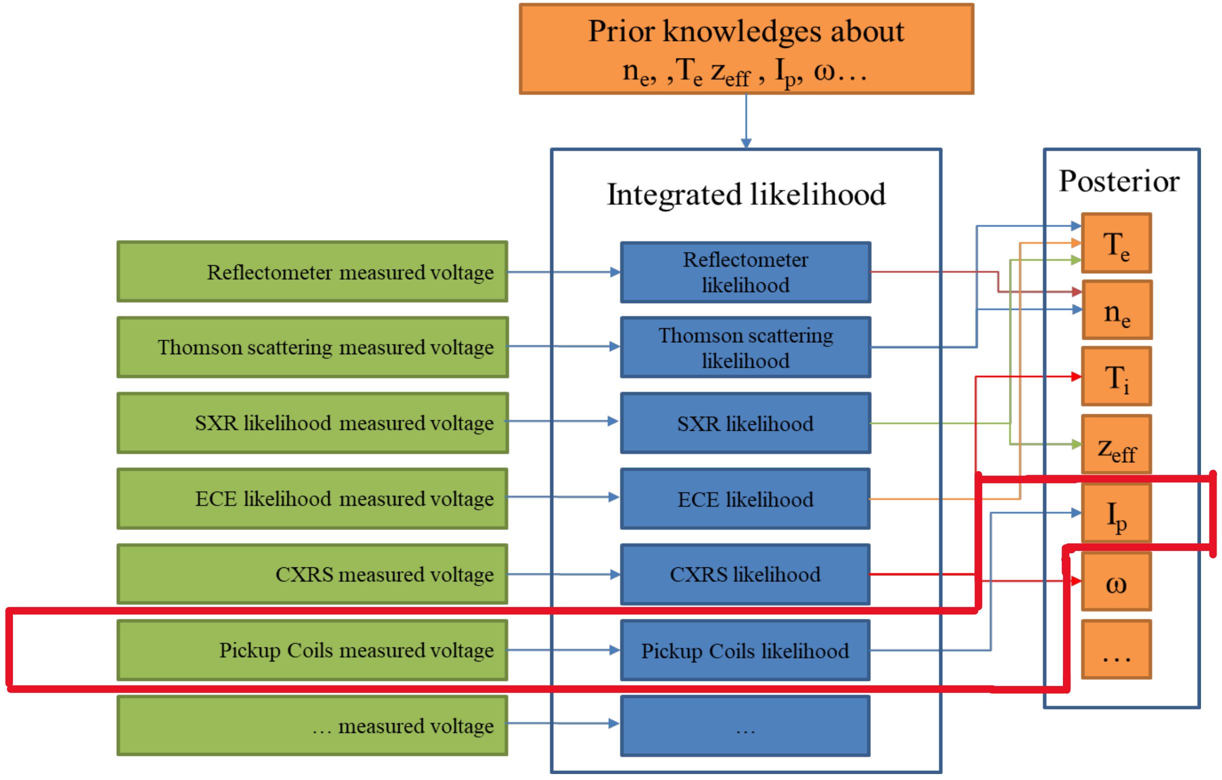




Robustness

Different devices HL-2A





RETINA, schematic diagram of the integrated data analysis platform on HL-2A

Neural network gives the reference discharge

Train Data

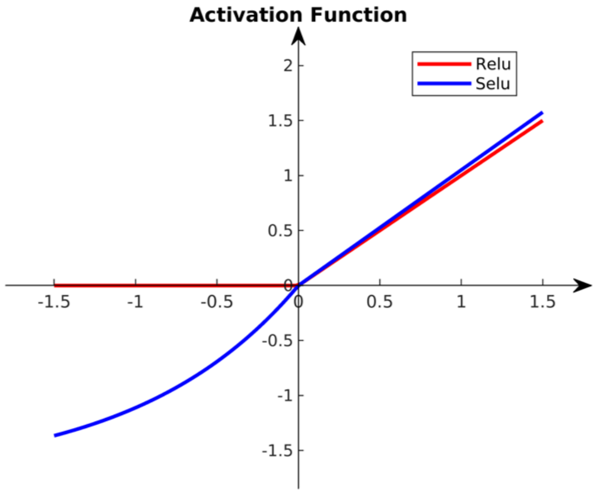
Table 4. Summary of the data samples to train and validate the networks

Parameter	*Definition*	*Datasize*	*No.of samples*
	Input		
I_{PF}	PF current (Rogowski loops)	14	
I_{Ip}	total plasma current (Rogowski loop)	1	276689
Ψ_{FL}	Poloidal magnetic flux (flux loops)	35	
	Output		
I	The current in different position	506	

Activation function

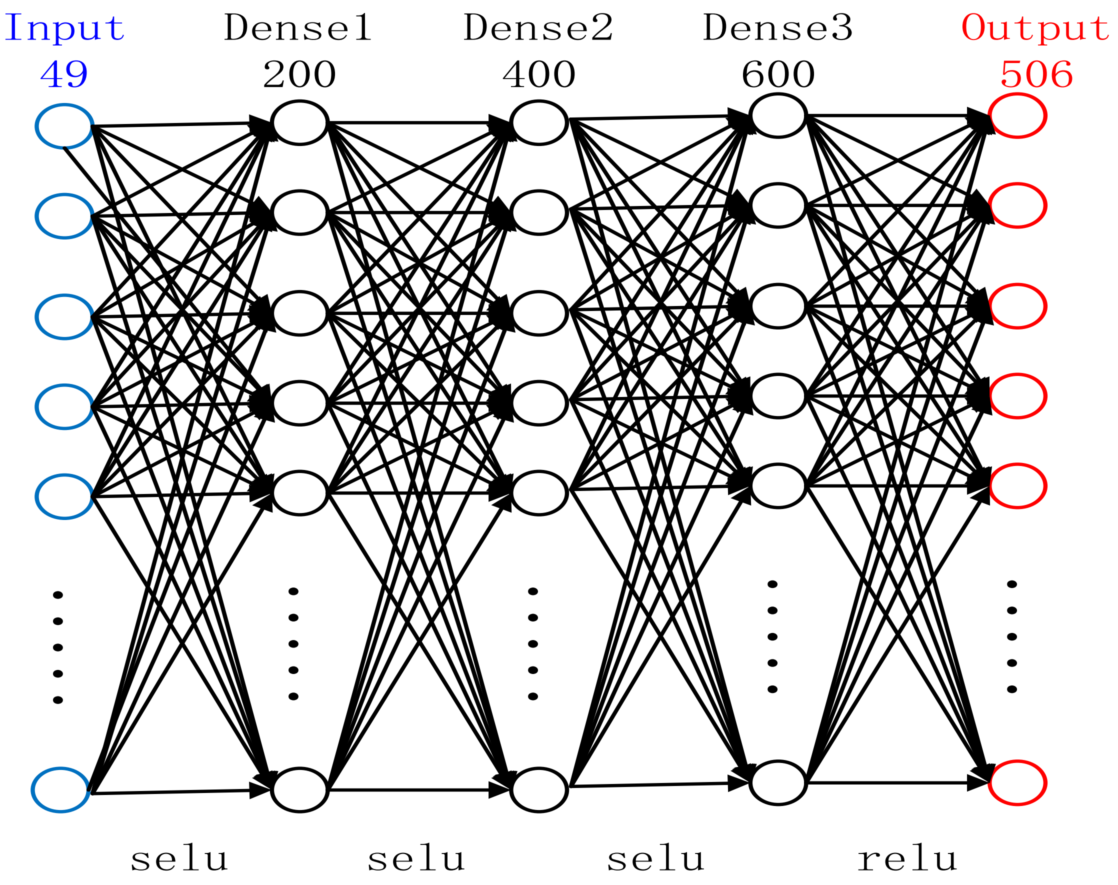
$$Selu = \lambda \begin{cases} x & if\ x > 0 \\ \alpha e^{x} - \alpha & if\ x \leq 0 \end{cases}$$

$$Relu = \max(0, x)$$



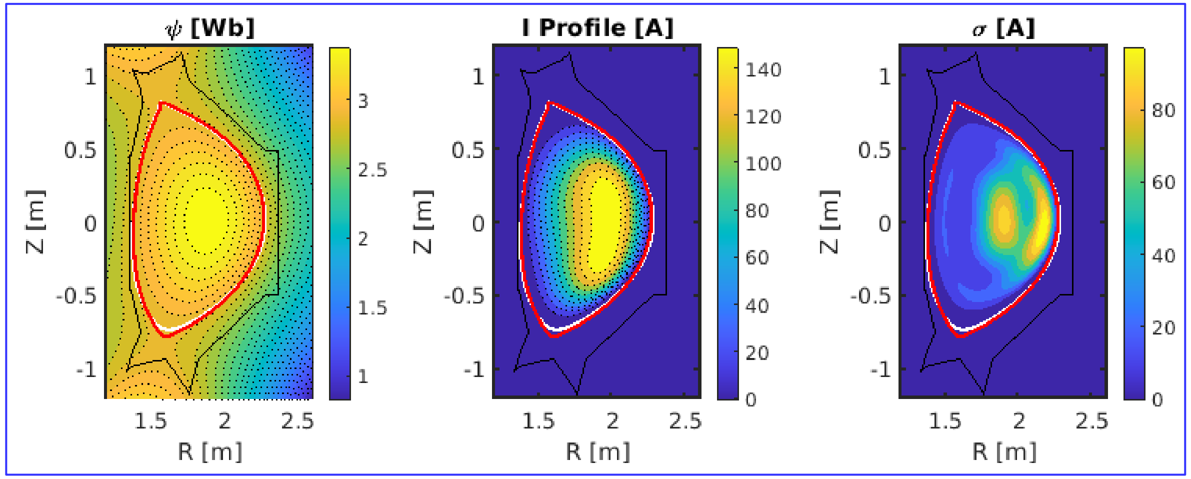
Loss function (MSE)

$$Loss = \frac{1}{N}\sum_{i=1}^{N}\left(I_i - I_i^{\text{Target}}\right)^2$$

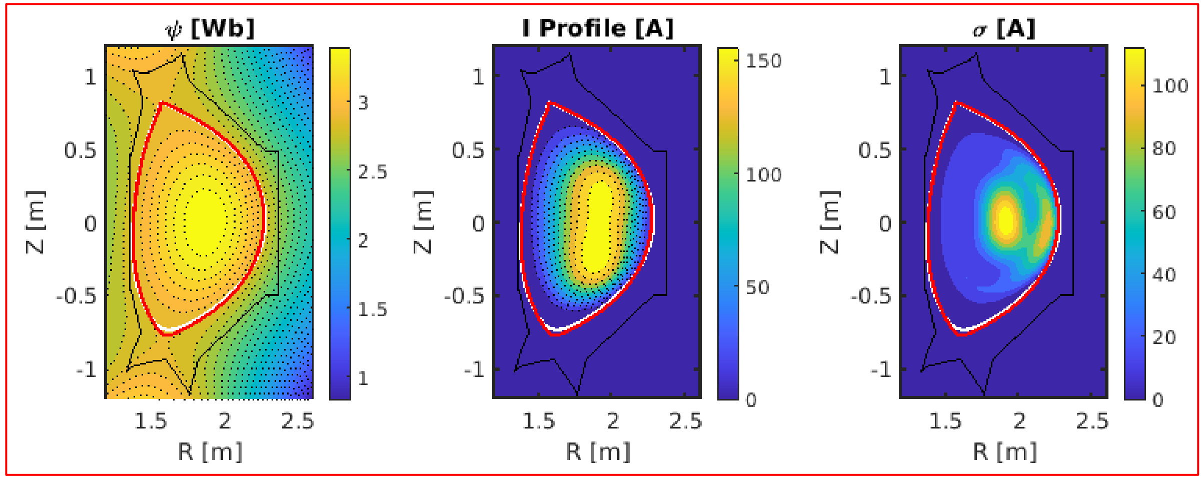


Experiment result from different reference discharge

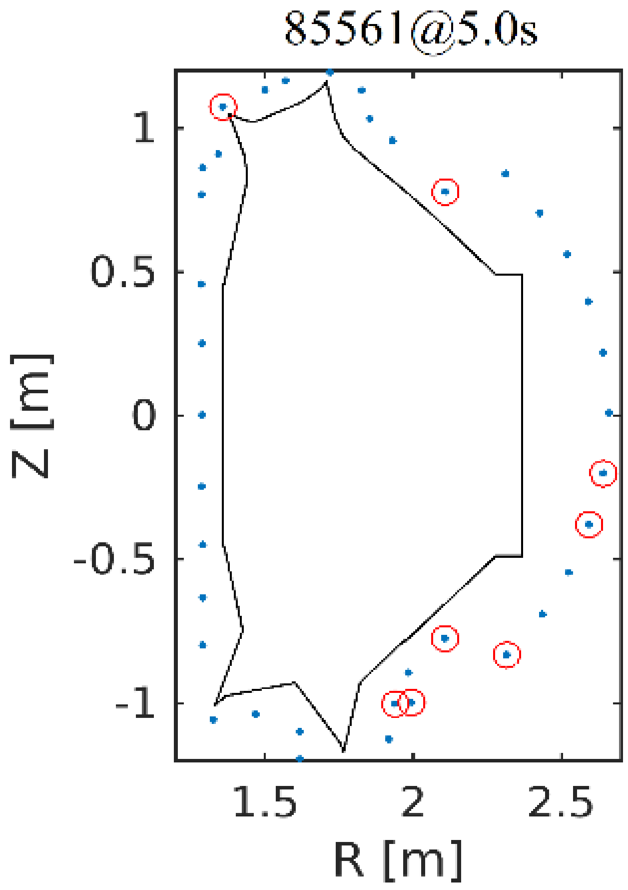
85559 # 5.0s



Reference discharge from 85559 # 5.0s



Reference discharge from Neural networks



Red line is the boundary from EFIT, Write Line is from the Bayesian model

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PART 03

Integrated data analysis

Diagnostic principle

POINT

$$Point_1 = \psi = \frac{\phi_R - \phi_L}{2} = 2.62 \times 10^{-13} \lambda^2 \int n_e \, B_{//} dl$$

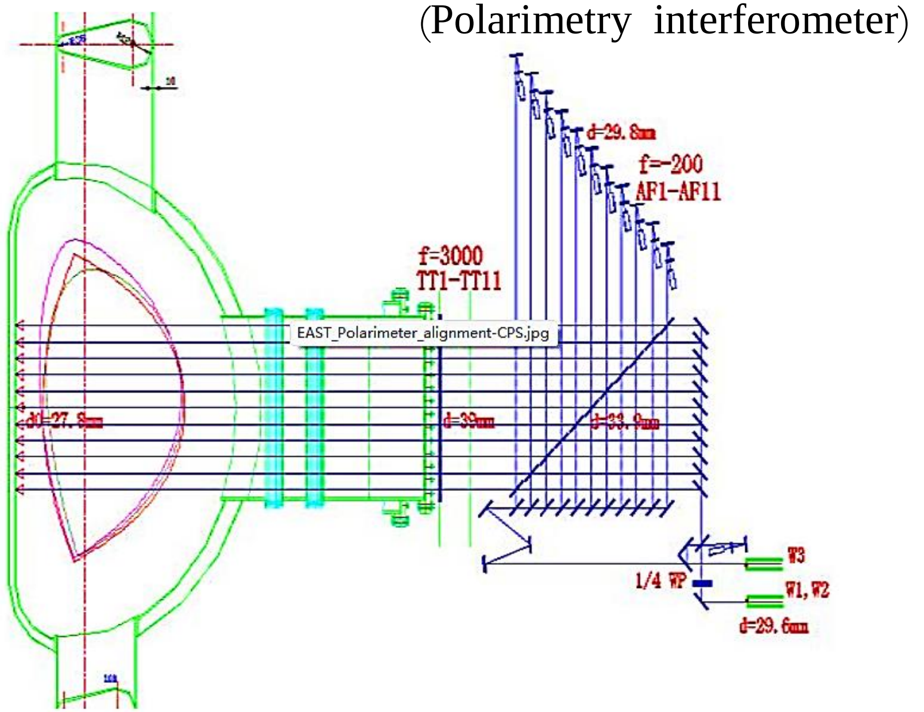
$$Point_2 = \phi = \frac{\phi_R + \phi_L}{2} = 2.82 \times 10^{-15} \lambda \int n_e \, dl$$

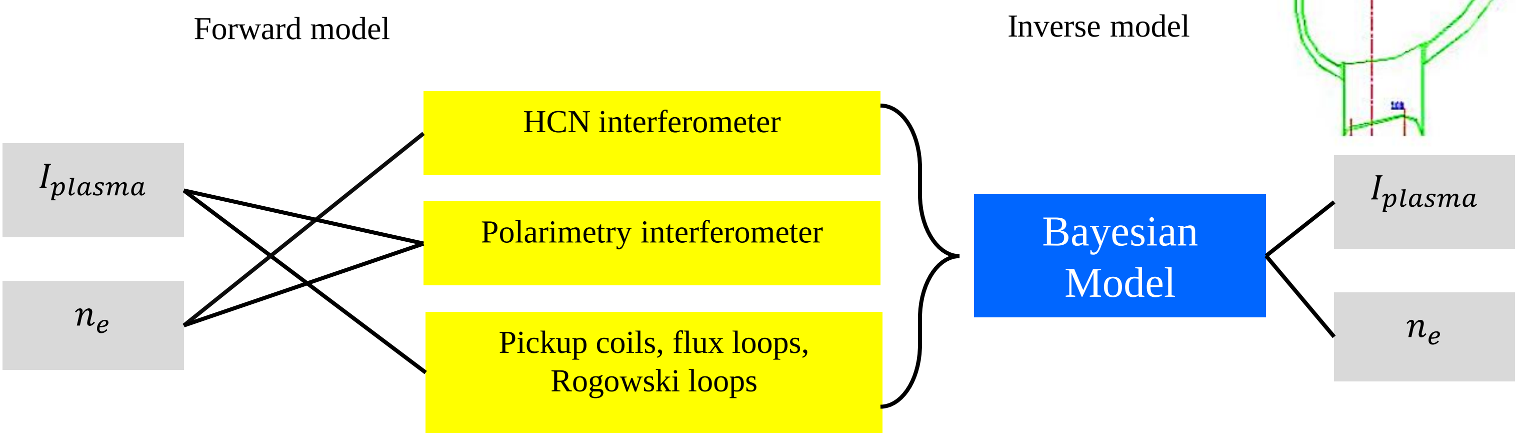
HCN

$$HCN = \frac{\pi}{\lambda n_c} \int n_e(z) \, dz$$

HCN interferometer

POINT
(Polarimetry interferometer)





Integration probability model

$$\overbrace{p(\bar{I},\bar{n}_e|\bar{d}_{point1},\bar{d}_{point2},\bar{d}_{HCN},\bar{d}_{Mag})}^{Posterior\ probability} \propto \overbrace{p(\bar{d}_{point1},\bar{d}_{point2},\bar{d}_{HCN},\bar{d}_{Mag}|\bar{I},\bar{n}_e)}^{Likelihood\ probability} \cdot \overbrace{p(\bar{I},\bar{n}_e)}^{Priori\ probability}$$

$$p(\bar{I},\bar{n}_e|\bar{d}_{point1},\bar{d}_{point2},\bar{d}_{HCN},\bar{d}_{Mag}) = p(\bar{d}_{point1}|\bar{I},\bar{n}_e)\cdot p(\bar{d}_{point2}|\bar{I},\bar{n}_e)\cdot p(\bar{d}_{HCN}|\bar{I},\bar{n}_e)\cdot p(\bar{d}_{Mag}|\bar{I},\bar{n}_e)$$

$$p(\bar{d}_{point2}|\bar{n}_e) \qquad p(\bar{d}_{HCN}|\bar{n}_e) \qquad p(\bar{d}_{Mag}|\bar{I})$$

✖

$$p(\bar{I},\bar{n}_e) = p(\bar{n}_e\mid\bar{I})\cdot p(\bar{I})$$

The isoelectron density surface and the isomagnetic surface are on the same surface, and the magnetic flux is related to the current

$$\boldsymbol{p(\bar{I},\bar{n}_e|\bar{d}_{point1},\bar{d}_{point2},\bar{d}_{HCN},\bar{d}_{Mag}) \propto p(\bar{d}_{point1}|\bar{n}_e)\cdot p(\bar{d}_{point2}|\bar{I},\bar{n}_e)\cdot p(\bar{d}_{HCN}|\bar{n}_e)\cdot p(\bar{d}_{Mag}|\bar{I})\cdot p(\bar{n}_e\mid\bar{I})\cdot p(\bar{I})}$$

Likelihood---Forward model

$$Point_1 = 2.62\times10^{-13}\lambda^2\int n_e\,B_{//}dl \approx 2.62\times10^{-13}\lambda^2\sum n_e\cdot(\bar{\bar{R}}_1\bar{I}+\bar{\bar{R}}_2\bar{I}_{pf})_i\cdot\triangle l = \bar{\bar{R}}_{piont2}\,diag(\bar{n}_e)\bar{I}+\bar{C}_1$$

$$\boldsymbol{p}(\bar{\boldsymbol{d}}_{\boldsymbol{point1}}\mid\bar{\boldsymbol{I}}) = \frac{1}{(2\pi)^{\frac{m}{2}}|\bar{\bar{\Sigma}}_{point1}|^{\frac{1}{2}}}exp\left[-\frac{1}{2}\left(\bar{\bar{R}}_{piont1}diag(\bar{n}_e)\bar{I}+\bar{C}_1-\bar{d}_{point1}\right)^T\bar{\bar{\Sigma}}_{point1}^{-1}\left(\bar{\bar{R}}_{piont2}diag(\bar{n}_e)\bar{I}+\bar{C}_1-\bar{d}_{point1}\right)\right]$$

$$Point_2 = 2.82\times10^{-15}\lambda\int n_e\,dl \approx 2.82\times10^{-15}\lambda\sum n_e\cdot\Delta l = \bar{R}_{piont2}\,\bar{n}_e$$

$$\boldsymbol{p}(\bar{\boldsymbol{d}}_{\boldsymbol{point2}}\mid\bar{\boldsymbol{n}}_{\boldsymbol{e}}) = \frac{1}{(2\pi)^{\frac{m}{2}}|\bar{\bar{\Sigma}}_{\boldsymbol{point2}}|^{\frac{1}{2}}}\boldsymbol{exp}\left[-\frac{1}{2}\left(\bar{\bar{R}}_{piont2}\cdot\bar{n}_e-\bar{d}_{point2}\right)^T\bar{\bar{\Sigma}}_{point2}^{-1}\left(\bar{\bar{R}}_{piont2}\cdot\bar{n}_e-\bar{d}_{point1}\right)\right]$$

$$HCN = \frac{\pi}{\lambda n_c}\int n_e(z)\,dz = \bar{\bar{R}}_{HCN}\cdot\bar{n}_e$$

$$\boldsymbol{p}(\bar{\boldsymbol{d}}_{\boldsymbol{HCN}}\mid\bar{\boldsymbol{n}}_{\boldsymbol{e}}) = \frac{1}{(2\pi)^{\frac{m}{2}}|\bar{\bar{\Sigma}}_{HCN}|^{\frac{1}{2}}}exp\left[-\frac{1}{2}\left(\bar{\bar{R}}_{HCN}\cdot\bar{n}_e-\bar{d}_{HCN}\right)^T\bar{\bar{\Sigma}}_{HCN}^{-1}\left(\bar{\bar{R}}_{HCN}\cdot\bar{n}_e-\bar{d}_{HCN}\right)\right]$$

Prior

SE kernel function

$$k_{SE}(\bar{x},\bar{x}') = \sigma^2 \exp(-\frac{(\bar{x}-\bar{x}')^2}{2\ell^2})$$

$$\bar{\bar{\Sigma}} = \begin{pmatrix} \mathcal{K}(\bar{x}_1,\bar{x}_1) & \cdots & \mathcal{K}(\bar{x}_1,\bar{x}_n) \\ \vdots & \ddots & \vdots \\ \mathcal{K}(\bar{x}_n,\bar{x}_1) & \cdots & \mathcal{K}(\bar{x}_n,\bar{x}_n) \end{pmatrix}$$

$p(\bar{n}_e \mid \bar{I})$

Convert Cartesian coordinates $\bar{x}, \bar{x}'$ to magnetic coordinates $\bar{\psi}, \bar{\psi}'$

$$\bar{\psi} = \bar{\bar{R}}_1\bar{I} + \bar{\bar{R}}_2\bar{I}_{pf}$$

$$k_{SE}(\bar{\psi},\bar{\psi}') = \sigma^2 \exp(-\frac{(\bar{\psi}-\bar{\psi}')^2}{2\ell^2})$$

$P(\bar{I})$

Construcr σ as a proportional function of current

If (i = j) $\sigma = k * \bar{I}'_i$

Else $\sigma = k * \sqrt{\bar{I}'_i * \bar{I}'_j}$

$$k_{SE}(\bar{x},\bar{x}') = (k * \sqrt{\bar{I}'_i * \bar{I}'_j})^2 \exp(-\frac{(\bar{x}-\bar{x}')^2}{2\ell^2})$$

$$p(\bar{n}_e \mid \bar{I}) = \frac{1}{(2\pi)^{N_{ne}/2}|\bar{\bar{\Sigma}}_{ne}|^{-1/2}} exp(-\frac{1}{2}\bar{n}_e{}^T\bar{\bar{\Sigma}}_{ne}\bar{n}_e)$$

$$P(\bar{I}) = \frac{1}{(2\pi)^{N_I/2}|\bar{\bar{\Sigma}}_I|^{1/2}} exp(-\frac{1}{2}\bar{I}^T\bar{\bar{\Sigma}}_I^{-1}\bar{I})$$

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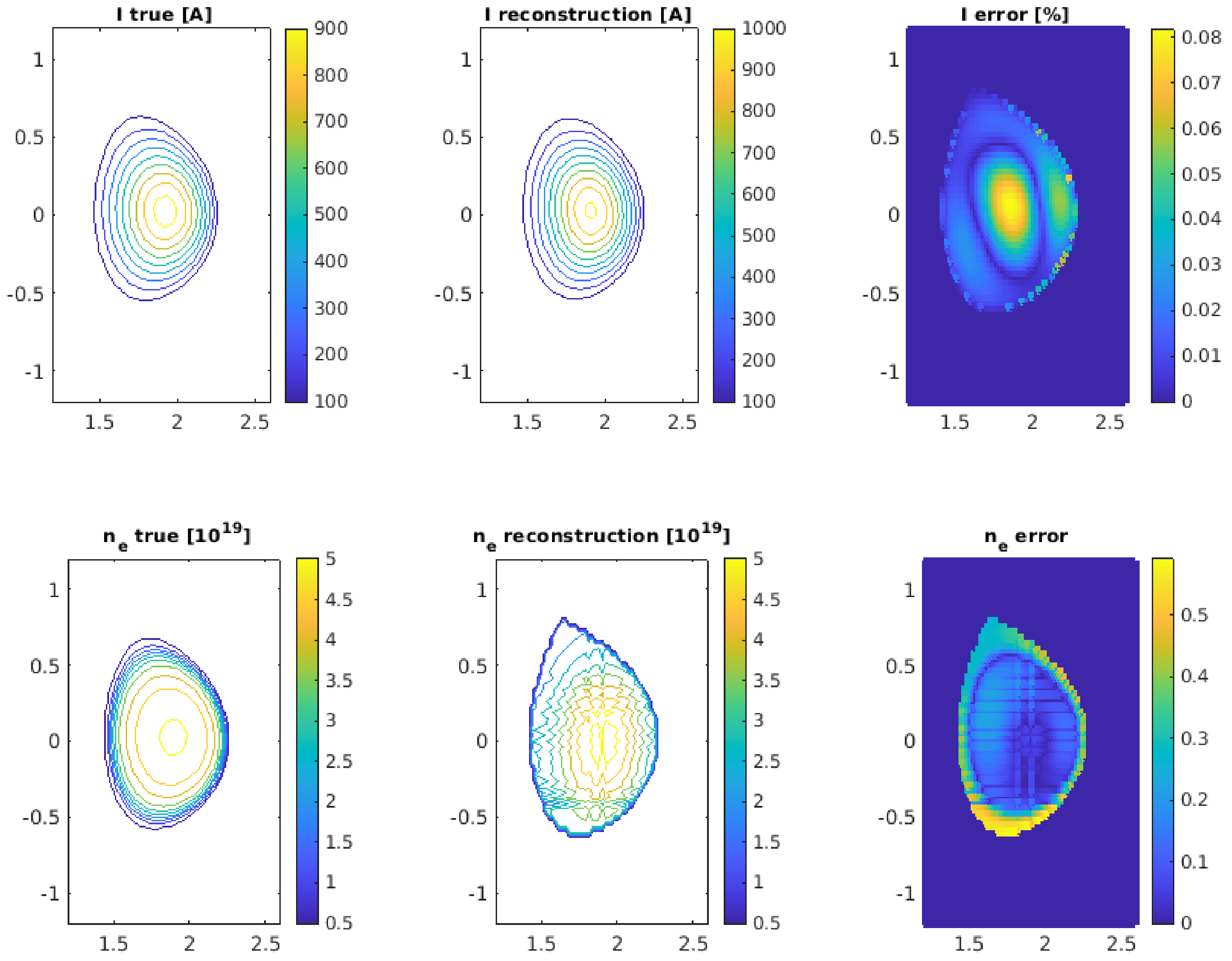
Result from simulation

CNNC ASIPP



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PART 04

Summary

Summary

- ASE Bayesian probability model performs well on reconstructing the plasma current
- For ASE Bayesian probability models, Bayesian probability have strong robustness, which can also achieve accurate plasma equilibrium reconstruction when there is a large difference between the reference discharge and the true discharge, and also be migrated to different devices.
- Neural network can automatically provide the appropriate reference discharge for our model

Next steps

- The simulation of the integrated analysis will be further improved
- The model will be tested using the experimental data
- Integrate more diagnostics, and build a large integrated analysis platform

Thanks for your attention!

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