A Bayesian model of filamentary dynamics in MAST

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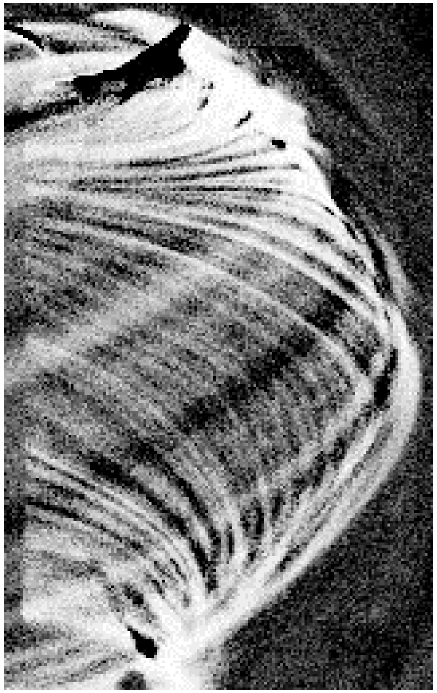
4th IAEA technical Meeting on Fusion Data Processing, Validation and Analysis,

30th November to 3rd December 2021

L C Appel et al, PPCF Volume 62, 125002, 2020

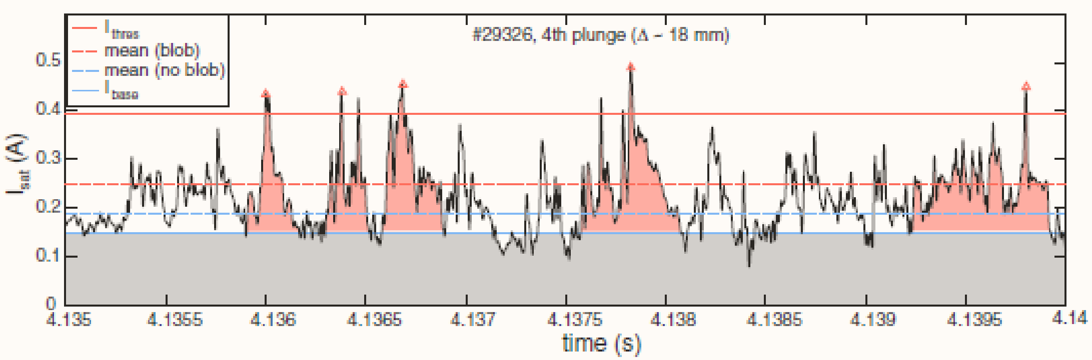
Background

- Fluctuating nature of plasma edge region has been long established from Langmuir probe data.
- From the 2000’s high-speed video photography identified these as large-scale coherent structures.
- Known as “blobs”, and later as “filaments”
- Structures are
  - 3-D
  - field-aligned
  - carry large flux of heat and energy.
  - *Dynamics strongly influenced by plasma parameters.*



Analysis of Langmuir Probe data

- Early work (eg Zweben 1983) applied auto- and cross-correlation analysis, moments of distributions, power spectra
- *Carerras 2000 applied multi-fractal analysis*
- *Conditional averaging technique (*Filippas 1985), became the most widely used analysis technique from 2000 onwards(eg Boedo 2001, Garcia 2007)
  - Fluctuations regarded as a superposition of coherent structures
  - Provides compelling evidence that ion saturation current signal is dominated by large amplitude bursts.
  - However…… Intrinsic variabilities between filaments not taken into account, and has "arbitrary" filament detection system.



Filament detection algorithm.

Carralero et al, Nucl. Fusion **54** (2014) 123005

Objective of current work

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- *To carry out statistical analysis of Langmuir probe data with "minimal" assumptions.*
- Following previous work of (Militello 2016, Garcia 2017), data is regarded as a superposition of filaments with asymmetric leading and trailing edges.
- In this approach,
  - ✓ no restrictions enforcing an "averaged" shape of filament.
  - ✓ no amplitude threshold for existence of filaments.
- *Approach uses a data-driven model describing filamentary dynamics, with the principles of Bayesian Inference.*

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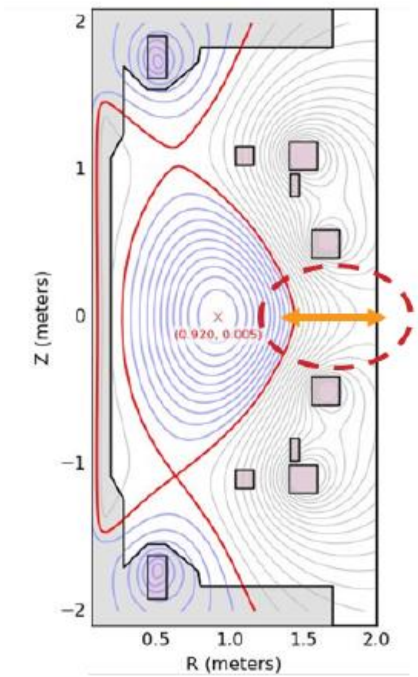
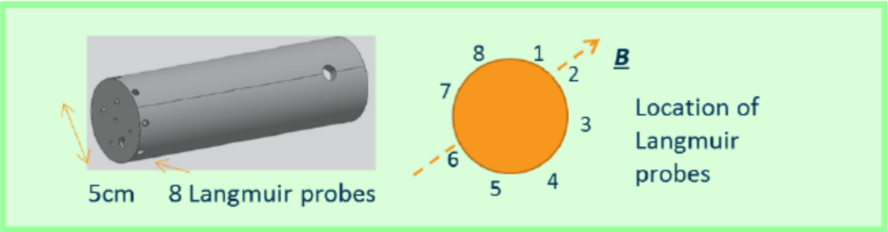
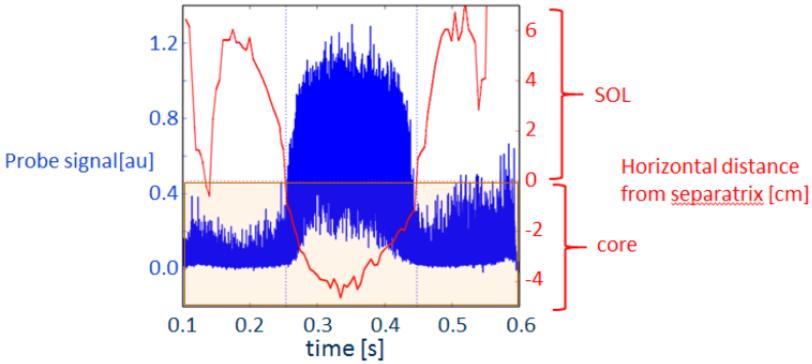
Experimental Description

- Fluctuation data $I_{sat} \sim n\sqrt{T_e}$ obtained from a Langmuir probe mounted on a reciprocating assembly.
- MAST Plasma discharge 21712 during ohmic L-mode phase.
- Data collected whilst probe moving from outer SOL into edge core plasma.





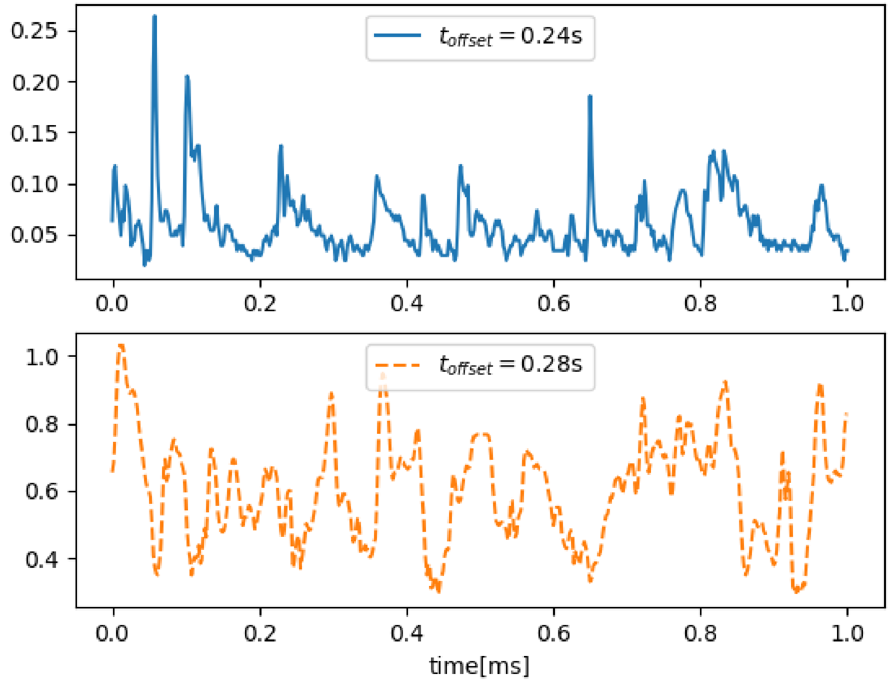


Experimental data

- Time-series data is split into 0.4ms subsets.

SOL

Plasma Core



- "Clear" evidence of Filaments
- Components exist on top of a background
- Both background and filament amplitude smaller in SOL

Filamentary model

- Filaments are represented by a superposition of *N* filaments *plus* background, B

$$f(t) = B + \sum_{i=1}^{N} A^{(i)} \left(\exp\left[-\frac{t_0^{(i)} - t}{\tau_1^{(i)}} \right] H(t_0^{(i)} - t) + \exp\left[-\frac{t - t_0^{(i)}}{\tau_2^{(i)}} \right] H(t - t_0^{(i)}) \right)$$

- Each Filament has
  - Amplitude, A
  - rise time, τ_1
  - fall time, τ_2
  - time at which filament has peak value, t_0

Bayesian Inference

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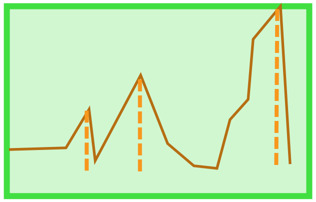
- If $\boldsymbol{H}$ are a set of free parameters, and $\boldsymbol{D}$ the data set, Bayes' formula can be written:

$$P(\boldsymbol{H}|\boldsymbol{D}) = \frac{P(\boldsymbol{D}|\boldsymbol{H})P(\boldsymbol{H})}{P(\boldsymbol{D})}$$

- Ignore evidence term as only interested in "shape" of posterior:

$$P(\boldsymbol{H}|\boldsymbol{D}) = K\, P(\boldsymbol{D}|\boldsymbol{H})P(\boldsymbol{H})$$



$\boldsymbol{H}$ are the set of free parameters. {A}, $\{\tau_1\}$, $\{\tau_2\}$, B
$\boldsymbol{D}$ are the data values in a data subset.
The set of times $\{t_0\}$ are the times corresponding to data $\{d_{i-i}, d_i, d_{i+i}\}$ in which d_i, is a local maximum

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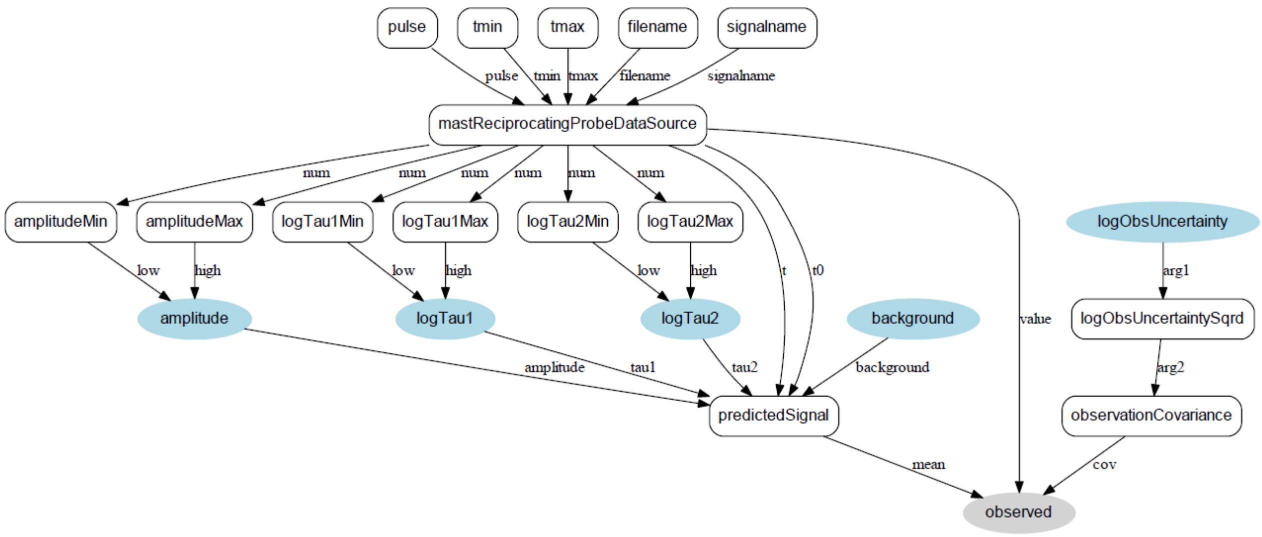
Assembling the Posterior

- The likelihood $P(\boldsymbol{D}|\boldsymbol{H})$ is a multiplication of *N* 1-D Normal distributions:
  - $P(D_i|\boldsymbol{H}) = \frac{\mathbf{1}}{\sqrt{\mathbf{2\pi\sigma^2}}}\exp\left[-\frac{(D_i - f(\boldsymbol{H}))^2}{\mathbf{2\sigma^2}}\right]$
  - σ treated as a free parameter: $\log_{10}\sigma$
- Prior distribution $P(\boldsymbol{H})$
  - All free parameters uncorrelated
  - Random uniform distribution for each parameter.
  - Restrict B>0 and A>0.
  - Use $\log_{10}\tau_1$, $\log_{10}\tau_2$ => enables all parameters to be of order 1.

Further modelling details

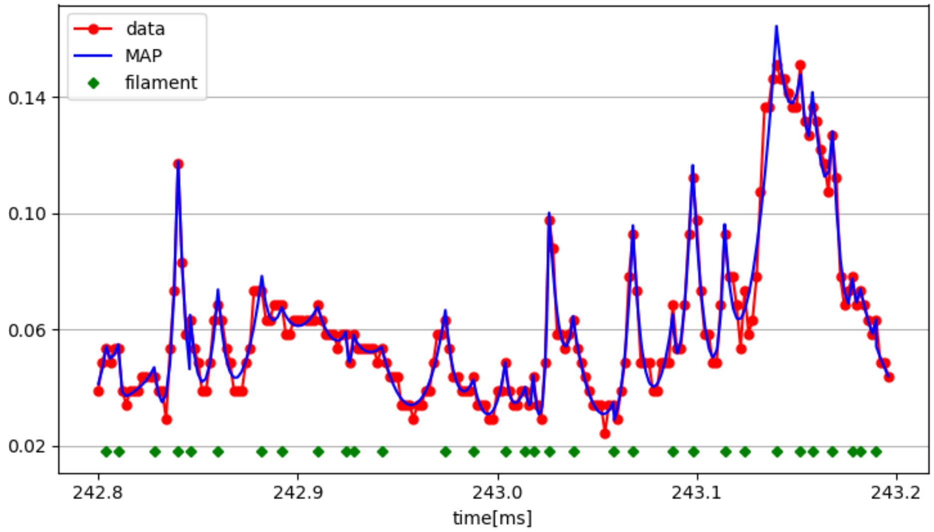
- Implement model in **minerva**
  - **MAP** solution obtained using the method of Hooke and Jeeves
  - Sampling of posterior obtained with MCMC.



Bayesian Graphical model from MINERVA

Bayesian inference for single data subset

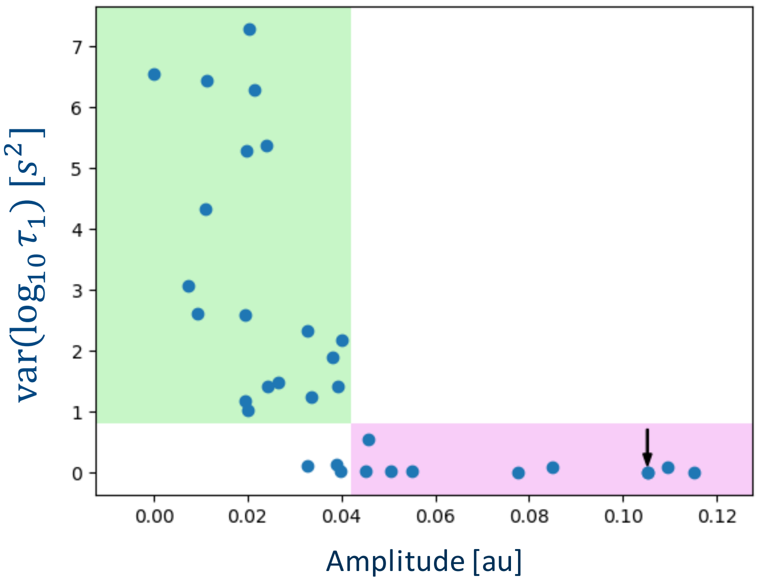
- Computed signals using the **MAP** parameters of 0.4ms data subset.
  - Probe is in SOL, *N=32, B=0,* $\sigma = 6 \times 10^{-3}$.
  - Summary for all data sets:
    - B=0 for 96% of cases, whereas $\sigma \approx 10\%$ of A.



Results of MCMC

- ✓ Variance in rise times ($\boldsymbol{\tau_1}$) is strongly correlated with amplitudes.
- ✓ Similar behaviour for fall times ($\boldsymbol{\tau_2}$)
  - filaments are symmetric (Concurs with previous analyses of filaments on MAST[Militello2013])

Variance of rise-times



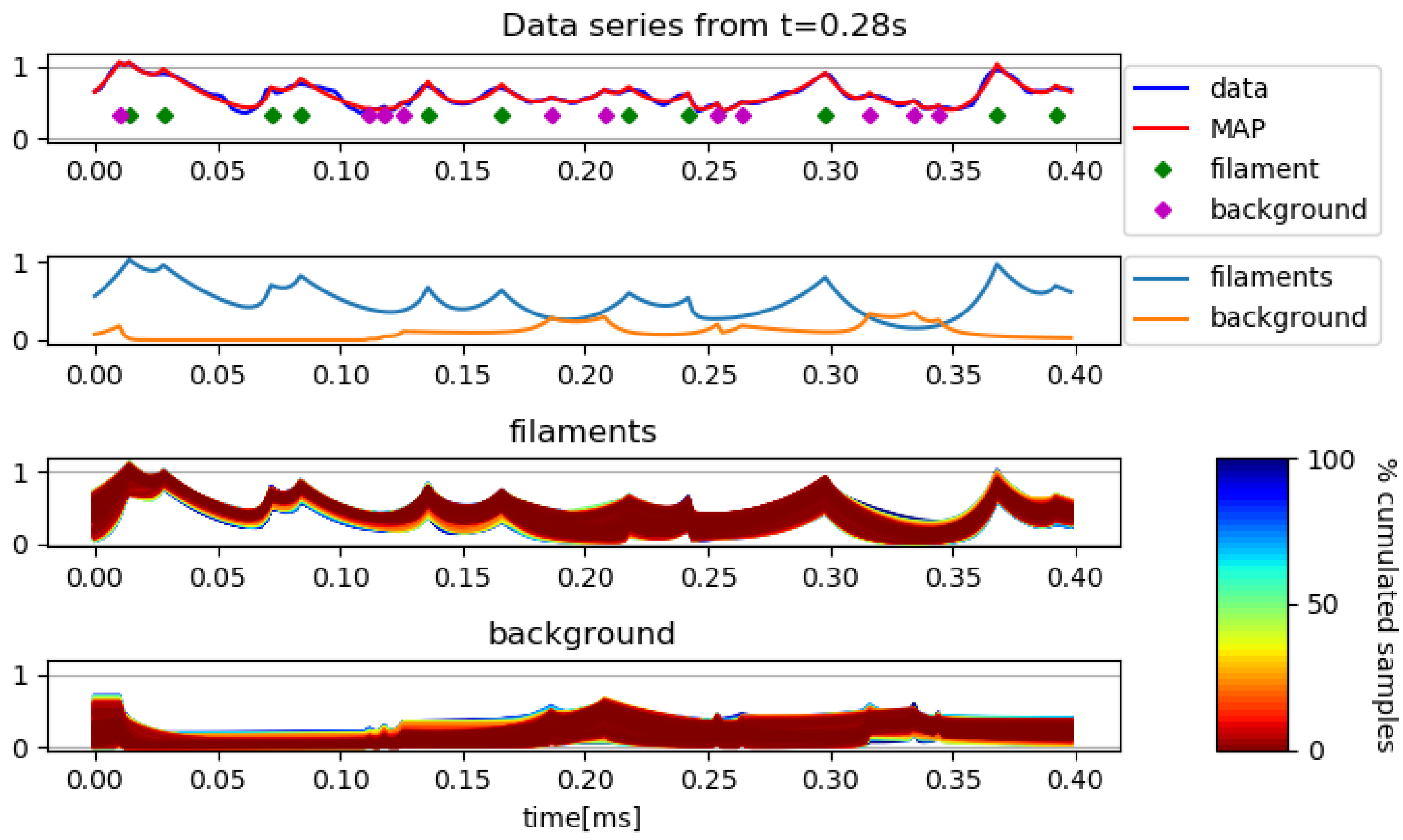
Discussion

Previous studies on MAST [Militello 2016] provides good evidence that

- filaments are structures, have been ejected from the plasma core and travel at a relatively constant velocity across flux surfaces.
- Filaments are presumed to pass through and interact with a quiescent plasma background [Easy2014].
- However, the interpretation of experimental data often involves applying a smoothing function, using time-window averaging [Militello2016]
- Results of Bayesian inference calculations **do not** provide compelling evidence that there is a **finite quiescent background**.
- Postulate a new definition: Filaments have $\delta \log_{10} \tau_2{}^{(i)} < -3$
  - "**Fluctuating background**" has constant B *plus* other small amplitude filaments

Filaments with a fluctuating background

- 0.28 < t[s] < 0.284 [plasma core]



Summary Filamentary dynamics

Results of analysing 0.4ms data-subsets:

- The MAP background is usually zero, and E[B] has a small finite value.
- The model uncertainty (σ) is approximately 10% of the average signal.
- Filaments are symmetrical ($\tau_1 \approx \tau_2$).

New postulate :

✓ *true* filaments are well defined structures that originate from the core

- Range of sampled τ_1 for each of these filaments is small

✓ remaining "filaments" form part of a fluctuating background.

- The *true* filaments have larger MAP amplitudes compared to those that form the background fluctuations.

Radial variation of filament dynamics

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Can use the new definition of plasma filaments and background fluctuations to explore statistical variations across the SOL and into the outer plasma core region.

We have explored radial variations of:

1. computed filament amplitudes and background fluctuations.
2. pdf distributions of filament amplitudes and background fluctuation amplitudes.
3. pdf distributions of filament waiting times and background fluctuation waiting times.

- *definition of pdf of signal (f) defined as* $(f - < f >)/\sigma_f$

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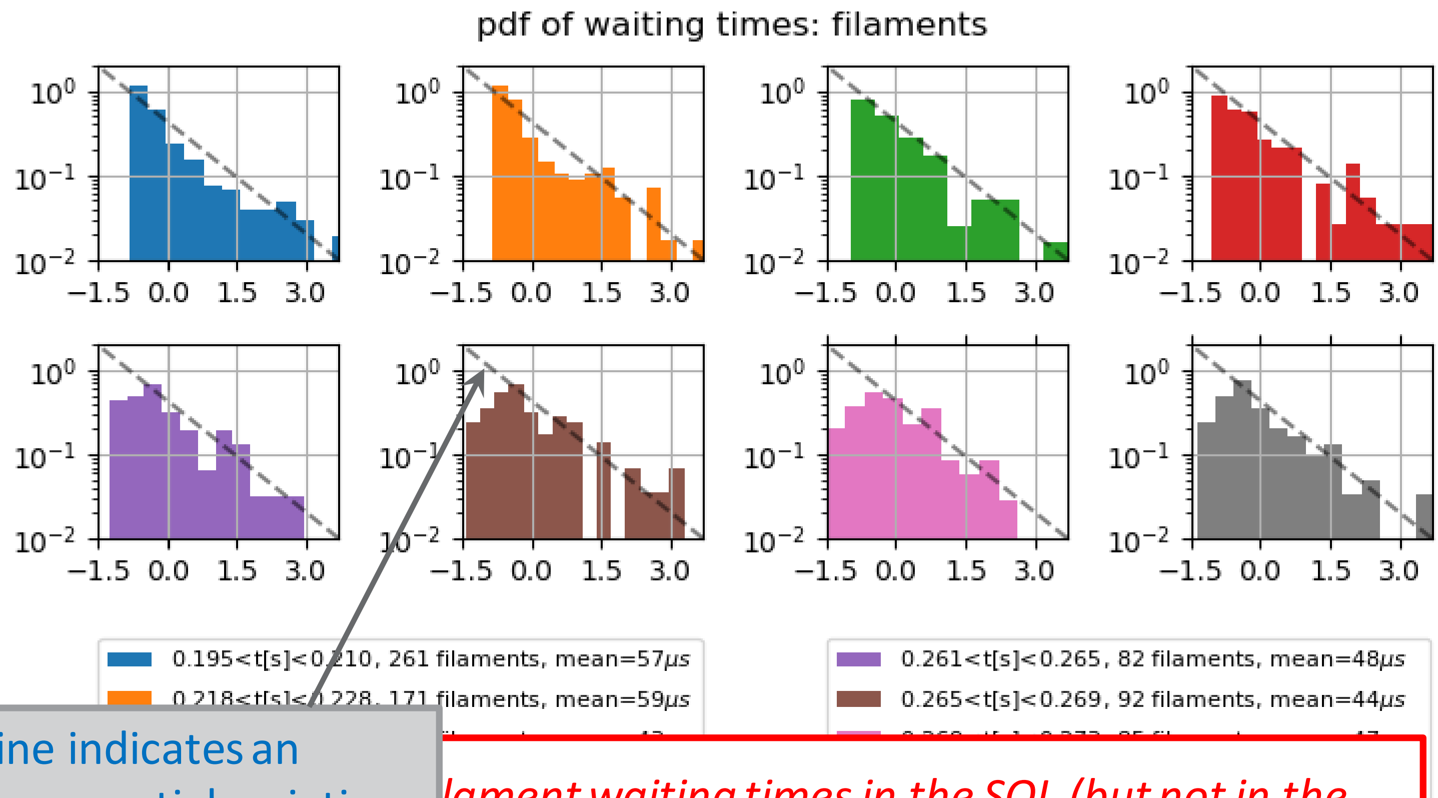
Radial variation of pdf waiting times

filament amplitudes



- line indicates an exponential variation indicative of a Poisson process

lament waiting times in the SOL (but not in the

e Poisson distributed.

Radial variation of pdf waiting times

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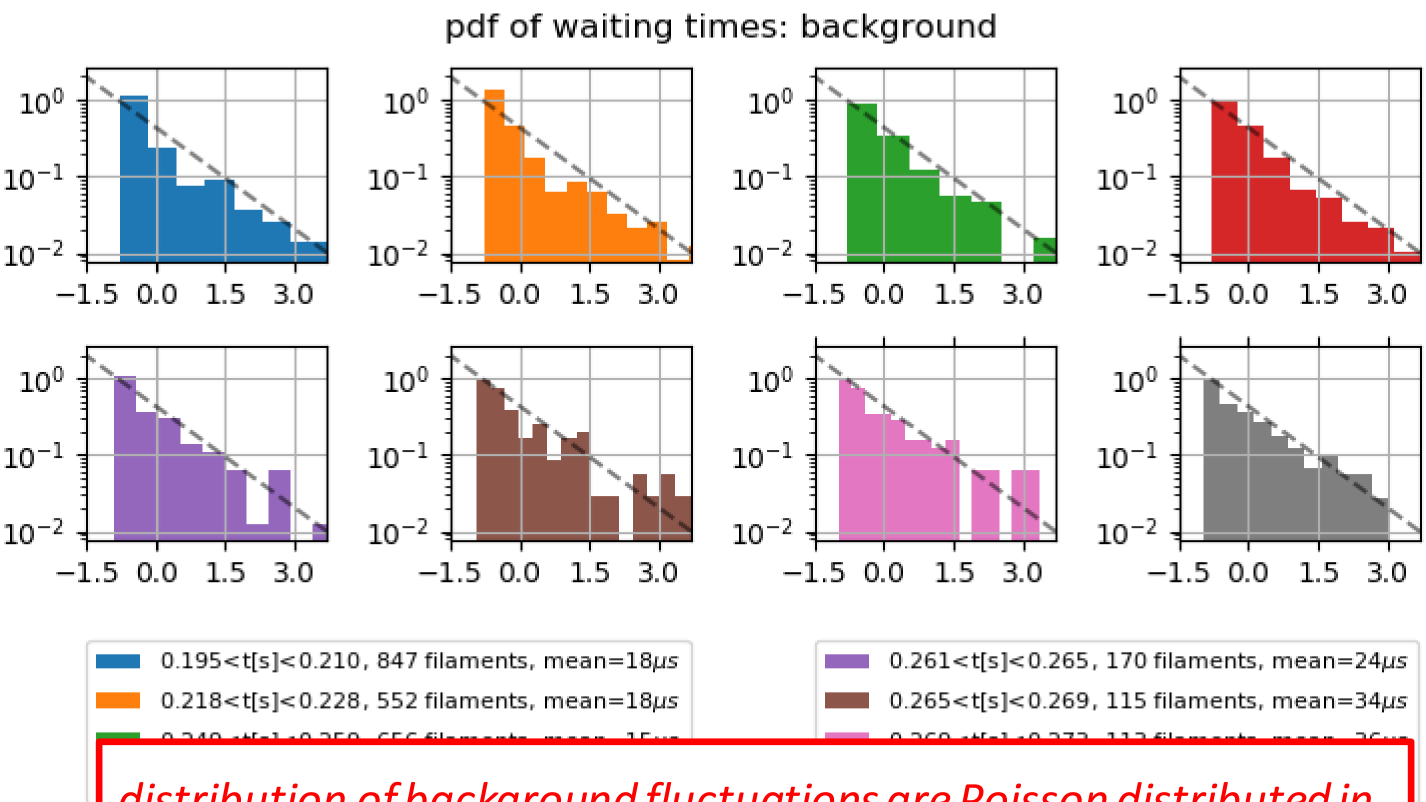
Background fluctuations



distribution of background fluctuations are Poisson distributed in both SOL and plasma core

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Conclusions

- A novel approach using Bayesian inference has been implemented to interpret the filamentary dynamics measured by a Langmuir probe fixed to a reciprocating assembly
- Distinctive feature of the approach is that no minimum threshold is set for the existence of filaments.
  - Also, the model uncertainty is provided as an additional free parameter.
- The results obtained achieve a fit to the model with an error of 10%.
- The MAP background signal is found to be zero in over 95% of subsample intervals.
- Results of Markov chain Monte Carlo sampling of the posterior distribution provide uncertainties on all model parameters.
- Whereas large amplitude filaments are well characterised in terms of rise/fall times, smaller amplitude filaments are not.

Conclusions

- Based on these findings, a new definition for the *plasma filaments* is proposed based on the uncertainty in the filament rise times. The remaining filaments together with the constant background component form a new time-dependent signal referred to as the *computed background fluctuation signal*.

- The characteristics of these signals are investigated.
- Computed plasma filaments have Poisson distributed waiting times and signal amplitudes in the SOL (but not in the core)
- Background fluctuations have Poisson distributed waiting times in both SOL and plasma core.

Final Remarks

- The current work is based on an ad hoc model. It is notable that fit of the model to the data is within 10%.
- Whereas it is plausible that the evolution of the explosive events that generate filaments could results in exponential signal components, extending this to include the background component is not substantiated.
- The separation of the signal components into two time-dependent fluctuating components is novel.
  - It permits the calculation of a fluctuating background signal that is consistent with the Bayesian model
- **Future work should establish a basis of the signals in terms of theoretically-based constraints.**

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End of talk

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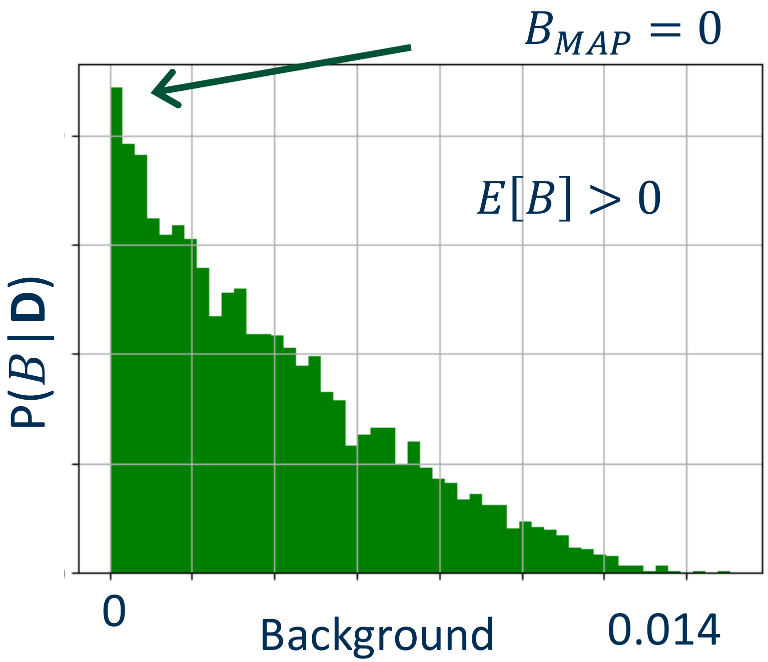
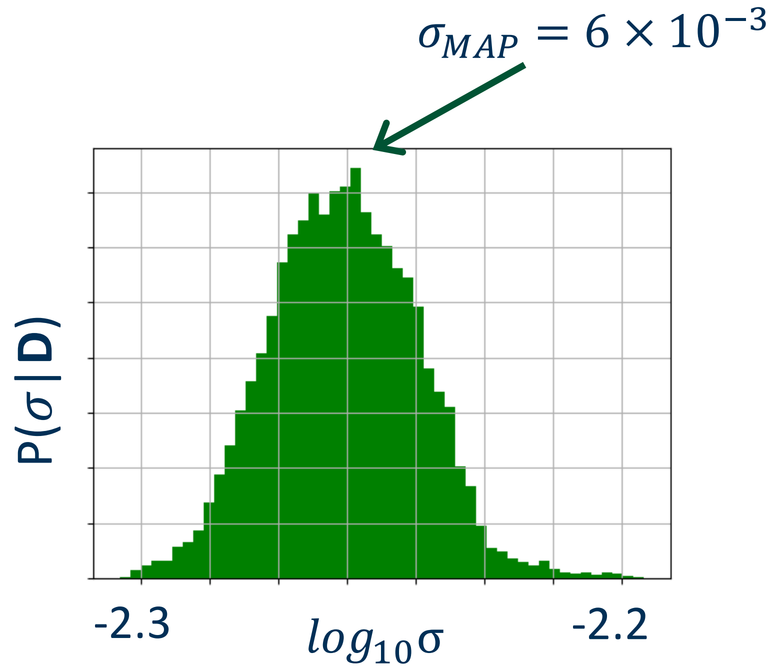
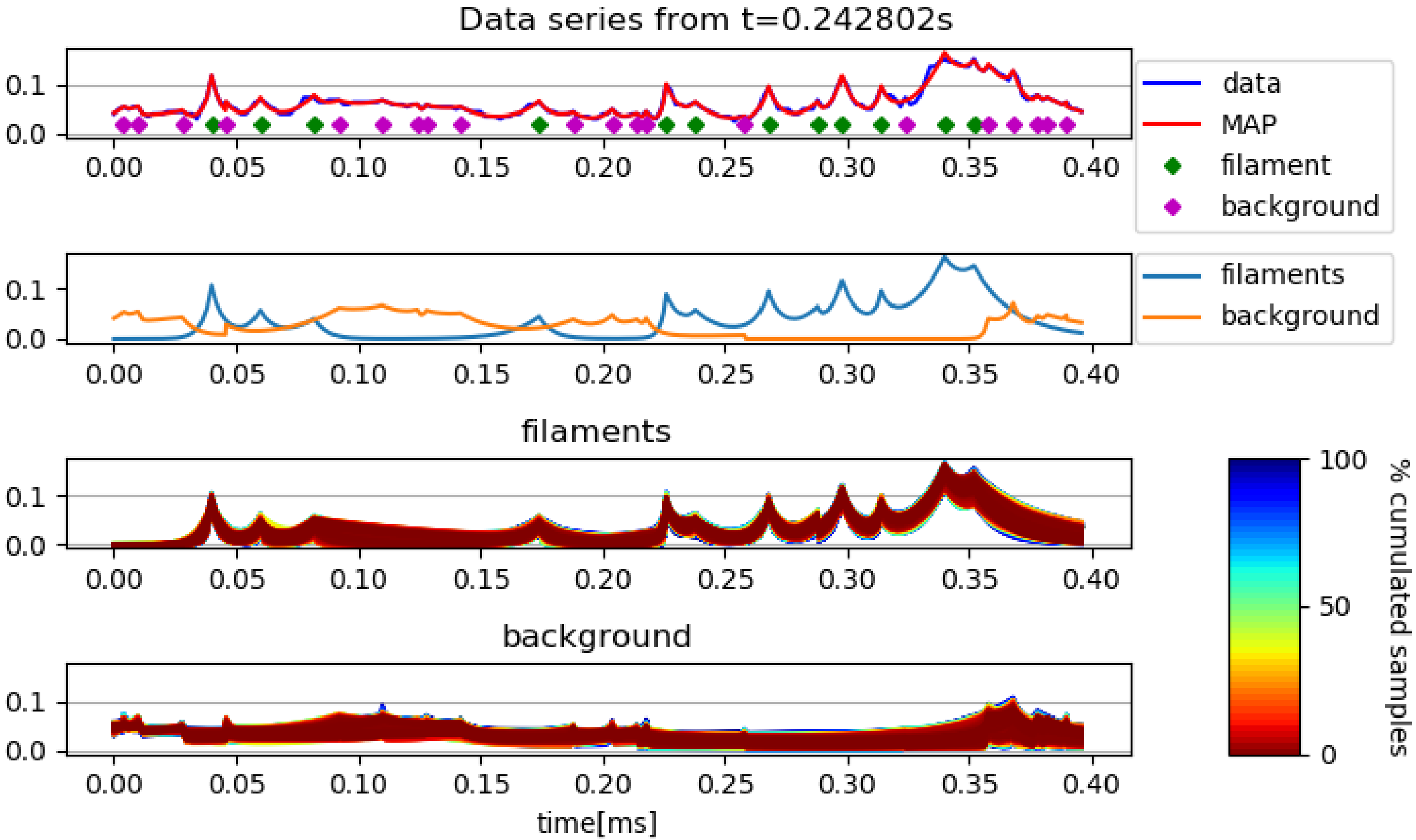
Marginal distributions: $\mathrm{P}(B|D), \mathrm{P}(\sigma|D)$





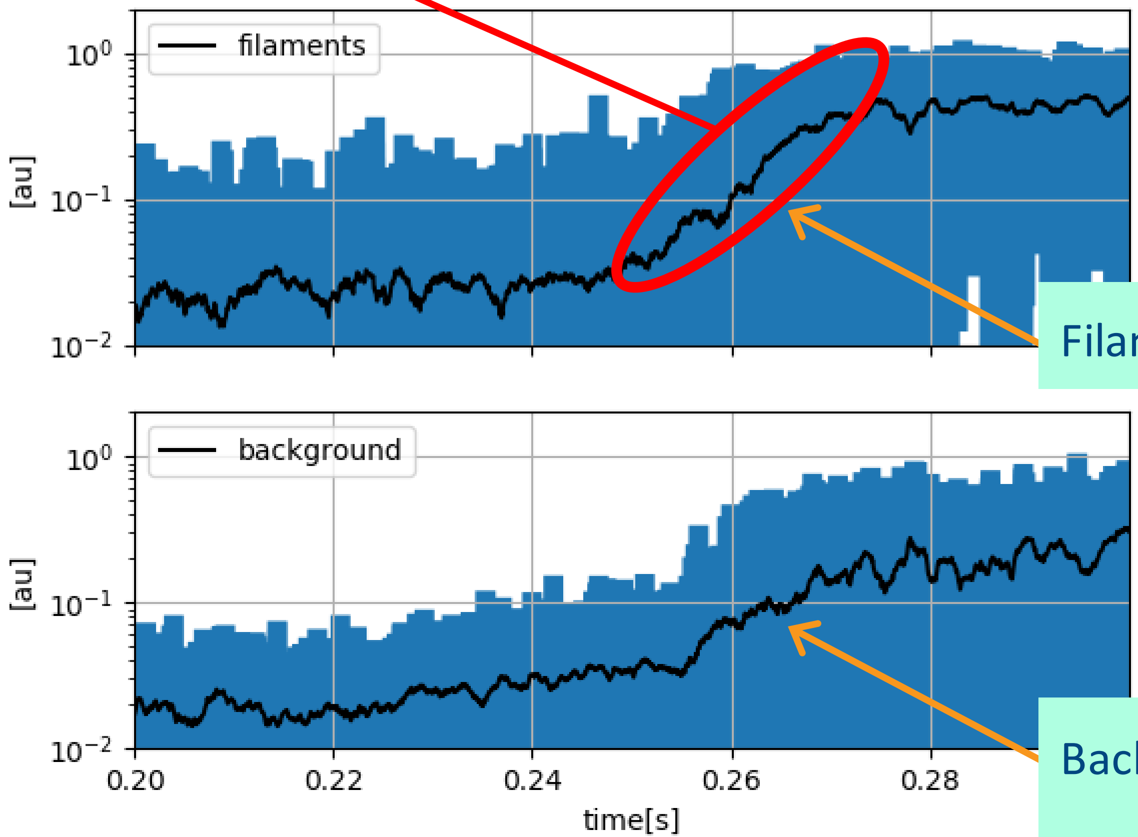
Filaments with a fluctuating background

- 0.242 < t[s] < 0.43 [SOL]



Radial variation of amplitudes

Could be explained by existence of a filament source,
- Uncertainties in magnetic geometry mean not possible to identify whether in SOL or plasma core



computed filament amplitude

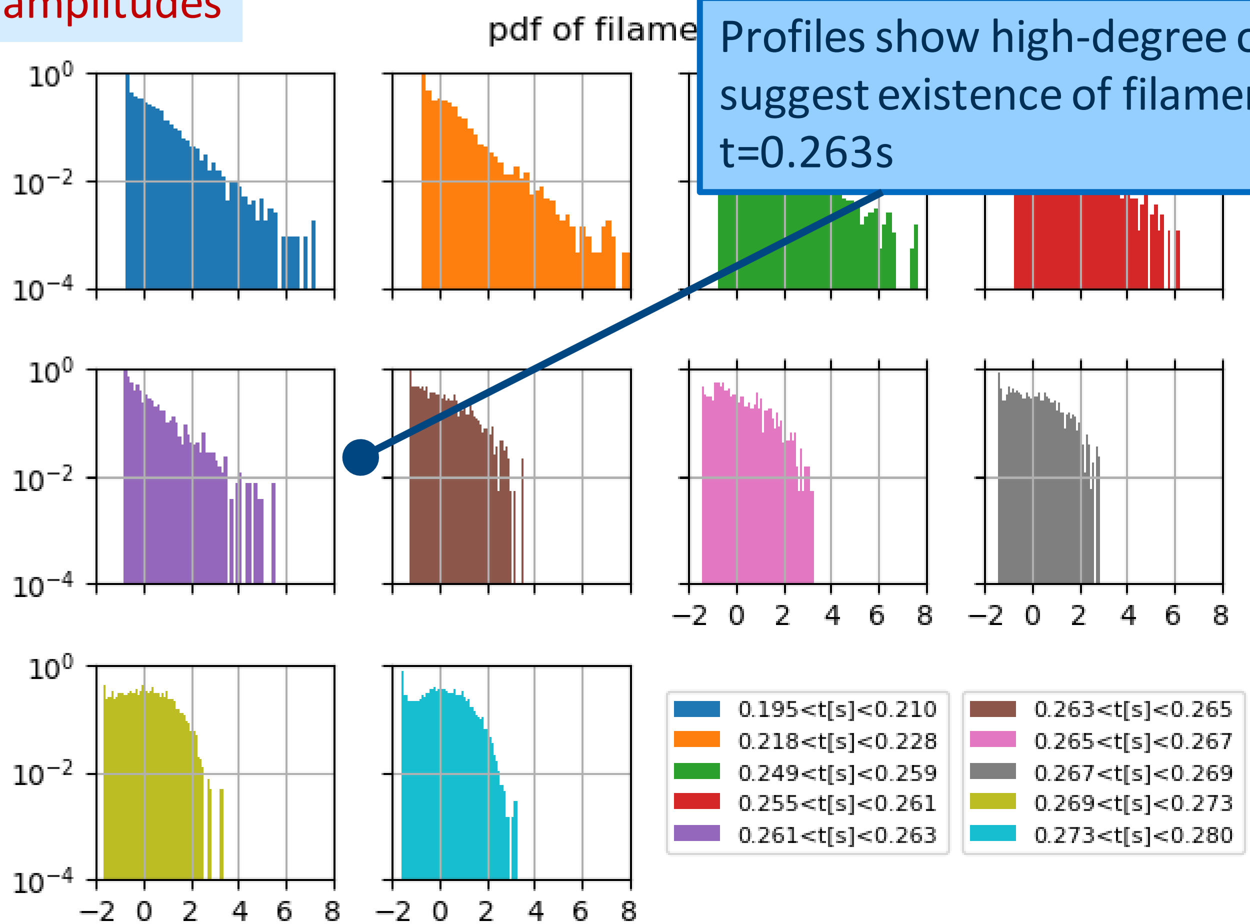
Filament amplitude rise faster

computed background fluctuations.

Background rise more gradual

Radial variation of pdf amplitudes

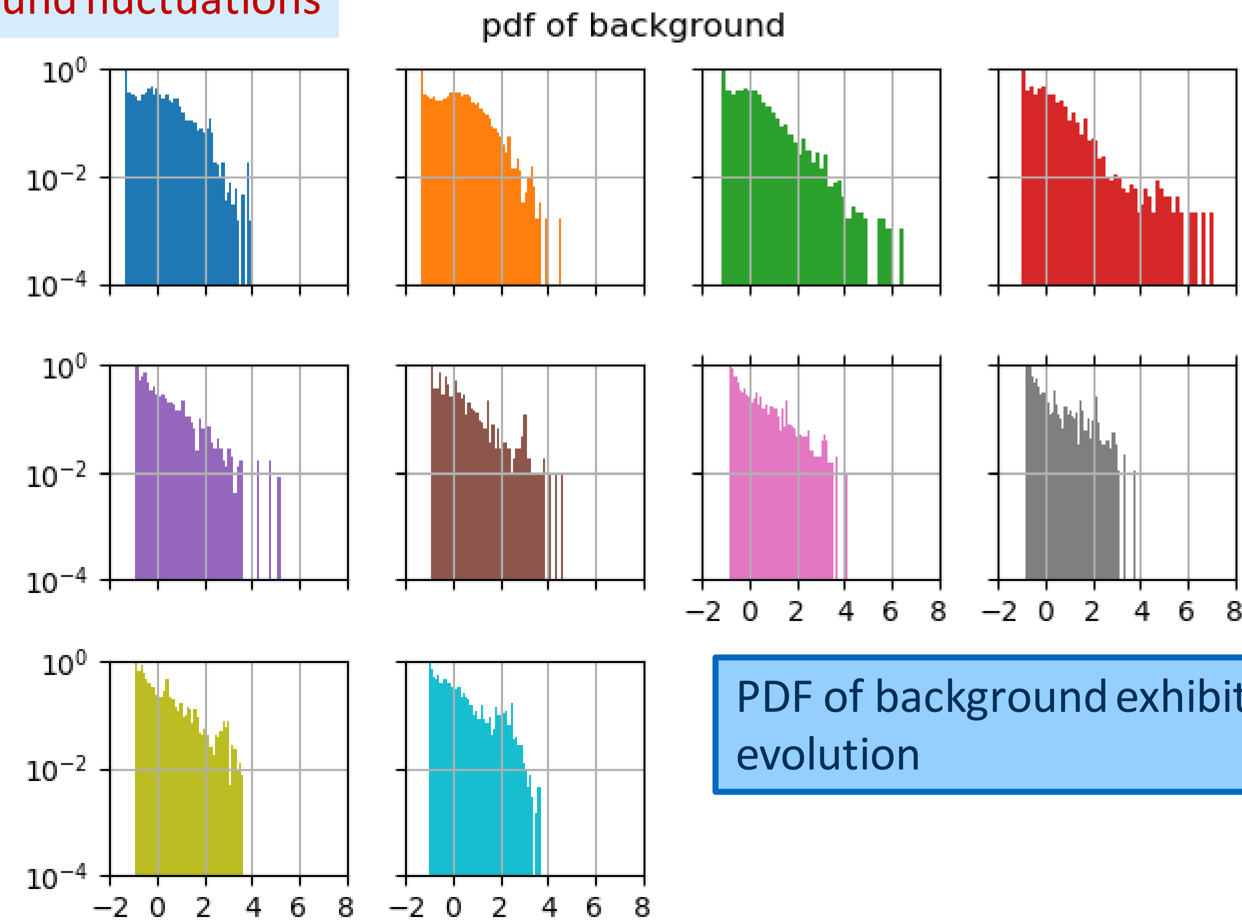
filament amplitudes



Profiles show high-degree of self-similarity, suggest existence of filament source at t=0.263s

Radial variation of pdf amplitudes

Background fluctuations



PDF of background exhibits more gradual evolution

Radial variation of waiting times

Small reduction

Maximum waiting times smaller
In SOL than plasma core

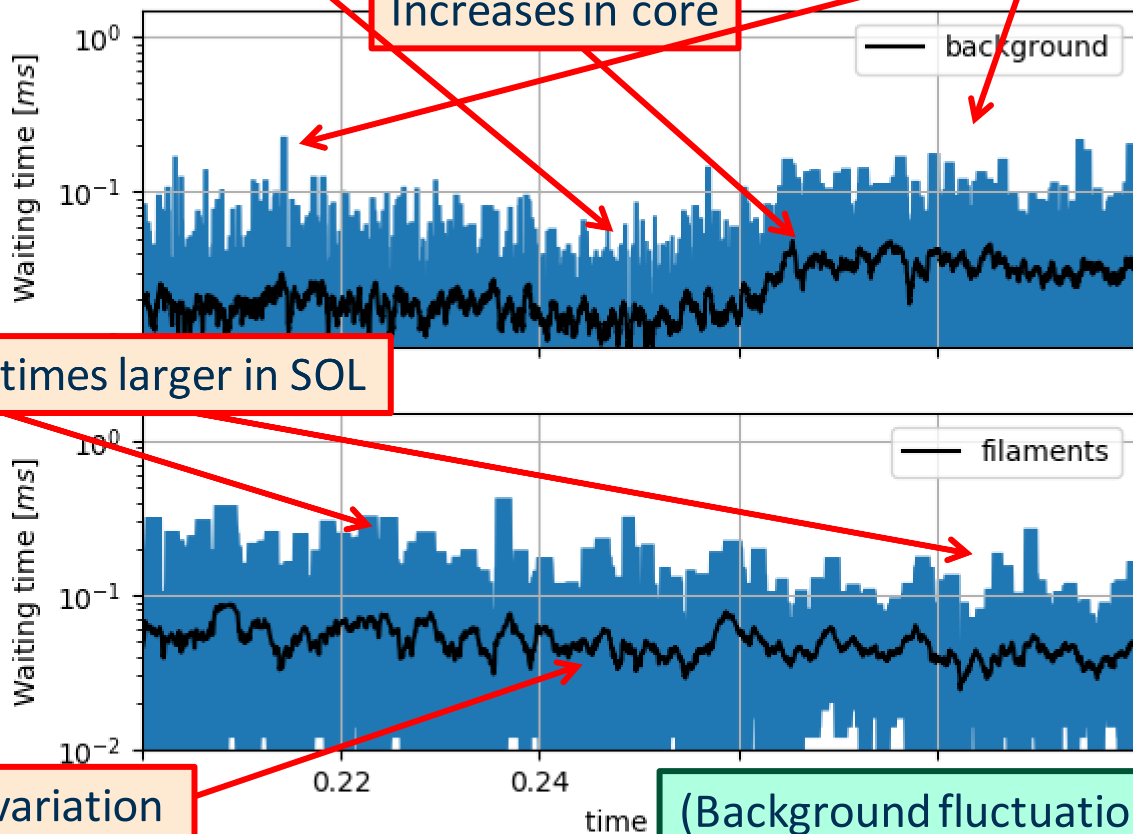
Increases in core

Background

fluctuations

Maximum waiting times larger in SOL

Filaments



Little variation

(Background fluctuation are
Poisson-distributed)