Orbit Weight Functions for Neutron Emission and One-step Reaction Gamma-ray Spectroscopy

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17th Technical Meeting on Energetic Particles and Theory of Plasma Instabilities in Magnetic Confinement Fusion

International Atomic Energy Agency, IAEA

**See the author list of 'Overview of JET results for optimising ITER operation' by J. Mailloux et al. to be published in Nuclear Fusion Special issue: Overview and Summary Papers from the 28th Fusion Energy Conference (Nice, France, 10-15 May 2021)*

Outline

- Introduction
- Orbit space (E, p_m, R_m)
- Weight Function Formalism
- Orbit weight functions
  - Perpendicular sightlines
    - TOFOR (NES)
    - $LaBr_3$ detector (GRS)
  - Oblique sightline (NE213 detector, NES)
- WF analysis
- Conclusion and outlook

Introduction

- Fast ions will play a vital role in future burning plasmas [1]
- Velocity-space tomography reconstructs the fast-ion distribution function at a single (R, z) point [2,3,4]
- 2D $(E, v_{||}/v)$ sensitivity can be mapped via velocity-space weight functions [5,6,7]
- 3D sensitivity can be mapped via orbit weight functions [8,9]
- Prior to this work, developed for some diagnostics [8,9,10]

[1] D Moseev et al 2018 Rev. Mod. Plasma Phys. **2** 7
[2] M Salewski et al 2013 Nucl. Fusion **53** 063019
[3] M Salewski et al 2017 Nucl. Fusion **57** 056001
[4] AS Jacobsen et al 2016 Plasma Phys. Control. Fusion **58** 045016
[5] W W Heidbrink et al 2007 Plasma Phys. Control. Fusion **49** 1457–1475
[6] M Salewski et al 2016 Nucl. Fusion **56** 046009
[7] B.S. Schmidt et al 2021 Rev. Sci. Instrum. **92** 053528
[8] Stagner L and Heidbrink W W 2017 Physics of Plasmas **24** 092505
[9] L. Stagner et al 2021 Nucl. Fusion, *at press*
[10] H Järleblad et al 2021 Rev. Sci. Instrum. **92** 043526

Introduction

- In this work, we map out how the orbit sensitivity varies with fast-ion and diagnostic energy
- Neutron emission spectroscopy (NES) orbit weight functions for TOFOR[10], an NE213-scintillator[11] and newly developed one-step reaction gamma-ray spectroscopy (GRS) orbit weight functions for a $LaBr_3$ detector[12] are used as examples
- We demonstrate that orbit weight functions are able to mimic forward model for computing synthetic signals
- Lastly, combine with example fast-ion distribution functions to split synthetic signals into orbit type constituents

[10] M Gatu Johnson et al 2008 Nucl. Instrum. Methods Phys. Res. A **591** 417–430
[11] F Binda et al 2014 Rev. Sci. Instrum. **85** 11E23
[12] M Nocente et al 2010 Rev. Sci. Instrum. **81** 10D321

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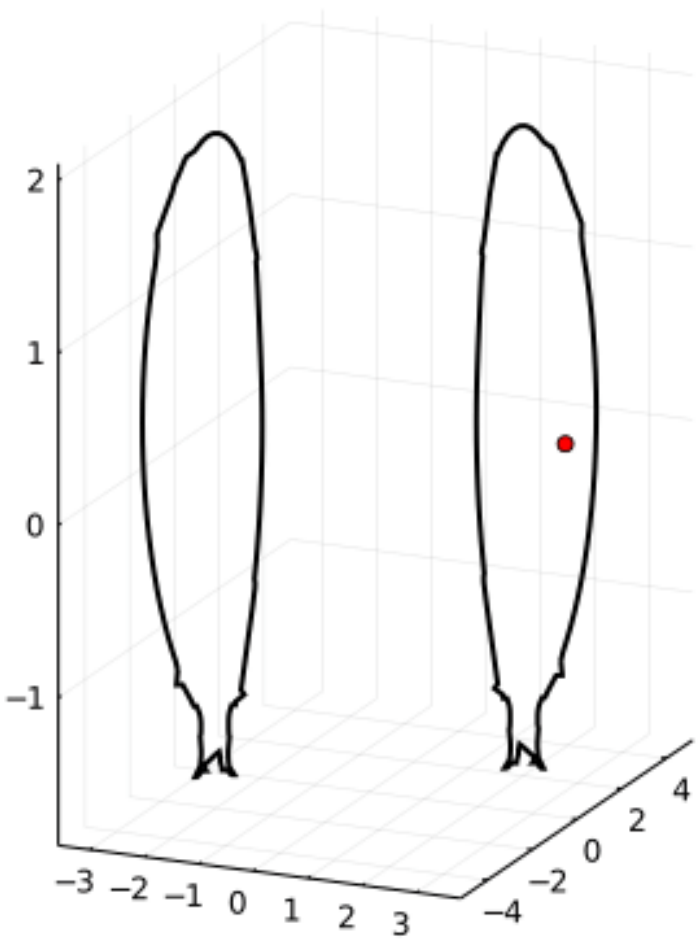
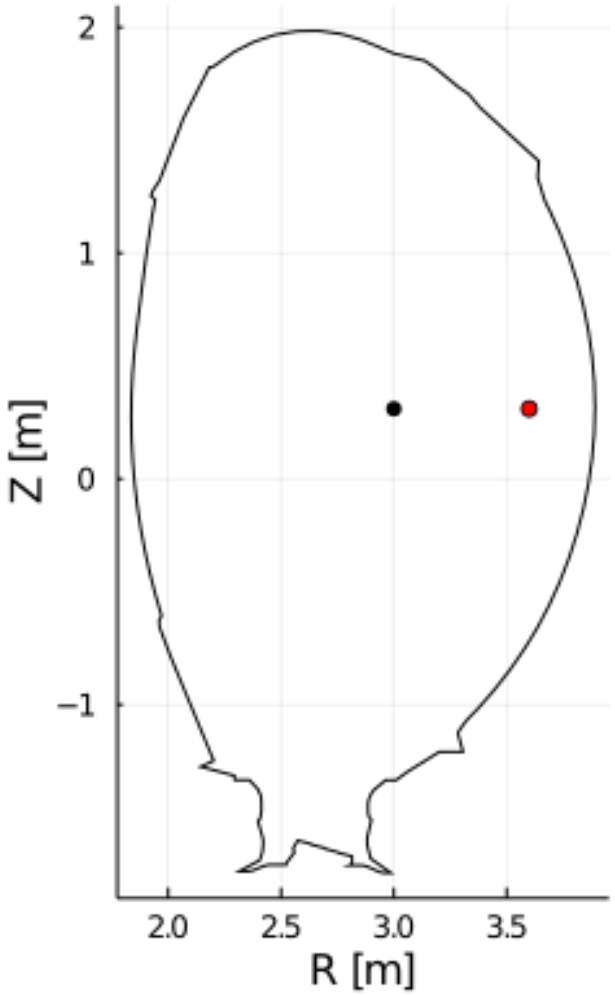
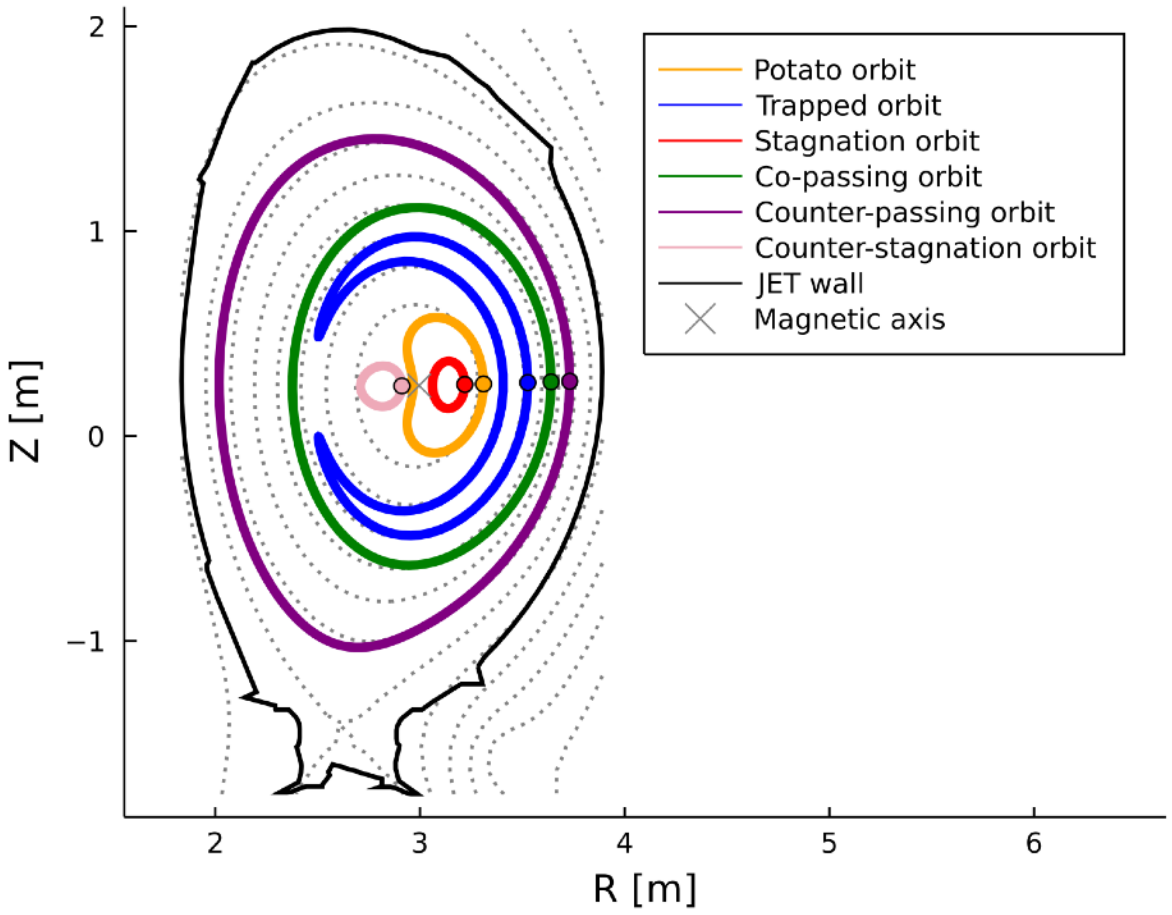
Orbit space (E, p_m, R_m)





Orbit space (E, p_m, R_m)

- The full six-dimensional charged particle motion in $\mathbf{x}$ and $\mathbf{v}$ can be dimensionally reduced to three
- Toroidal symmetry, guiding-centre-picture and $\nu\tau_p << 1$
- Well-filled, clear boundaries and no mixing of position and velocity space [13]
- (E, p_m, R_m)

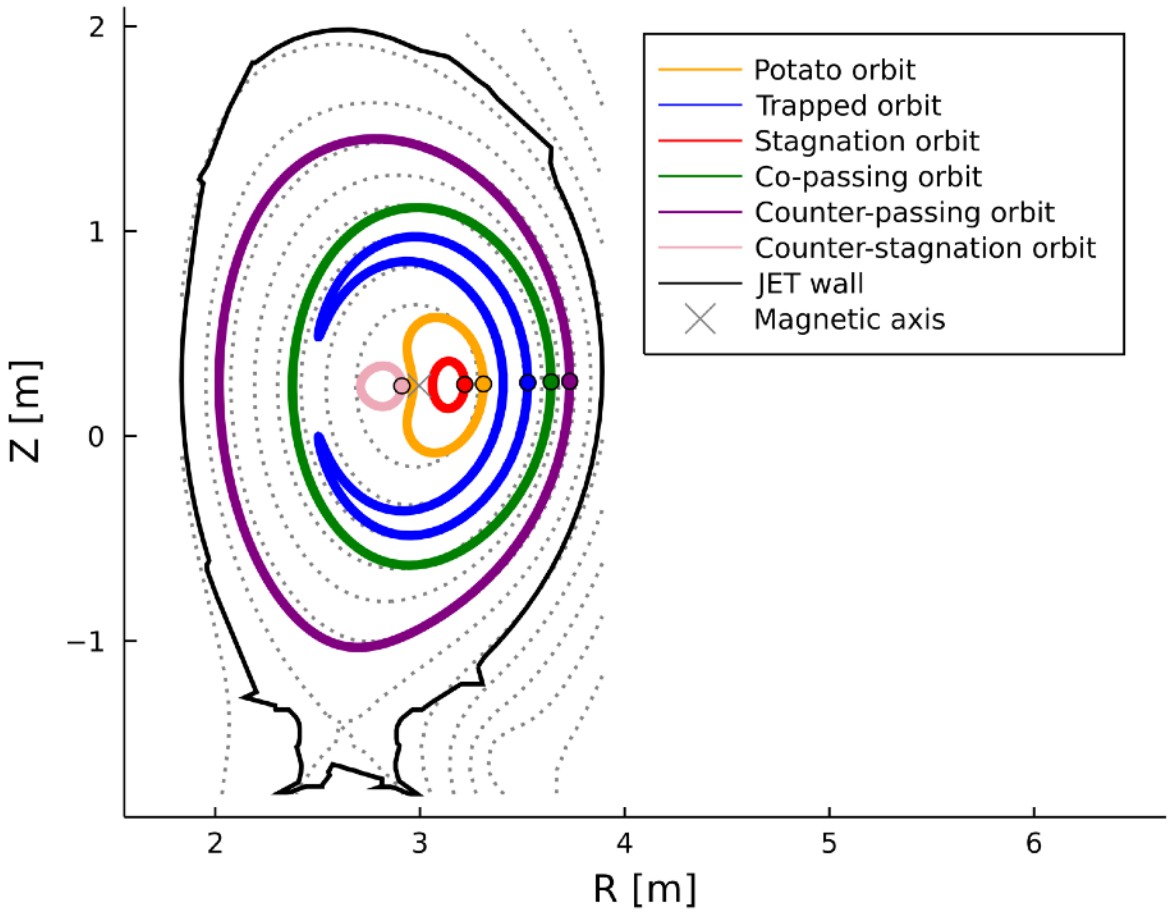


JET shot No. 94701 at 10.7932 s

[13] JA Rome and YK M Peng 1979 Nucl. Fusion **19** 1193

Orbit space (E, p_m, R_m)

- E is the energy, p_m is the pitch at the maximum major radius position R_m of the orbit
- Every realisable (valid) orbit corresponds to a unique (E, p_m, R_m) triplet



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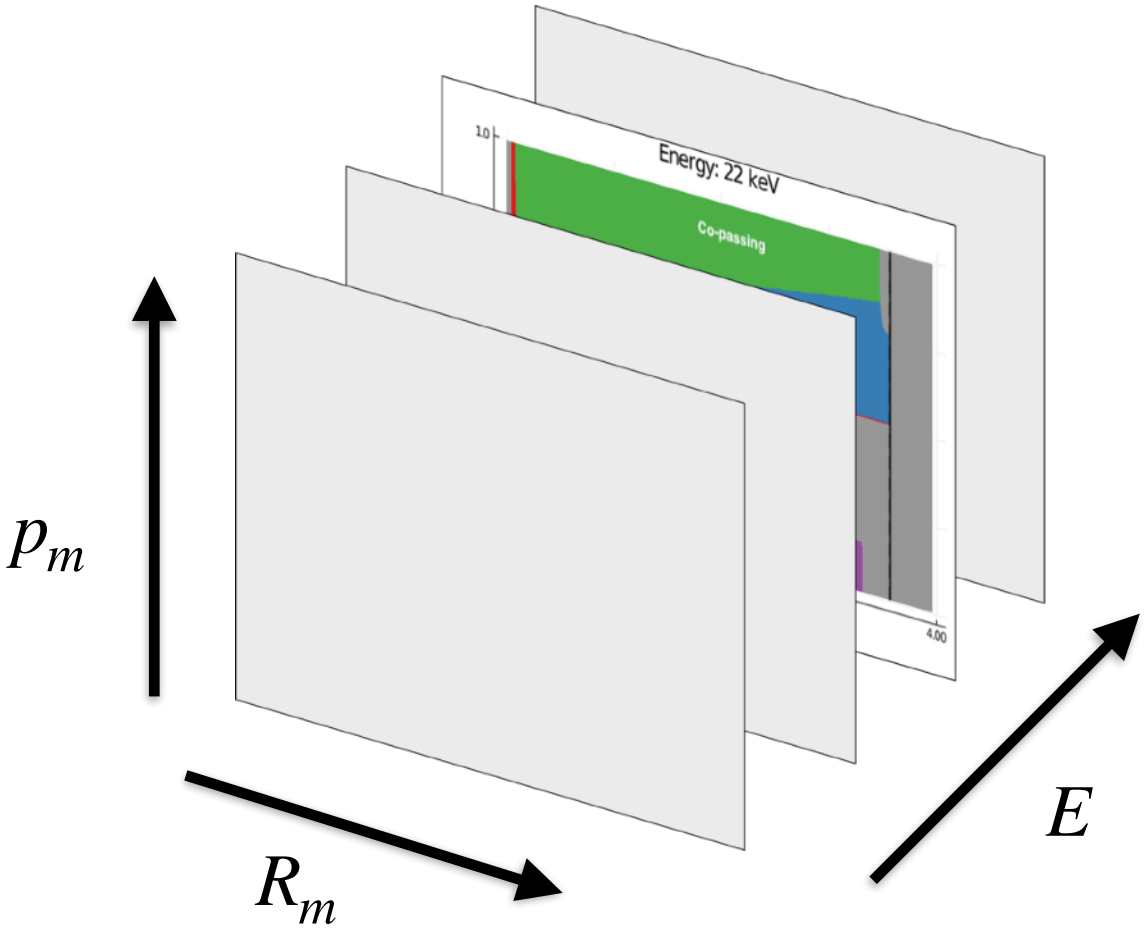
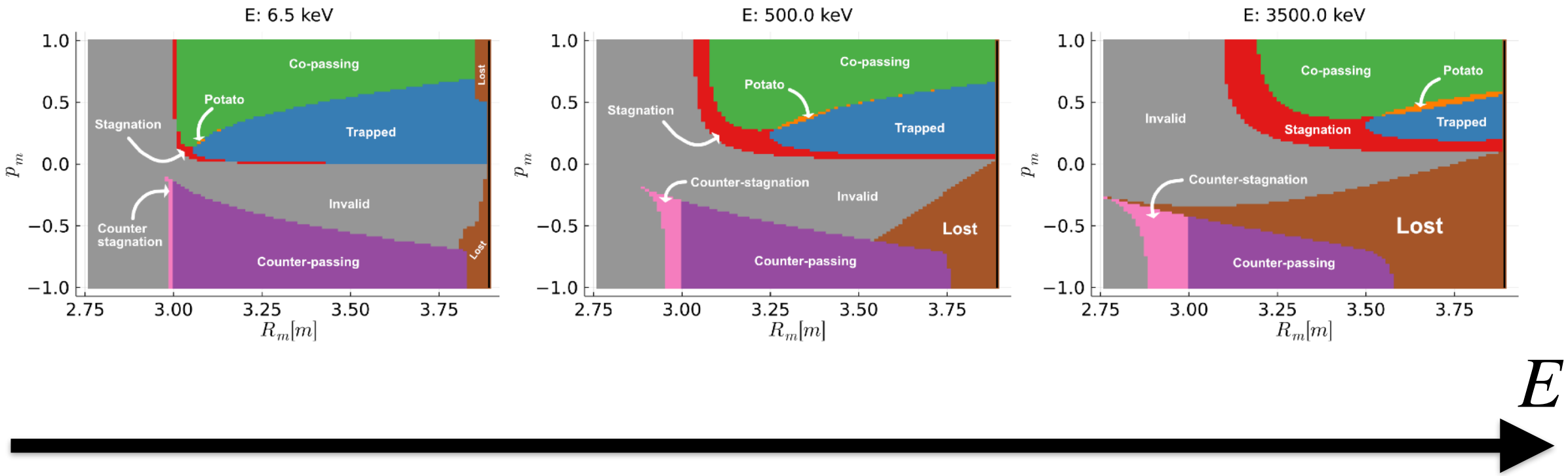


Orbit space (E, p_m, R_m)



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Weight function formalism

$$s(E_{1,d}, E_{2,d}) = \int w(E_{1,d}, E_{2,d}, \mathbf{x}, \mathbf{v}) f(\mathbf{x}, \mathbf{v}) d\mathbf{x} d\mathbf{v}$$

$$\downarrow$$

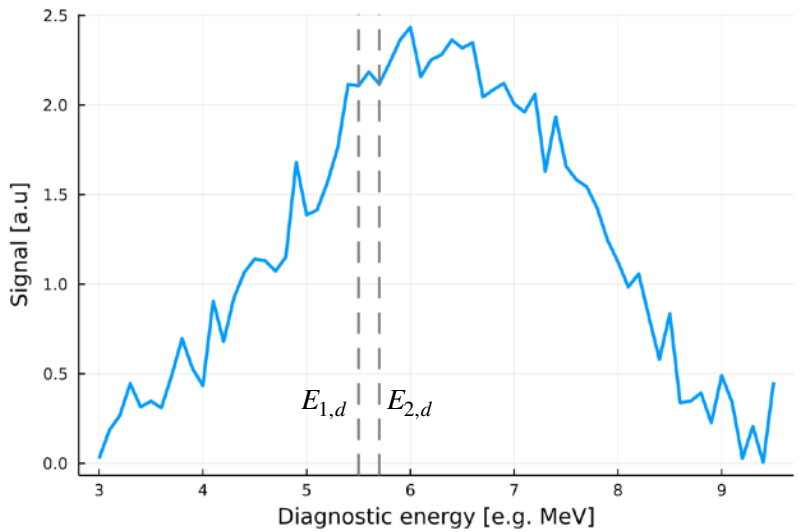
$$s(E_{1,d}, E_{2,d}) = \int w(E_{1,d}, E_{2,d}, E, p_m, R_m) f(E, p_m, R_m) dE dp_m dR_m$$

$$\downarrow$$

$$s(E_{1,d}, E_{2,d}) = \sum_{i,j,k} w(E_{1,d}, E_{2,d}, E_i, p_{m,j}, R_{m,k}) f(E_i, p_{m,j}, R_{m,k}) \Delta E \Delta p_m \Delta R_m$$

$$\downarrow$$

$$S = WF^*$$



* See also for example J. Rueda-Rueda (Tuesday)

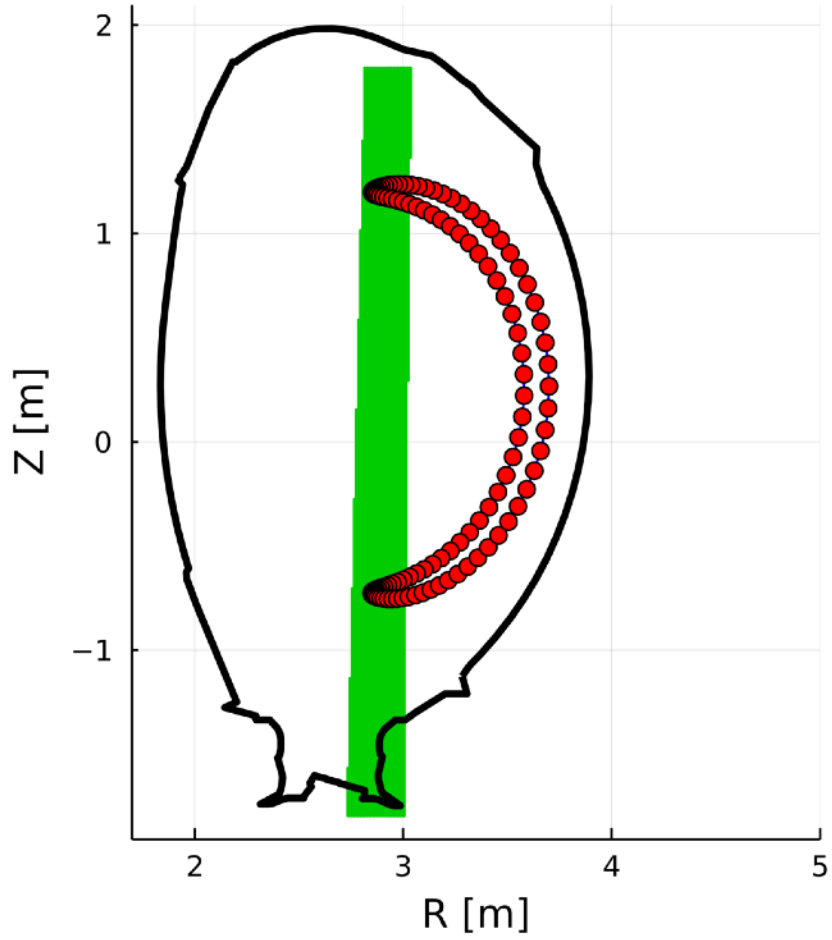
Weight function formalism

$$
\begin{bmatrix} 0.0 \\ 0.138 \\ 0.142 \\ 0.167 \\ \vdots \\ 0.171 \\ 0.143 \\ 0.092 \\ 0.0 \end{bmatrix}
=
\begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & \dots & 0.0 & 0.0 & 0.0 \\
0.0 & 0.002 & 0.007 & 0.001 & \dots & 0.013 & 0.005 & 0.001 \\
0.002 & 0.010 & 0.007 & 0.006 & \dots & 0.012 & 0.009 & 0.0 \\
0.003 & 0.010 & 0.006 & 0.008 & \dots & 0.013 & 0.008 & 0.002 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\
0.004 & 0.009 & 0.008 & 0.002 & \dots & 0.020 & 0.005 & 0.09 \\
0.003 & 0.005 & 0.008 & 0.07 & \dots & 0.0 & 0.006 & 0.009 \\
0.0 & 0.006 & 0.0 & 0.0 & \dots & 0.0 & 0.0 & 0.004 \\
0.0 & 0.0 & 0.0 & 0.0 & \dots & 0.0 & 0.0 & 0.1
\end{bmatrix}
\begin{bmatrix}
\text{valid orbit 1} \\
\text{valid orbit 2} \\
\text{valid orbit 3} \\
\text{valid orbit 4} \\
\vdots \\
\text{valid orbit n-3} \\
\text{valid orbit n-2} \\
\text{valid orbit n-1} \\
\text{valid orbit n}
\end{bmatrix}
$$

S m x 1 **W** m x n **F** n x 1

Weight function formalism

- Split orbit into its (E, p, R, Z) points
- Weigh each point by $\Delta t/\tau_p$
- This ‘distribution’ is sent into the forward model [14]
- $s(E_{1,d}, E_{2,d}) = \int w(E_{1,d}, E_{2,d}, E, p_m, R_m)\delta(E - E_i)\delta(p_m - p_{m,j})\delta(R_m - R_{m,k})dEdp_m dR_m$
- $\Rightarrow s(E_{1,d}, E_{2,d}) = w(E_{1,d}, E_{2,d}, E_i, p_{m,j}, R_{m,k})$
- Put signals as columns in matrix
- The rows are the discretised weight functions $w(E_{1,d}, E_{2,d}, E, p_m, R_m)$
- The matrix is the weight matrix W



[14] J Eriksson et al 2016 CPC **199** 40–46

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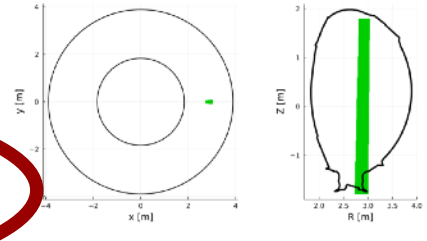
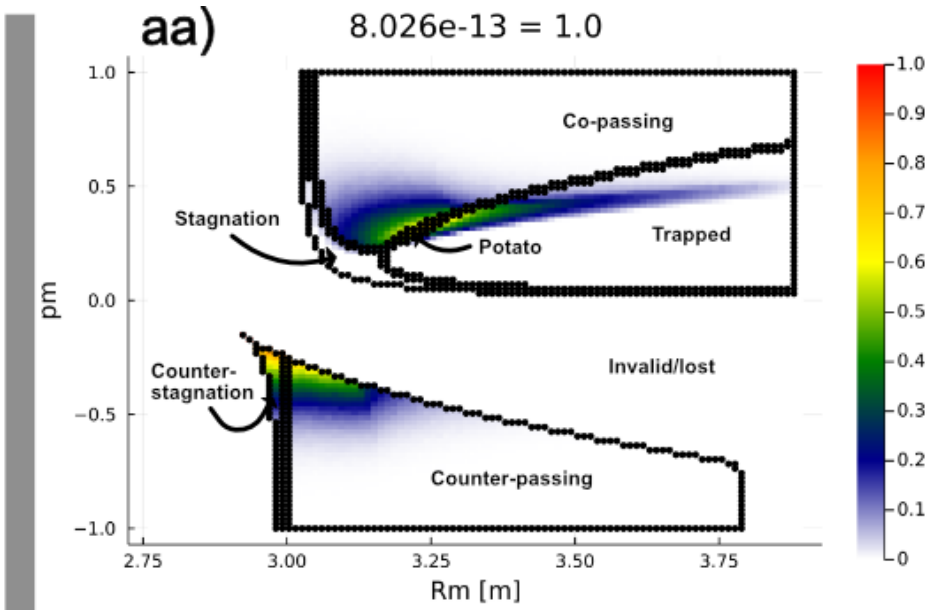
Orbit weight functions

Perpendicular sightlines - TOFOR - D(D,n)^{3}He



Neutron energy:

3,0 MeV



Fast-ion energy (keV)

150.0

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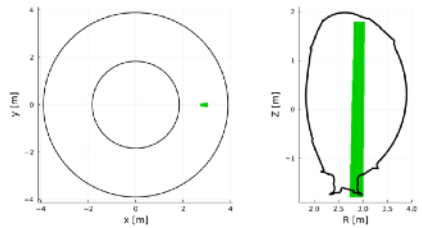

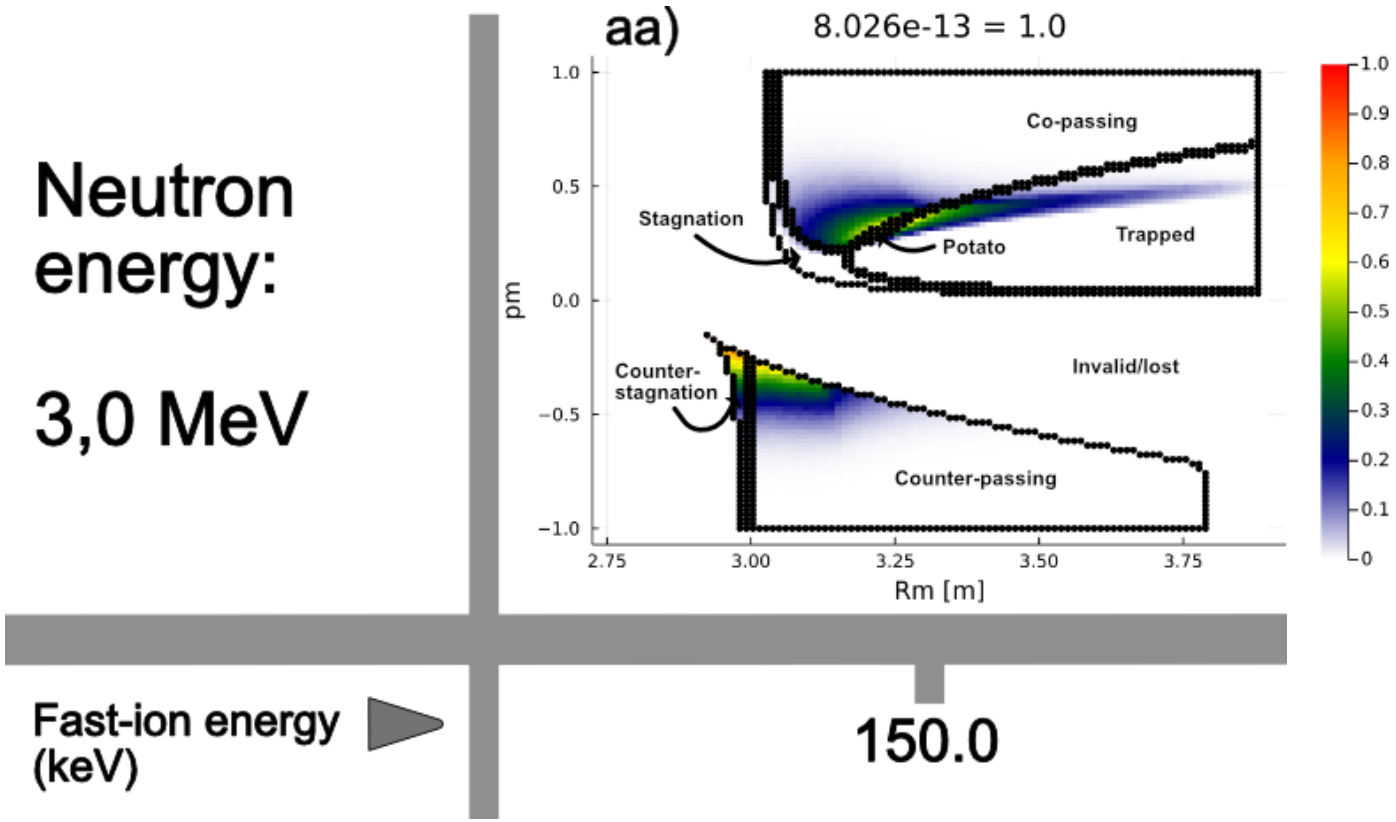

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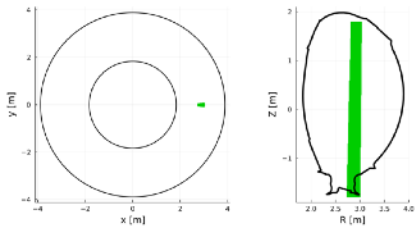
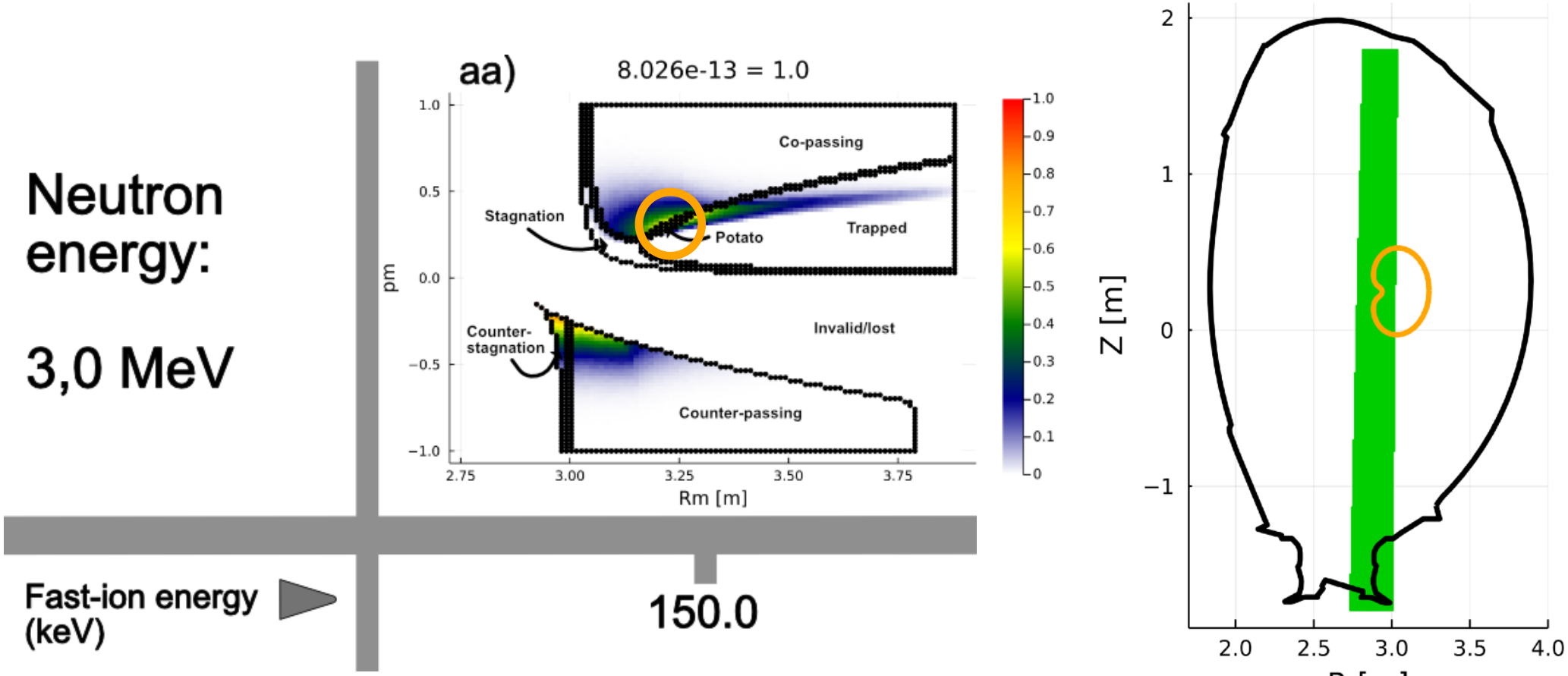
Slide keywords: TOFOR high sensitivity potato

Orbit weight functions

Perpendicular sightlines - TOFOR - D(D,n)^{3}He





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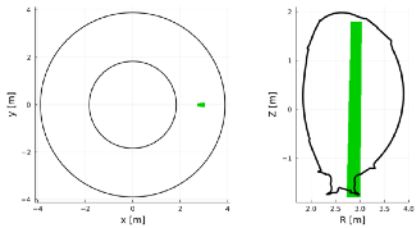

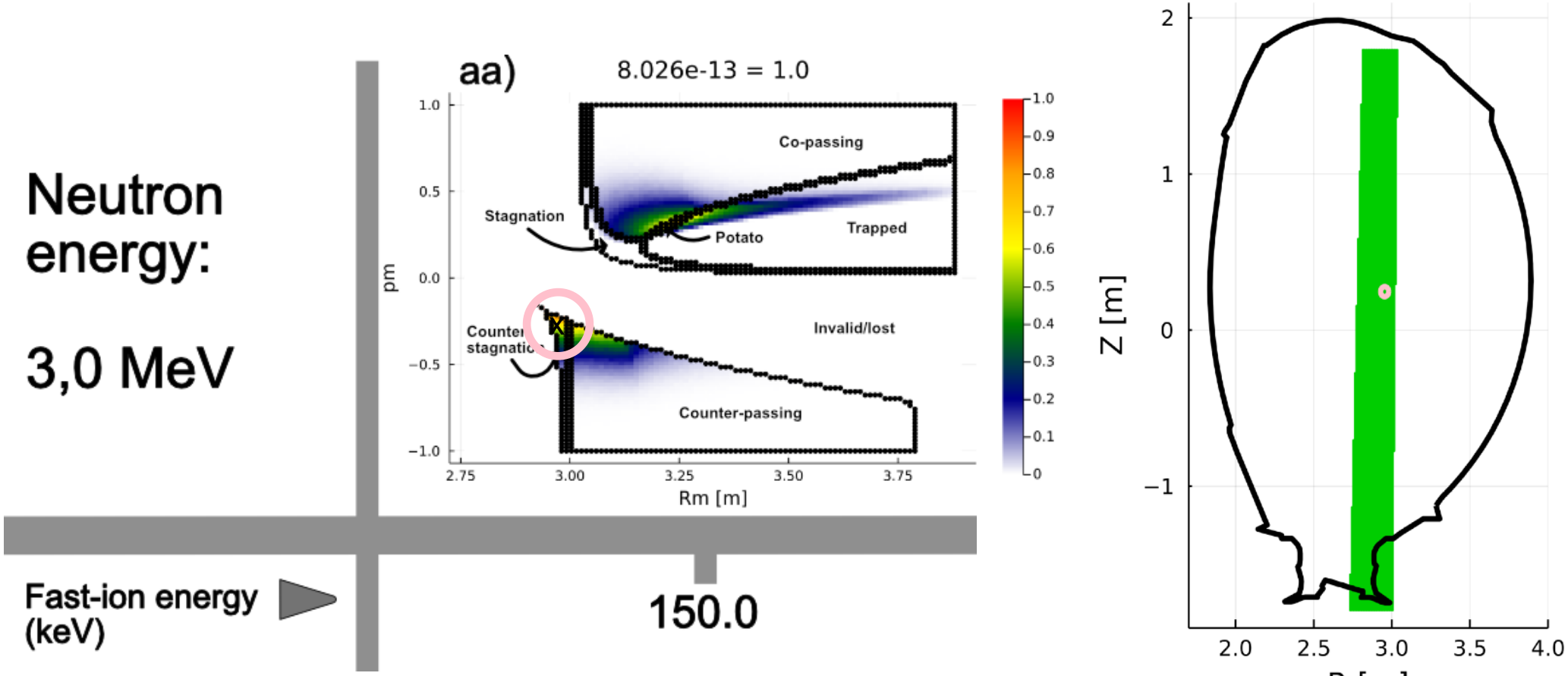

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Slide keywords: TOFOR high sensitivity trapped orbit banana tip

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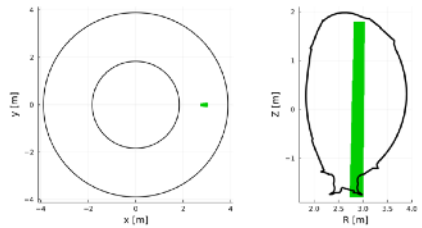

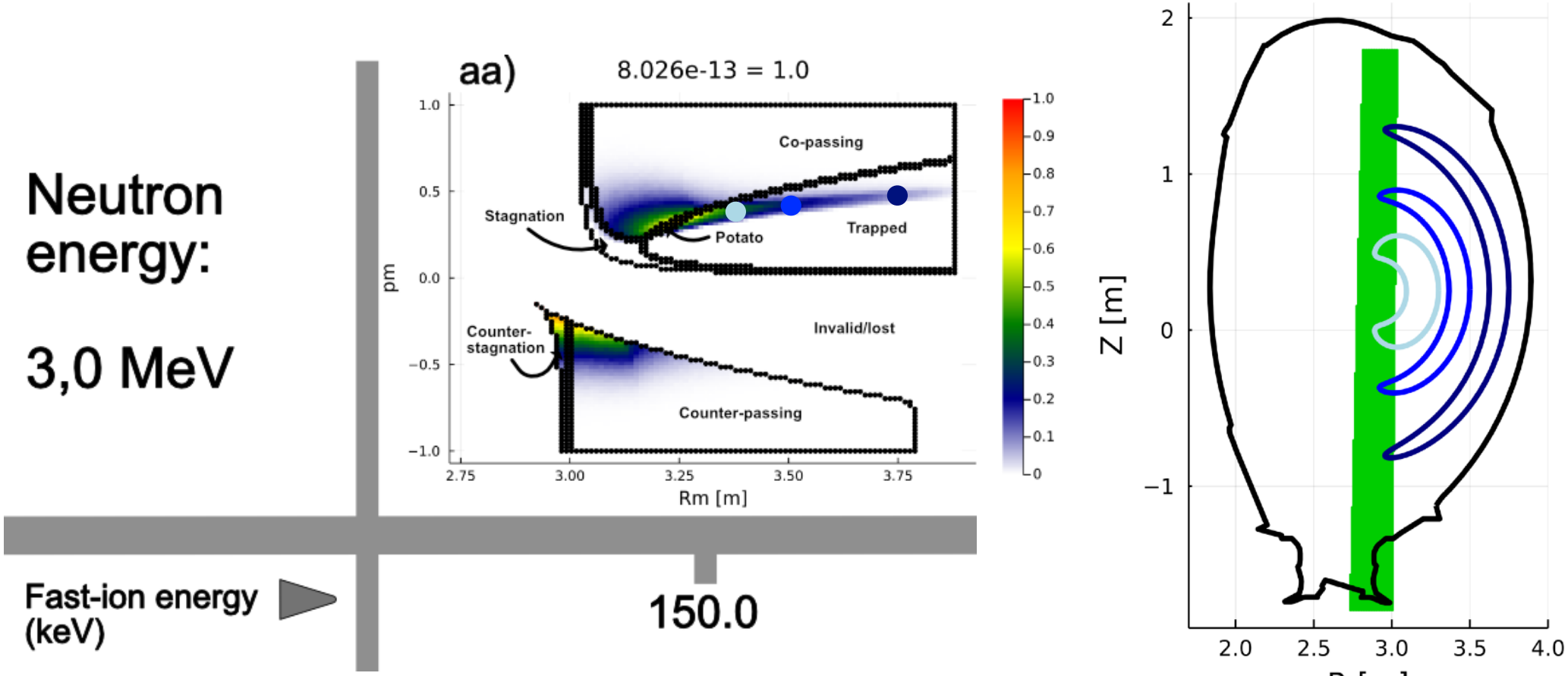

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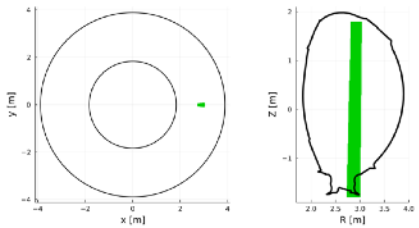

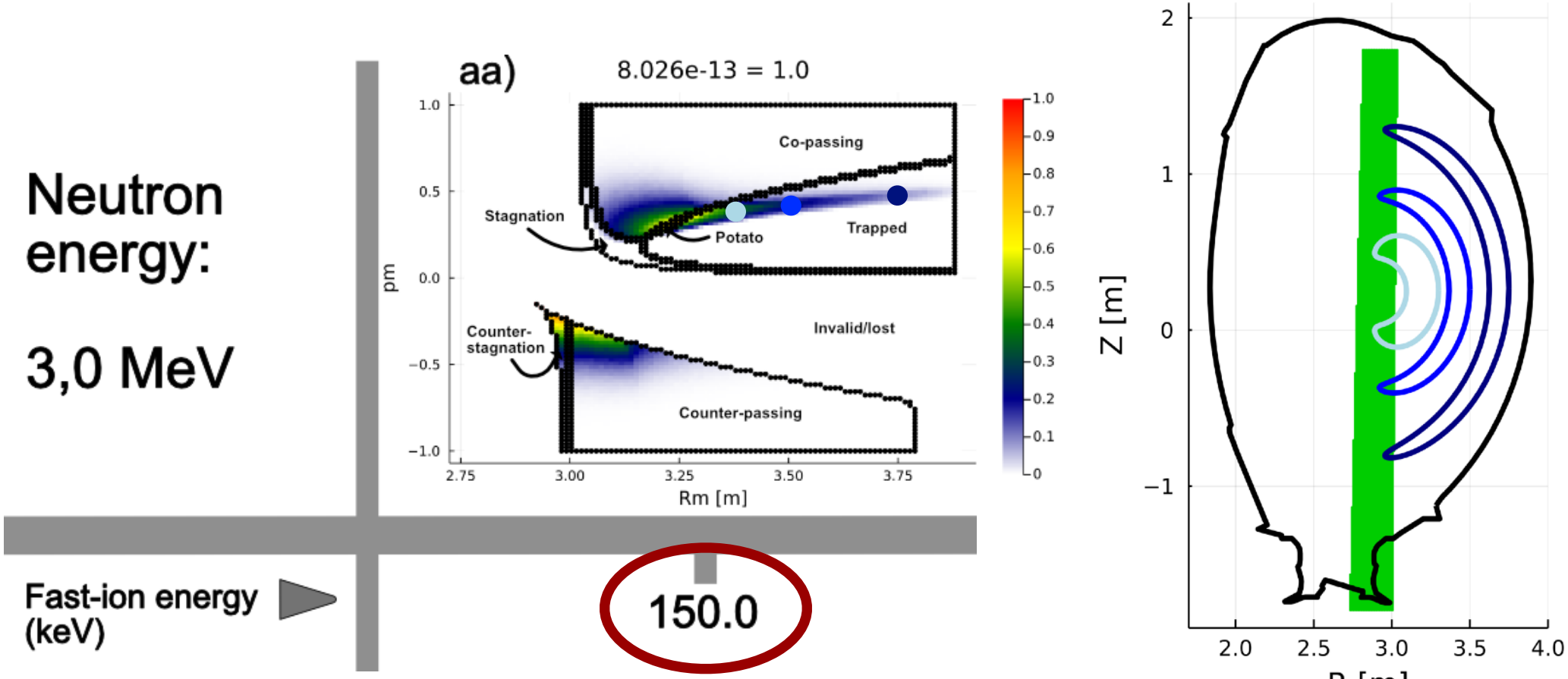

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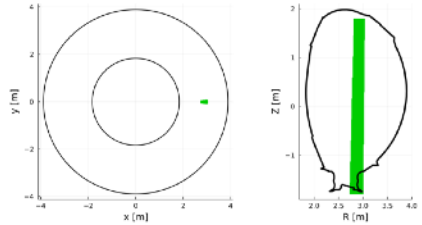

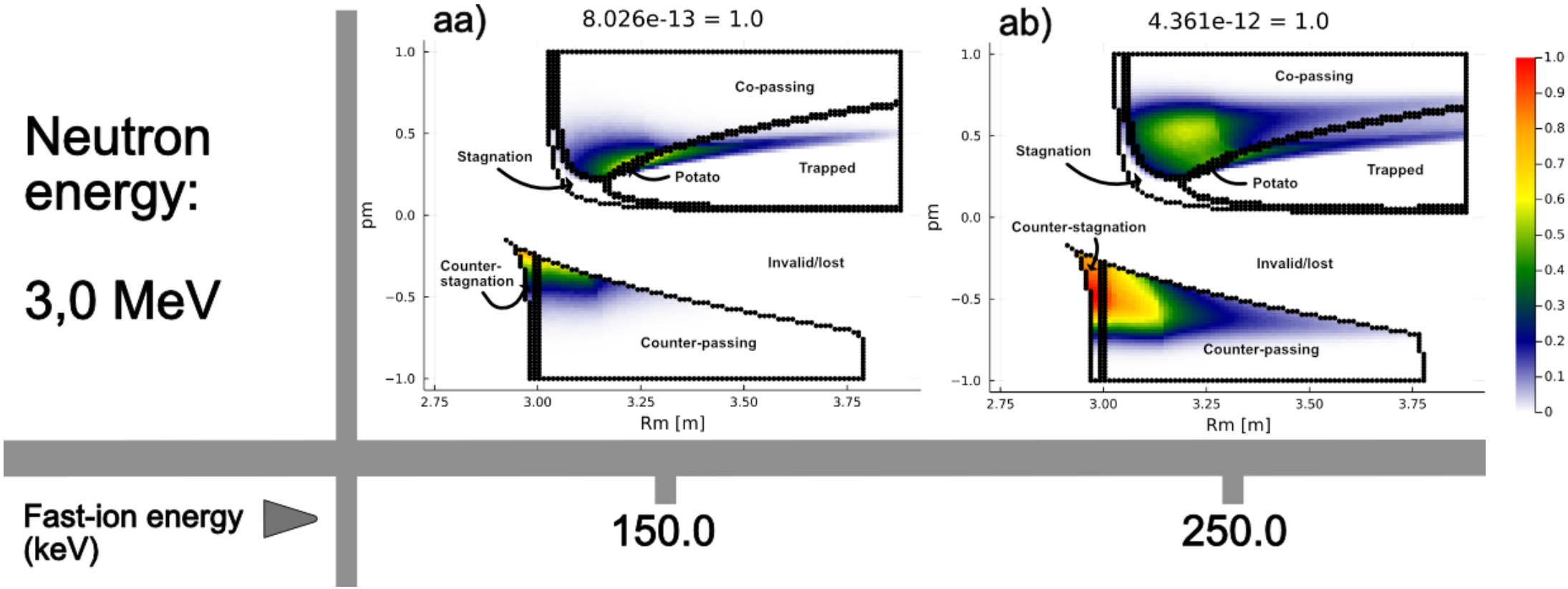

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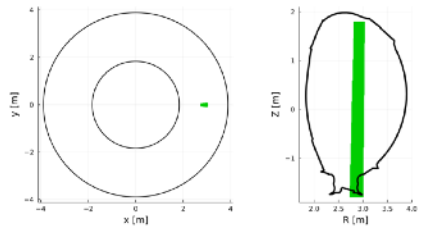

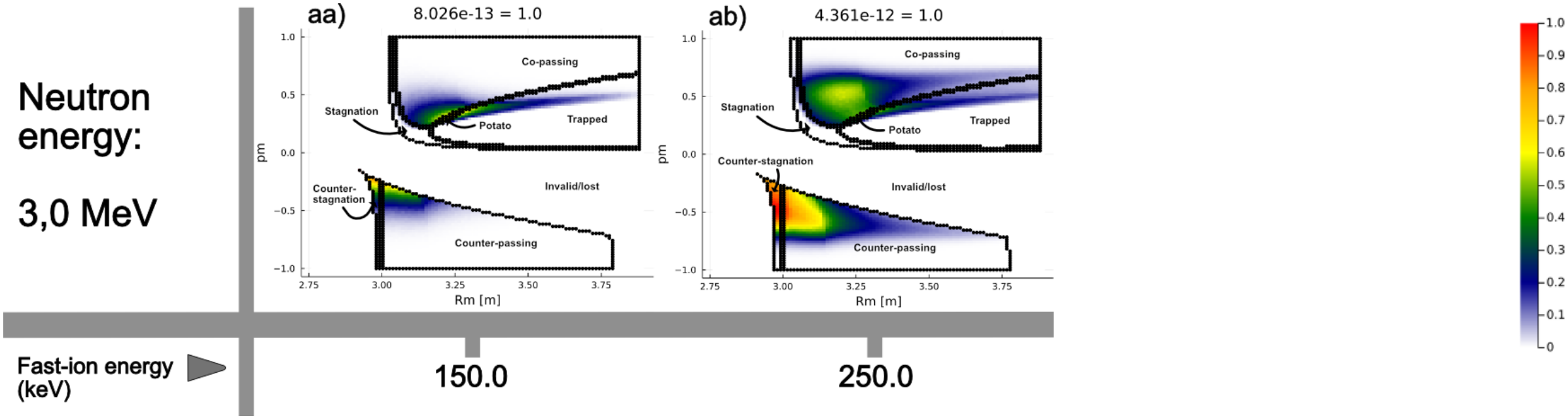

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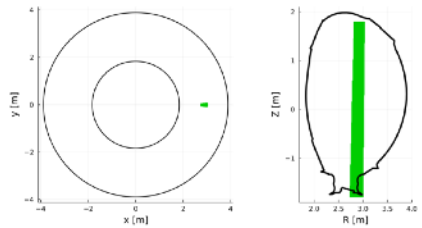

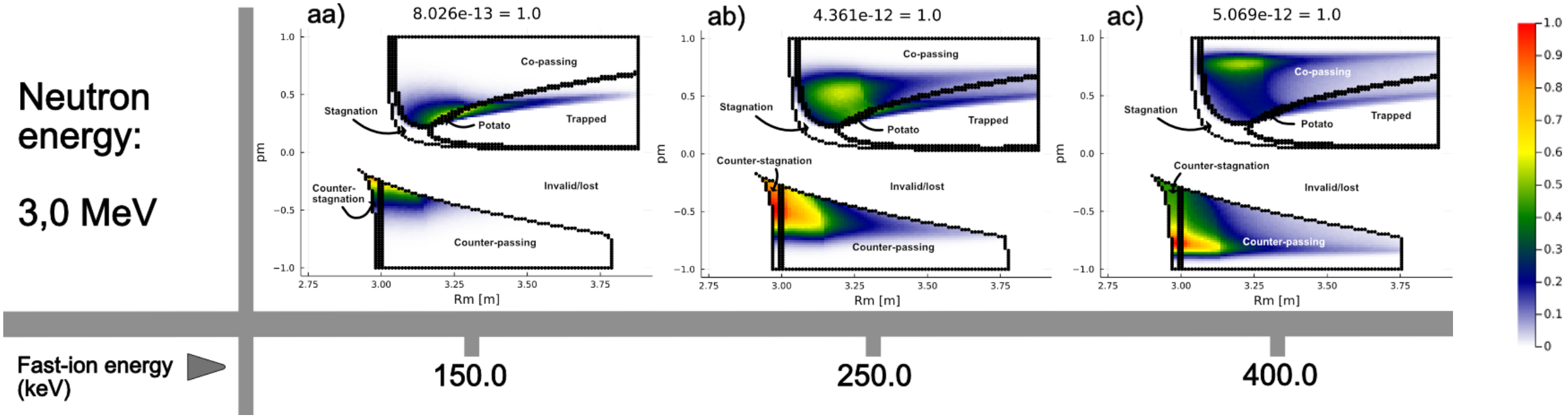

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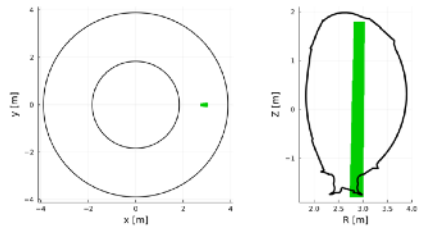

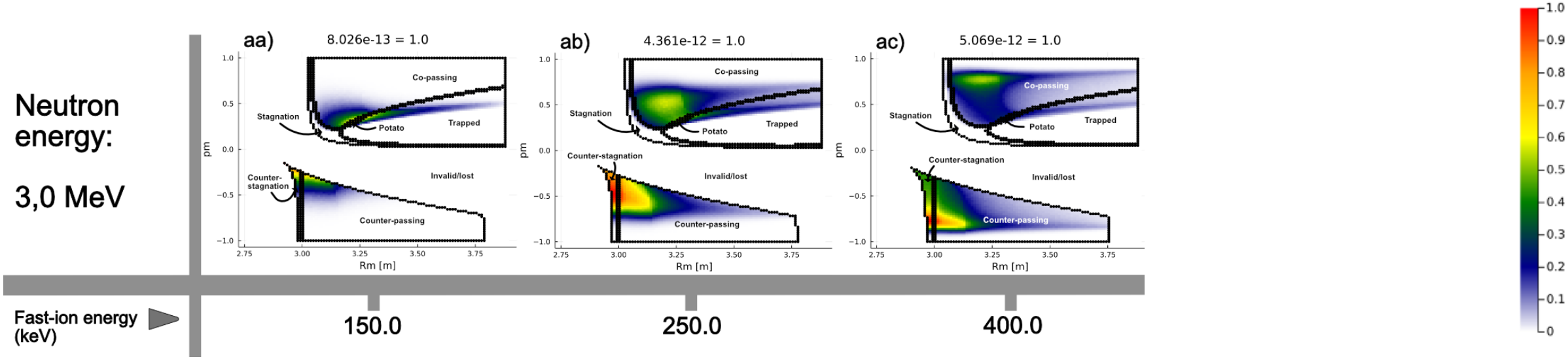

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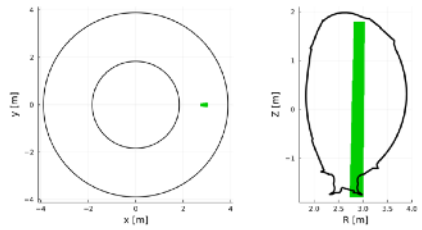

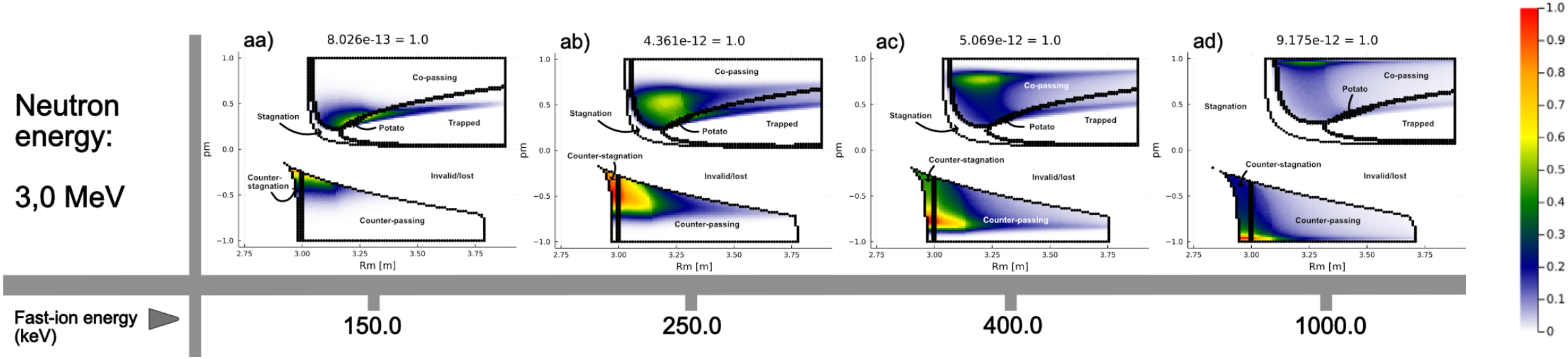

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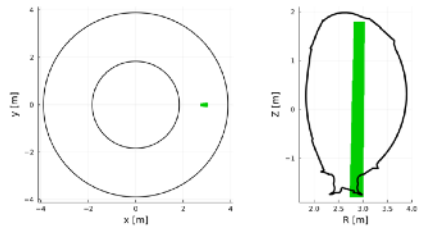

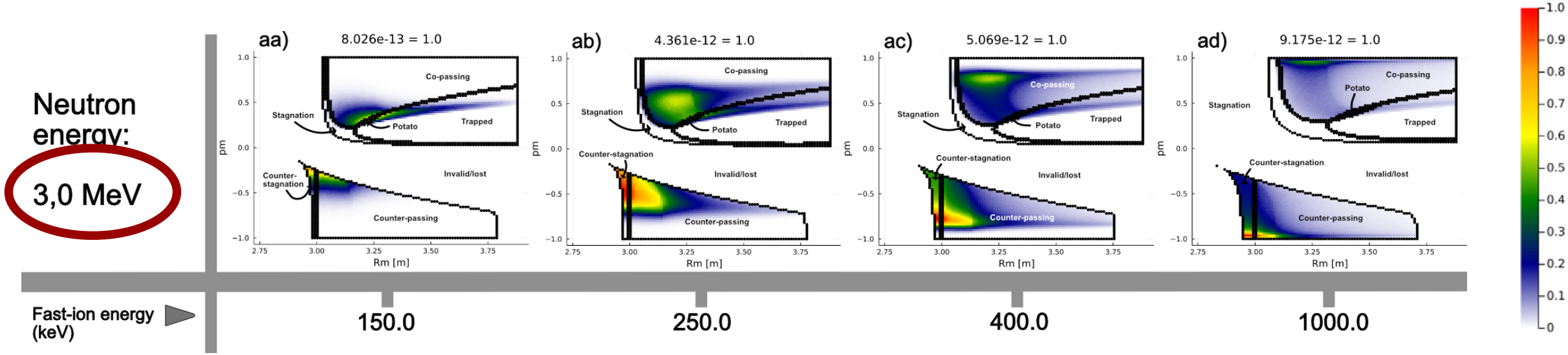

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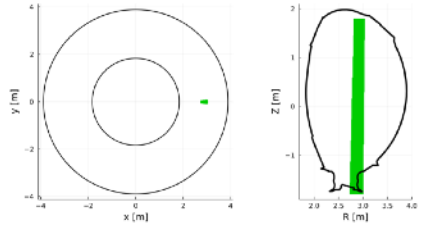

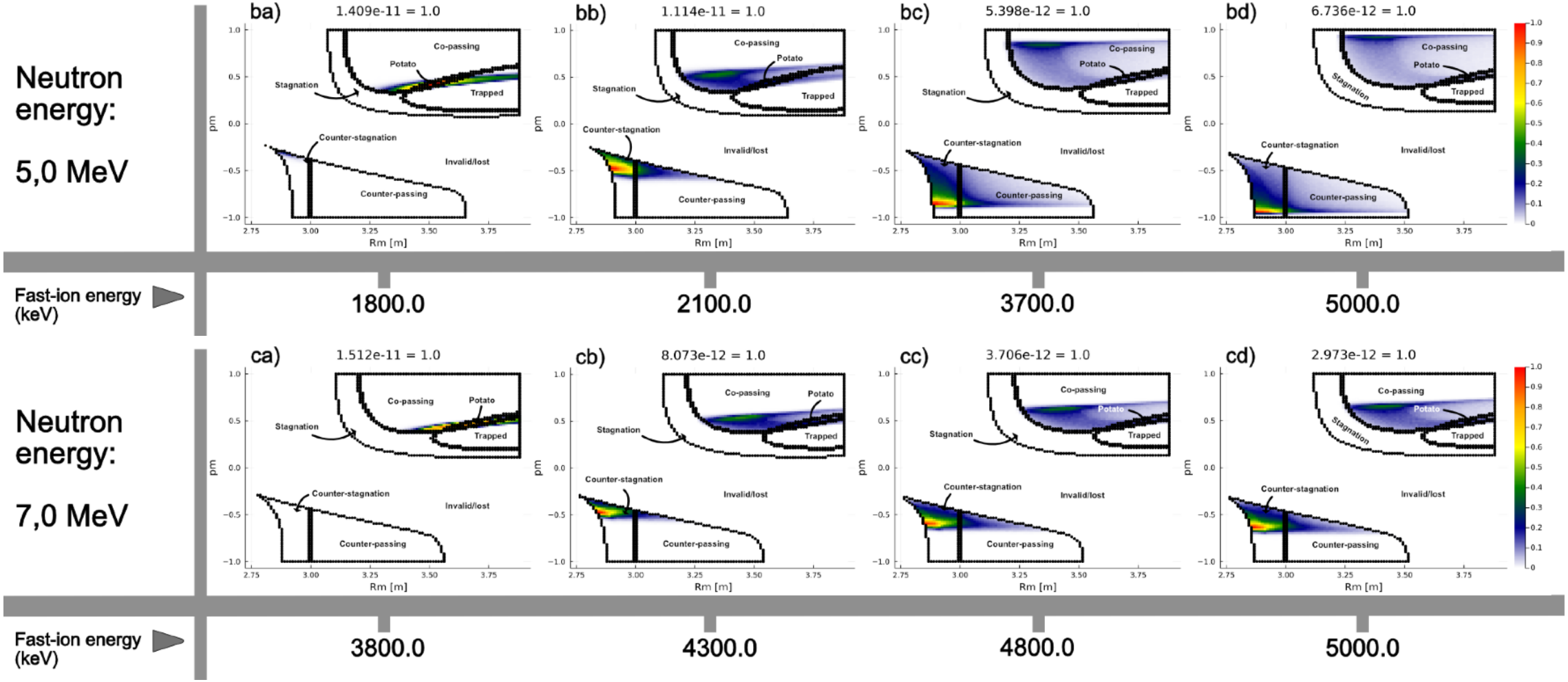

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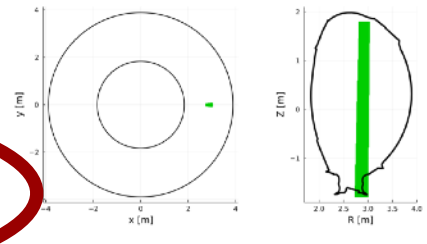

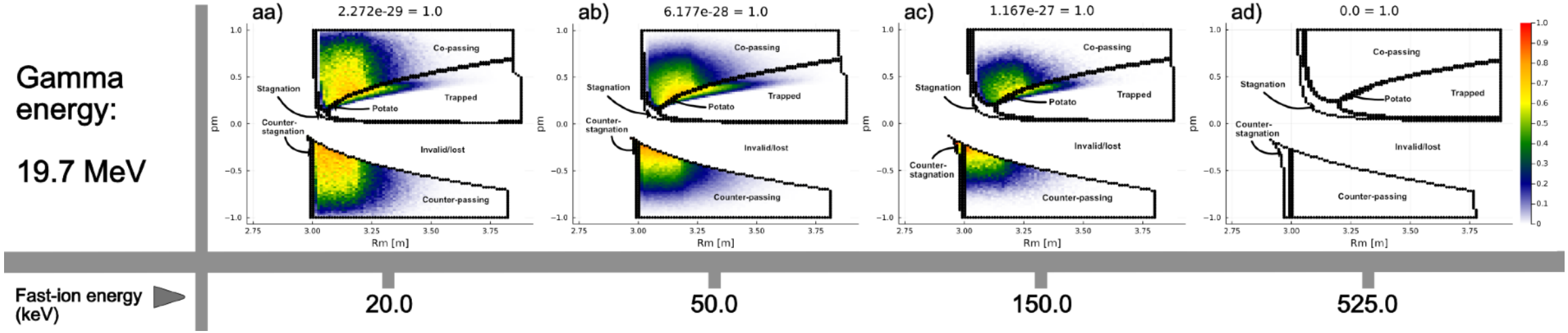

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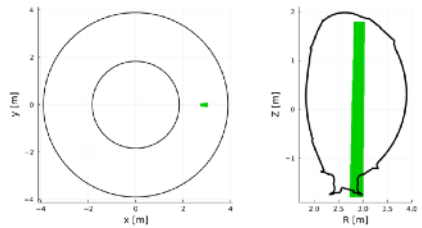
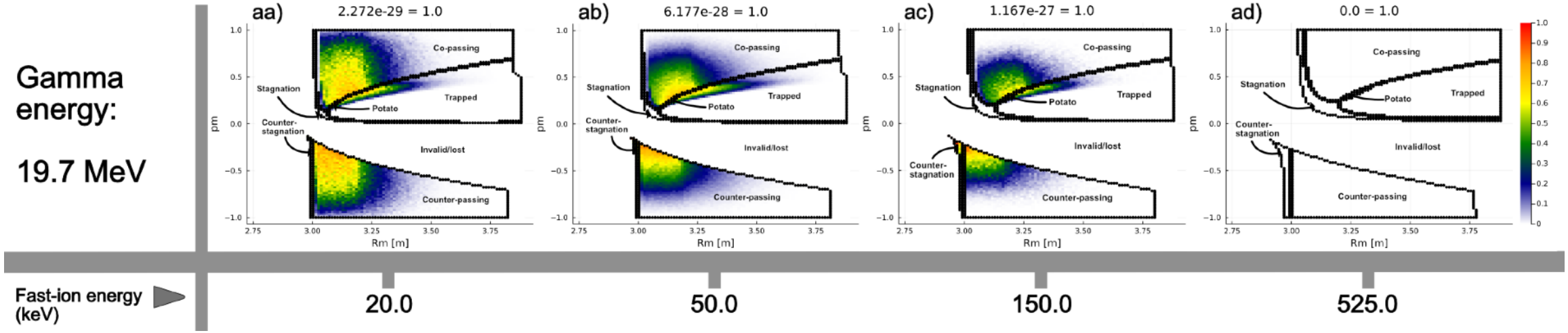
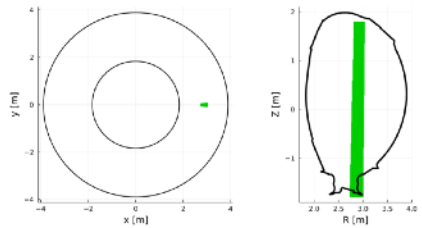


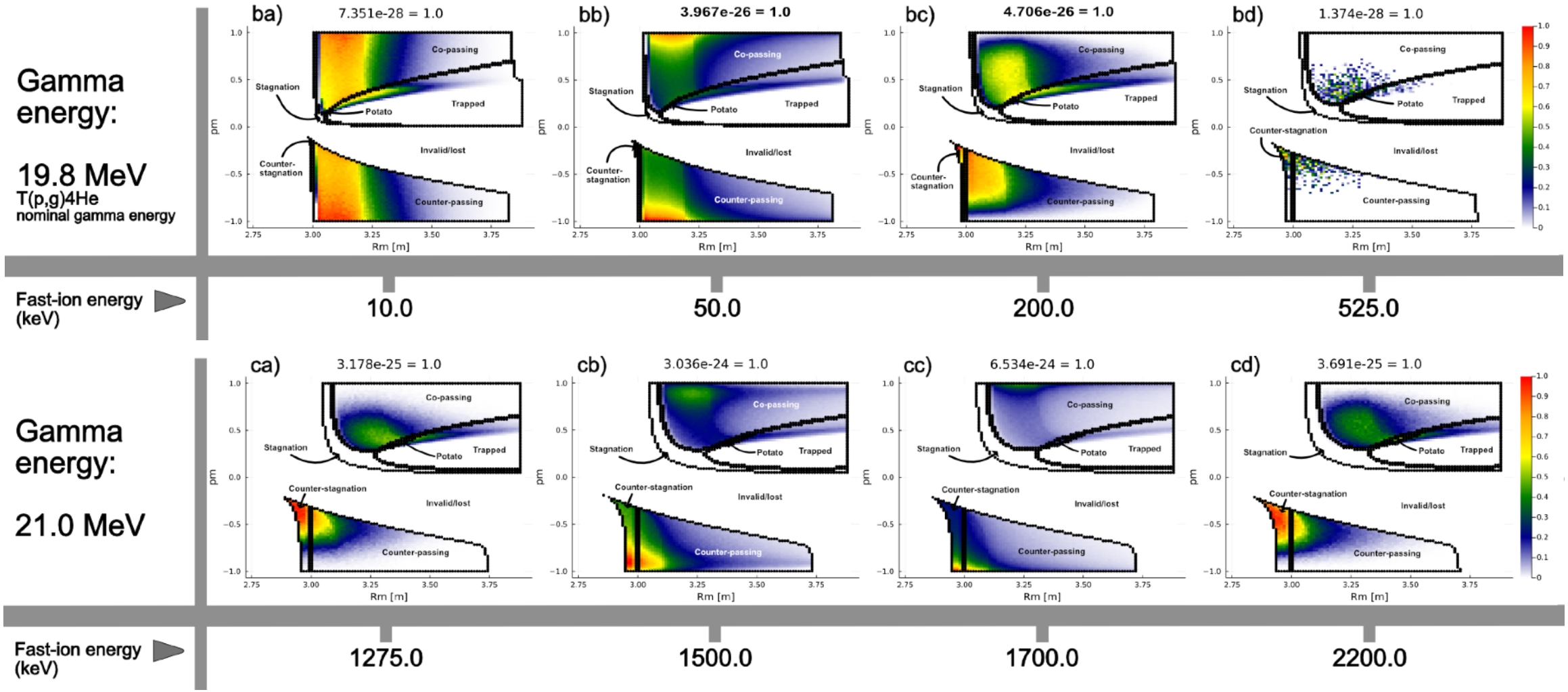


JET shot No. 94701 at 10.7932 s

Orbit weight functions

Perpendicular sightlines - GRS - T(p,γ)^{4}He





Outline

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  - Oblique sightline (NE213 detector, NES))
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- Conclusion and outlook

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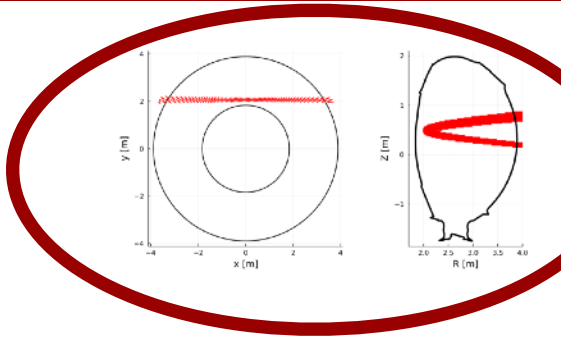
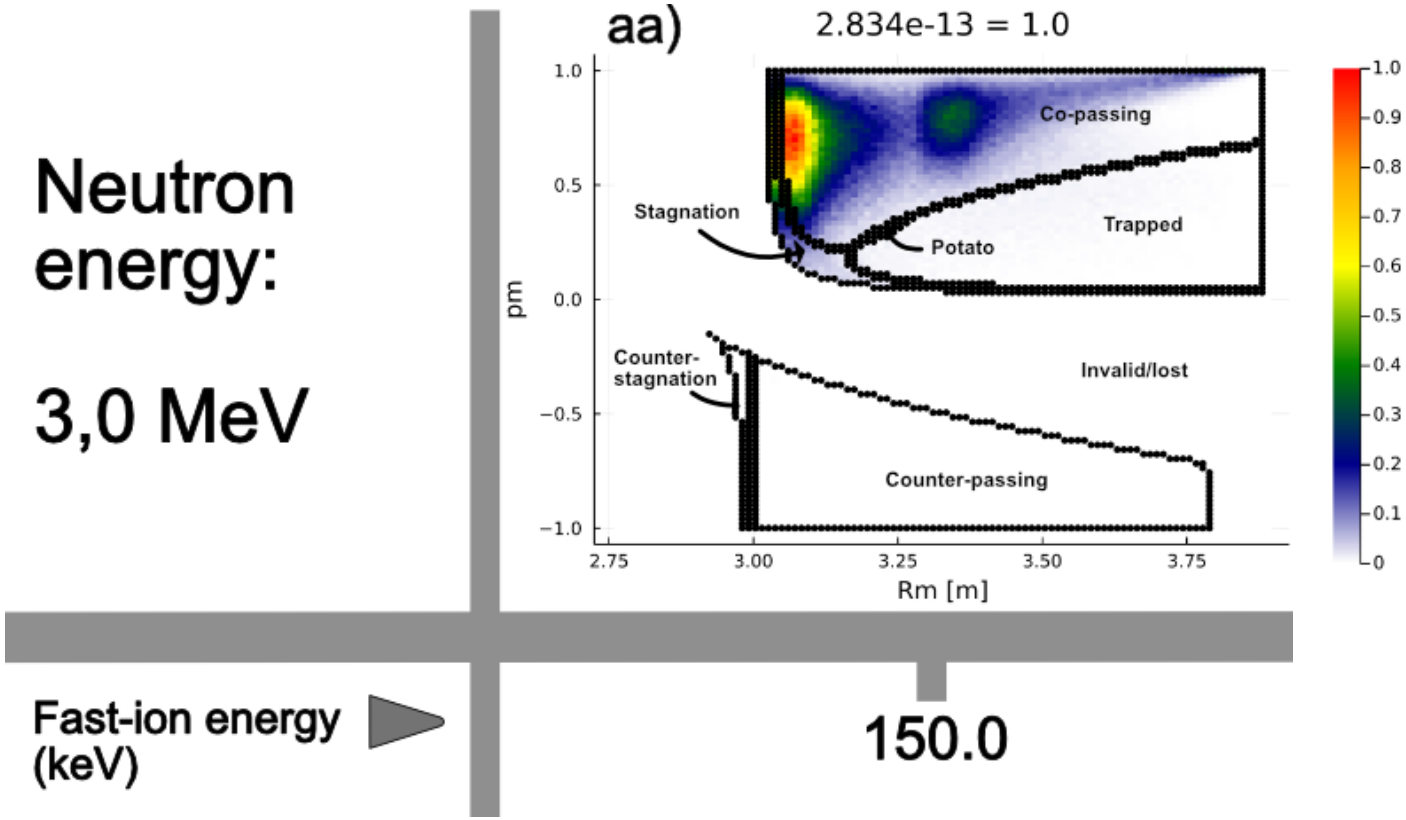
Oblique sightline - NE213 - D(D,n)^{3}He





JET shot No. 94701 at 10.7932 s

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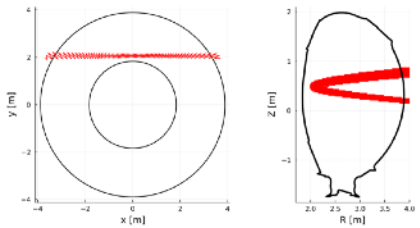
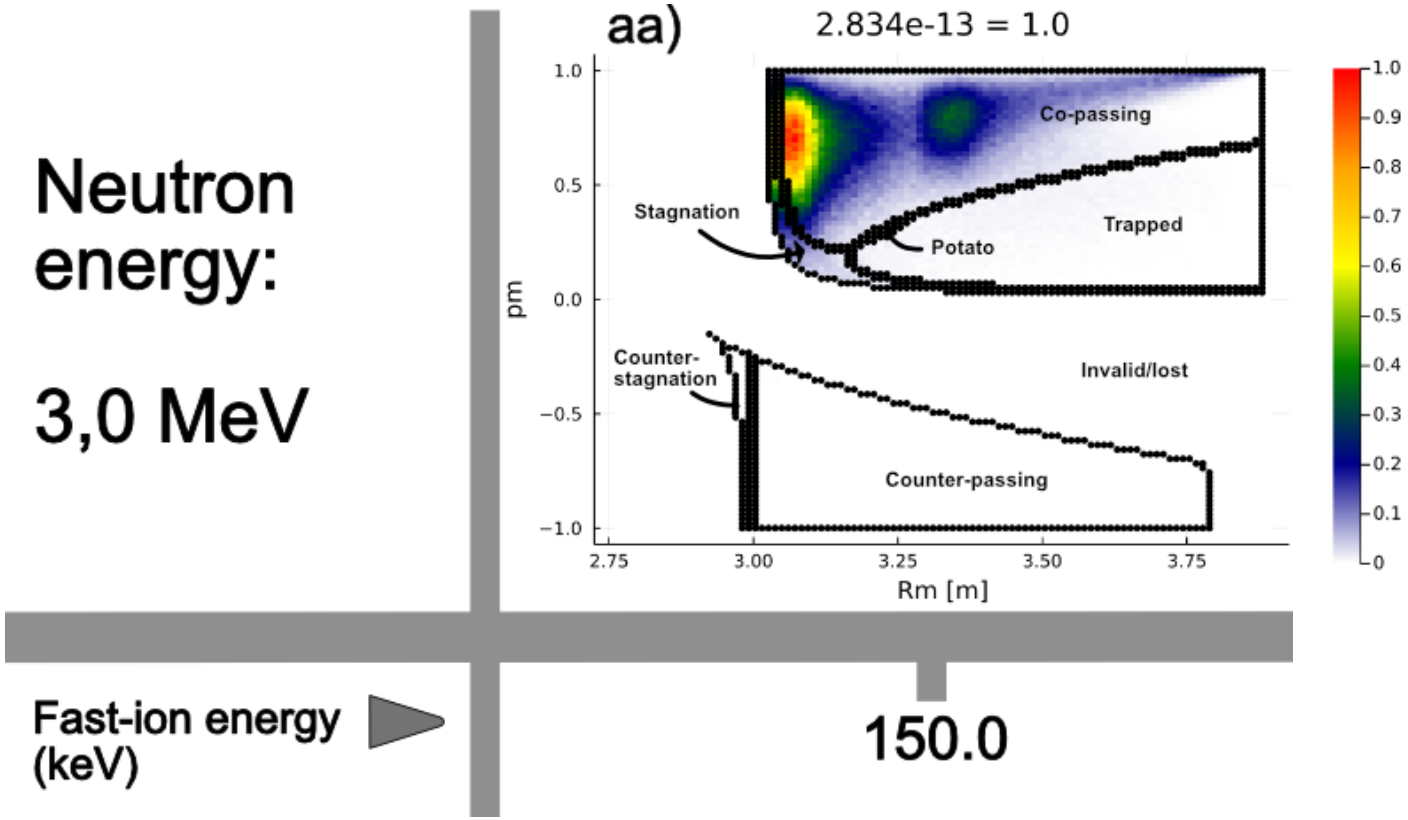
Oblique sightline - NE213 - D(D,n)^{3}He





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Orbit weight functions
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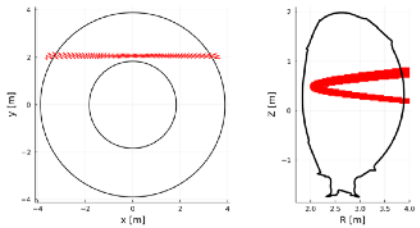

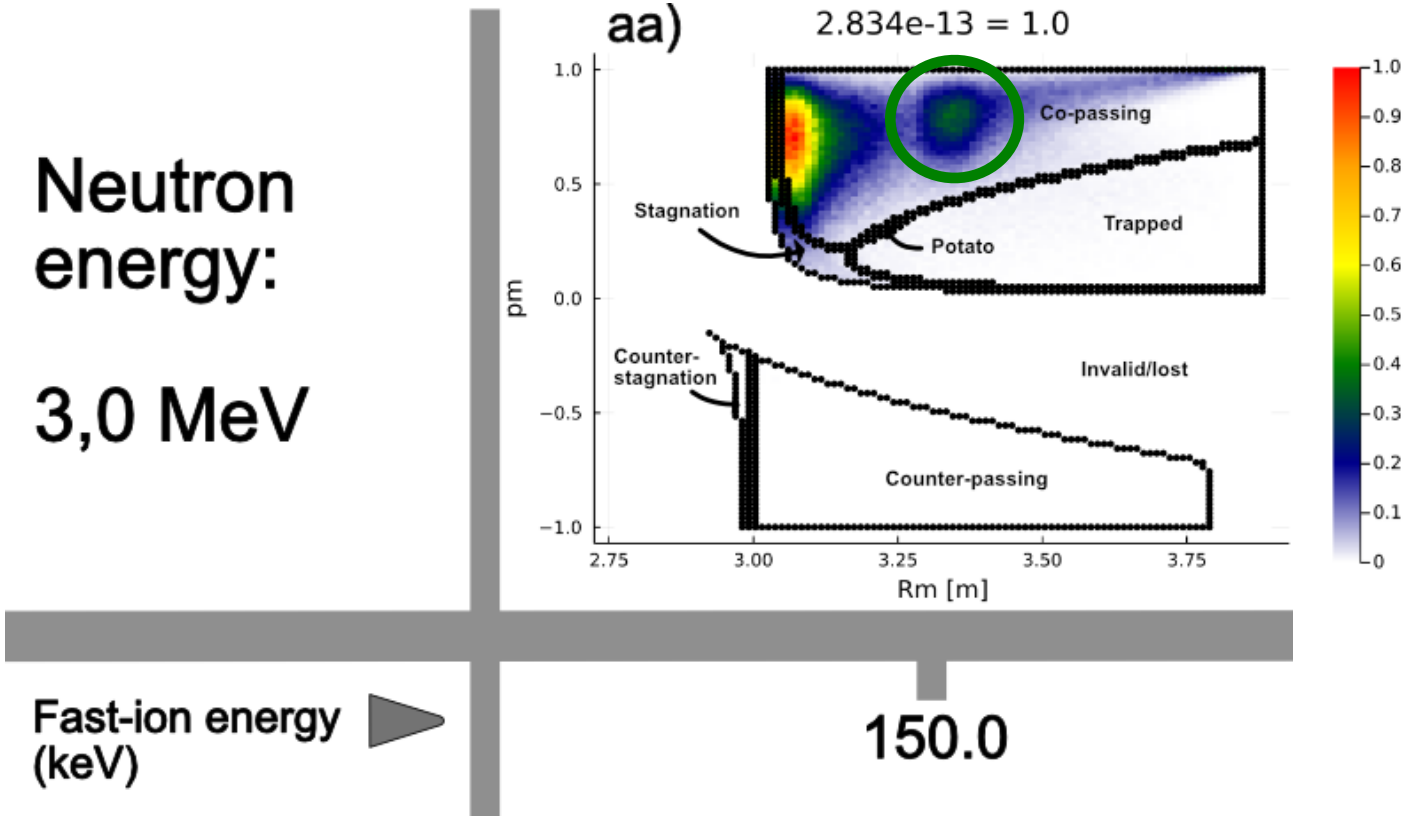

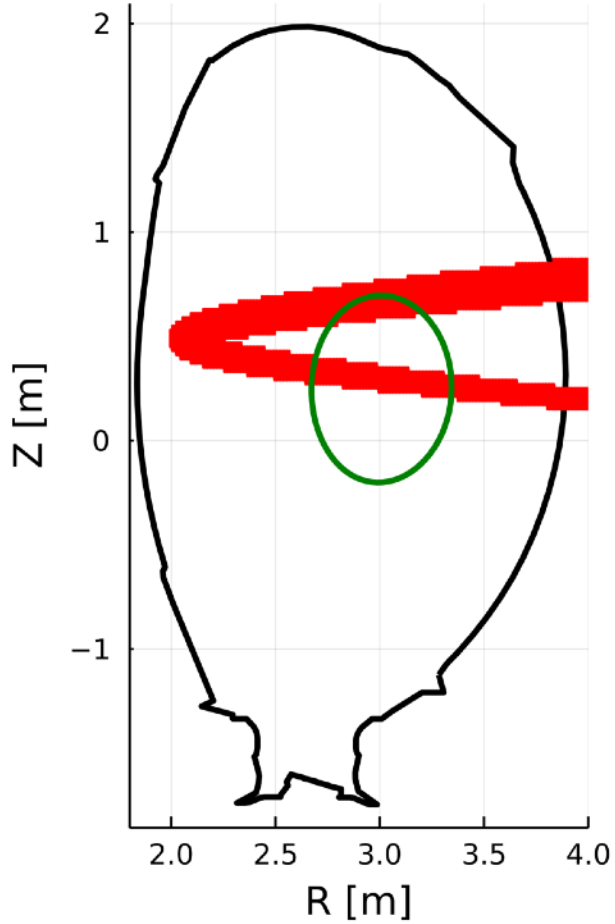

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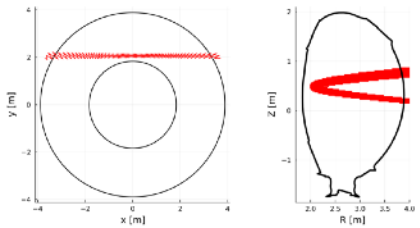
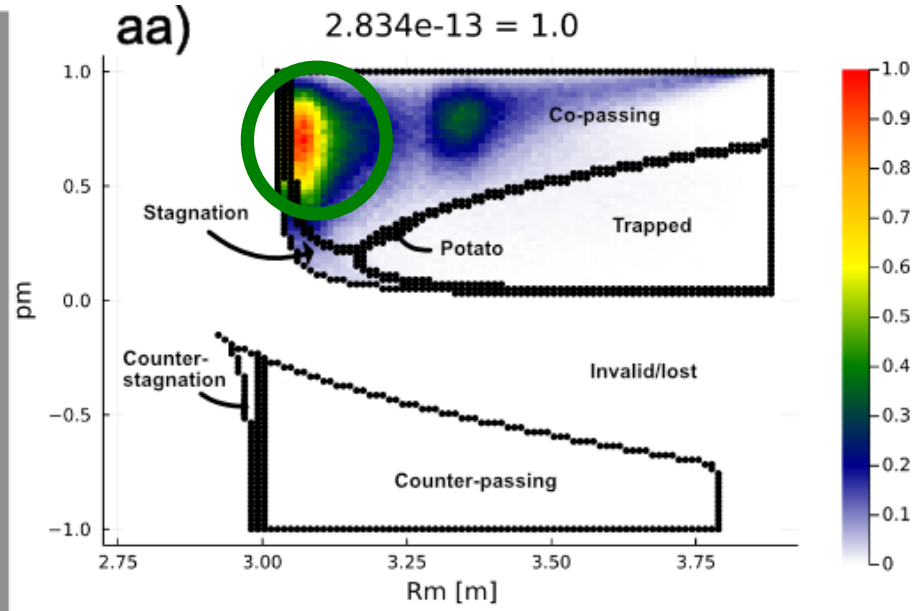
Orbit weight functions
Oblique sightline - NE213 - D(D,n)^{3}He



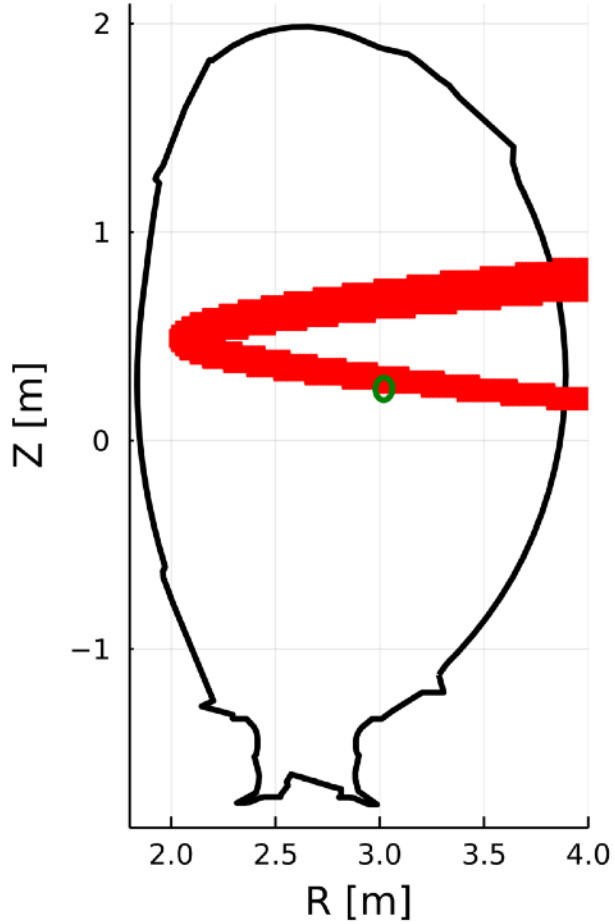
Neutron energy:

3,0 MeV



Fast-ion energy (keV)

150.0



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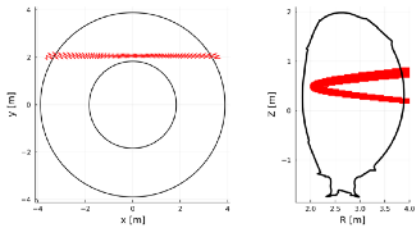

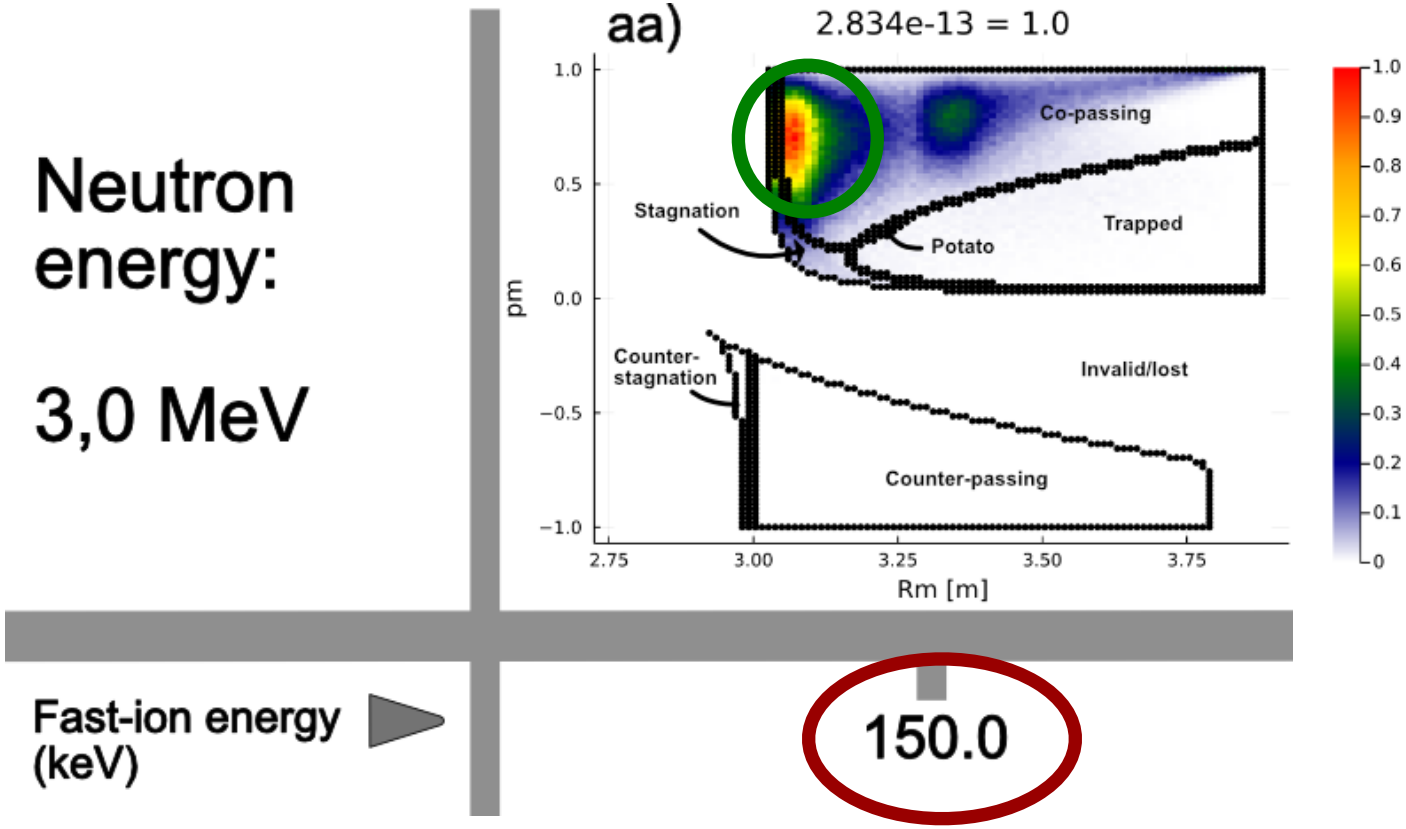

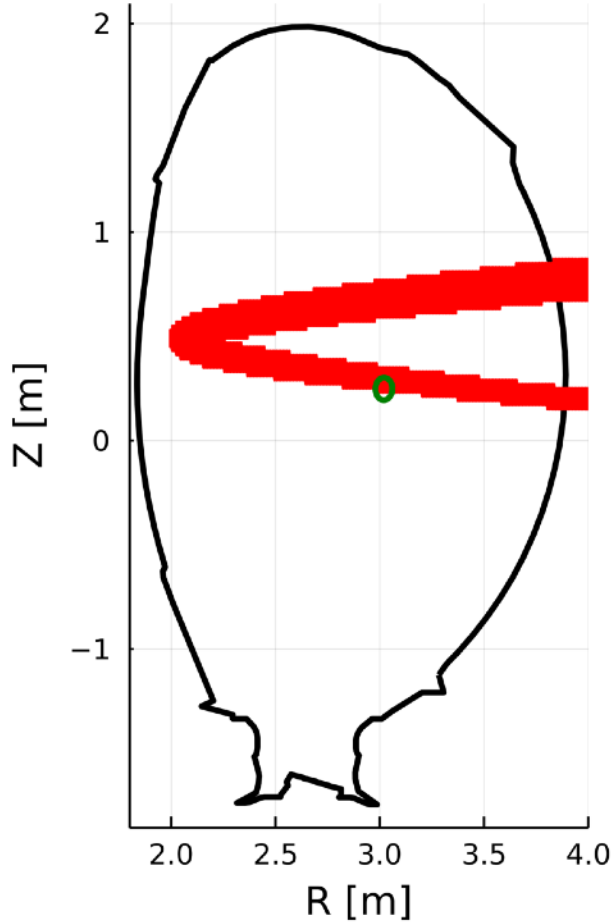

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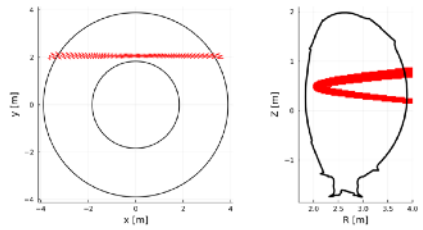

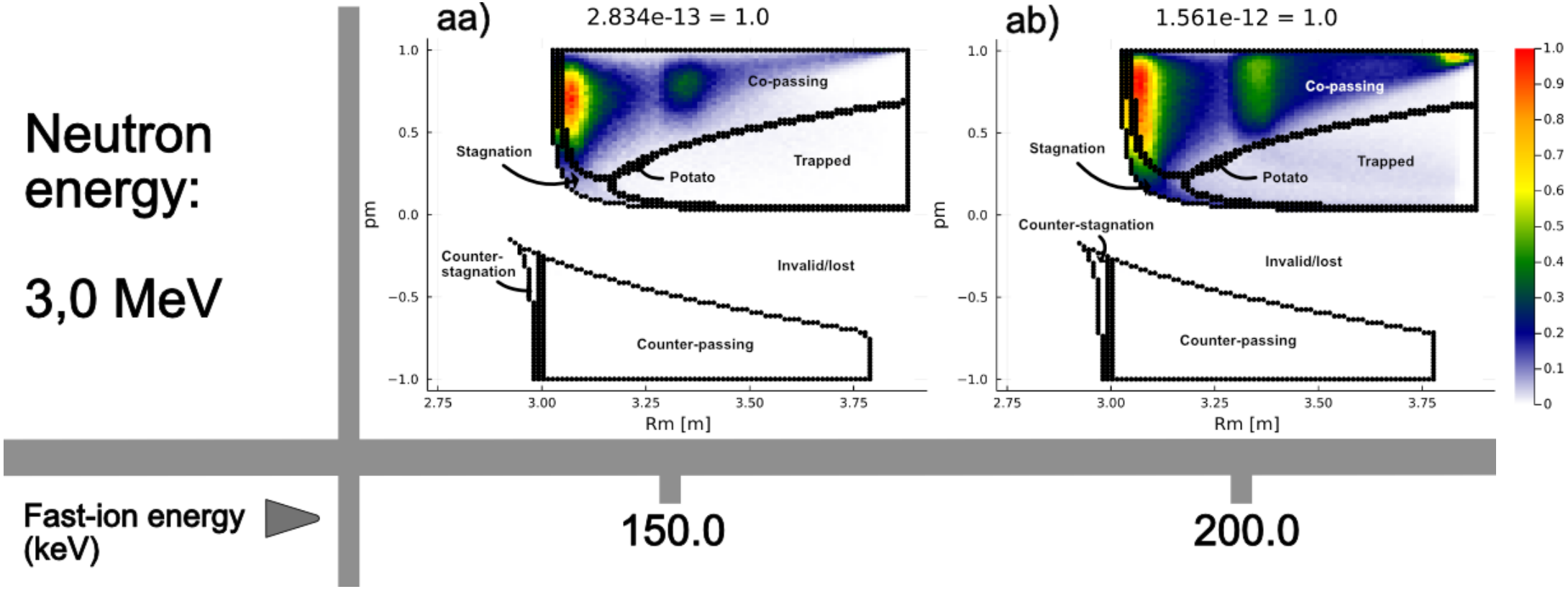

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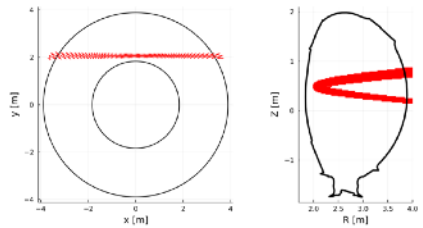

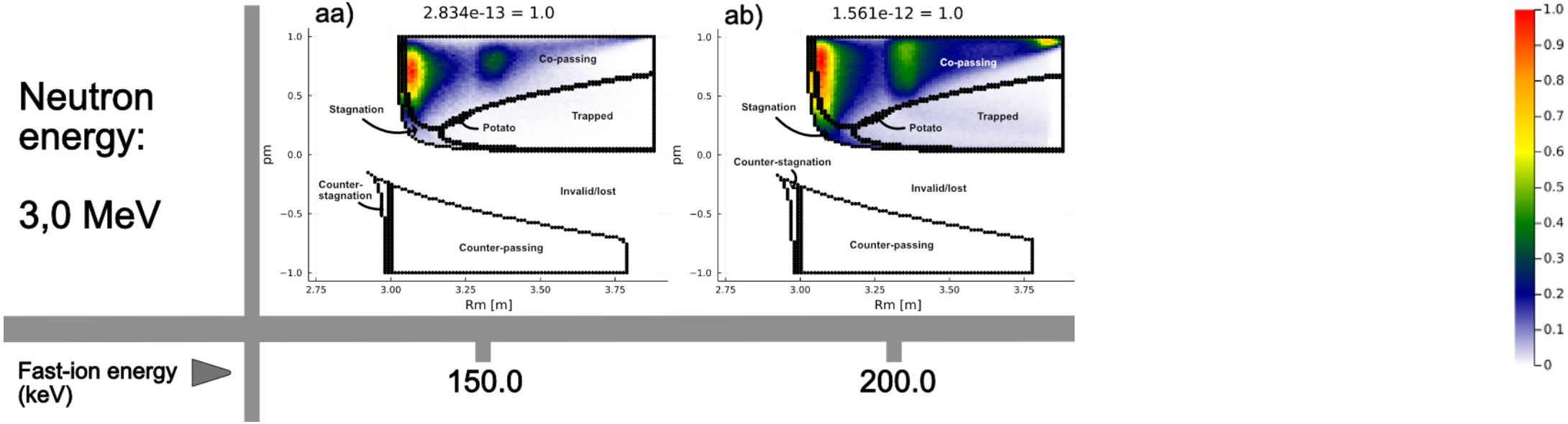

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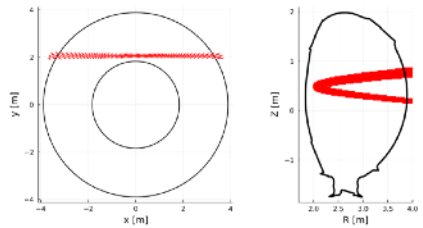

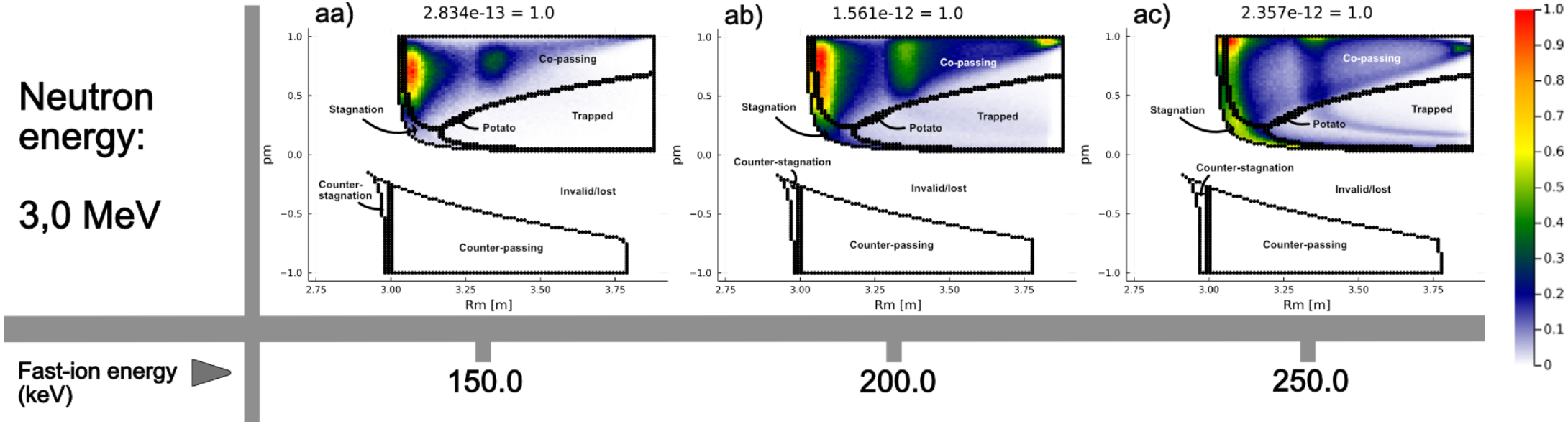

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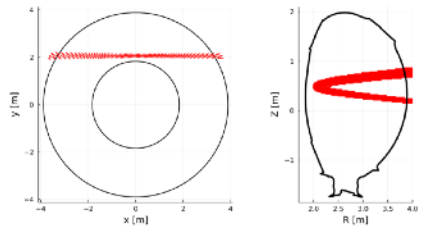

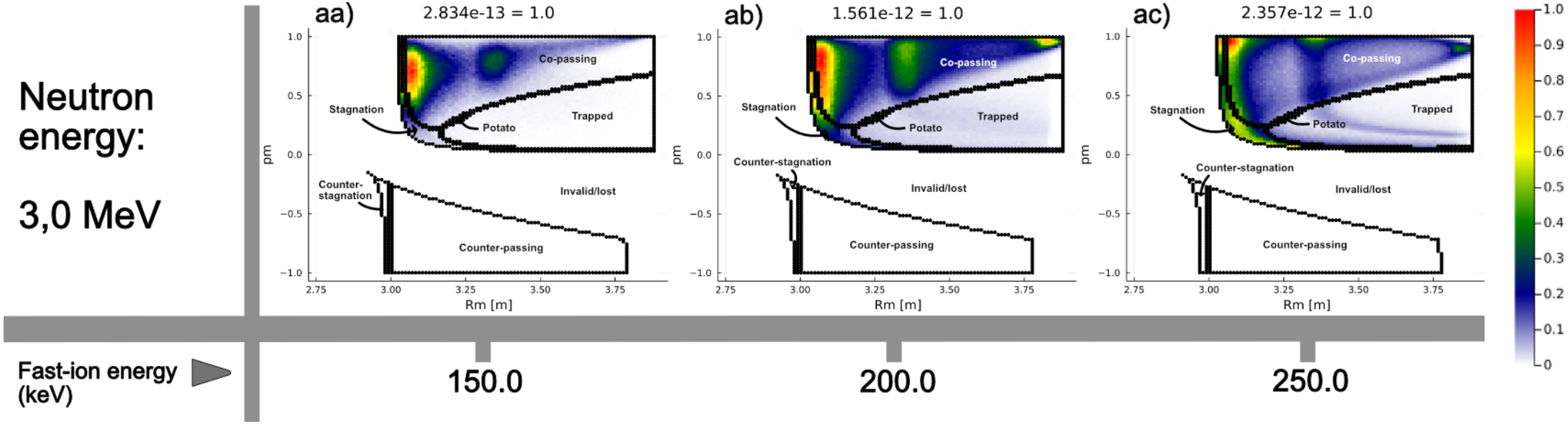

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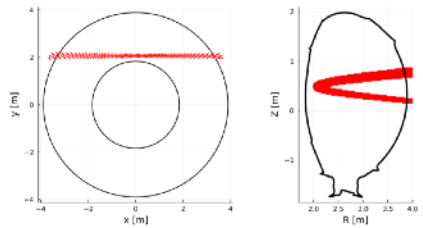

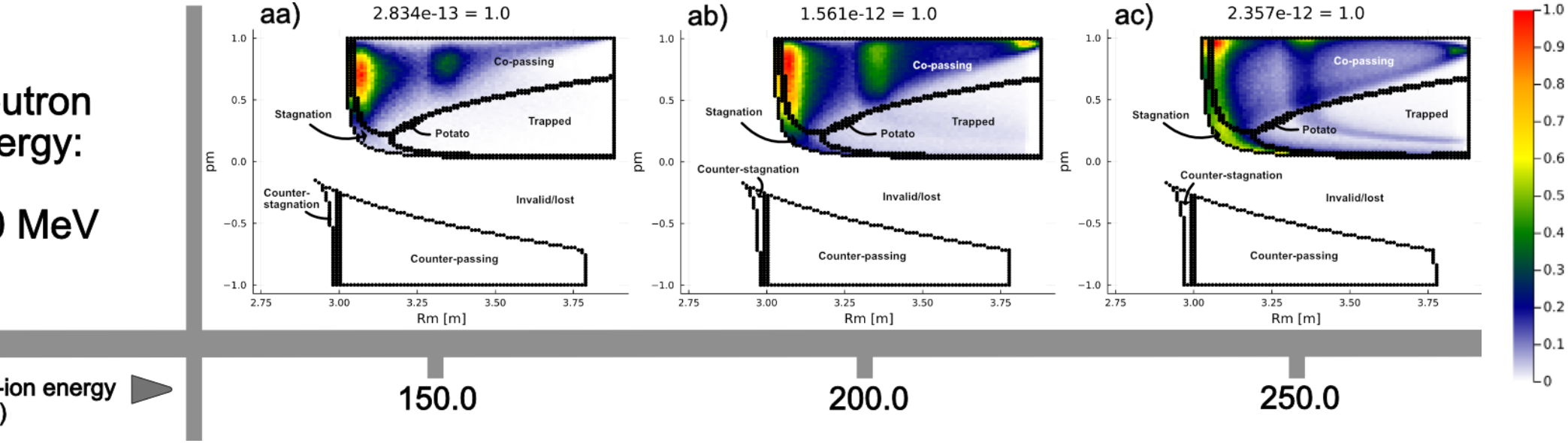

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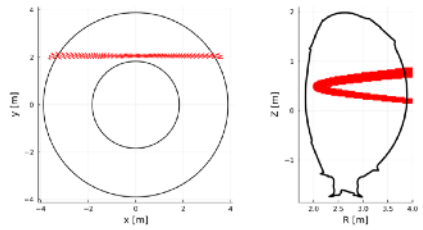

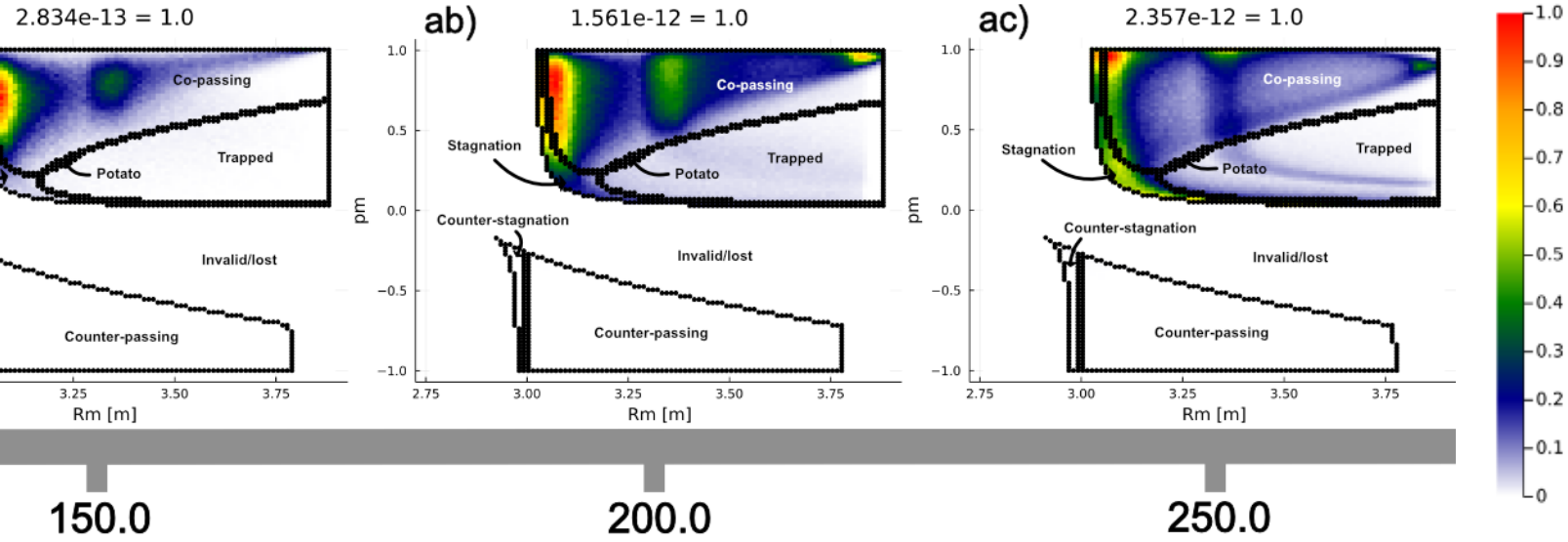

JET shot No. 94701 at 10.7932 s

Slide keywords: NE213 high sensitivity trapped orbits banana tip

Orbit weight functions

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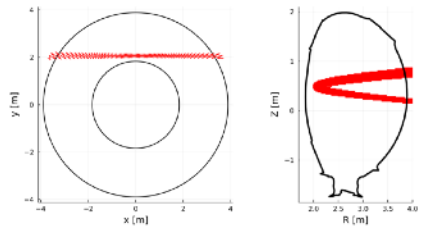

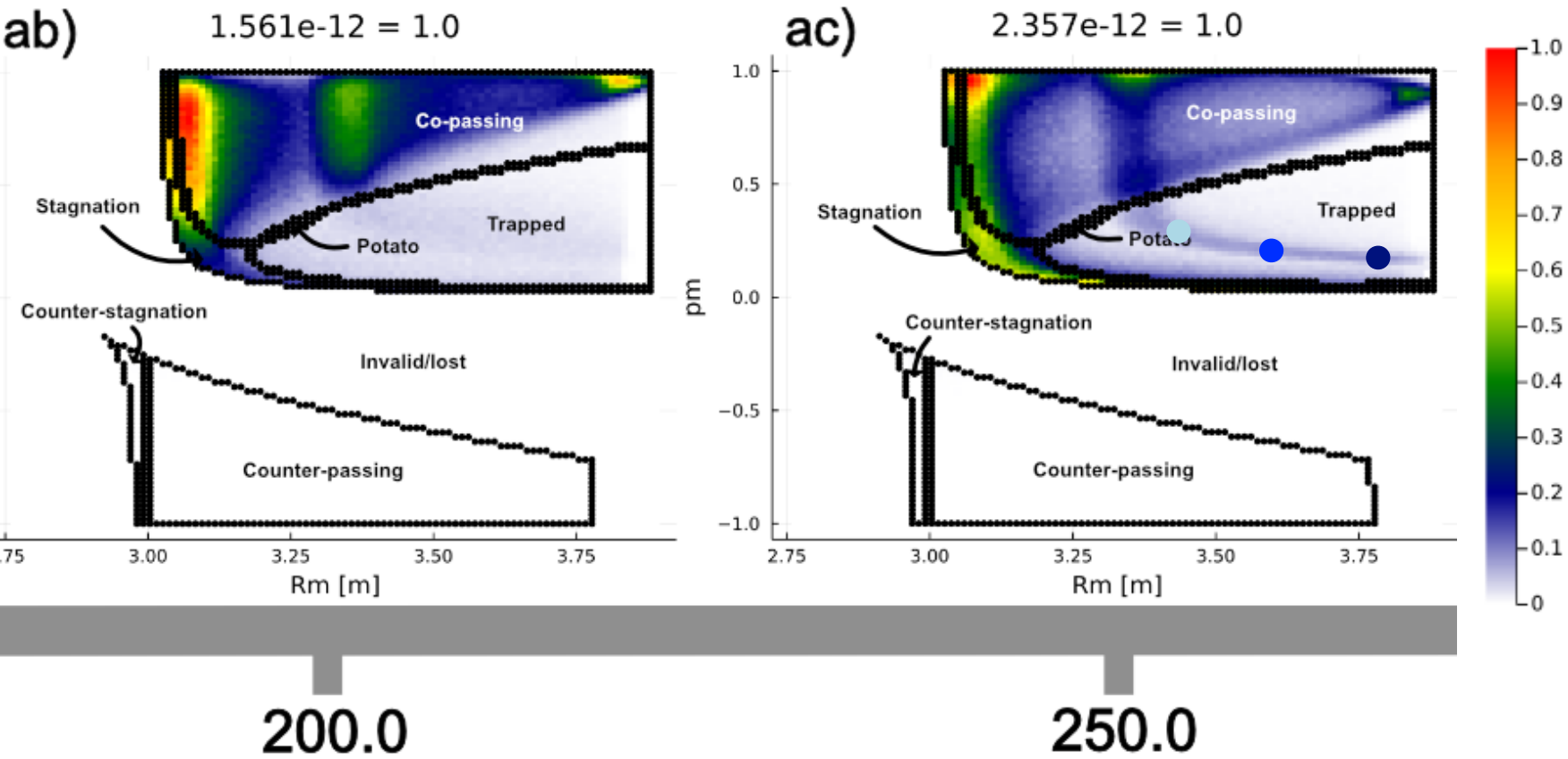

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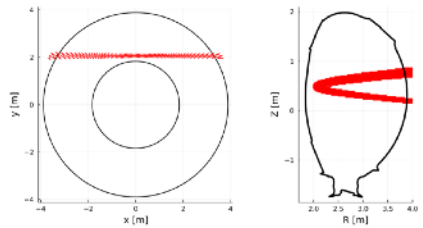

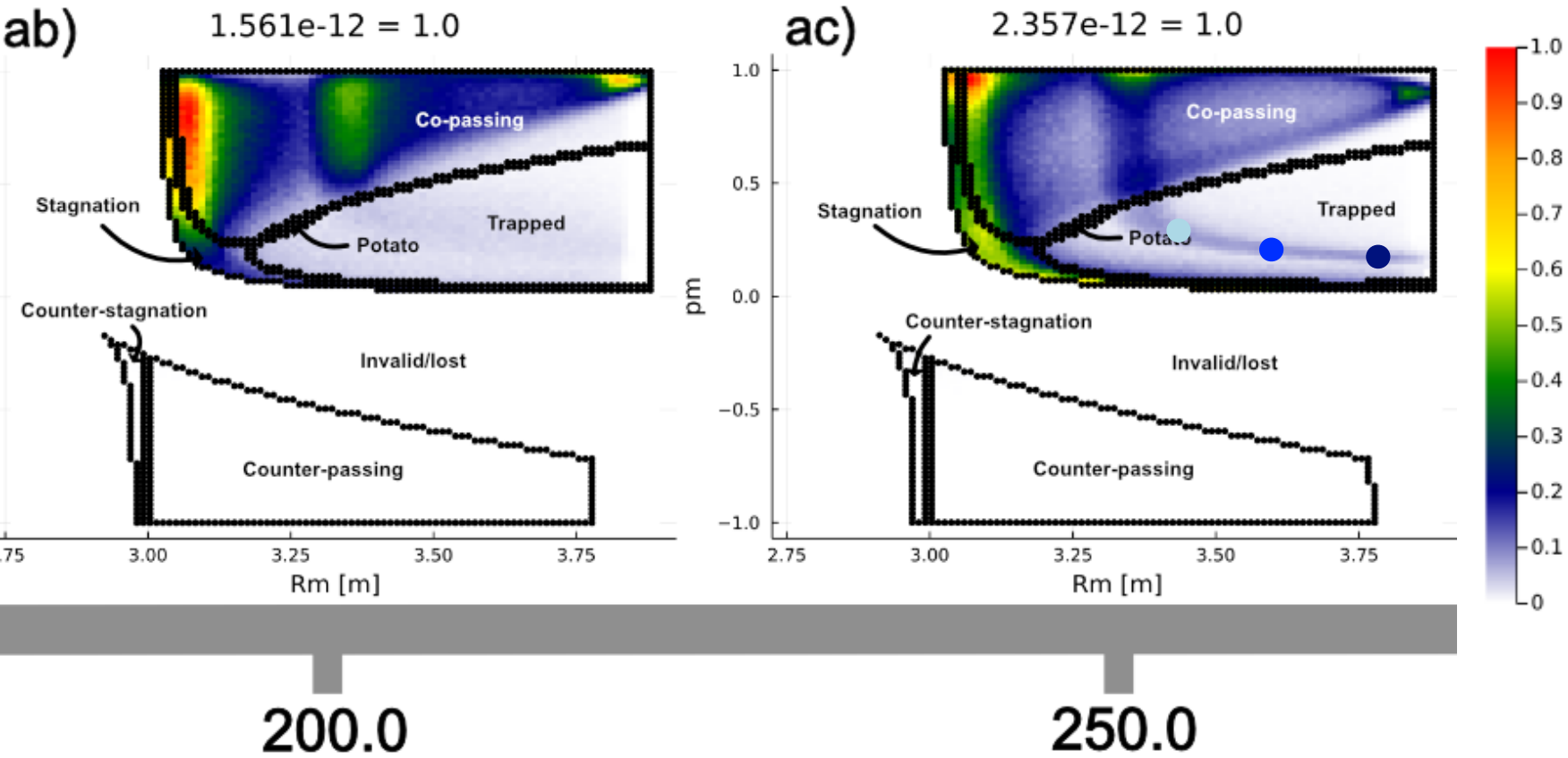

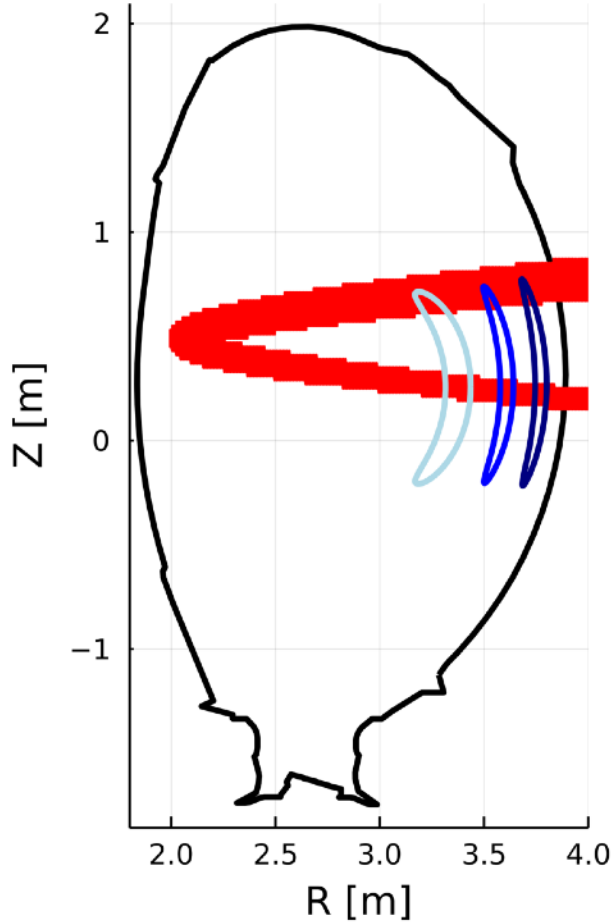

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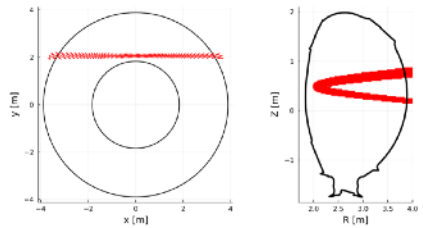
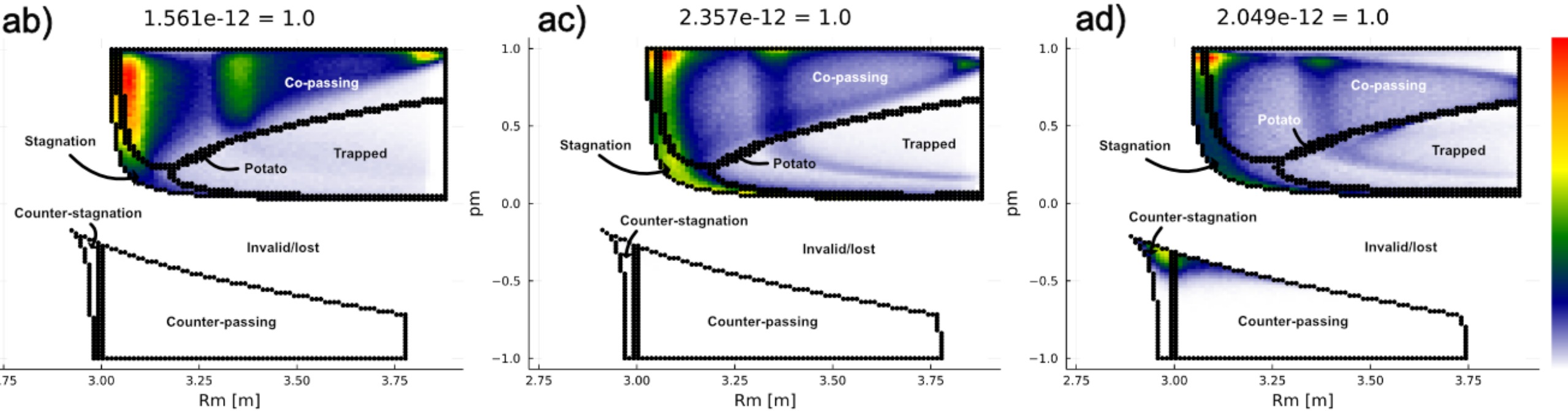
Orbit weight functions
Oblique sightline - NE213 - D(D,n)^{3}He





200.0 250.0 600.0

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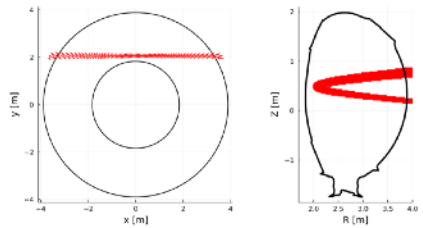

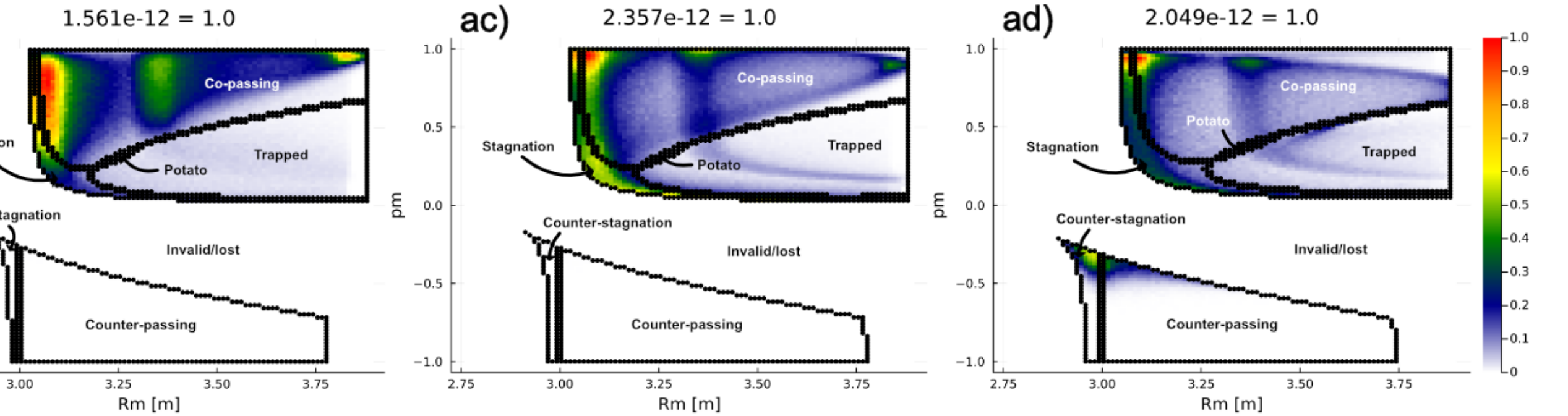

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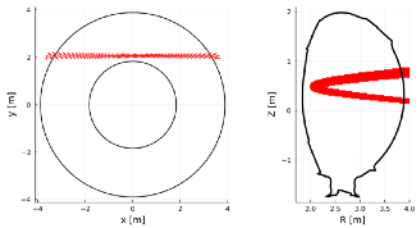

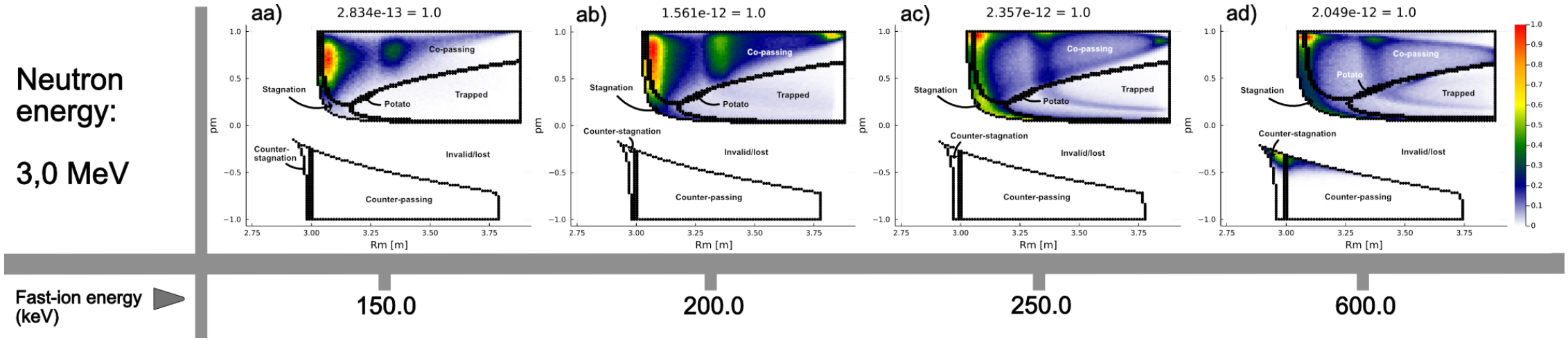

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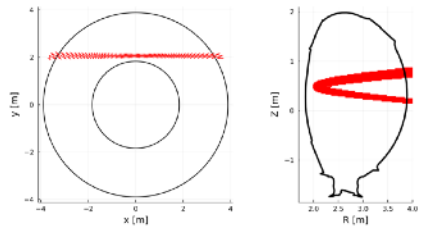

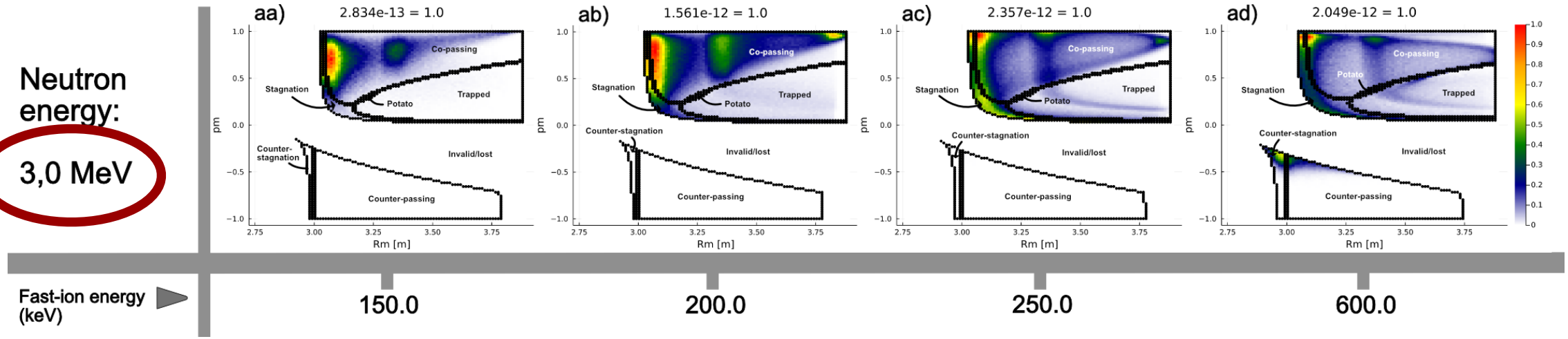

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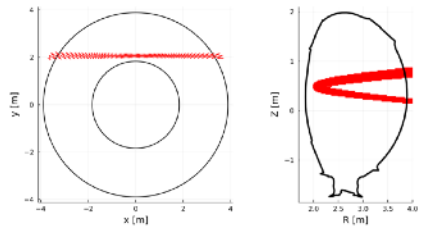

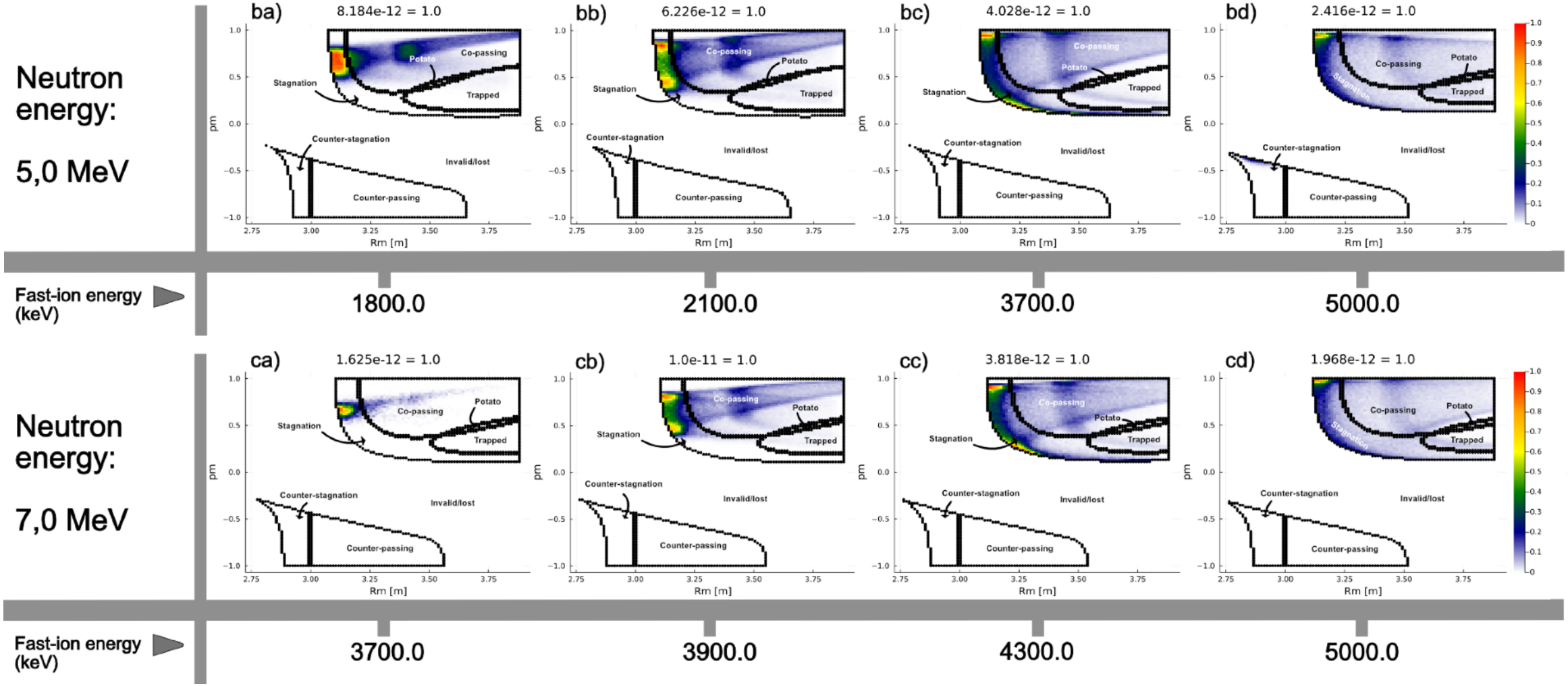

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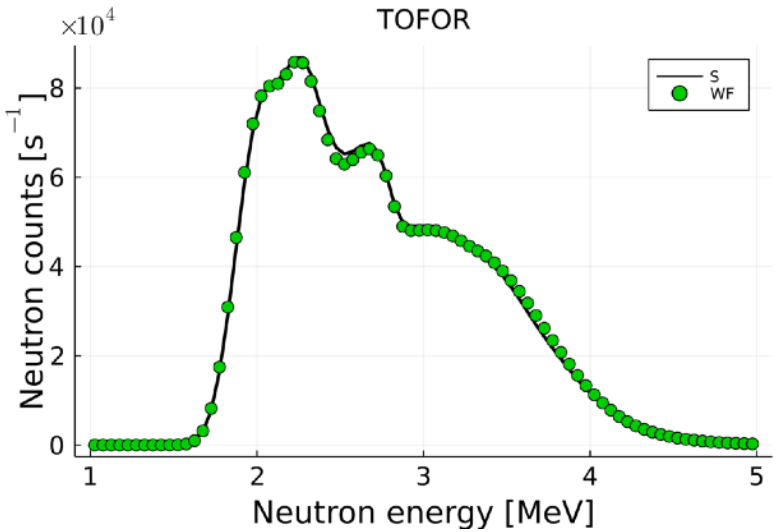
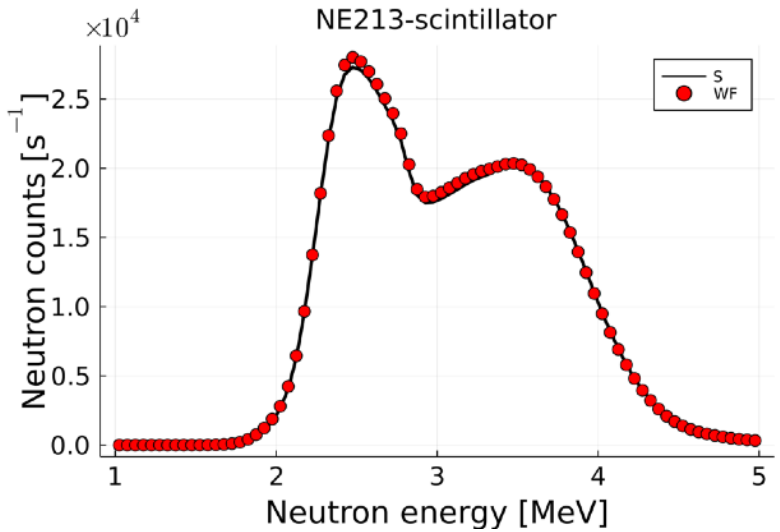
WF analysis

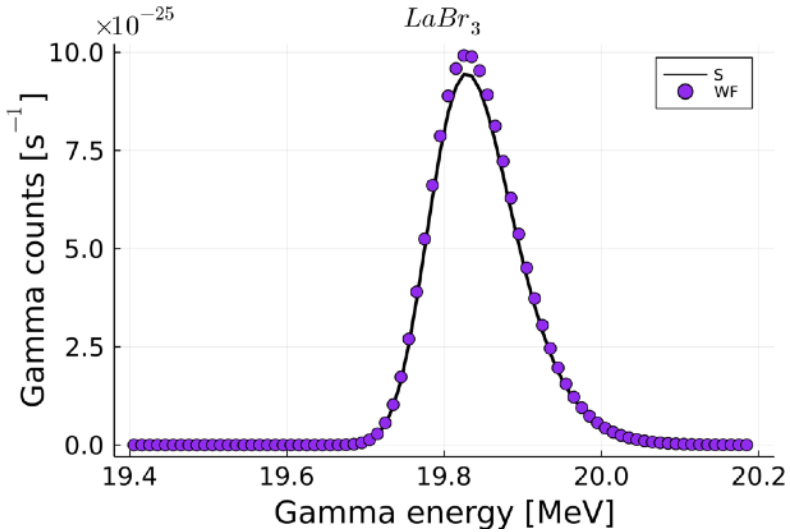
$$S = WF$$

WF analysis

Computed with Maxwellian test distribution for the purpose of illustration







$$S = WF$$

JET shot No. 94701 at 10.7932 s

WF analysis

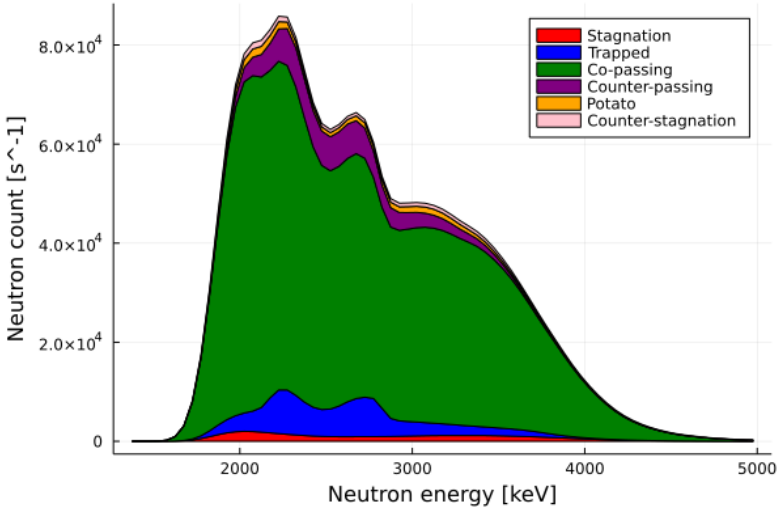
$$WF = \sum_h W_h F_h$$

$h =$ **co-passing, trapped, counter-passing,…**

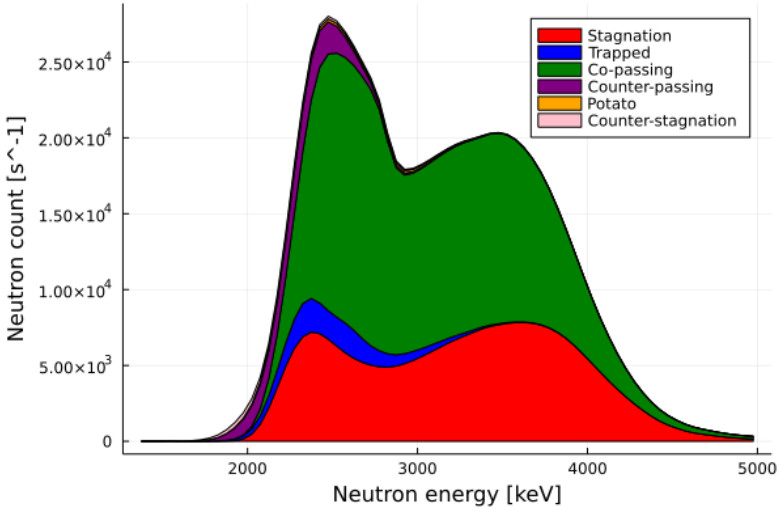
WF analysis

Computed with Maxwellian test distribution for the purpose of illustration

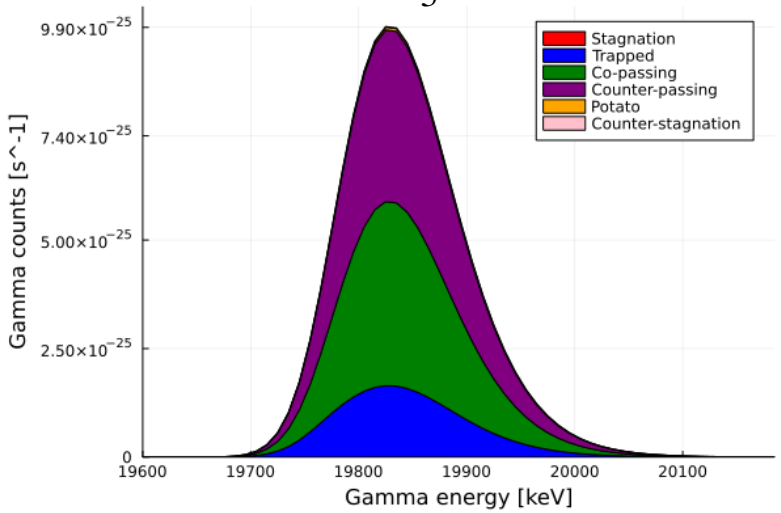
TOFOR



NE213



$LaBr_3$



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WF analysis

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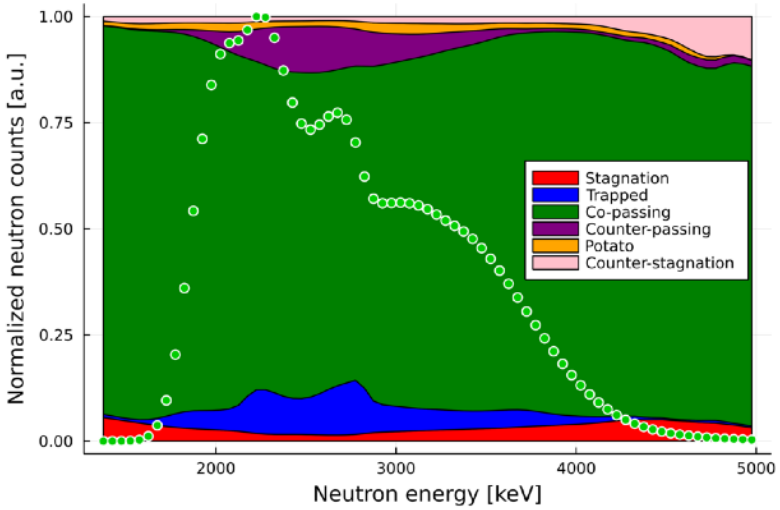
TOFOR



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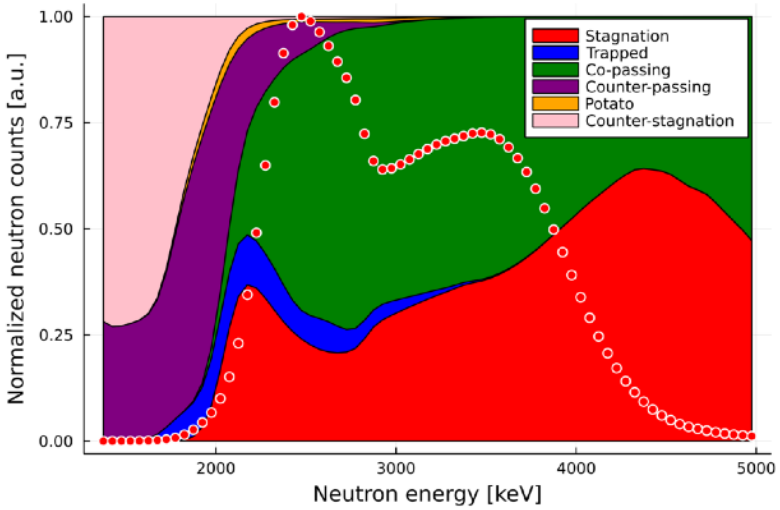
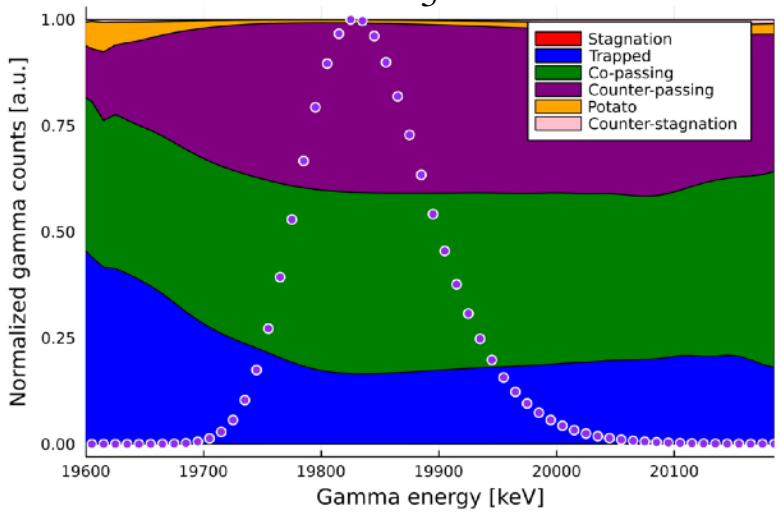
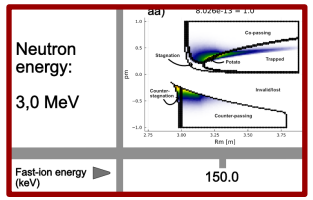


$LaBr_3$

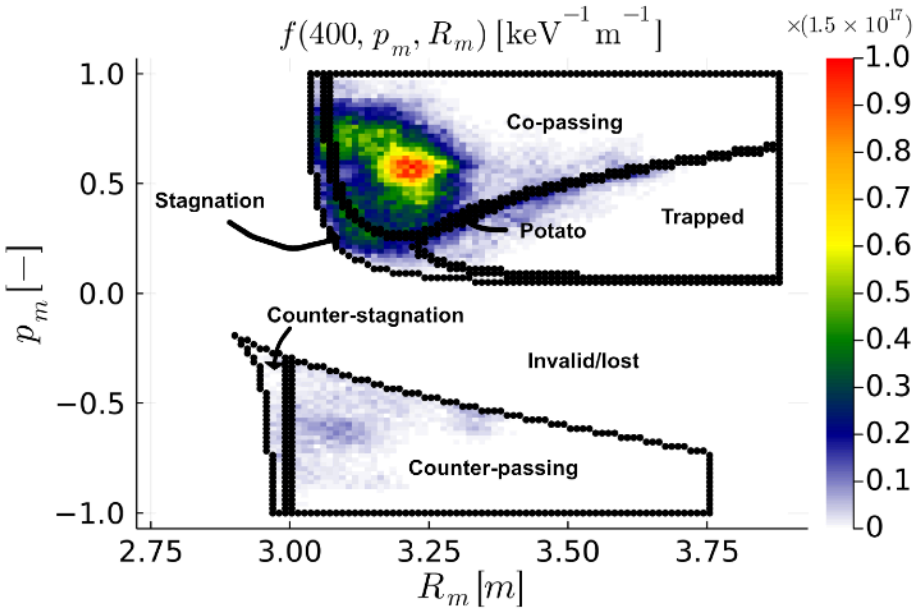


JET shot No. 94701 at 10.7932 s

WF analysis

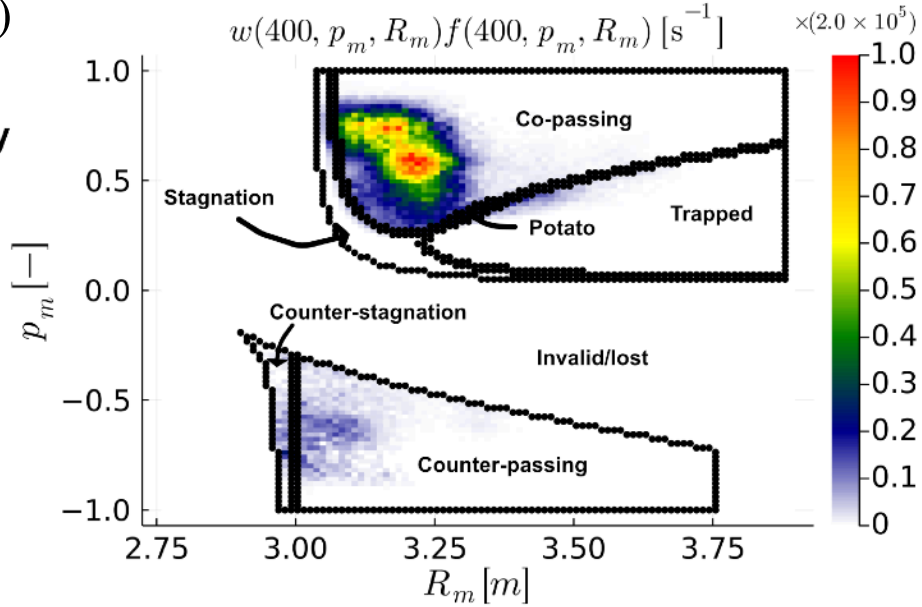


$w(400, p_m, R_m)$



a)

Pointwise multiply



b)

JET shot No. 94701 at 10.7932 s

Outline

- ~~Introduction~~
- ~~Orbit space (E, p_m, R_m)~~
- ~~Weight Function Formalism~~
- ~~Orbit weight functions~~
  - ~~Perpendicular sightlines~~
    - ~~TOFOR (NES)~~
    - ~~$LaBr_3$ detector (GRS)~~
  - ~~Oblique sightline (NE213 detector, NES))~~
- ~~WF analysis~~
- **Conclusion and outlook**

Conclusion and outlook

- Orbit weight functions for NES and one-step reaction GRS have been developed
- Orbit weight functions map out the sensitivity of a diagnostic to fast-ion orbits
- The orbit sensitivity has patterns that can be identified by scanning 3D orbit space slice-by-slice in terms of fast-ion energy while superimposing topological boundaries between different orbit types
- Orbit weight functions are able to mimic forward models for computing synthetic signals
- Lastly, combine with fast-ion distribution functions to split synthetic signals into orbit type constituents
- In future work, orbit weight functions will be used to reconstruct the fast-ion distribution in terms of orbits, which can be transformed to (E, p, R, Z)

Orbit Weight Functions for Neutron Emission and One-step Reaction Gamma-ray Spectroscopy

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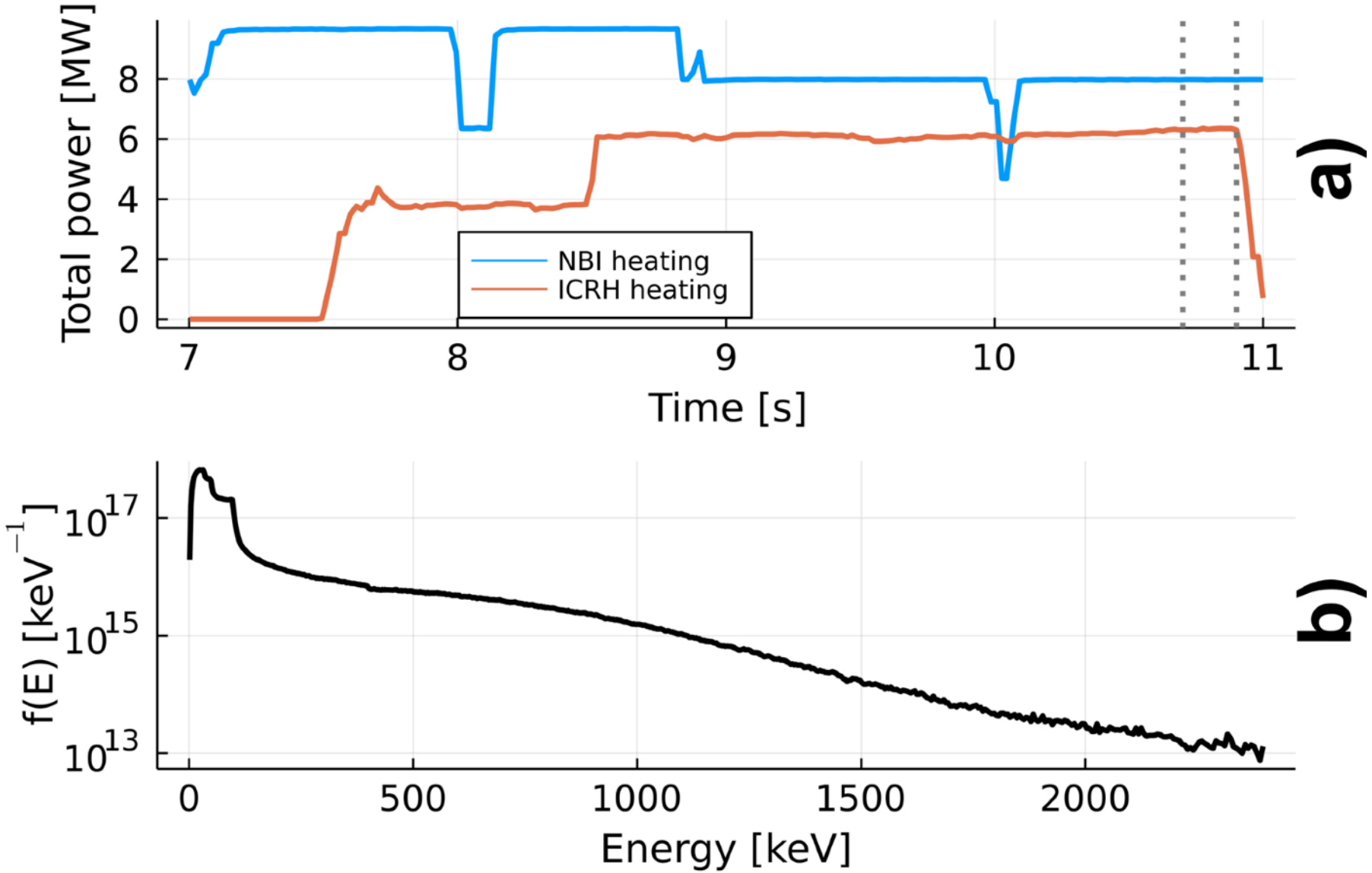
Thank you for your attention!

**See the author list of 'Overview of JET results for optimising ITER operation' by J. Mailloux et al. to be published in Nuclear Fusion Special issue: Overview and Summary Papers from the 28th Fusion Energy Conference (Nice, France, 10-15 May 2021)*


Total power [MW]
0
2
4
6
8
7
8
9
10
11
Time [s]
NBI heating
ICRH heating
a)
f(E) [keV^{-1}]
10^{17}
10^{15}
10^{13}
0
500
1000
1500
2000
Energy [keV]
b)

Hamiltonian theory of adiabatic motion of relativistic charged particles

Cite as: Phys. Plasmas **14**, 092107 (2007); https://doi.org/10.1063/1.2773702
Submitted: 13 June 2007 . Accepted: 30 July 2007 . Published Online: 18 September 2007

Xin Tao, Anthony A. Chan, and Alain J. Brizard