An advanced plasma current tomography based on Bayesian inference and neural networks

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PART 01  Background
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PART 03  Integrated data analysis
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PART 01

Background
Advantages:

- The Bayesian inference takes the prior information $p(I)$, the likelihood probability $p(D_{Mag}|I)$ into consideration to give the current distribution in a probabilistic manner.
- The error is visually represented by a given uncertainty
- Other diagnostics were easily integrated by joint probabilities

Previous works:

plasma current tomography based on CAR(conditional autoregressive) prior\(^1\)

- The model was severely affected by the diagnostics (damaged)
- The current at the core is always underestimated

\(^1\) Z. J. Liu and et al. Plasma current profile reconstruction for east based on Bayesian inference. Fusion Engineering and Design
PART 02

Plasma current tomography
Plasma current tomography

Assume: The fitting error satisfies with the Gaussian distribution

\[ p(B|A) = p(B) \frac{p(A|B)}{p(A)} \propto p(B) \cdot p(A|B) \]

\[ p(I|D^{Mag}) \propto p(I) \cdot p(D^{Mag}|I) \]

Likelihood \( p(D^{Mag}|I) \)

Biot-Savart Law: \( \vec{B} = \int \vec{dB} = I \oint \frac{\mu_0}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3} = GI \)

\( D^{Mag} = GI + C \)

The contribution of the PF coil current to the diagnostic signal

Covariance matrix, determined by the diagnostics

\[ p(D^{Mag}|I) = \frac{1}{(2\pi)^{N^2/2}|\Sigma_D|^{1/2}} \exp \left( -\frac{1}{2} \left( \vec{G}I + \vec{C} - D^{Mag} \right)^T \Sigma_D^{-1} \left( \vec{G}I + \vec{C} - D^{Mag} \right) \right) \]

Magnetic diagnostics on EAST
- Pickup coils: 38 (red dot)
- Magnetic flux loops: 35 (blue asterisk)
- Rogowski loop: 1

The yellow grids contain the plasma
Construcr $\sigma$ as a proportional function of current (The current is from reference discharge)

If \( i = j \) \quad $\sigma = k \cdot \bar{I}_i$

Else $\sigma = k \cdot \sqrt{\bar{I}_i \cdot \bar{I}_j}$

Where $k$ is a constant

$$p(\bar{I}) = \frac{1}{(2\pi)^{N/2} |\Sigma_I|^{1/2}} \exp\left(-\frac{1}{2} \bar{I}^T \Sigma_I^{-1} \bar{I}\right)$$
\( p(\overline{I}|\overline{D^{Mag}}) \)

- Prior probability:
  \[
p(\overline{I}) = \frac{1}{(2\pi)^{N_I/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2} (\overline{I})^T \Sigma^{-1} (\overline{I}) \right]
  \]

- Likelihood probability:
  \[
p\left(\overline{D^{Mag}}|\overline{I}\right) = \frac{1}{(2\pi)^{N_D/2} |\Sigma_D|^{1/2}} \exp\left[-\frac{1}{2} (\overline{G}\overline{I} + \overline{C} - \overline{D^{Mag}})^T \Sigma_D^{-1} (\overline{G}\overline{I} + \overline{C} - \overline{D^{Mag}}) \right]
  \]

Posterior probability:
\[
p\left(\overline{I}|\overline{D^{Mag}}\right) = \frac{1}{(2\pi)^{N_I/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2} (\overline{I} - \overline{m})^T \Sigma^{-1} (\overline{I} - \overline{m}) \right]
\]

Covariance matrix:
\[
\overline{\Sigma} = (\overline{G}^T \Sigma_D^{-1} \overline{G} + \overline{\Sigma}_I^{-1})^{-1}
\]

Mean:
\[
\overline{m} = (\overline{G}^T \Sigma_D^{-1} \overline{G} + \Sigma_I^{-1})^{-1} \Sigma_D^{-1} (\overline{D^{Mag}} - \overline{C} - \overline{G}\overline{m}_I)
\]
Result from simulation

Relative error:
\[ \xi_i = \frac{|I_{i}^{\text{rec}} - \bar{I}_i|}{\max(\bar{I})} \]

Root-mean-square deviation:
\[ \text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n}(I_{i}^{\text{rec}} - \bar{I}_i)^2}{n}} \]
Robustness

Add 3% noise

**Configuration**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>USN</th>
<th>LSN</th>
<th>DN</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 (cm)</td>
<td>0.0545</td>
<td>0.0476</td>
<td>0.0850</td>
</tr>
<tr>
<td>C2 (cm)</td>
<td>0.0953</td>
<td>0.0093</td>
<td>0.0372</td>
</tr>
<tr>
<td>C3 (cm)</td>
<td>0.1100</td>
<td>0.0474</td>
<td>0.1600</td>
</tr>
<tr>
<td>C4 (cm)</td>
<td>0.0107</td>
<td>0.0652</td>
<td>0.0820</td>
</tr>
<tr>
<td>C5 (cm)</td>
<td>0.0535</td>
<td>0.0305</td>
<td>0.0539</td>
</tr>
<tr>
<td>C6 (cm)</td>
<td>0.0161</td>
<td>0.0589</td>
<td>0.0561</td>
</tr>
<tr>
<td>X_{U}(cm)</td>
<td>0.0668</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X_{L}(cm)</td>
<td></td>
<td>0.0289</td>
<td>0.1580</td>
</tr>
<tr>
<td>Max Error(cm)</td>
<td>0.4800</td>
<td>0.1700</td>
<td>0.7800</td>
</tr>
<tr>
<td>Max (\xi) Current (%)</td>
<td>5.47%</td>
<td>3.34%</td>
<td>5.95%</td>
</tr>
<tr>
<td>RSMD Current (A)</td>
<td>2.2524</td>
<td>1.1884</td>
<td>1.7880</td>
</tr>
<tr>
<td>Max (\xi) Flux (%)</td>
<td>1.30%</td>
<td>0.97%</td>
<td>0.86%</td>
</tr>
<tr>
<td>RSMD Flux (Wb)</td>
<td>0.0021</td>
<td>0.0017</td>
<td>0.0018</td>
</tr>
</tbody>
</table>
The different current distributions were obtained by changing the $q$.

Select $q = 2$ as reference discharge and reconstruct plasma current at $q = 1.5$, 2.5 and 3.
Different devices  HL-2A

RETINA, schematic diagram of the integrated data analysis platform on HL-2A
Neural network gives the reference discharge

Train Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Datasize</th>
<th>No.of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{PF}$</td>
<td>PF current (Rogowski loops)</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>$I_{P0}$</td>
<td>total plasma current (Rogowski loop)</td>
<td>1</td>
<td>276689</td>
</tr>
<tr>
<td>$\Psi_{FL}$</td>
<td>Poloidal magnetic flux (flux loops)</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>The current in different position</td>
<td>506</td>
<td></td>
</tr>
</tbody>
</table>

Activation function

$Selu = \lambda \begin{cases} 
  x & \text{if } x > 0 \\
  \alpha e^x - \alpha & \text{if } x \leq 0 
\end{cases}$

$Relu = \max(0, x)$

Loss function (MSE)

$Loss = \frac{1}{N} \sum_{i=1}^{N} (I_i - I^{\text{Target}}_i)^2$
Experiment result from different reference discharge

Red line is the boundary from EFIT, Write Line is from the Bayesian model.
PART 03

Integrated data analysis
Diagnostic principle

POINT

\[ \text{Point}_1 = \psi = \frac{\phi_R - \phi_L}{2} = 2.62 \times 10^{-13} \lambda^2 \int n_e B_{//} dl \]

\[ \text{Point}_2 = \phi = \frac{\phi_R + \phi_L}{2} = 2.82 \times 10^{-15} \lambda \int n_e \, dl \]

HCN

\[ HCN = \frac{\pi}{\lambda n_c} \int n_e(z) \, dz \]

Forward model

Inverse model

<table>
<thead>
<tr>
<th>I_{plasma}</th>
<th>HCN interferometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>n_e</td>
<td>Polariometry interferometer</td>
</tr>
<tr>
<td></td>
<td>Pickup coils, flux loops, Rogowski loops</td>
</tr>
</tbody>
</table>

Bayesian Model

\[ I_{plasma} \]

\[ n_e \]
The isoelectron density surface and the isomagnetic surface are on the same surface, and the magnetic flux is related to the current.
Likelihood---Forward model

\[
P_{\text{point}} = 2.62 \times 10^{-13} \lambda^2 \int n_e B_{\parallel} dl \approx 2.62 \times 10^{-13} \lambda^2 \sum n_e \cdot (\bar{R}_1 \bar{I} + \bar{R}_2 \bar{I}_{pf}) \cdot \Delta l = \bar{R}_{\text{point}2} \text{diag}(\bar{n}_e) \bar{I} + \bar{C}_1
\]

\[
p(\bar{d}_{\text{point}1} | \bar{I}) = \frac{1}{(2\pi)^{\frac{m}{2}} |\bar{\Sigma}_{\text{point1}}|^\frac{1}{2}} \exp\left[-\frac{1}{2} (\bar{R}_{\text{point1}} \text{diag}(\bar{n}_e) \bar{I} + \bar{C}_1 - \bar{d}_{\text{point1}})^T \bar{\Sigma}_{\text{point1}}^{-1} (\bar{R}_{\text{point1}} \text{diag}(\bar{n}_e) \bar{I} + \bar{C}_1 - \bar{d}_{\text{point1}})\right]
\]

\[
P_{\text{point}} = 2.82 \times 10^{-15} \lambda \int n_e dl \approx 2.82 \times 10^{-15} \lambda \sum n_e \cdot \Delta l = \bar{R}_{\text{point}2} \bar{n}_e
\]

\[
p(\bar{d}_{\text{point}2} | \bar{n}_e) = \frac{1}{(2\pi)^{\frac{m}{2}} |\bar{\Sigma}_{\text{point2}}|^\frac{1}{2}} \exp\left[-\frac{1}{2} (\bar{R}_{\text{point2}} \cdot \bar{n}_e - \bar{d}_{\text{point2}})^T \bar{\Sigma}_{\text{point2}}^{-1} (\bar{R}_{\text{point2}} \cdot \bar{n}_e - \bar{d}_{\text{point2}})\right]
\]

\[HCN = \frac{\pi}{\lambda n_c} \int n_e(z) dz = \bar{R}_{\text{HCN}} \cdot \bar{n}_e
\]

\[
p(\bar{d}_{\text{HCN}} | \bar{n}_e) = \frac{1}{(2\pi)^{\frac{m}{2}} |\bar{\Sigma}_{\text{HCN}}|^\frac{1}{2}} \exp\left[-\frac{1}{2} (\bar{R}_{\text{HCN}} \cdot \bar{n}_e - \bar{d}_{\text{HCN}})^T \bar{\Sigma}_{\text{HCN}}^{-1} (\bar{R}_{\text{HCN}} \cdot \bar{n}_e - \bar{d}_{\text{HCN}})\right]
\]
Prior

**SE kernel function**

\[
k_{SE}(\bar{x}, \bar{x}') = \sigma^2 \exp(-\frac{(\bar{x}-\bar{x}')^2}{2\ell^2})
\]

\[
\Sigma = \begin{pmatrix}
K(\bar{x}_1, \bar{x}_1) & \cdots & K(\bar{x}_1, \bar{x}_n) \\
\vdots & \ddots & \vdots \\
K(\bar{x}_n, \bar{x}_1) & \cdots & K(\bar{x}_n, \bar{x}_n)
\end{pmatrix}
\]

\[
P(\bar{I})
\]

Construct \(\sigma\) as a proportional function of current

- If \(i = j\), \(\sigma = k \cdot \bar{I}_i\)
- Else, \(\sigma = k \cdot \sqrt{\bar{I}_i \cdot \bar{I}_j}\)

\[
k_{SE}(\bar{x}, \bar{x}') = (k \cdot \sqrt{\bar{I}_i \cdot \bar{I}_j})^2 \exp(-\frac{(\bar{x}-\bar{x}')^2}{2\ell^2})
\]

\[
p(\bar{n}_e \mid \bar{I})
\]

Convert Cartesian coordinates \(\bar{x}, \bar{x}'\) to magnetic coordinates \(\bar{\psi}, \bar{\psi}'\)

\[
\bar{\psi} = \bar{R}_1 \bar{I} + \bar{R}_2 \bar{I}_{pf}
\]

\[
k_{SE}(\bar{\psi}, \bar{\psi}') = \sigma^2 \exp(-\frac{(\bar{\psi}-\bar{\psi}')^2}{2\ell^2})
\]

\[
p(\bar{n}_e \mid \bar{I}) = \frac{1}{(2\pi)^{N_{ne}/2} |\Sigma_{ne}|^{-1/2}} \exp(-\frac{1}{2} \bar{n}_e^T \Sigma_{ne}^{-1} \bar{n}_e)
\]

\[
P(\bar{I}) = \frac{1}{(2\pi)^{N_{I}/2} |\Sigma_{I}|^{-1/2}} \exp(-\frac{1}{2} \bar{I}^T \Sigma_{I}^{-1} \bar{I})
\]
Result from simulation
PART 04

Summary
Summary

➢ ASE Bayesian probability model performs well on reconstructing the plasma current
➢ For ASE Bayesian probability models, Bayesian probability have strong robustness, which can also achieve accurate plasma equilibrium reconstruction when there is a large difference between the reference discharge and the true discharge, and also be migrated to different devices.
➢ Neural network can automatically provide the appropriate reference discharge for our model

Next steps

◆ The simulation of the integrated analysis will be further improved
◆ The model will be tested using the experimental data
◆ Integrate more diagnostics, and build a large integrated analysis platform
Thanks for your attention!

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