CONDITIONAL RECURRENT PLOTS AND TRANSFER ENTROPY FOR OBSERVATIONAL CAUSALITY DETECTION

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Authors are grateful to Dani Gallart and David Taylor

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement number 633053.
The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Outline

- Introduction
- Sawteeth pacing experiment, brief description
- Milestone already achieved with TE and JRP
- The idea behind and the need of new tools: CTE and CJRP
- Analysis of H mode discharges from M18-05 pulses
  - Results from PION
  - CTE and CJRP results
  - Comparing the results
- Conclusions

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Causality and Time Series

- Quantifying causality for time series is a difficult issue (*see A.Murari talk*)

- Just remind the following assessment (Granger Causality), please:

if the knowledge of the past of signal j helps predicting signal i, then j is considered causal to i.

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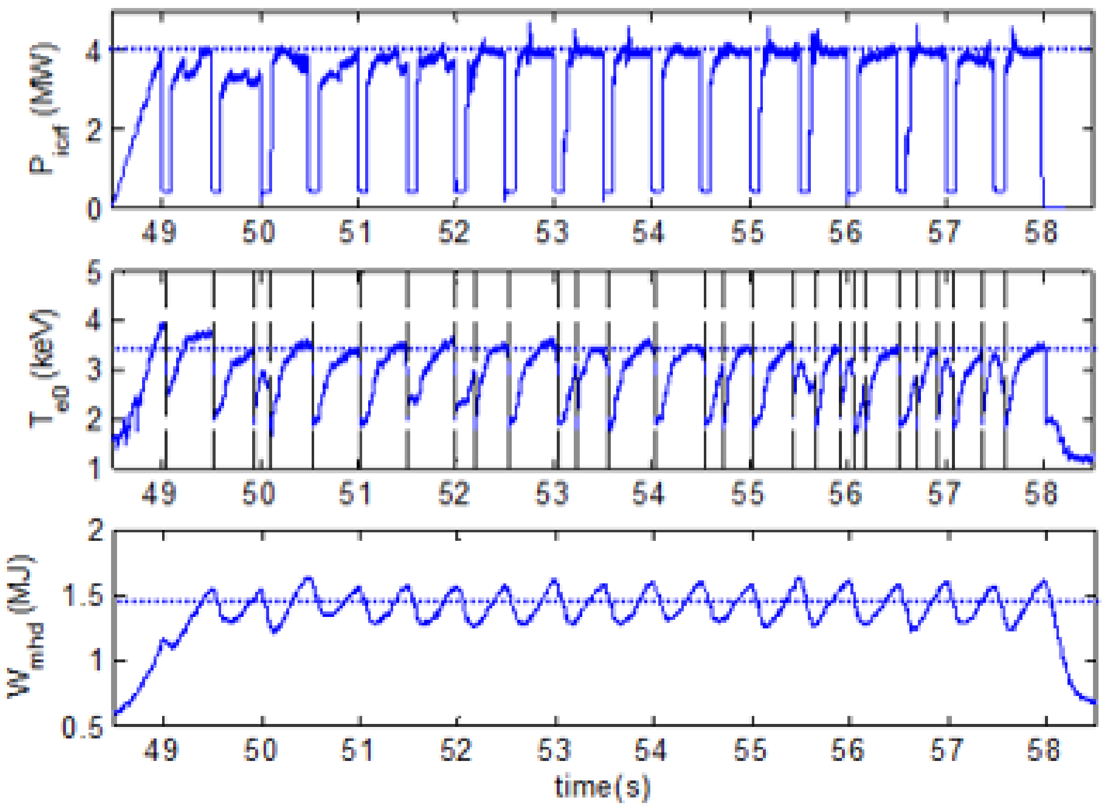
SAWTEETH PACING with ICRH MODULATION [1]

2Hz RF B=2.75T; Ip=2MA X[H]=2%



In these experiments the fast ions population is reduced by notches in the RF power, this reduces their stabilising effect and triggers a sawtooth crash.

One of the main related issues is that sawteeth are quasiperiodic and therefore, in case of pulsed actions, after a while a sawtooth is *naturally* going to occur.

Consequently, to better asses the efficiency of the modulation a statistical approach is suggested.

[1] E.Lerche et al. *"Sawteeth pacing with on-axis ICRH modulation in JET-ILW"* , Nuclear Fusion, 57 036027

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Milestones already achieved with statistics

Two main questions:

1. Is the ICRH modulation efficient?
2. Are there any further pieces of information that we can obtain by analysing these data?

We tested independent methods [2] and have been able to provide:

- evidences that the ICRH modulation actually paces sawteeth within a time interval we called the “causality horizon” (CH);

- Our CH is in line with the estimated slowing down time of the fast ions both in H and L mode.

- The tools used are:

1. Transfer Entropy (TE) [3].
2. Joint Recurrence [4]

[2] A.Murari et al Nucl. Fusion 56 026006 (11pp).
[3] T. Schreiber, PRL, 2000, 85(2), 461-464
[4] Marwan et al., Phys. Rev. E, 2002, 66(2), 026702”

Transfer Entropy

- Transfer Entropy quantifies the information flow from one time series to another.

- Introducing the formalism of the Markov processes and consequently the transition probabilities, the two observables to be analyzed can be written as $i_n^{(k)} = (i_n, \dots, i_{n-k+1})$ and $j_n^{(l)} = (j_n, \dots, k_{n-l+1})$.

$$T_{J\to I} = \sum p\left(i_{n+1}, i_n^{(k)}, j_n^{(l)}\right) \log\left(\frac{p\left(i_{n+1} | i_n^{(k)}, j_n^{(l)}\right)}{p\left(i_{n+1} | i_n^{(k)}\right)}\right)$$

RP and JRP

- Recurrence plot (RP)[4] ⇨
  - a plot showing the times at which the phase space trajectory of a dynamical system visits roughly the same area in the phase space.

$$R_{\boldsymbol{i}} = \Theta\left(\varepsilon_{\boldsymbol{i}} - \left\|\overrightarrow{\boldsymbol{i}_p} - \vec{\boldsymbol{i}}_q\right\|\right) \text{,p,q=1,…,N}$$

- $\overrightarrow{i_{(i=1,\ldots,N)}}$, $\overrightarrow{j_{(j=1,\ldots,N)}}$ are points in the phase space at different times,
- ε is a threshold
- ||°|| is a distance

- Joint Recurrence Plots (JRP) [4] ⇨
  - To assess the simultaneous occurrence of recurrences in two (or more) systems

$$JRP_{\boldsymbol{ij}} = \Theta\left(\varepsilon_{\boldsymbol{i}} - \left\|\overrightarrow{\boldsymbol{i}_p} - \overrightarrow{\boldsymbol{i}_q}\right\|\right) \cdot \Theta\left(\varepsilon_{\boldsymbol{j}} - \left\|\overrightarrow{y_{\boldsymbol{p}}} - \overrightarrow{y_{\boldsymbol{q}}}\right\|\right)$$

- Recurrence quantification analysis (RQA) ⇨
  - Allows quantifying very important properties of a system phase space besides visualization of the periodicities of dynamical systems

[4] Marwan et al., Phys. Rev. E, 2002, 66(2), 026702.

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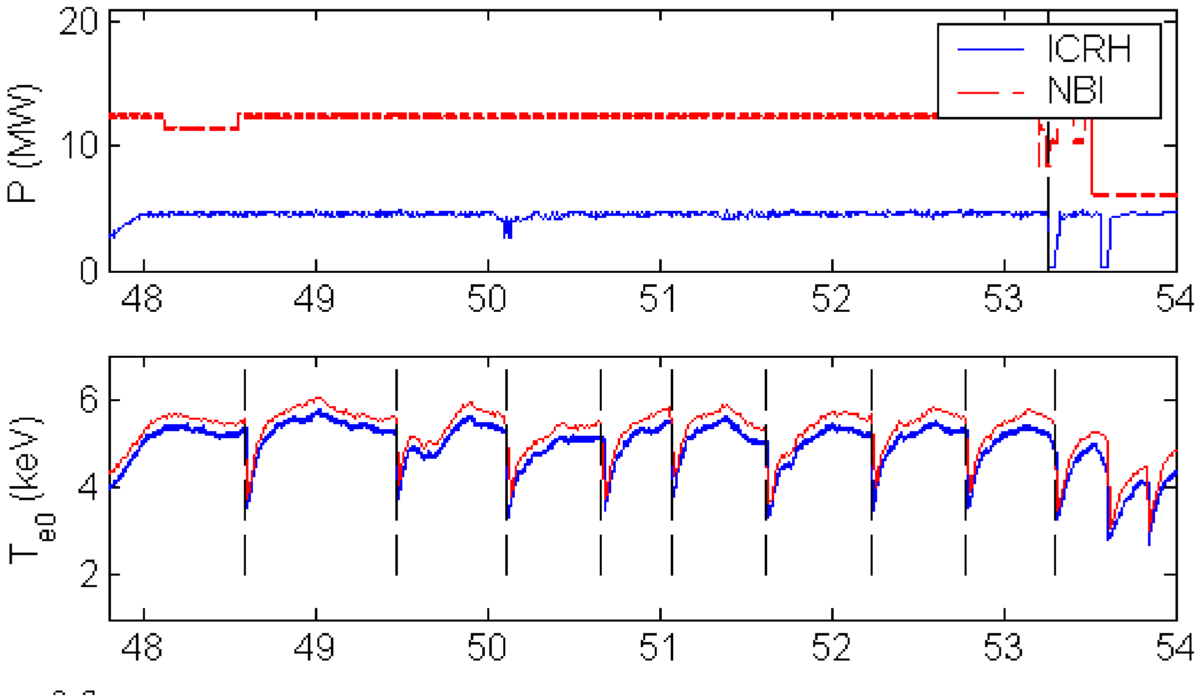
How to improve the analysis?

- The plasma is a non-linear and complex dynamical system. Consequently, when studying the interaction between two quantities, **the hypothesis that all the information flow is due by the considered single driver, i.e excluding any other mechanism for the analysis, is quite a strong one**.

- When new experiments are planned and performed, a few pulses are dedicated to repeat specific and already studied "scenarios" of plasma and are called "references pulses".

- Implicitly it means that they have hidden useful information and that can be used in dedicated analysis to better quantify the actual "information flow".

How to improve the analysis



- Considering the Satweeth paced experiment, a possibility lays in **conditioning the information transfer with the specific knowledge that sawteeth can occur also natually during a discharge**.

- Following the notation already used then, the idea has been to condition the information flow between the driver (ICRH, $\overrightarrow{i_i}$ or $i_n^{(k)}$) and the response system, (Te during the ICRH notching, $\overrightarrow{j_i}$ or $j_n^{(l)}$) with natural sawtheet (Te, $\overrightarrow{z_i}$ or $z_n^{(m)}$) of a reference pulse using two new tools:
  1. The Conditional Tranfer Entropy (CTE)
  2. The Conditional Joint Recurrence Plots

CTE

- The TE extension, the CTE[5] can be synthetized in terms of conditional mutual information as:

$$TE_{i\to j} = I\left(i_n^{(l)}; j_{n+1} | j_n^{(k)}\right) \quad \Rightarrow \quad \boldsymbol{CTE_{i\to j|z} = I\left(i_n^{(l)}; j_{n+1} | j_n^{(k)}, z_n^{(m)}\right)}$$

- Applied Methodology: ⇨
  - Two delays have to be considered at the same time:
    1. d_{ij} (Between the driver and the target)
    2. d_{zj} (Between the coupled system and the one used to condition the information flow)

The latter (d_{zj}) is choosen looking at the local maxima observed from the TE between "i" and "j"

In general, a clearer description of the interaction between the driver and the response variable emerges.

[5] J.T.Lizier, Chaos 20, 037109 (2010)

CJRP[6]

Conditional Joint Recurrence plot (CJRP)[6]

- To assess the occurrences between two systems (a driver "*i*" and a response observable "*j*"), sifting out spurious couplings between the response and a third one (an external perturbation "*z*") .

$$CJRP_{\boldsymbol{ij}|z} = \mathrm{JRP}_{\mathbf{ij}} \odot \left(1 - \mathrm{JRP}_{\mathbf{z}\boldsymbol{j}}\right)$$

where

$$JRP_{\boldsymbol{ij}} = \Theta\left(\varepsilon_{\#\mathbf{1}} - \left\|\vec{\boldsymbol{i}}_i - \vec{\boldsymbol{i}}_j\right\|\right) \cdot \Theta\left(\varepsilon_{\#\mathbf{2}} - \left\|\vec{\boldsymbol{j}}_i - \vec{\boldsymbol{j}}_j\right\|\right)$$

- $\mathrm{JRP}_{\mathbf{ij}}, \mathrm{JRP}_{\mathbf{zj}}$ are JRP bewteen considered quantities;
- "$\odot$" is the Hadamard product between JRPs

- $\boldsymbol{\epsilon}_{\#\mathbf{1,2}}$ in JRPs are key parameters:

The higher their values, the more elements equal "1" in the matrices of the JRPs. The lower the thresholds, the closer the points on the attractors have to be in order to count as recurrences.

[6] E.Peluso, "*A Refinement of Recurrence Analysis to Determine the Time Delay of Causality in Presence of External Perturbations*" *Entropy*, *22*(8), 865

$$CJRP_{ij|z} = \mathrm{JRP}_{\mathbf{ij}} \odot \left(1 - \mathrm{JRP}_{\mathbf{z}j}\right)$$

where

$$JRP_{ij} = \Theta\left(\varepsilon_{\#1} - \left\|\vec{\boldsymbol{i}}_i - \vec{\boldsymbol{i}}_j\right\|\right) \cdot \Theta\left(\varepsilon_{\#2} - \left\|\vec{\boldsymbol{j}}_i - \vec{\boldsymbol{j}}_j\right\|\right)$$

- Applied Methodology: ⇨
  - Reducing progressively $\varepsilon_{\#2}$, the number of occurrences between "*z*" and "*j*" decreases.
  - Only the points on the attractors of "*z*" that are close to those of "*j*" cause the Heaviside function to output a "1" and consequently a "0" in the matrix of $CJRP_{xy|z}$.
  - CJRP are easier to apply with respect to CTE;
  - RQA estimators (RR,L, DET,…) have to be used

[6] E.Peluso, "*A Refinement of Recurrence Analysis to Determine the Time Delay of Causality in Presence of External Perturbations*" *Entropy*, *22*(8), 865

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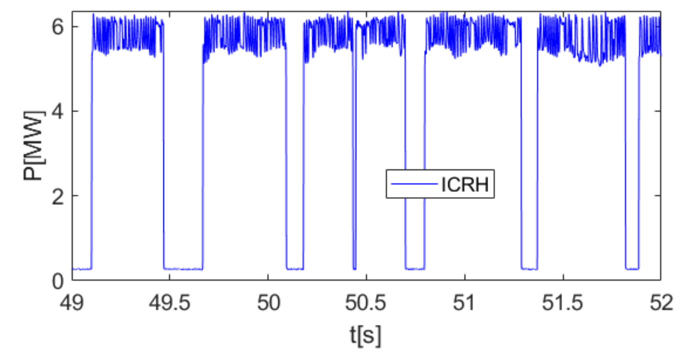
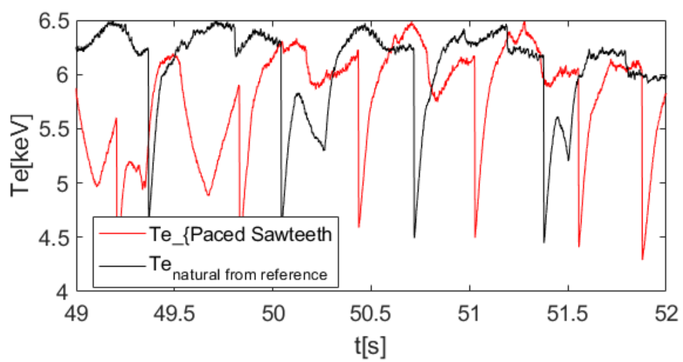
Analysis from M18-05 (*) in H mode phase

2.2MA

- 94087 (not optimal pacing, X[H] 4%);
  Almost pacing at 3Hz, natural S. observed; ICRH at 3Hz
- 94088 (not optimal pacing X[H]1.5%);
  Poor pacing. Most between natural (1.4Hz) and 3Hz; ICRH at 3Hz
- 94089 (excellent pacing X[H] 1.5%);
  Paced at ICRH of 2.5Hz
- 94090 (reference pulse X[H] 1.5%)
- 94091 (reference pulse X[H] 4%)





(*)E.Lerche et al, "M18-05 (1/5): Sawtooth control with ICRH modulation in H-mode (B15-06 cont.)", TF meeting 27th June 2019

Analysis from M18-05 in H mode: PION analysis

- For these three pulses the **1st H harmonic**, the **2nd D one** and electrons adds up to 99.9% of the damping power of the ICRH. Both resonances overlap at R=3m, i.e $\psi_{\text{norm}} \sim 0; 0.05$ (*)

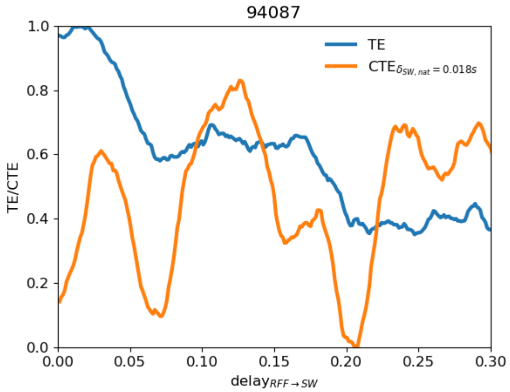
- The following table reports the PION estimates of the slowing down times of the Deuterium/Hydrogen fast ions on electrons

shot	$\boldsymbol{\tau_{eD}^{sd}}$ [ms]	$\boldsymbol{\tau_{eH}^{sd}}$ [ms]
94087 (ICRH 3Hz)	(531 ± 80)	(266 ± 40)
94088 (ICRH 3Hz)	(519 ± 50)	(260 ± 25)
94089 (ICRH 2.5Hz)	(468 ± 52)	(234 ± 26)

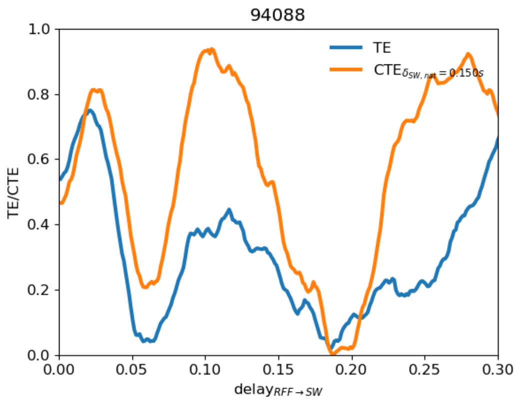
- The analysis that follows for the CTE and the CJRP has been done over one period of the ICRH, i.e $t_{delay} \in \left[0, \frac{1}{\nu_{ICRH}}\right]$

(*) Thanks to PION RO and code maintainers: David Taylor and Dani Gallart

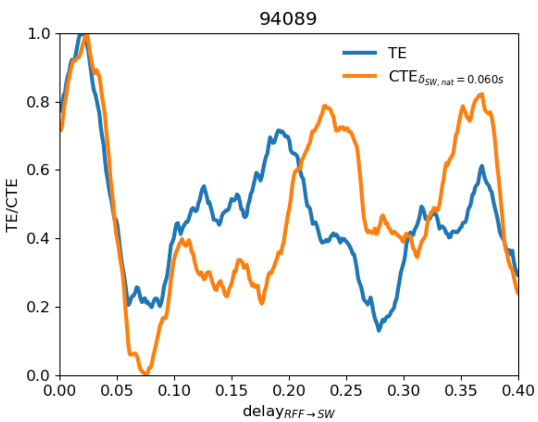
Results from CTE



- 94087: peaks at 240ms and 127ms emerge more clearly w.r.t the TE analysis where only the ~20ms one can be observed.

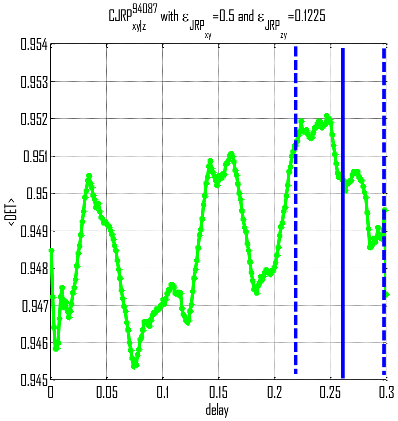


- 94088 peaks at 100ms and 255ms emerge more clearly w.r.t the TE analysis where only the ~20ms one can be observed;

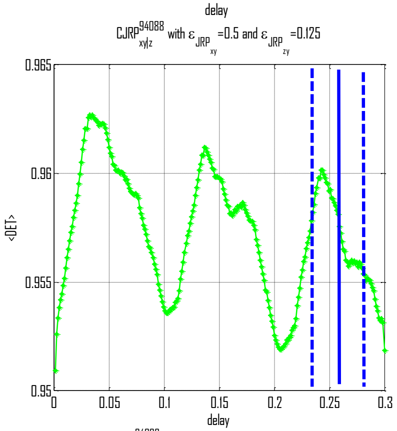


- 94089: peaks at 236 ms and 400 ms emerge more clearly w.r.t the TE analysis where only the ~20ms one can be observed and the peak at the H resonance is downshifted in time;

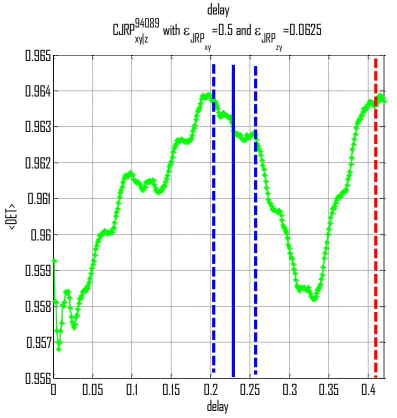
- Blue for τ_{eH}^{sd}; red for τ_{eD}^{sd}. Dashed lines for boundaries estimates ($\tau_{eH,eD}^{sd} \pm \sigma_{\tau_{eH,eD}^{sd}}$)



- 94087: the relevant peak the 1st harmonic has higher relevance w.r.t the previous ones detected.



- 94088: the relevant peak for the 1st harmonic has lower relevance w.r.t previous ones detected;



- 94089 shows the importance of the 2nd harmonic of D **and** of the 1st harmonic of H;

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Conclusions

- The CTE and the new CJRP can be used to extend a «Granger causality» analysis from a «bi-» to a «multi-dimensional» one.
- Especially the CJRP can be easily extended to more than three dimensions with a simple matricial calculation, allowing the inclusion of further quantities to condition the analysis

- CJRP and the CTE have been applied to sawteeth pacing with ICRH in H mode plasmas (JET M18-05 experiment) at JET.
- CTE and CJRP are in good agreement, but CTE results are in general more difficult to infer than the CJRP ones.

- Considering the different behaviours of the pulses analyzed:
- **94089**: the better coupling is thought to be due to the ICRH able to influence both the D fast ion population by the 2nd harmonic and the H one by the 1st harmonic, confirming [1][2].

- **94087 and 94088**: the not excellent coupling is thought to be due to the ICRH influencing only the H fast ion species;
- 94088: a worse coupling w.r.t to 94087 can be due to the different concentration used for the minorities (1.5% of 94088 w.r.t to 4% of 94087).

[1] E.Lerche et al, "M18-05 (1/5): Sawtooth control with ICRH modulation in H-mode (B15-06 cont.)", TF meeting 27th June 2019
[2] E.Lerche et al. "Sawteeth pacing with on-axis ICRH modulation in JET-ILW" , Nuclear Fusion, 57 036027

Thank you for your attention