On the Potential of Time Delay Neural Networks to identify Causality Graphs

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Introduction to Causality Detection

Causality is the influence that one time series (or an event) has on another time series (or another event)

Granger's Causality:

"We say that a variable X that evolves over time Granger-causes another evolving variable Y if predictions of the value of Y based on its own past values and on the past values of X are better than predictions of Y based only on Y's own past values."

Note: Correlation does not imply causality!

Example:

"If data shows that as the number of fires increase, so does the number of fire fighters. Therefore, to cut down on fires, you should reduce the number of fire fighters." - Pearl, Book of Why 2018

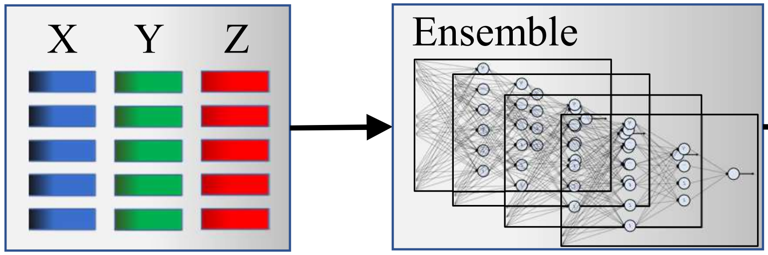
Causality detection by Time Delay Neural Network

- Concept of Causality used:

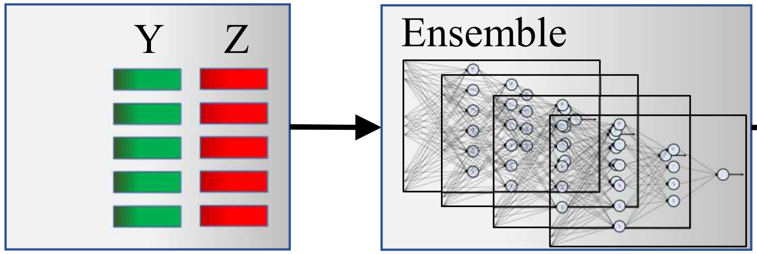
"A time series X causes another time series Y if when the past of X is removed from the possible drivers of Y, the prediction performances on Y degrade"

- The networks are run to predict the next time point

Algorithm logic



Prediction error with all possible drivers:
Median: $\overline{X_{all}}$
Standard deviation: σ_{all}



Prediction error removing the analysed driver:
Median: $\overline{X_k}$
Standard deviation: σ_k

Causality detection:

$$Z_{score} = \frac{\overline{X_k} - \overline{X_{all}}}{\sqrt{\sigma_k^2 + \sigma_{all}^2}}$$

$$if\ Z_{score} > Z_{threshold}$$

$$\downarrow$$

$$Causality = 1$$

Causality quantification:

$$R_{sigma} = \frac{Median(E_{yz,z})}{Median(E_{xyz,z})}$$

Types of influence tests (1)

The capability to detect different types of influences have been analysed.

General formulation of bivariate case:

$$x(i) = 0.5\,x(i-1) + \sigma_x(i)$$

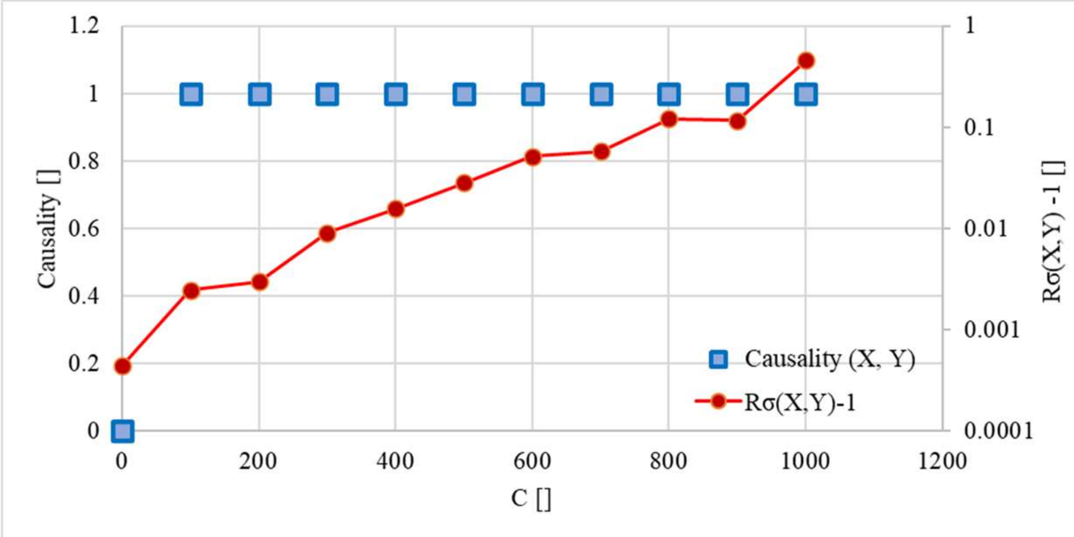
$$y(i) = \underline{f\big(x(i-1), y(i-1)\big)} + 0.7\,y(i-1) + \sigma_y(i)$$

Example: Multiplicative Quadratic Influence

$$x(i) = 0.5\,x(i-1) + \sigma_x(i)$$

$$y(i) = \underline{Cx(i-1)^2 y(i-1)} + 0.7\,y(i-1) + \sigma_y(i)$$

C: coupling constant



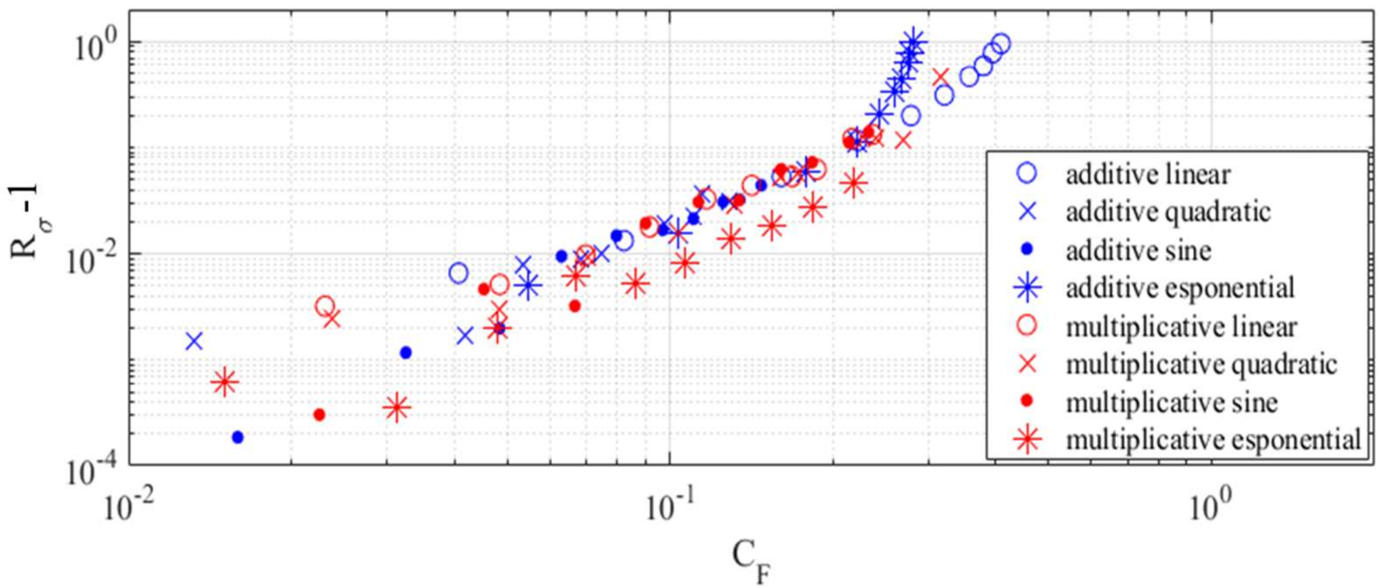
Types of influence tests (2)

The capability to detect different types of influences have been analysed.

Tests have been performed on: additional and multiplicative linear, quadratic, exponential and sine functions

Overall Causality detection and quantification performances:

$$\boldsymbol{R_{sigma} - 1\ vs\ dimensionless\ C\ (C_F)}$$



Comparison with the literature

Comparison with the methods analysed in the review "*A. Krakovská, Physical Review E 97, 042207 (2018)*"

Methods investigated:

1. Granger's vector autoregressive test (G)
2. Extended Granger test (EG)
3. Kernel Granger test (KG)
4. Conditional mutual information (CMI)
5. Cross mappings (CCM)
6. Predictability improvements (PI)

Systems investigated:

1. Coupled autoregressive models (AR models)
2. Hénon-Hénon
3. Rössler-Lorenz
4. Rössler-Rössler
5. Bidirectional two species
6. Fishery model
7. Mediated Link

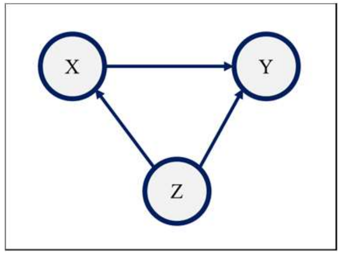
		G	EG	KG	CMI	CCM	PI	CauseNet - Ensemble
AR models	False Negative	0%	0%	0%	0%	0%	71%	0%
	False Positive	0%	0%	0%	0%	100%	0%	0%
Hénon-Hénon	False Negative	0%	0%	0%	0%	0%	0%	0%
	False Positive	65%	100%	0%	0%	88%	0%	0%
Rössler-Lorenz	False Negative	0%	0%	0%	0%	0%	0%	0%
	False Positive	87%	100%	87%	60%	87%	0%	6%
Rössler-Rössler	False Negative	0%	0%	0%	0%	0%	22%	0%
	False Positive	45%	100%	64%	18%	82%	0%	0%
Two-species	False Negative	0%	0%	0%	0%	0%	0%	0%
	False Positive	\\\	\\\	\\\	\\\	\\\	\\\	\\\
Fishery Model	False Negative	\\\	\\\	\\\	\\\	\\\	\\\	\\\
	False Positive	100%	0%	100%	100%	100%	0%	0%
Mediated Link	False Negative	0%	0%	0%	0%	0%	100%	0%
	False Positive	100%	100%	100%	0%	0%	0%	0%
Mean performances (averages of all systems)	False Negative	0%	0%	0%	0%	0%	32%	0%
	False Positive	66%	67%	59%	30%	76%	0%	1%

Some critical cases

We may have an unsteady state causality (causality changing with time)

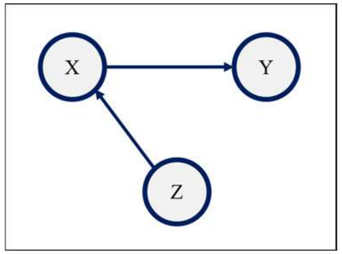
Confounder

$$x(i) = 0.5\,x(i-1) + 0.2\,z(i-2) + \sigma_x(i)$$
$$y(i) = Cx(i-1) + 0.7\,y(i-1) + 0.3\,z(i-2) + \sigma_y(i)$$
$$z(i) = 0.6z(i-1) + \sigma_z(i)$$



Mediator

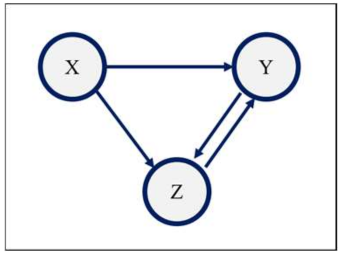
$$x(i) = 0.5\,x(i-1) + 0.2\,z(i-1) + \sigma_x(i)$$
$$y(i) = Cx(i-1) + 0.7\,y(i-1) + \sigma_y(i)$$
$$z(i) = 0.6z(i-1) + \sigma_z(i)$$



No errors, all these systems have been correctly detected

Feedback loop

$$x(i) = \sigma_x(i)$$
$$y(i) = 0.3x(i-1) + 0.25\,z(i-1) + \sigma_y(i)$$
$$z(i) = 0.6x(i-1) + 0.44\,y(i-1) + \sigma_z(i)$$

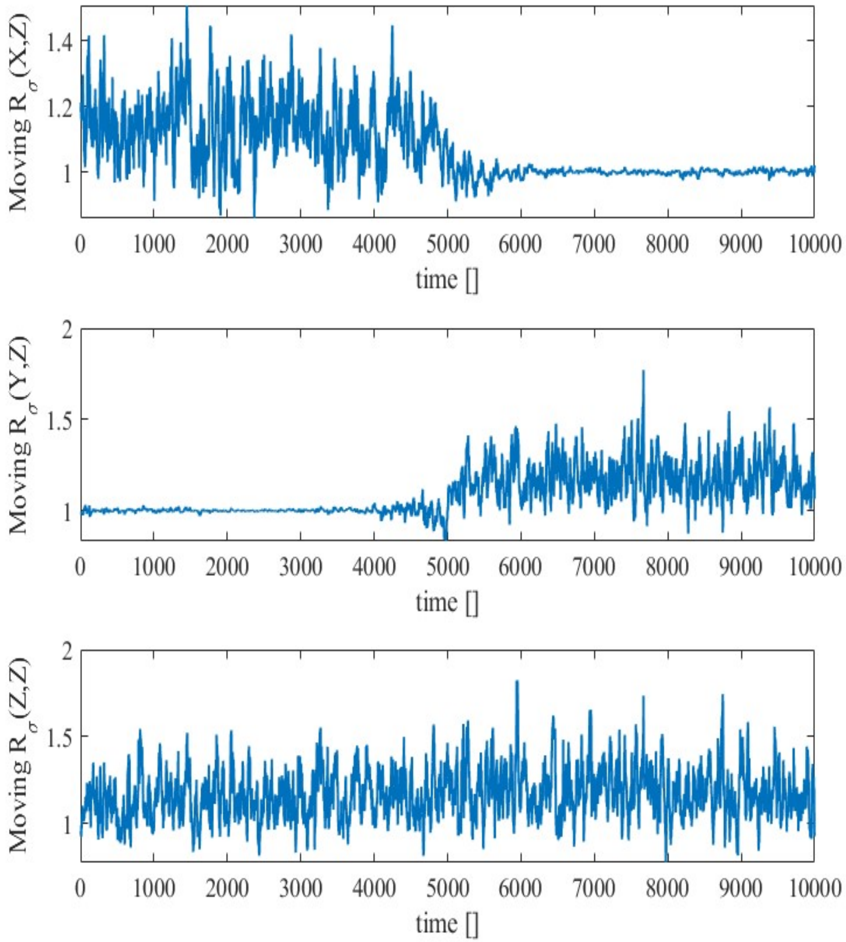


Critical cases: time varying causal influnces

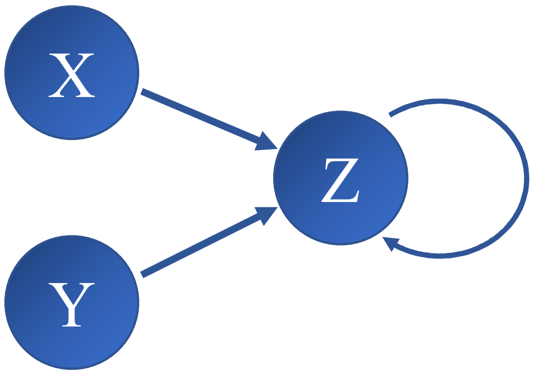
We may have causal influences changing with time (structural changes)

Example:

$$X(i) = 0.3\,X(i-1) + \sigma_X(i)$$
$$Y(i) = 0.7\,Y(i-1) + \sigma_Y(i)$$
$$Z(i) = 0.5\,Z(i-1) + \sigma_Z(i) + \begin{cases} 0.5\,X(i-1) & if\ i \le i_{change} \\ 0.5\,Y(i-1) & if\ i > i_{change} \end{cases}$$



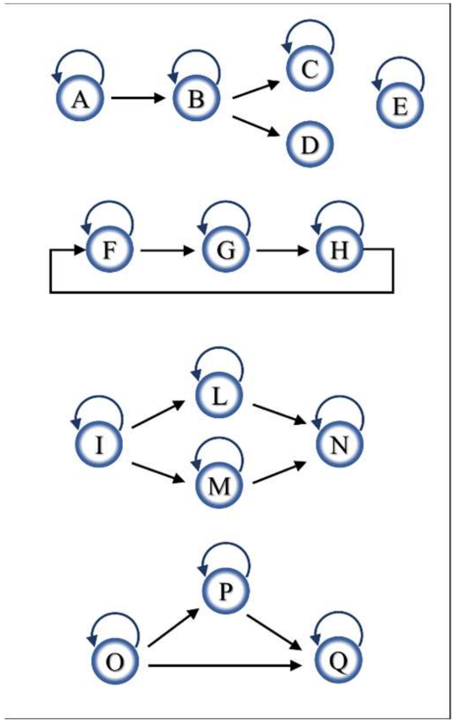
Causality is correctly identified:



A system of 15 time series:

$$A(i) = 0.9A(i-1) + \sigma_A(i)$$
$$B(i) = 0.8B(i-1) + 0.2A(i) + \sigma_B(i)$$
$$C(i) = 0.7C(i-1) + B(i-1)^2 + \sigma_c(i)$$
$$D(i) = e^{100B(i-1)} + \sigma_D(i)$$
$$E(i) = 0.5E(i-1) + \sigma_E(i)$$
$$F(i) = 0.5F(i-1) + 0.7H(i-1) + \sigma_F(i)$$
$$G(i) = 0.5G(i-1) - 0.2F(i-2) + \sigma_G(i)$$
$$H(i) = 0.7H(i-1) + 0.3|G(i-1)| + \sigma_H(i)$$
$$I(i) = 0.7I(i-1) + \sigma_I(i)$$
$$L(i) = 0.7L(i-1) - L(i-1)I(i-1) + \sigma_L(i)$$
$$M(i) = 0.7M(i-1) - 0.3I(i-1) + \sigma_M(i)$$
$$N(i) = 0.7N(i-1) + 10L(i-1)M(i-1) + \sigma_N(i)$$
$$O(i) = 0.5O(i-1) + \sigma_O(i)$$
$$P(i) = 0.5P(i-1) + 0.2O(i-1) + \sigma_P(i)$$
$$Q(i) = 0.5Q(i-1) + 0.3O(i-1) + 0.5P(i-2) + \sigma_Q(i)$$

Influenced \ Influencing	A	B	C	D	E	F	G	H	I	L	M	N	O	P	Q
A	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
C	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
D	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
F	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
G	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
H	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
I	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
L	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
M	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
N	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
O	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
Q	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1



Conclusions

Time Delay Neural Network Ensembles have clearly the capability to:

1. Correctly detect causality in several cases, with (potentially) all types of functionality
2. Detect feedback loops, confounders, mediators
3. Quantify causality
4. Provide information about the causality trend (as a function of time)
5. Deal with large number of time series

Thank you for your attention!

Appendix A – Z threshold

Parametric analyses have been performed to find the best Z threshold:

$$Z_{threshold,best} \sim 2$$