International Atomic Energy Agency

Application of Tomography methods in EAST Line-Integrated Radiation Diagnostics

L. Q. Xu[1], Y. M. Duan[1], Y. Chao[1,2], C.W. Mai[1,2], K.Y.Chen[1], E.Z.Li[1], H.X.Qu[1,2], S.Y.Lin[1], T.B.Wang[3] and L. Q. Hu[1]

[1]*Institute of Plasma Physics, Chinese Academy of Sciences, Hefei 230031, China*

[2]*University of Science and Technology of China, Hefei 230026, China*

[3]*Southwestern Institute for Physics, CNNC, Chengdu 610200, China*

30th Nov – 3th Dec 2021

Topical issues on Inverse Problem

4th IAEA Technical Meeting on Fusion Data Processing, Validation and Analysis

Outline

I. 2-D tomographic reconstruction

II. Tomography methods on EAST

- Abel method
- Fourier-Bessel method (SXR)
- Constrained optimization method (divertor XUV)
- L-curve Tikhonov method (GEM)
- Gaussian progress method (SXR, HX and XUV)
- Neural network method (CNN, FCNN and VGGNet)

III. Applications

- MHD mode structure analysis (by SVD)
- Impurity density estimation

IV. Summary and Outlook

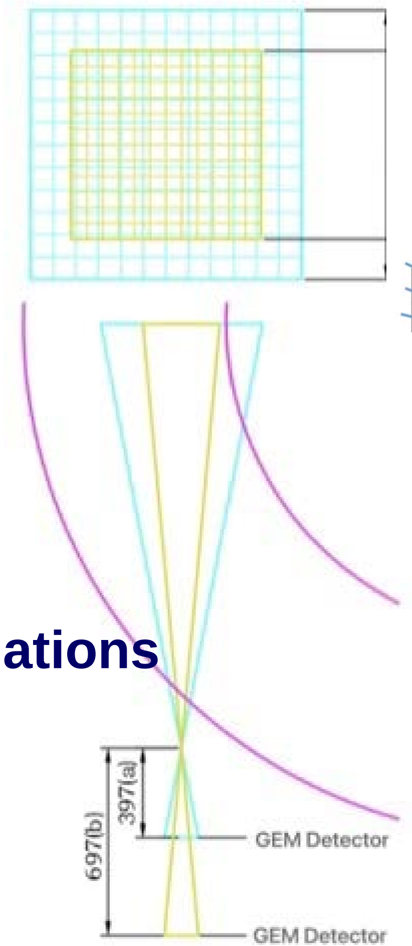
International Atomic Energy Agency

2-D tomographic reconstruction

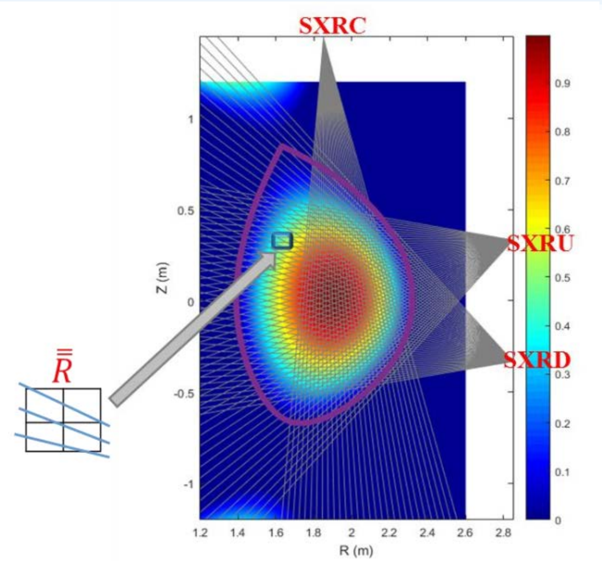
- **Line-Integrated diagnostics**
  - Soft X-ray
  - XUV/bolometer
  - Gas Electron Multiplier
  - Hard X-ray
  - Point
  - EUV, XCS…

- **Forward model**
  - More unknowns than equations
  - ill-posed problem

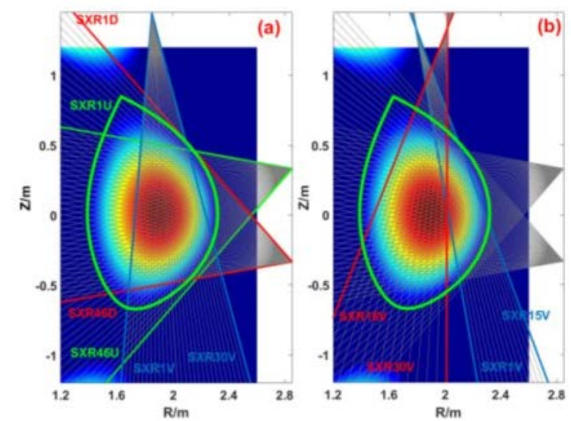
$$\bar{d}_M = \bar{\bar{R}}\bar{E}_N$$



GEM Tangential camera



LOS of SXR system



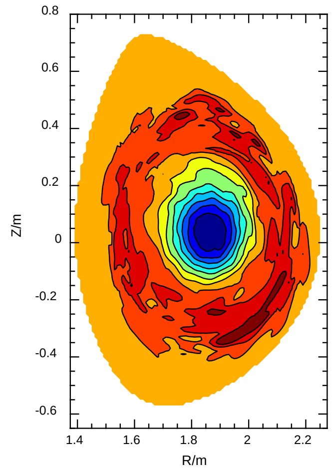
LOS of XUV system

Forward and inverse problems

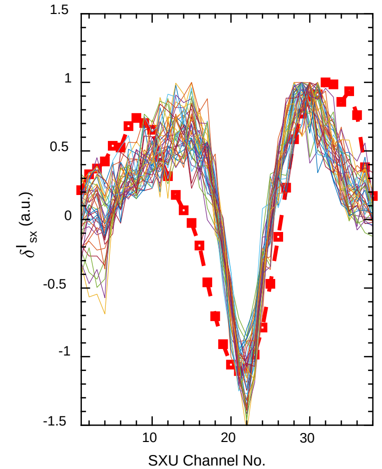
- **Forward problem**
  - From $\bar{\boldsymbol{E}}_N$ to $\bar{\boldsymbol{d}}_M$
  - Forward calculation
  - Possible distribution verification

$$\bar{d}_M = \bar{\bar{R}}\bar{E}_N$$

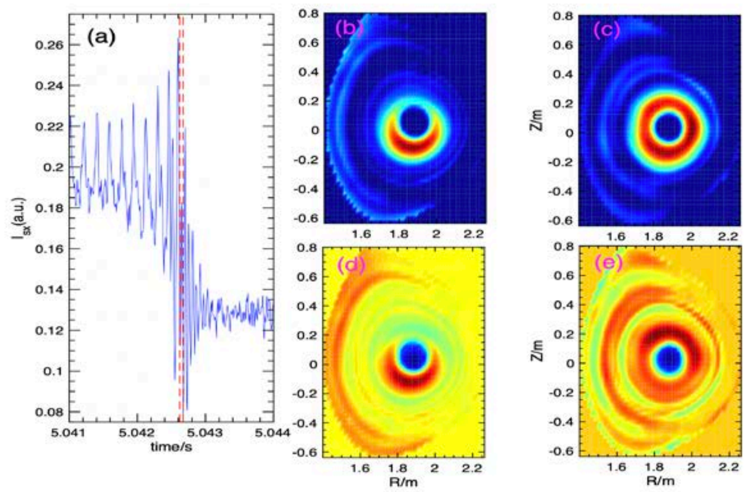
- **Inverse problem**
  - From $\bar{\boldsymbol{d}}_M$ to $\bar{\boldsymbol{E}}_N$
  - Indirect method
  - ill-posed problem, multi-solutions



possible distribution



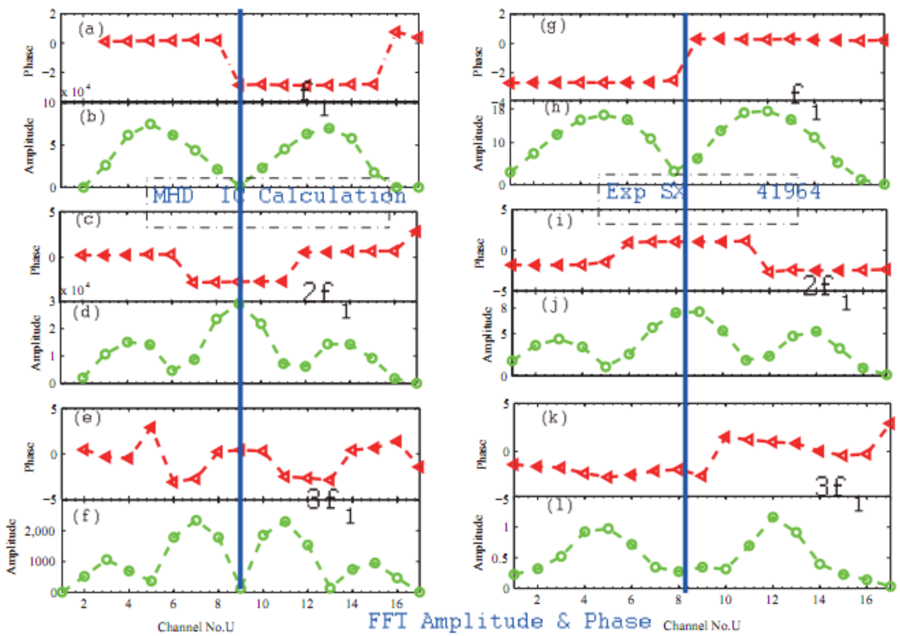
measurements & calculations



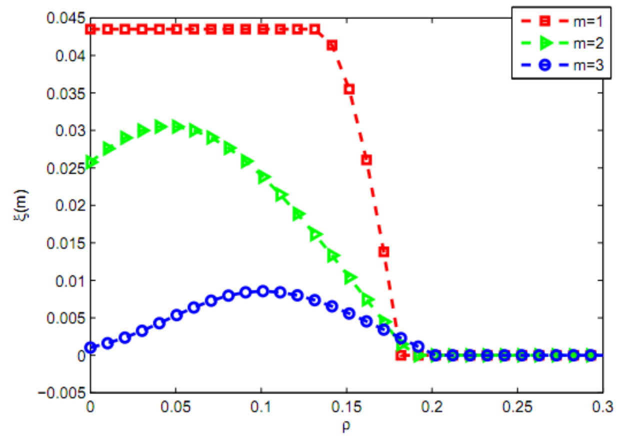
SXR reconstructions

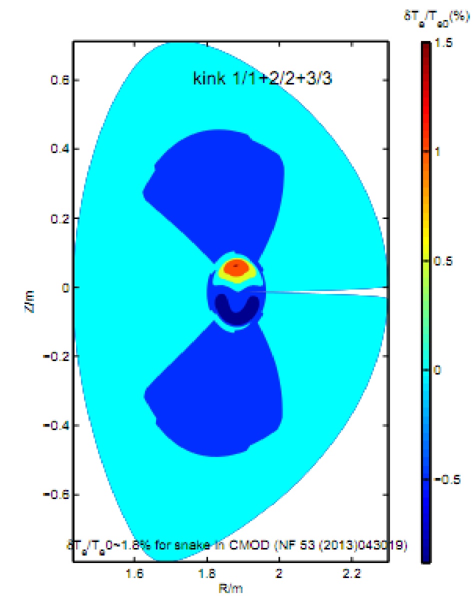
Forward method

- **Key issues**
  - **Plenty prior information** **(predicted by codes)**
  - **Plenty comparation of measurements and calculations** **(multi-arrays detection, perturbation amplitude and phase)**



amplitude vs. phase





hypothetical distribution(1-D/2-D)

Outline

I. 2-D Tomographic reconstruction

II. Tomography methods on EAST
- Abel method
- Fourier-Bessel method (SXR)
- Constrained optimization method (divertor XUV)
- L-curve Tikhonov method (GEM)
- Gaussian progress method (SXR, HX and XUV)
- Neural network method (CNN, FCNN and VGGNet)

III. Applications
- MHD mode structure analysis (by SVD)
- Impurity density estimation

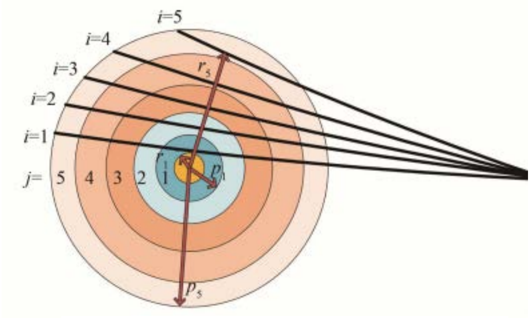
IV. Summary and Outlook

(1) Abel method

Abel model: $$E_j = \sum_i L_{ij}^{-1} B_i$$

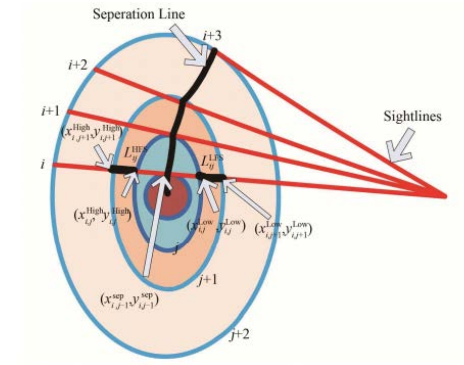
circular:

$$L_{ij} = \begin{cases} 2\sqrt{p_j^2 - r_i^2} & (p_j > r_i, i = j) \\ 2(\sqrt{p_j^2 - r_i^2} - \sqrt{p_{j-1}^2 - r_i^2} & (p_j > r_i, i \neq j) \\ 0 & (p_j > r) \end{cases}$$



non-circular:

$$L_{ij} = \begin{cases} \sqrt{(x_{i,j}^{\mathrm{High}} - x_{i,j-1}^{\mathrm{High}})^2 + (y_{i,j}^{\mathrm{High}} - y_{i,j-1}^{\mathrm{High}})^2} + \sqrt{(x_{i,j}^{\mathrm{Low}} - x_{i,j-1}^{\mathrm{Low}})^2 + (y_{i,j}^{\mathrm{Low}} - y_{i,j-1}^{\mathrm{Low}})^2} & (i < j) \\ \sqrt{(x_{i,j}^{\mathrm{High}} - x_{i,j}^{\mathrm{Low}})^2 + (y_{i,j}^{\mathrm{High}} - y_{i,j}^{\mathrm{Low}})^2} & (i = j) \\ 0 & (i > j) \end{cases}$$



Processes the 2-D problem to 1-D, too simple to applicate to SXR, XUV and HX complex tomography in Tokamak.

(2) Fourier-Bessel method (SXR)

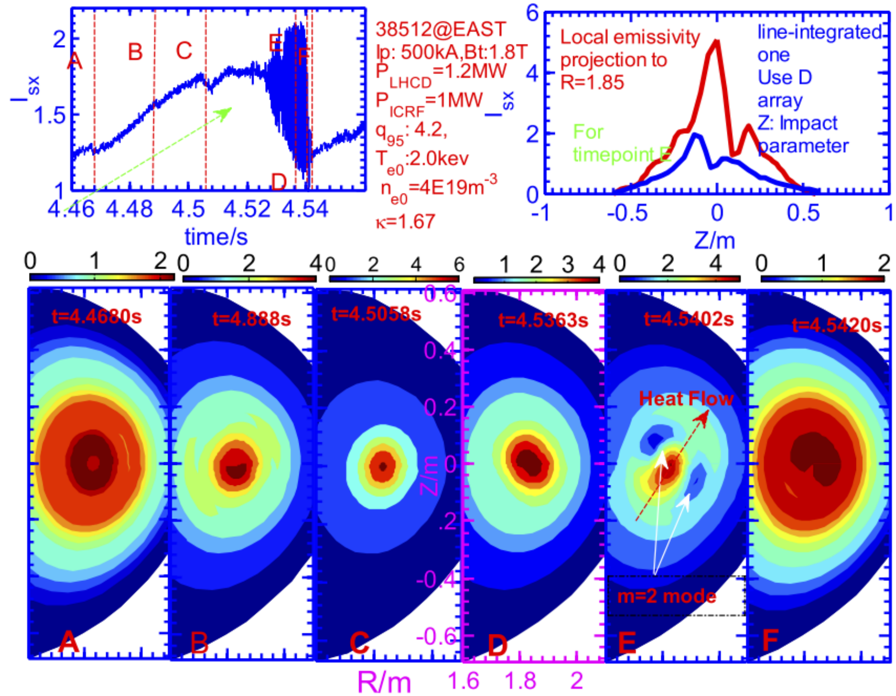
Angular Fourier expansion of emissivity:

$$g(r,\theta^*) = \sum_{m=0}^{M} [g_m^c(\rho)\cos(m\theta^*) + g_m^s(\rho)\sin(m\theta^*)]$$

Radial Bessel expansion of emissivity:

$$f = \sum_{m=0}^{M}\sum_{l=0}^{L}\int_L [a_{ml}^c(\rho)\cos(m\theta^*) + a_{ml}^s(\rho)\sin(m\theta^*)] J_m(\lambda_m^{l+1}\rho)dL$$

- **LFCS as boundary** **(emissivity=0)**
- **Suitable for SXR** **(core SXR emissivity and core MHD perturbation reconstruction)**
- **Unsuitable for XUV**
- **Smooth reconstructions**

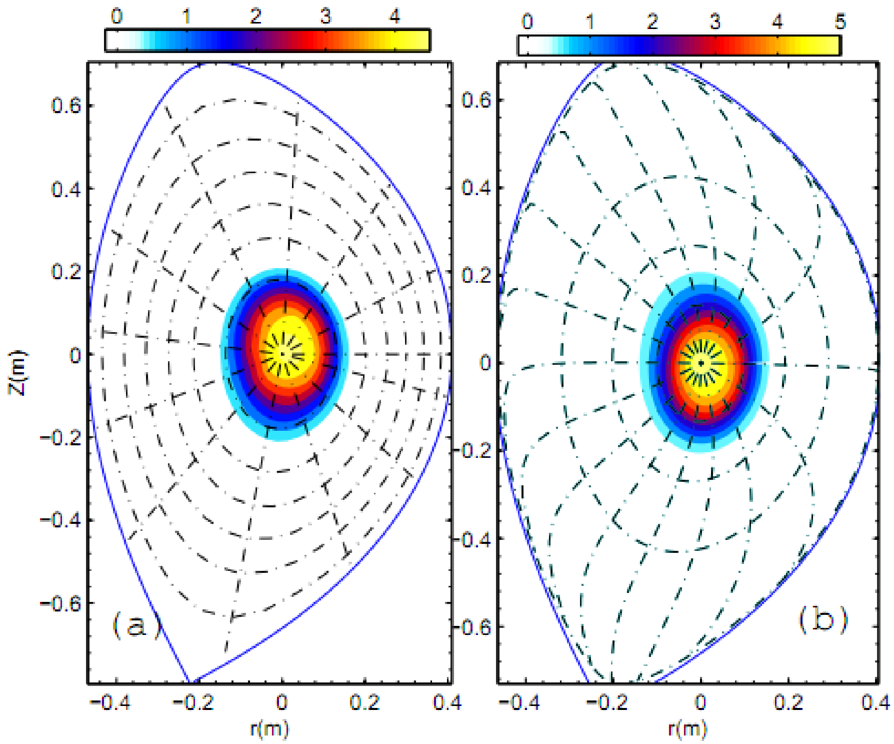


sawteeth

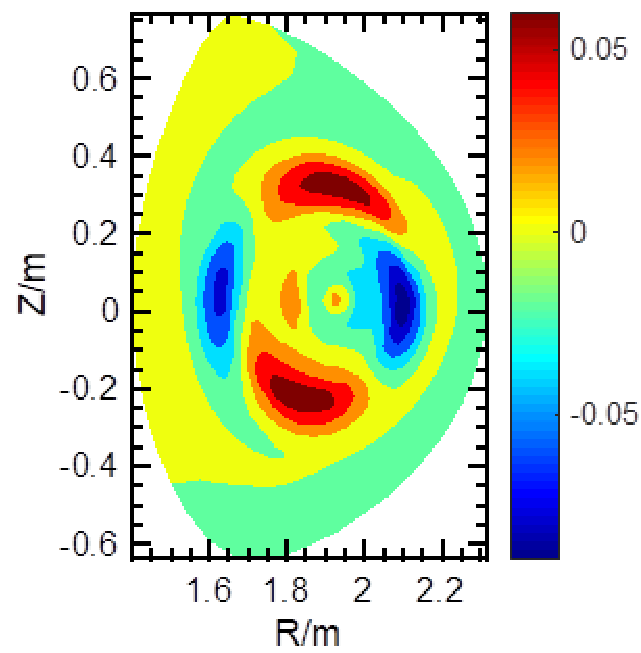
K. Chen, L. Xu#, et al., Rev. Sci. Instrum. 87 (2016) 063504

(2) Fourier-Bessel method (SXR)

- **Magnetic flux coordinates for non-circular configuration** **(to characterize polar angle and radius accurately).**
- **Assuming emissivity is a function of ψ.**
- **Suitable for reconstructing core SXR and peeking XUV emissivity.**



magnetic flux vs. Cartesian polar coordinates



mode m=2

(3) Constrained optimization method (divertor XUV)

Minimum problem:

$$\chi^2 = \left(\boldsymbol{Kg} - \boldsymbol{f}\right)^{T} \boldsymbol{W} \left(\boldsymbol{Kg} - \boldsymbol{f}\right) = 0$$

Regular matrix:

$$\phi = \frac{1}{2}\chi^2 + \alpha \mathfrak{R}$$

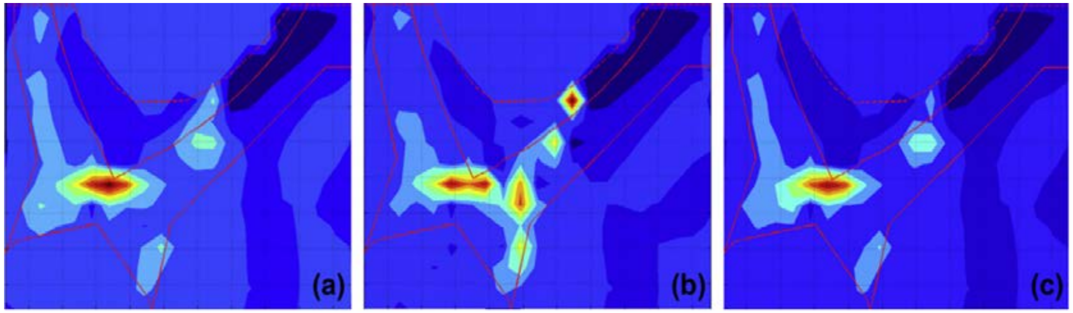
$$\mathfrak{R}(\boldsymbol{g}) = \iint \left\{ \nabla \bullet \left[\boldsymbol{D} \bullet \nabla g(x,y) \right] \right\}^2 dxdy$$
$$= \iint \left[\nabla \bullet \left(\boldsymbol{n} D_{\perp} \boldsymbol{n} \bullet \nabla g(x,y) + \boldsymbol{t} D_{//} \boldsymbol{t} \bullet \nabla g(x,y) \right) \right]^2 dxdy$$

$$\boldsymbol{H} = \left(\boldsymbol{C}_x \boldsymbol{D}_x + \boldsymbol{C}_y \boldsymbol{D}_y + \boldsymbol{C}_{xx} \boldsymbol{D}_{xx} + 2\boldsymbol{C}_{xy} \boldsymbol{D}_{xy} + \boldsymbol{C}_{yy} \boldsymbol{D}_{yy} \right)^{\mathrm{T}} \boldsymbol{F} \left(\boldsymbol{C}_x \boldsymbol{D}_x + \boldsymbol{C}_y \boldsymbol{D}_y + \boldsymbol{C}_{xx} \boldsymbol{D}_{xx} + 2\boldsymbol{C}_{xy} \boldsymbol{D}_{xy} + \boldsymbol{C}_{yy} \boldsymbol{D}_{yy} \right) \Delta x \Delta y + \boldsymbol{C}_0$$

$$F_{ij}^{(n)} = \begin{cases} \left(\bar{g}^{(n)} / g_j^{(n)} \right)^{\beta} \delta_{ij} & , \quad g_j^{(n)} \geq \gamma \bar{g}^{(n)} , \quad n > 0 \\ \delta_{ij} & , \quad g_j^{(n)} < \gamma \bar{g}^{(n)} , \quad n > 0 \end{cases}$$

C: diagonal matrix   D: matrix derivative

C_0: Constant matrix, to adjust the emissivity of a specific area



XUV reconstructions of diverter plasma during ELMs

Y. Duan, et al., J. Nucl. Meter.. 438 (2013) S228-S341

- **XUV radiation outside LFCS can be reconstructed (reconstructions will not be negative like the maximum entropy method).**
- **Regular matrix introduces two free parameters (complicated regular matrix)**
- **Equilibrium needed**
- **No large radial radiation gradient**

(4) L-curve Tikhonov method (GEM)

Introduce $\|\lambda C\varepsilon\|$:

$$(\boldsymbol{L^T L} + \lambda \boldsymbol{C^T C})\boldsymbol{\varepsilon} = \boldsymbol{L^T g}$$

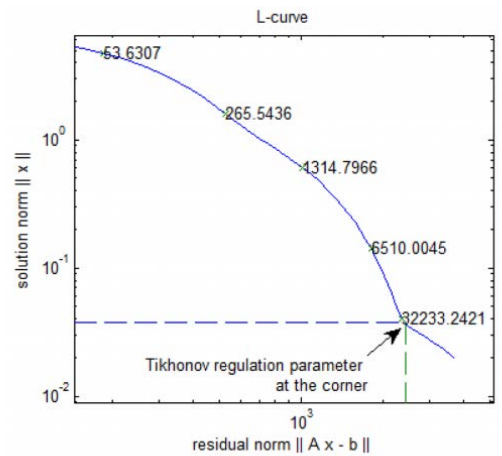
$$C_{i,j} = \begin{cases} -4 & (j = i) \\ 1 & (j = i \pm 1, j = i \pm nR) \\ 0 & \end{cases}$$

$$\rho^2 = \|\boldsymbol{L\varepsilon_\lambda} - \boldsymbol{g}\| \qquad \eta^2 = \|\boldsymbol{C\varepsilon_\lambda}\|$$

λ: free parameter   C: positive definite matrix

ρ^2: error caused by Positive definite matrix

η^2: error caused by ill-posed problem



L-curve method to determine λ

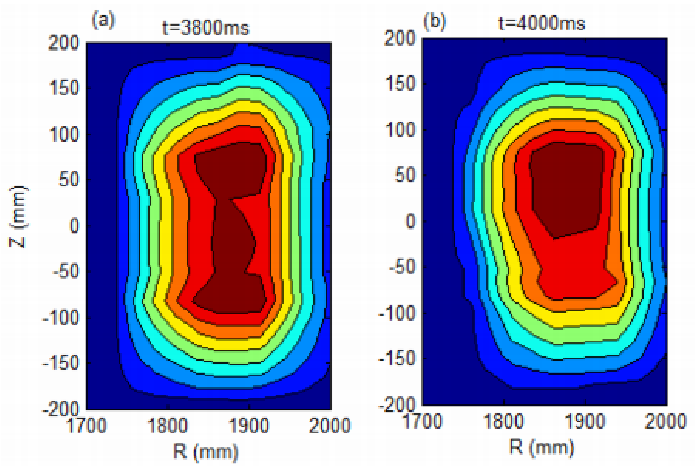
Positive definite term error: $\|Ax - b\|^2 = \lambda^4 \sum^n (\beta_i/(\gamma_i^2 + \lambda^2))^2$

Residual error: $\|x\|^2 = \sum_{i=1}^n (\gamma_i \beta_i/(\gamma_i^2 + \lambda^2))^2.$

$$(\boldsymbol{L} \cdot \boldsymbol{C}^{-1}) = \boldsymbol{U} \cdot \boldsymbol{\gamma} \cdot \boldsymbol{V}^T, \qquad \boldsymbol{\beta} = \boldsymbol{U}^T \cdot \boldsymbol{I},$$

(β and γ can be obtained by SVD)

- **Simple positive definite matrix (no need for equilibrium)**
- **λ determined by L-curve technology**
- **Radial radiation symmetry assumed (mapping 3D measurements to 2D cross- section)**



2-D core Fe impurities(GEM)

H. Qu, et al., Rev. Sci. Instrum. 90 (2019) 093507

(5) Gaussian progress method (SXR, HX and XUV)

Bayesian inference:

$$p(\boldsymbol{E}_{\mathrm{n}}|\boldsymbol{d}_{\mathrm{m}},\boldsymbol{\theta}) = \frac{p(\boldsymbol{d}_{\mathrm{m}}|\boldsymbol{E}_{\mathrm{n}},\boldsymbol{\theta})\cdot p(\boldsymbol{E}_{\mathrm{n}}|\boldsymbol{\theta})}{p(\boldsymbol{d}_{\mathrm{m}}|\boldsymbol{\theta})} \sim p(\boldsymbol{d}_{\mathrm{m}}|\boldsymbol{E}_{\mathrm{n}},\boldsymbol{\theta}) \cdot p(\boldsymbol{E}_{\mathrm{n}}|\boldsymbol{\theta})$$

posterior probability likelihood probability prior probability

Gaussian Process modelling:

prior probability

$$p(\boldsymbol{E}_{\mathrm{n}}|\boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{n}{2}}\left|\boldsymbol{\Sigma}_E{}^{prior}\right|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\left(\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{\mu}_E^{prior}\right)^T \boldsymbol{\Sigma}_E{}^{prior^{-1}}\left(\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{\mu}_E^{prior}\right)\right]$$

likelihood probability

$$p(\boldsymbol{d}_{\mathrm{m}}|\boldsymbol{E}_{\mathrm{n}},\boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{m}{2}}|\boldsymbol{\Sigma}_d|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(\boldsymbol{R}\cdot\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{d}_{\mathrm{m}})^T\, \boldsymbol{\Sigma}_d^{-1}\,(\boldsymbol{R}\cdot\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{d}_{\mathrm{m}})\right]$$

posterior probability

$$p(\boldsymbol{E}_{\mathrm{n}}|\boldsymbol{d}_{\mathrm{m}},\boldsymbol{\theta}) \sim \exp\begin{bmatrix} -\frac{1}{2}(\boldsymbol{R}\cdot\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{d}_{\mathrm{m}})^T\, \boldsymbol{\Sigma}_d{}^{-1}\,(\boldsymbol{R}\cdot\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{d}_{\mathrm{m}}) \\ -\frac{1}{2}\left(\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{\mu}_E^{prior}\right)^T \boldsymbol{\Sigma}_E{}^{prior^{-1}}\left(\boldsymbol{E}_{\mathrm{n}} - \boldsymbol{\mu}_E^{prior}\right) \end{bmatrix}$$

Uncertainty:

$$\boldsymbol{\Sigma}_E{}^{post} = \left(\boldsymbol{R}^T\boldsymbol{\Sigma}_d\boldsymbol{R} + \boldsymbol{\Sigma}_E{}^{prior^{-1}}\right)^{-1}$$

(covariance matrix of posterior)

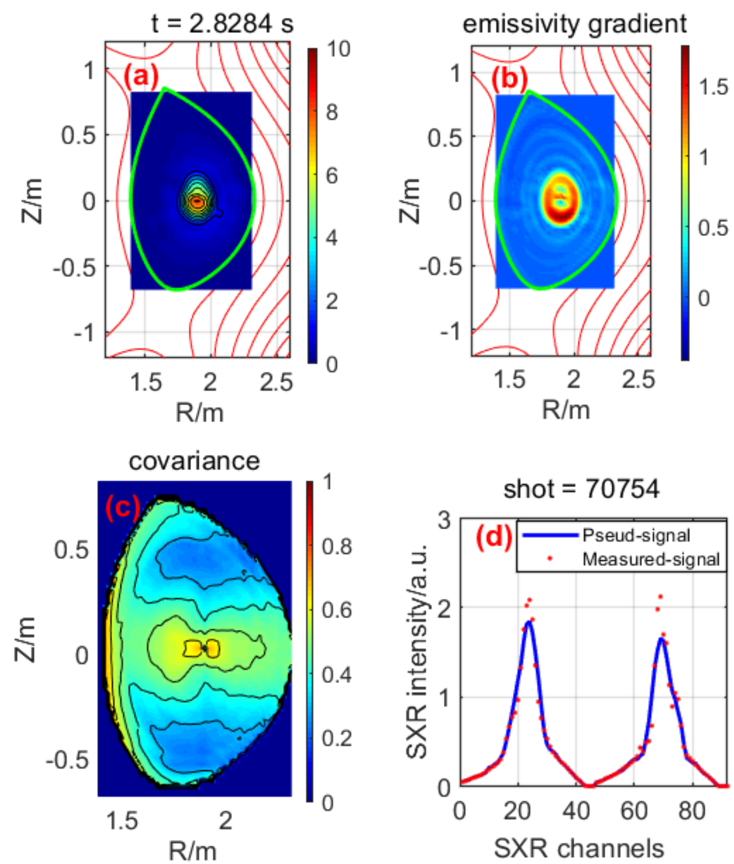
Emissivity inferred:

$$\boldsymbol{\mu}_E^{post} = \boldsymbol{\mu}_E^{prior} + \left(\boldsymbol{R}^T\boldsymbol{\Sigma}_d\boldsymbol{R} + \boldsymbol{\Sigma}_E{}^{prior^{-1}}\right)^{-1}\boldsymbol{R}^T\,\boldsymbol{\Sigma}_d{}^{-1}\left(\boldsymbol{d}_{\mathrm{m}} - \boldsymbol{R}\cdot\boldsymbol{\mu}_E^{prior}\right)$$

(mean vector of posterior)

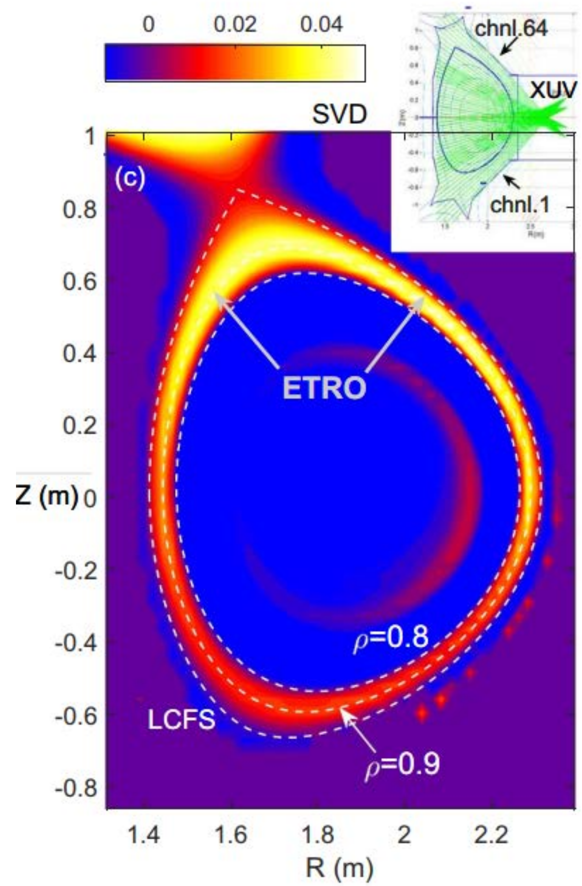
(5) Gaussian progress method (SXR, HX and XUV)

- **Uncertainty analysis & potential to real-time application**



MHD-SXR-GPT reconstruction

Y. Chao, L. Xu#, et al., Chin. Phys. B 29 (2020) 095201

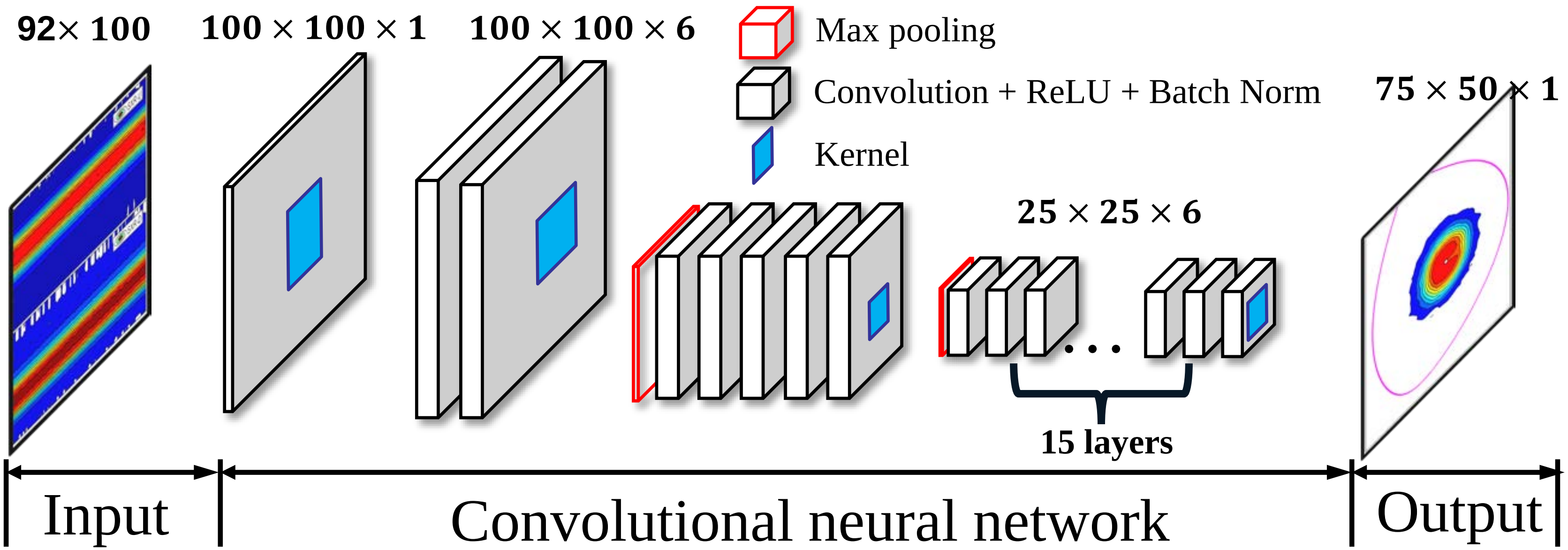


ETRO-XUV-GPT reconstruction

A. D. Liu et al., Nucl. Fusion 60 (2020) 126016

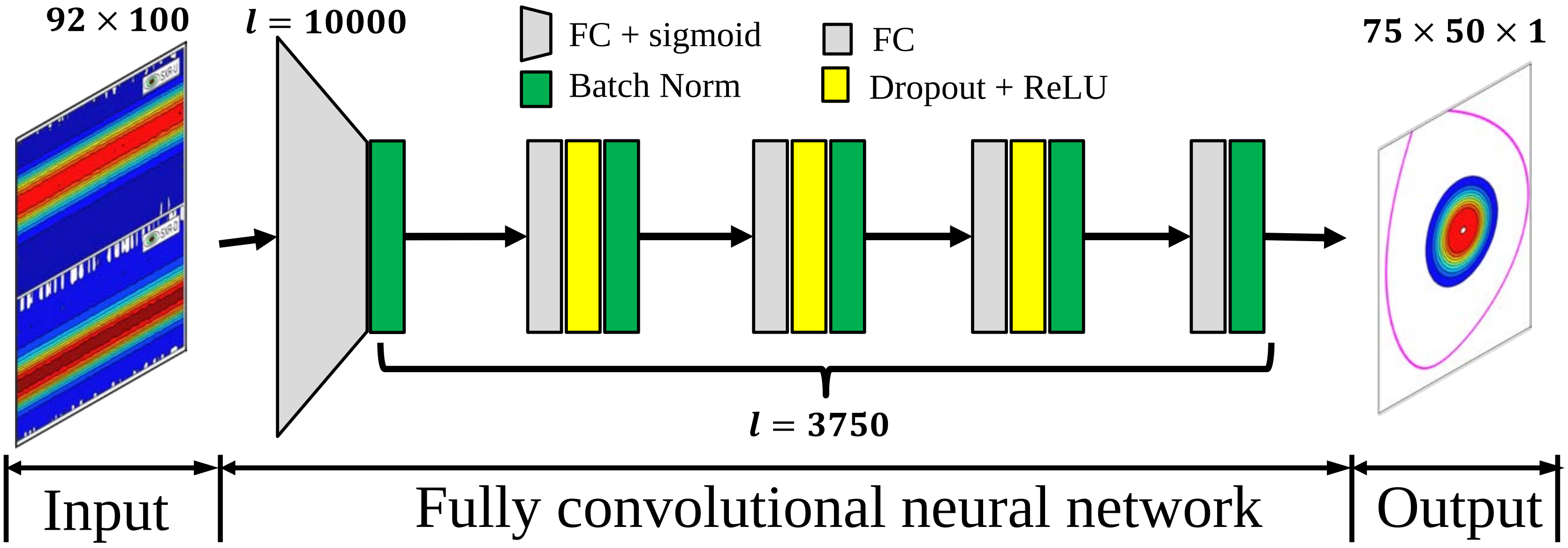
(6) Convolutional Neural Network (CNN)

- **CNN only consists of convolutional layers**
- **The number of parameters is $O(l)$** (l is the size of data)
- **First three layers: (1,1), (7,7), (7,7) others: (3,3) kernels**



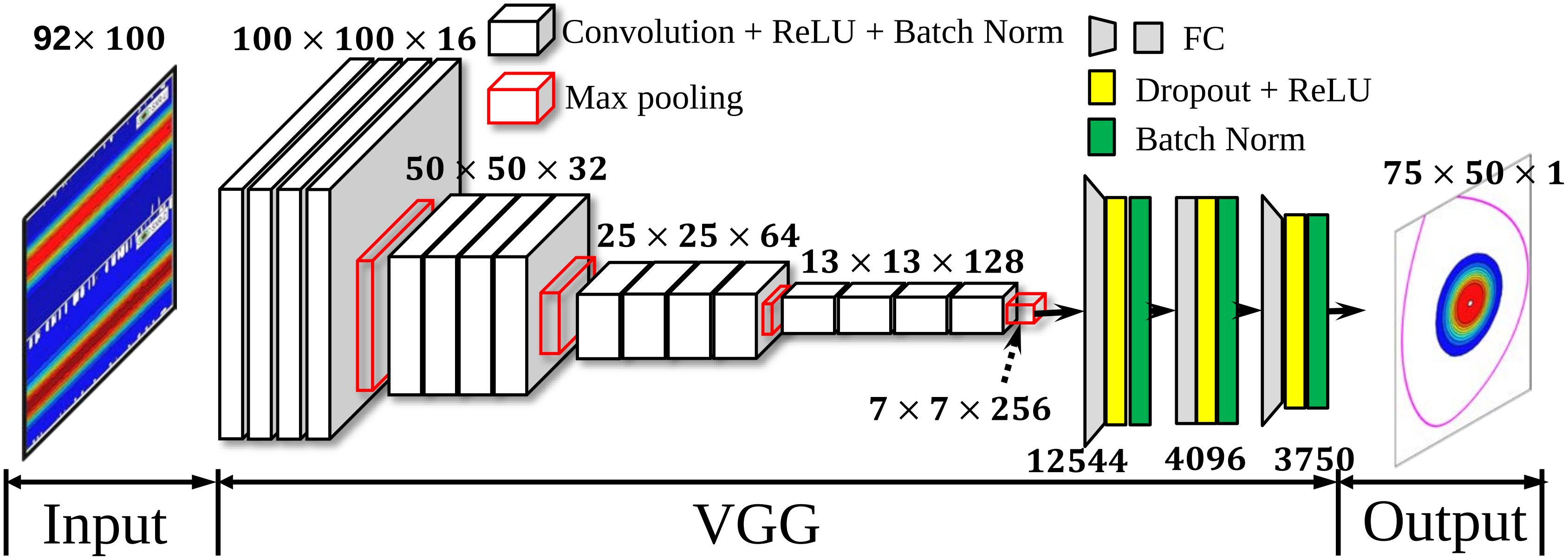
(6) Fully Convolutional Neural Network (FCNN)

- **FCNN consists of fully convolutional (FC) layers**
- **The number of parameters is $O(l^2)$** **(l is the size of data)**
- **Dropout layers are added after FC for optimization**



(6) Visual Geometry Group Net (VGGNet)

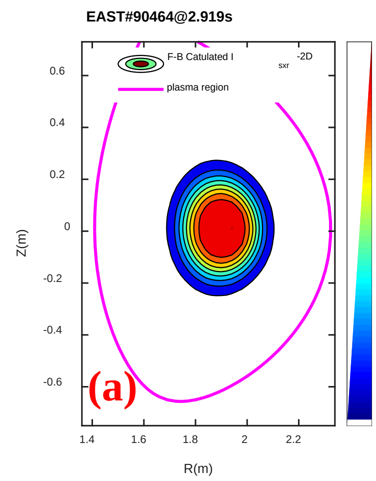
- VGGNet consists of FC and convolutional layers
- For convolution layers: (3,3) kernel (every 4 layers, pooling halve and the channel double)
- For FC layers, Dropout layers are added for optimization



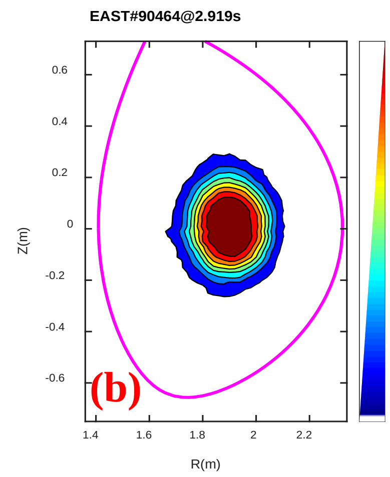
(6) Reconstruction results

- **Input layer: 92 SXR LOS measurements** **(with interaction dimension expansion)**
- $ReLU = \begin{cases} x, x > 0 \\ 0, x \leq 0 \end{cases}$ **(Rectified linear unit (ReLU))**
- **Output layer: 75×50 matrix** **(with ReLU activation)**, **SXR emissivity map**

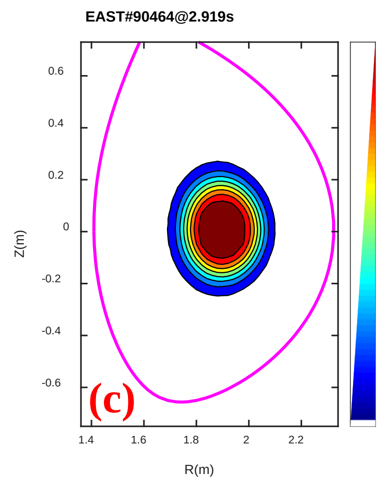
	CNN	FCNN	VGGNet	GPT
Tests	1000	1000	1000	\
Averaged Maximum Error	**29.3%**	**57.03%**	**46.96%**	\
RMSD	1.49×10^{-4}	4.086×10^{-5}	2.784×10^{-5}	\
Execution time	**3 ms**	**25.5 ms**	**34.9ms**	$\geq 47s$



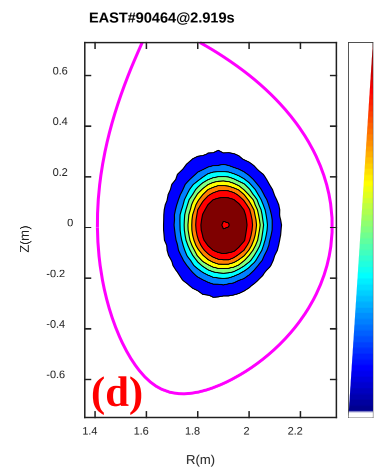
GPT



CNN



FCNN



VGGNet

Outline

I. 2-D Tomographic reconstruction

II. Tomography methods on EAST

- Abel method
- Fourier-Bessel method (SXR)
- Constrained optimization method (divertor XUV)
- L-curve Tikhonov method (GEM)
- Gaussian progress method (SXR, HX and XUV)
- Neural network method (CNN, FCNN and VGGNet)

III. Applications

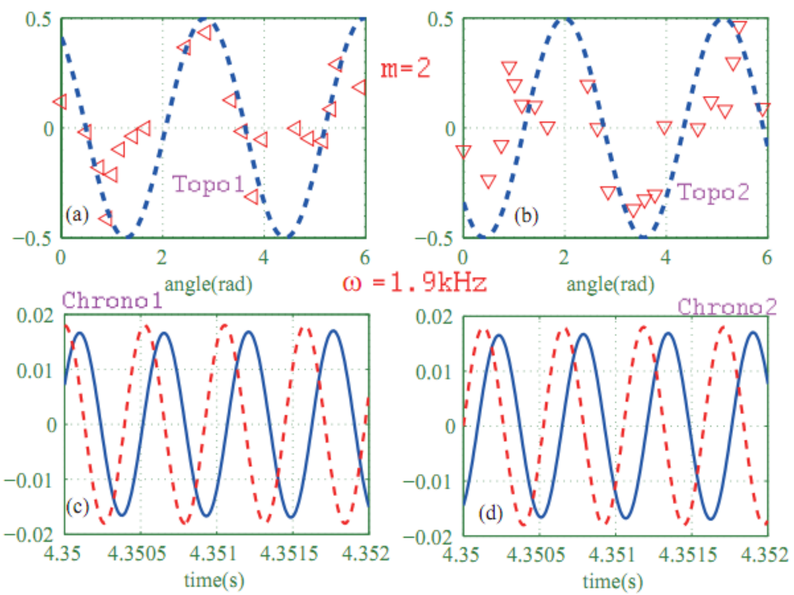
- MHD mode structure analysis (by SVD)
- Impurity density estimation

IV. Summary and Outlook

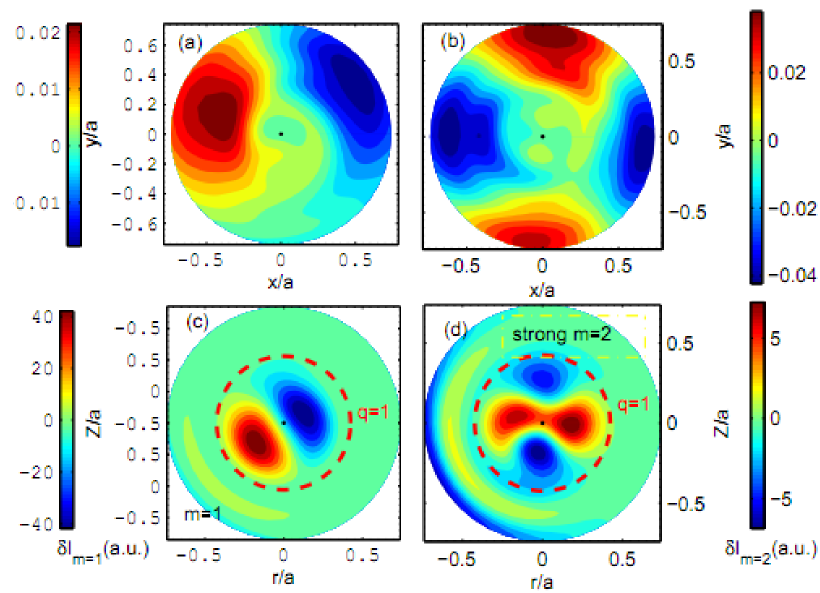
International Atomic Energy Agency

MHD mode structure analysis (by SVD)

- $A_{n\times p} = U_{n\times n} \cdot S_{n\times p} \cdot V^T_{p\times p}$ **($U_{n\times n}$ and $V_{p\times p}$ are orthogonal, $S_{n\times p}$ represents the weight of each topo/chrono)**
- **Applications: blurred image recognition/restoration, signal denoising and MHD mode structure analysis.**



spatial-temporal signals decomposed by SVD



MHD coupling components decomposed by SVD

L. Xu, et al., Chin. Phys. B 21 (2012) 055208

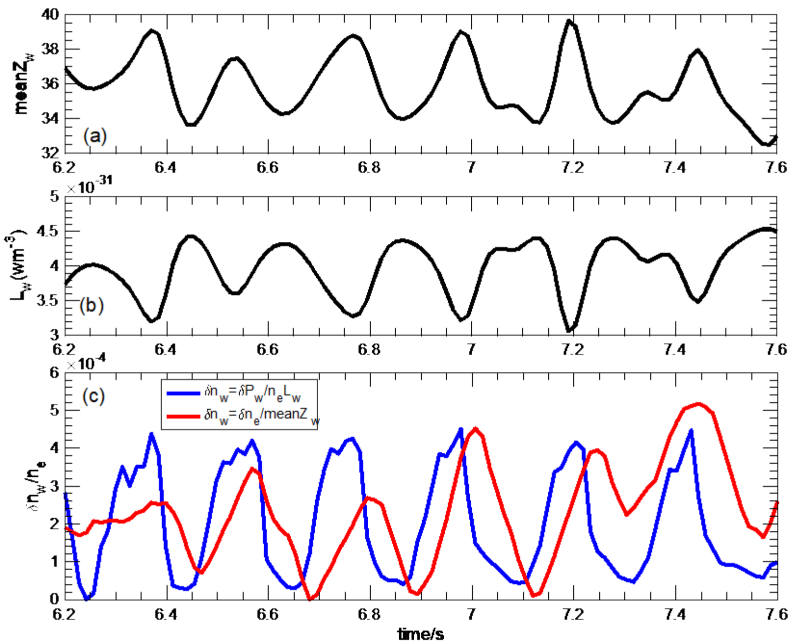
Impurity density estimation

- $\Delta n_W = \Delta n_e / Z_W$
- $\Delta n_W = \Delta p_W / (L_W n_e)$

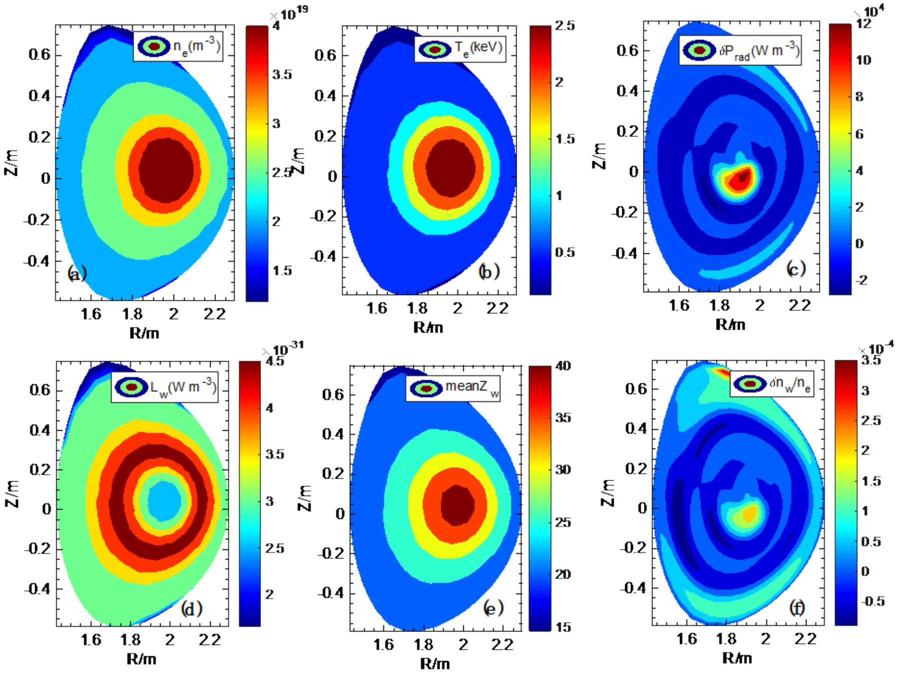
Δn_e measured by point

Δp_W reconstructed from XUV

Z_W and L_W from ADAS



impurity density evolution



2-D impurity density distribution

Outline

I. **2-D Tomographic reconstruction**

II. **Tomography methods on EAST**

- **Abel method**
- **Fourier-Bessel method (SXR)**
- **Constrained optimization method (divertor XUV)**
- **L-curve Tikhonov method (GEM)**
- **Gaussian progress method (SXR, HX and XUV)**
- **Neural network method (CNN, FCNN and VGGNet)**

III. **Applications**

- **MHD mode structure analysis (by SVD)**
- **Impurity density estimation**

IV. **Summary and Outlook**

International Atomic Energy Agency

Summary

- **A variety tomography methods have been developed on EAST (including Fourier-Bessel, constrained optimization, L-curve Tikhonov, Gaussian progress and neural network methods). The pros&cons and the application scope of these methods are analyzed.**

- **Developed MHD mode structure analysis method based on SVD, combined with tomography can recognize complicated MHD. The impurity density can be obtained by combining XUV and SXR reconstructions.**

- **The GPT method, which runs fast with uncertainty analysis, has been implemented to HX, SXR and XUV on EAST, supporting MHD and transport research.**

Outlook

- **Implement the GPT method to more diagnostics (such as neutron imaging diagnostic), and further optimize it for the SXR real-time tomography.**

- **Realize real-time core heavy impurity density estimation to support EAST H-mode long-pulse operation.**

- **Develop new tomography method for SXR and XUV (based on machine training and big data analytics, such as neural networks), to avoid the large radial XUV radiation gradient of reconstructions.**

Thank you for the attention!