EUROfusion

Observational causality detection for time series: from theory to practice

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Objectives of Scientific Research

- In general the objectives of any scientific endeavour are Prediction, Explanation and Control (PEC).

- Causality is an essential conceptual element of human understanding of the world and therefore one would expect it to be also central to the scientific process.

- On the contrary, it is difficult to find the word cause in most scientific and statistical books (certain statisticians such as K Pearson fought explicitly to ban the term and the concept).

Lack of Causality in Physics

Even the basic scientific notation does not make explicit allowance for causality.

Second Newton law can be written indifferently:

$$F=ma \qquad m=F/a \qquad a=F/m$$

- Statistical and machine learning tools have become very powerful at detecting associations (correlations) but their paradigms are blind to the distinction between correlation and causality.
- This because association is easier to detect than causation.

Correlation is not Causation

This talk is focussed on the epistemic of causation i.e. how we determine causal relations.

Association is descriptive: $P(Y|X)$

Causation is intrinsic linked to interventions: $P(Y|do(x))$

Correlation is not causation because in general :

$$P(Y|X) \neq P(Y|do(x))$$

Assume that N is needle of a thermometer and T the temperature of the environment.

$P(T|N=n)$ and $P(N|T=t)$ obey Bayes formula but $P(N|do(T=t))$ and $P(T|do(N=n))$ do not

Interventions do not obey Bayes law.

Causality and control

Statistics and Machine Learning detect only correlations. Neglecting the difference between association and causation can have catastrophic consequences for understanding and control.

Investigation of Y given X and Z (assuming values between 0 and 1)

$Y \leftarrow 0.5X + E_y$ where E_y is the additive error affecting Y

$Z \leftarrow Y + E_z$ where E_Z is the additive error affecting Z

If Z has comparable SNR to X, the best regression is:

$$Y = 0.25\,X + 0.5\,Z$$

Observational Causality Detection

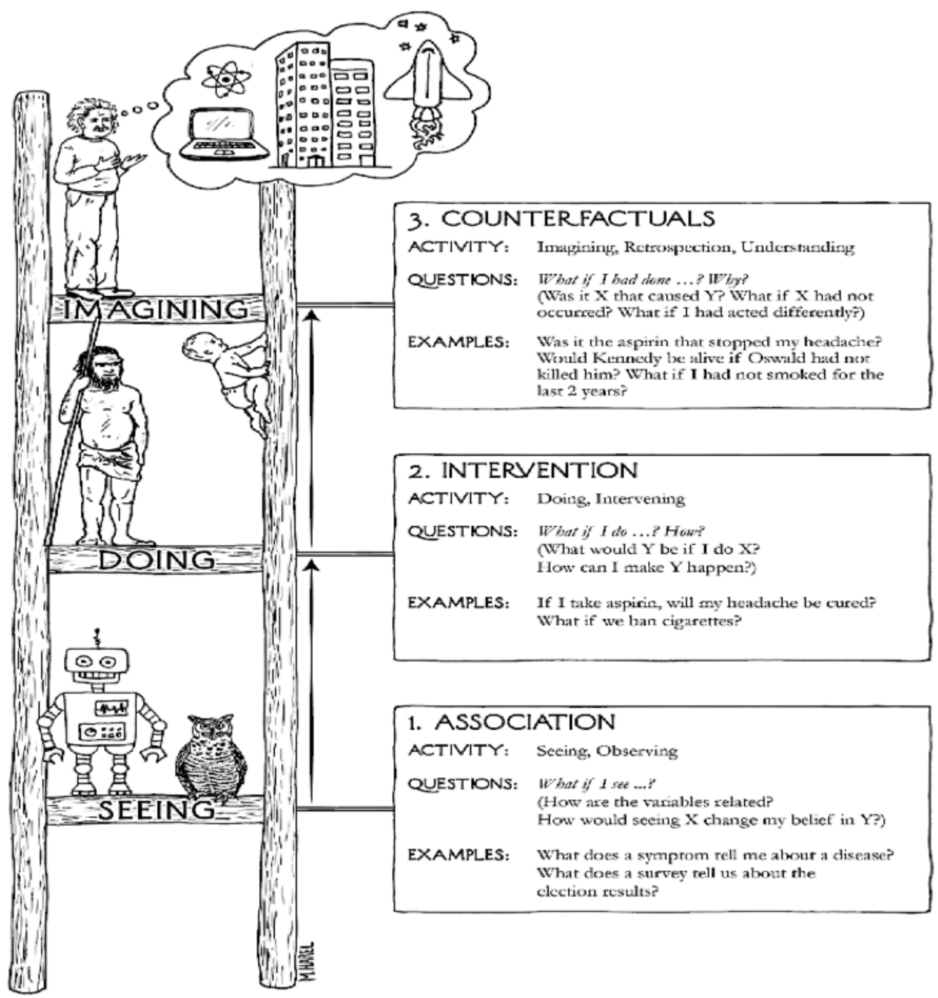
- Observational Causality Detection (OCD) is meant to derive as much information as possible about the causal relations between quantities from the analysis of data.

- It becomes essential when experiments are impossible, impractical or unethical. Moreover, OCD can also contribute to their design.

- In the last decades a lot of work has produced fantastic results about observational causality detection for cross sectional data. In many fields of science though the data are typically time series.

Causality and Data Analysis

<u>The ladder of causality (J.Pearl The book of why)</u>



It is not possible to determine causality only from data analysis: interventions (experiments) are indispensable.

On the other hand, good analysis tools are essential.

Association techniques to extract robust indications about causality have been recently developed (C.Granger)

Given the difficulties of the task a <u>multipath approach is proposed: convergence of different numerical methods</u>

Causality Detection for Time series

- Detecting causal influences between time series can be very challenging. On the other hand, explicit time ordering is very important and increases enormously the potential of OCD.
- The main techniques belong to the following categories:
    - Granger Causality
    - Causal Graphical Models
    - Statistical Analysis including memory
    - Analysis of the recurrences in phase space
    - Properties of the attractors
    - Conversion of time series into complex networks
    - Neural networks of specific topology

Kullback-Leibler divergence

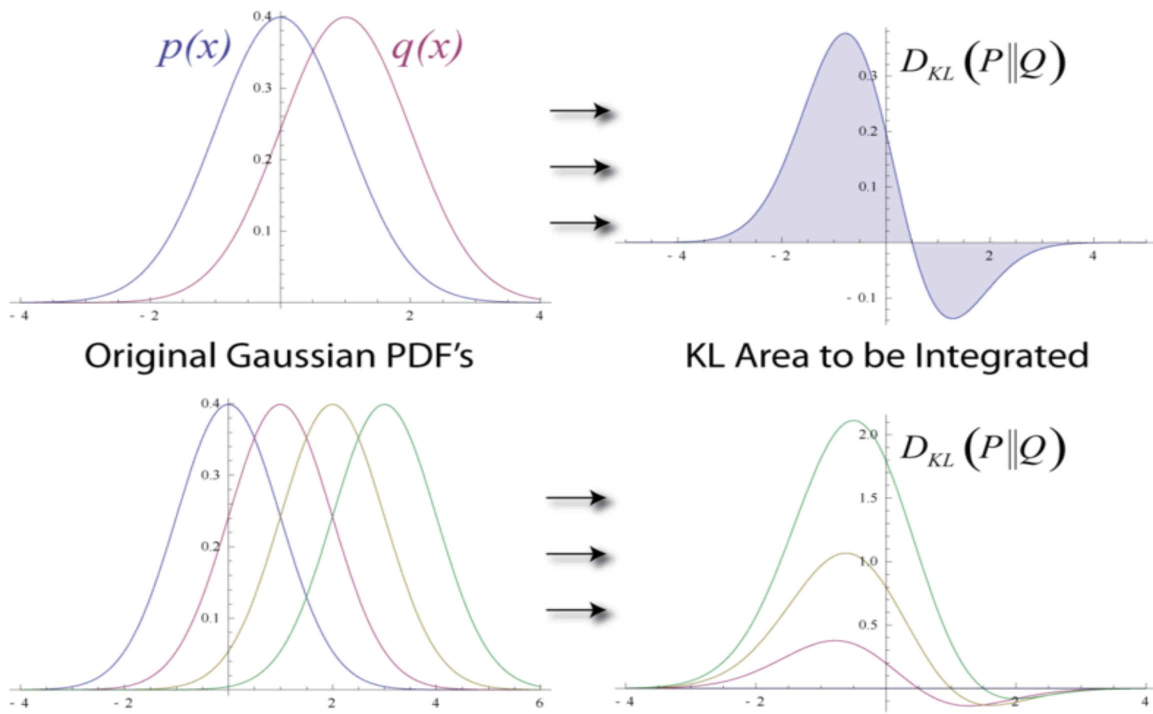
- For distributions *P* and *Q* of two continuous random variables KL-divergence is defined to be the integral:

$$KLD(P||Q) = \int p(x) \cdot \ln\left(\frac{p(x)}{q(x)}\right) dx$$

where *p* and *q* denote the densities of *P* and *Q*.

- The Kullback–Leibler divergence is always non-negative and zero if and only if *p* = *q*.

- The smaller the KLD the closer the densities p and q



Conditional Mutual Information, Transfer Entropy

- Introducing the formalism of the Markov processes, the two observables to be analysed can be written as
  $y_n^{(k)} = (y_n, \dots, y_{n-k+1})$ and $x_n^{(l)} = (x_n, \dots, x_{n-l+1})$.

- Then the dynamical structure of the relationship between the two can be investigated using the Transfer Entropy, defined as:

$$T_{X\to Y} = \sum_n p\left(y, y, x_n^{(l)}\right) \ln\left(\frac{p\left(y_{n+1}|y_n^{(k)}, x_n^{(l)}\right)}{p\left(y_{n+1}|y_n^{(k)}\right)}\right)$$

The main characteristic of the TE are:

- The more *X* influences *Y*, the higher the TE
- It is time asymmetric, i.e the $TE_{X\to Y} \neq TE_{Y\to X}$.
- TE takes into account <u>the past history of the signals</u>

See talks by T. Craciunescu and E. Peluso

Recurrence Plots

A recurrence plot (RP) is a plot showing the times at which the phase space trajectory of a dynamical system visits roughly the same area in the phase space

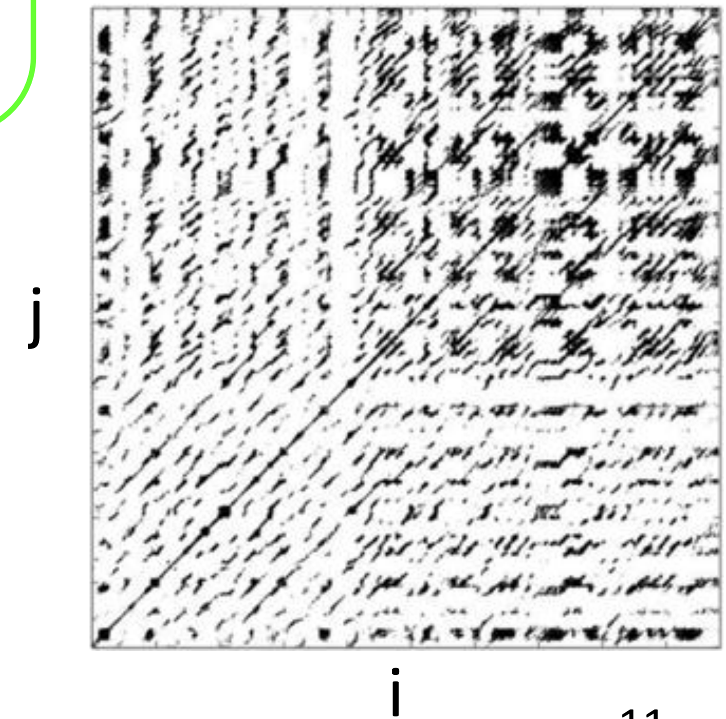
$$R_{ij} = \Theta\left(\varepsilon - \left\|\vec{x_i} - \vec{x_j}\right\|\right), \quad \text{i,j=1,…,N}$$

$\vec{x_i}$, $\vec{x_j}$ - points in phase space at times $\boldsymbol{i}$ and $\boldsymbol{j}$

ε is a predefined threshold

$\Theta(x)$ is the Heaviside function

RPs are based on the matrix representation



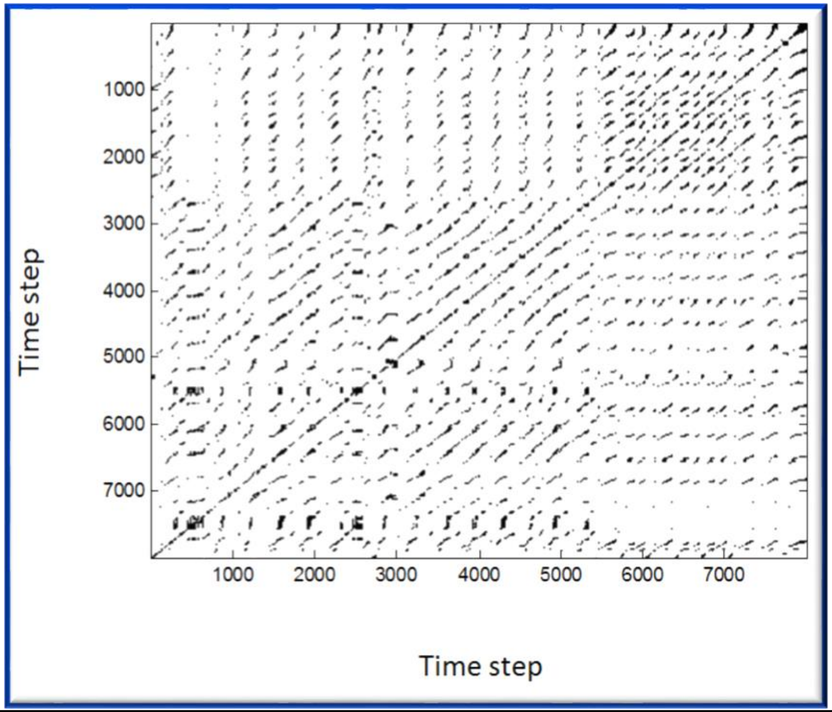
Hadamard product of the recurrence plots of the considered sub-systems

$$JR(i,j) = \Theta\left(\varepsilon_x - \left\|\vec{x_i} - \vec{x_j}\right\|\right) \cdot \Theta\left(\varepsilon_y - \left\|\vec{y_i} - \vec{y_j}\right\|\right)$$

- Compare the simultaneous occurrence of recurrences in two (or more) systems

- Joint recurrence plots can be used to detect causality and phase synchronisation.

Recurrence Quantification Analysis (RQA)



- The ***entropy of the diagonal lengths***:

$$ENTR = -\sum_{l=l_{min}}^{N} p(l) \cdot \ln[p(l)]$$

➢ Gives a measure of how much information is needed to recover the system and it reflects the complexity of the RP with respect to the diagonal lines.

- Single isolated points correspond to states with a rare occurrence, do not persist of they are characterized by high fluctuation
- Vertical/Horizontal lines correspond to states which do not change significantly during a certain period of time
- Diagonal lines occur when the trajectory visits the same region at different times and a segment of the trajectory runs parallel to another segment.
- Long diagonal structures correspond to similar time evolution of the two processes.

See talk by E. Peluso

Granger Causality: causality as predictivity

For time series, Norbert Wiener proposed a concept of causality based on prediction: <u>if the knowledge of the past of signal X helps predicting signal Y beyond what can be done knowing only the past of Y, then X is considered causal to Y</u>.

This approach was formalised by Clive Granger and is called Granger causality. It is based on increased predictability

The original Granger formalism is only valid for linear systems. (system for which the autoregressive models shown before are appropriate models of the time evolution).

TDNNs extend the application of the concept of causality to non linear systems and multiple causality.

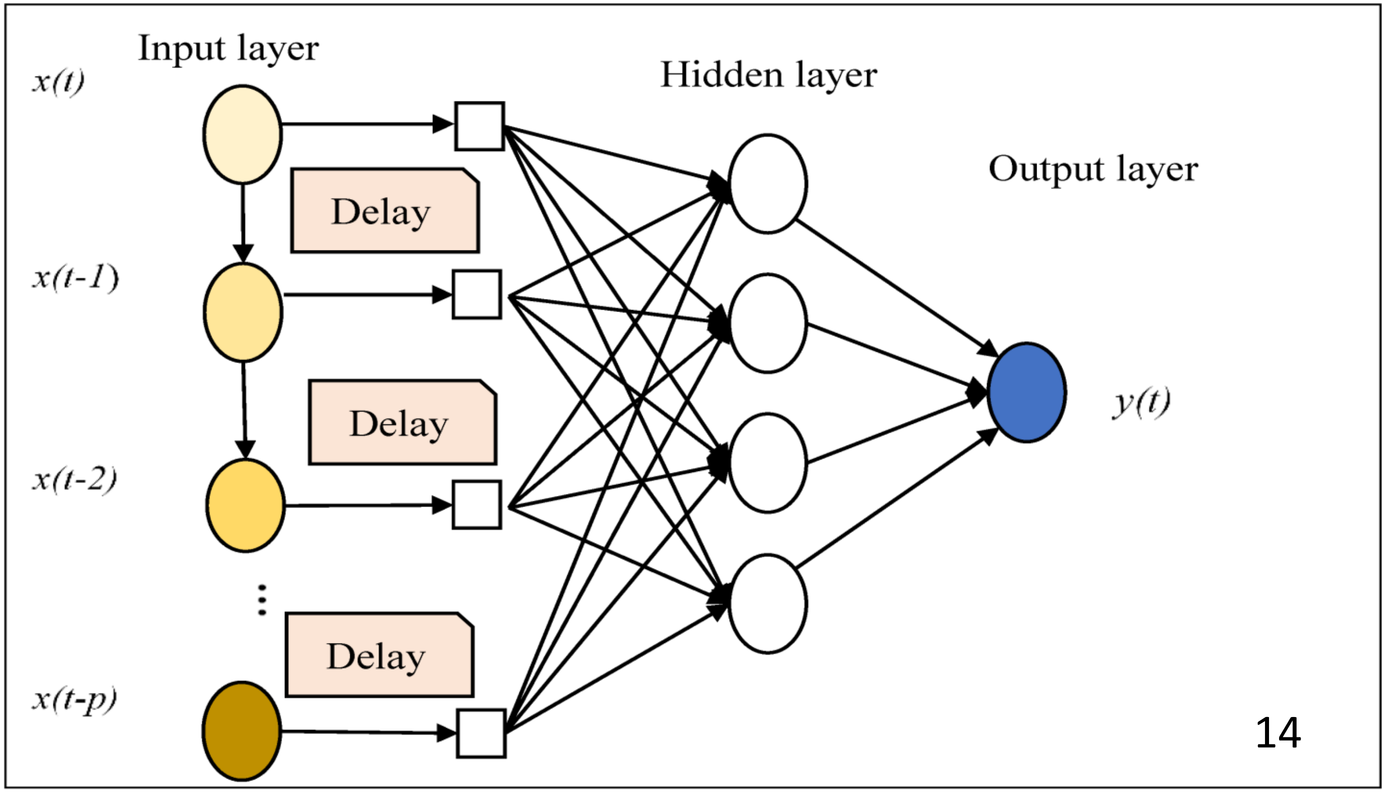
Time Delay Neural Networks (TDNN)

Time delay neural network (TDNN) is a multilayer artificial neural network architecture whose purpose is to 1) classify patterns with shift-invariance, and 2) model context at each layer of the network.

In our context they are used to take into account the past by being provided with inputs which are time sequences and not independent time slices

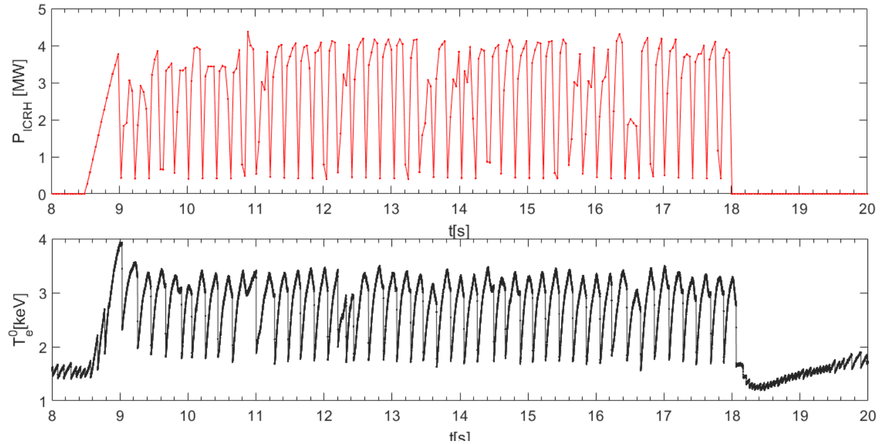
Ensembles of TDNNs have proved to be very effective in detecting causality.

See talk by L.Spolladore



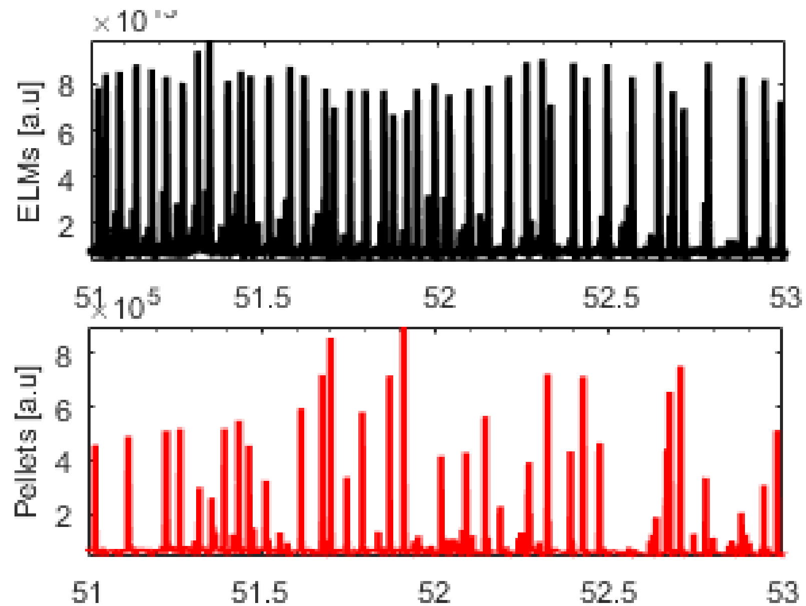
Examples of applications: synchronisation experiments

Let us consider two examples: the problem is to determine which external preturbation (bottom plot) triggers the reaction (top plot)



a) Sawteeth triggering with ICRH (top) *what's the average time interval between the ICRH modulation and the saweteeth crashes?*

b) ELMs pacing (bottom) *how many ELMs have been triggered in the most efficient coupling time?*



How to determine the efficiency and how to discriminate the effects of phase and amplitude synchronization?

Conclusions

Observational causality detection is becoming an essential ingredient in evidence based science.

1. The methodologies being developed try to mathematize causality detection starting directly from observations and data.

2. They can therefore be a complement and a preliminary step to experiments.

3. The relative strengths and weaknesses of the various methods have to be still fully analysed.

4. In the application to plasma physics the main limitation is typically the lack of data.

Many Thanks for Your Attention!

QUESTIONS?