NSTX-U

NSTX-U is sponsored by the
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Theory for control of Alfvén instabilities and implications for anomalous electron energy transport

Jeff Lestz (*UCI*)
Elena Belova, Nikolai Gorelenkov, Eric Fredrickson (*PPPL*)
Neal Crocker, Shawn Tang (*UCLA*)

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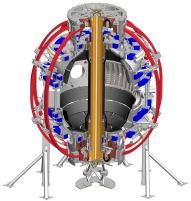

Motivation and Main Results

- **Motivation**: sub-cyclotron Alfvén Eigenmodes (AEs) have been experimentally linked to anomalous electron temperature flattening in NSTX
  - No theory quantitatively reproduces the observations
- **Goal**: predict instability conditions for realistic neutral beam (NBI) distributions using analytic theory and numerical simulations
- **Main result**: simple theory describes high frequency AE excitation and demonstrates how to stabilize modes with additional NBI source
  - Explains NSTX-U suppression of AEs with new beam source
  - Provides insight to control and study the associated electron energy transport

Outline

- Introduction: Alfvén Eigenmodes Linked to Anomalous Electron Transport
- Hybrid Simulations Reveal Complicated Stability Boundaries
- Simple Analytic Theory Explains Simulations
- Theory Yields Experimental Insights
- Injecting Multiple Beams Can Control Alfvén Eigenmodes

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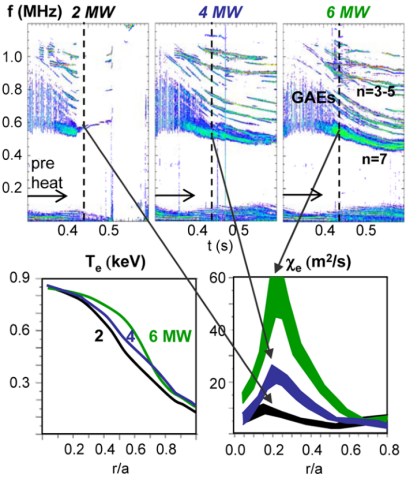
Anomalous T_e Flattening in NSTX Correlates with CAE/GAEs

- Beam-driven compressional (CAE) and global (GAE) Alfvén eigenmodes have been excited in NSTX(-U), MAST, DIII-D, AUG, and may be present in ITER
- Temperature profiles can not be explained by turbulence in gyrokinetic simulations
- Methods to control CAEs/GAEs are essential to studying and predicting the electron energy transport that they induce[1]

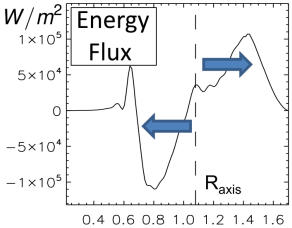


[1] D. Stutman *et al.* Phys. Rev. Lett. **102**, 115002 (2009)

How Can CAEs/GAEs Affect Temperature Profiles?

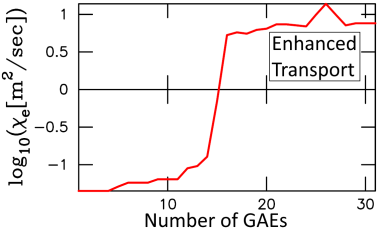
Energy Channeling

- AE in core can mode convert to KAW near edge, damping on electrons
- Modifies effective beam energy deposition profile[2,3]



Orbit Stochastization

- Sufficiently many unstable AEs can stochasticize electron orbits
- Enhances diffusion, transporting energy away from the core[4]



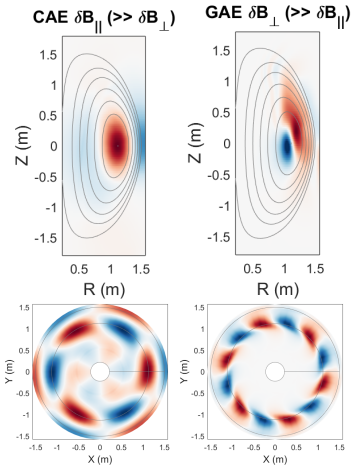
[2] Y.I. Kolesnichenko *et al.* Phys. Rev. Lett. **104**, 075001 (2010)
[3] E.V. Belova *et al.* Phys. Rev. Lett. **115**, 015001 (2015)
[4] N.N. Gorelenkov *et al.* Nucl. Fusion **50**, 084012 (2010)

Sub-cyclotron Alfvén Eigenmodes in NSTX(-U)

- **Compressional Alfvén eigenmode** (CAE): ideal magnetosonic mode: $\omega \approx k v_A$
- **Global Alfvén eigenmode** (GAE): discrete shear Alfvén eigenmode existing below minimum of Alfvén continuum: $\omega \leq \left[k_{\|}(r) v_A(r)\right]_{\min}$
- CAEs/GAEs interact with fast ions through Doppler-shifted cyclotron resonance

$$\omega - k_{\|} \left\langle v_{\|} \right\rangle = \ell \left\langle \omega_{ci} \right\rangle$$

- Observed to propagate both **co-** ($k_{\|} > 0, \ell = 0$) and **cntr-** ($k_{\|} < 0, \ell = 1$) to the beam/plasma current with $|n| = 3 - 15$, $\omega/\omega_{ci} \approx 0.1 - 1.2$



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Hybrid Simulation Method

- Hybrid MHD and Particle code (HYM)
- Initial value code in tokamak geometry
- Thermal plasma: single fluid resistive MHD model
- Beam ions: full orbit kinetic particles with δf scheme
  - Captures Doppler-shifted cyclotron resonance which drives the modes
- Equilibrium includes fast ion effects self-consistently[5]
- Thermal plasma and beam ions coupled through current in momentum equation

$$\rho \frac{d\boldsymbol{V}}{dt} = -\nabla P + \underbrace{(\boldsymbol{J} - \boldsymbol{J_b}) \times \boldsymbol{B} - e n_b (\boldsymbol{E} - \eta \delta \boldsymbol{J})}_{\text{current coupling}} + \mu \Delta \boldsymbol{V}$$

[5] E.V. Belova *et al.* Phys. Plasmas **10**, 3240 (2003)

Fast Ion Distribution Model

- Equilibrium distribution $F_0 = F_1(v)F_2(\lambda)F_3(p_\phi)$
  - Trapping parameter $\lambda = \mu B_0/\mathcal{E} \approx v_\perp^2/v^2$

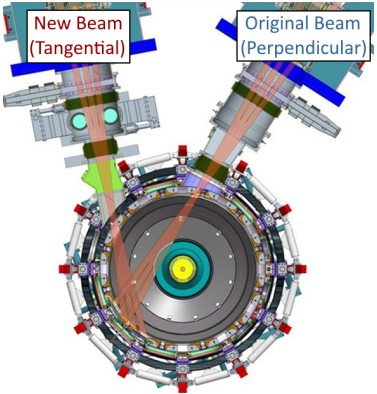
$$F_1(v) = \frac{1}{v^3 + v_c^3}$$

$$F_2(\lambda) = \exp\left(-(\lambda - \lambda_0)^2/\Delta\lambda^2\right)$$

$$F_3(p_\phi) = \left(\frac{p_\phi - p_{\min}}{m_i R_0 v - q_i \psi_0 - p_{\min}}\right)^\sigma$$

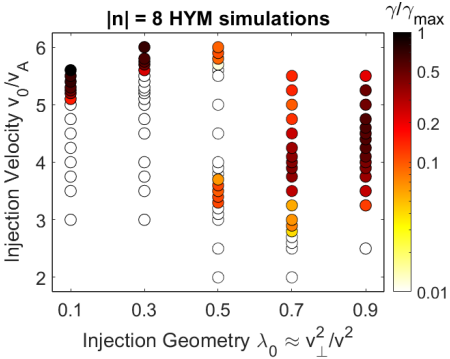
$$\text{for } v < v_0 \quad \text{and} \quad p_\phi > p_{\min}$$

- NSTX: $v_0/v_A \lesssim 5$, $\lambda_0 = 0.5 - 0.7$ for original beam
- NSTX-U: $v_0/v_A \lesssim 2$, $\lambda_0 = 0$ for new beam
- Parameters matched to TRANSP ($\Delta\lambda \approx 0.3$)



Hybrid Simulations Predict Rich Mixture of CAEs/GAEs

- Parameter scan of injection geometry (λ_0) and velocity (v_0/v_A) reveals complicated stability boundaries for different mode types[6]
  - Simulated $|n| = 1 - 12$ separately
- GAEs excited at lower beam energy ($v_0/v_A \gtrsim 2.5$) than CAEs ($v_0/v_A \gtrsim 4$), typically with larger growth rates
- co-GAEs excited with very tangential beams
  - Anomalous cyclotron resonance ($\ell = -1$)
  - May exist in future NSTX-U experiments



- Colored dot: growth rate of most unstable mode in each simulation
- White dot: no unstable modes

[6] J.B. Lestz *et al.* Nucl. Fusion **61**, 086016 (2021)

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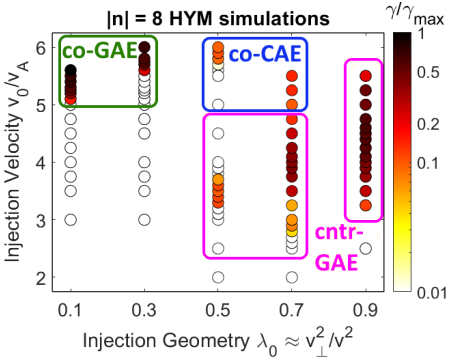

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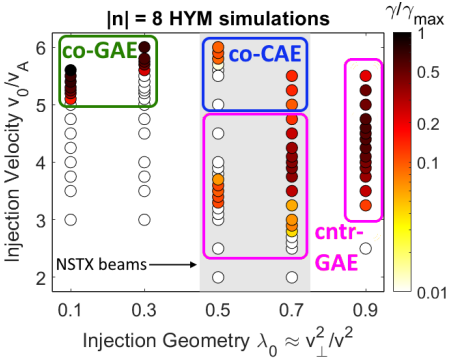

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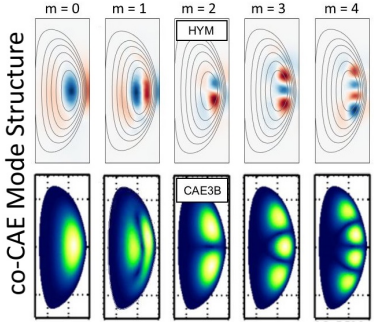
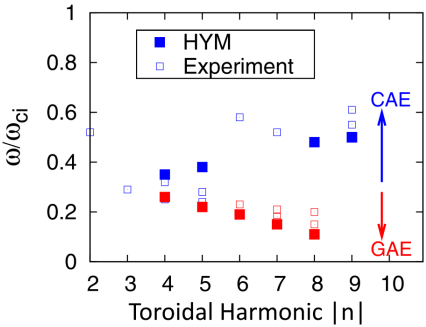
[6] J.B. Lestz *et al.* Nucl. Fusion **61**, 086016 (2021)

Realistic Simulations Motivate Development of Theory

- HYM simulations accurately model CAEs/GAEs in NSTX(-U) experiments and recover mode structures from CAE3B eigensolver
- Theory is needed to interpret simulation results and improve understanding to develop predictive capability





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Simple Conditions Derived for Net Drive

- Fast ion drive depends on gradients of the distribution

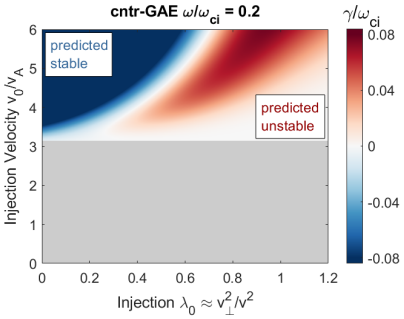
$$\gamma \propto \int h(\lambda, v) \left[\left(\frac{\ell \omega_{ci}}{\omega} - \lambda \right) \frac{\partial}{\partial \lambda} + \frac{v}{2} \frac{\partial}{\partial v} \right] f_b(\lambda, v)\, d^2\boldsymbol{v} > 0 \text{ for instability}$$

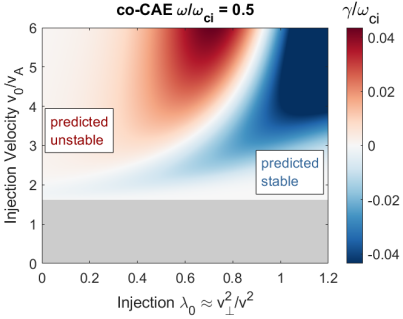
- cntr-propagating CAE/GAEs $(\ell = +1)$ driven by $\partial f_b / \partial \lambda > 0$
  - $v_0 < v_{\|,\mathrm{res}} / (1 - \lambda_0 \langle \bar{\omega}_{ci} \rangle)^{3/4}$ necessary for instability[7]
- co-propagating CAEs $(\ell = 0)$ driven by $\partial f_b / \partial \lambda < 0$
  - $v_0 > v_{\|,\mathrm{res}} / \left(1 - \frac{\langle \bar{\omega}_{ci} \rangle}{2} \left[\lambda_0 + \sqrt{\lambda_0^2 + 8\Delta\lambda^2/3} \right] \right)^{5/8}$ necessary for instability[8]
- Due to resonance condition, $v_{\|,\mathrm{res}}(\omega/\omega_{ci}, k_\|/k_\perp) = (\omega - \ell \langle \omega_{ci} \rangle) / k_\|$

[7] J.B. Lestz *et al.* Phys. Plasmas **27**, 022513 (2020)
[8] J.B. Lestz *et al.* Phys. Plasmas **27**, 022512 (2020)

Analytic Bounds Explain Simulation Results

- Numerically integrate full analytic expression for growth rate to predict instability
  - red: net fast ion drive, blue: net fast ion damping
  - gray: insufficient beam velocity for resonant interaction





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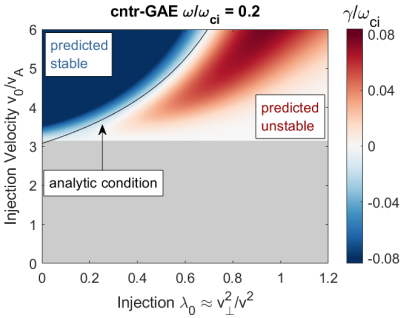

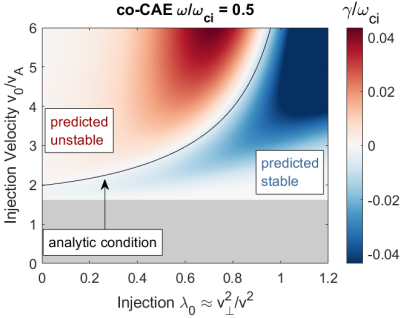

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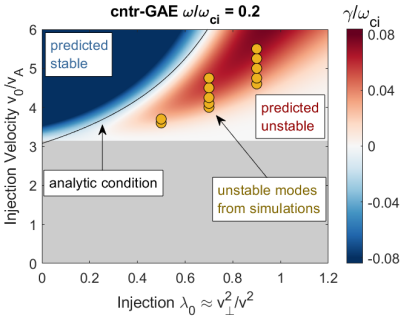
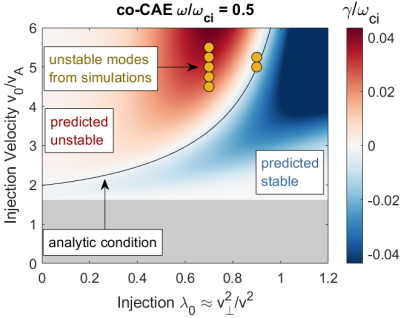
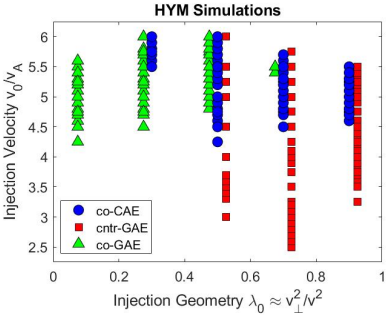
- black curve: approximate analytic conditions reproduce numerical calculation
- gold: unstable modes from HYM simulations agree with theory





Beam Parameters Determine Most Unstable Modes

- cntr-GAEs prefer $\lambda_0 \to 1$, whereas co-GAEs require small λ_0 for instability
  - Driven by opposite sign of $\partial f_0/\partial\lambda$
- co-CAEs are less unstable due to a smaller coefficient multiplying growth rate
  - $\gamma_{\ell=0} \sim (\omega/\omega_{ci})\gamma_{\ell=\pm1}$
- cntr-GAEs can be destabilized at small v_0/v_A
  - co-GAEs require large Doppler shift
  - co-CAEs suffer relatively large $\partial f_0/\partial v$



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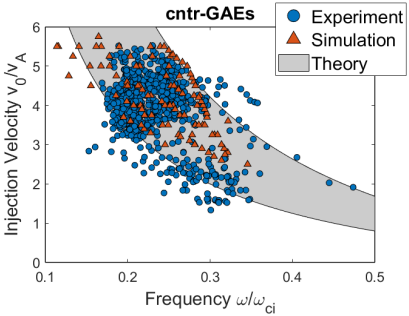
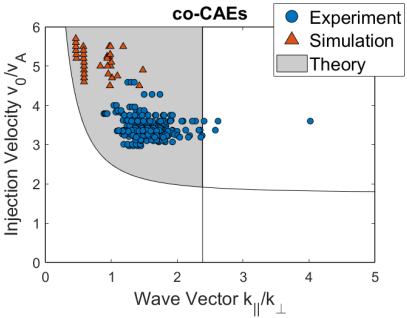
Agreement Between NSTX Experiments, Theory, and Simulations

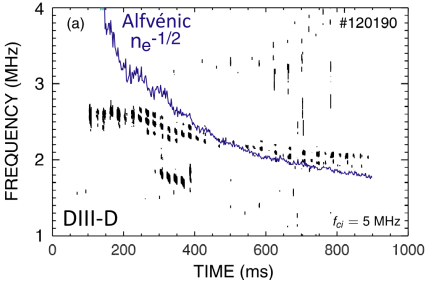
- For fixed NBI parameters, instability conditions constrain the spectrum of modes
- Cross-comparison with NSTX database of cntr-GAEs and co-CAEs demonstrates greater than 80% agreement with theory
- blue: NSTX observations, red: HYM simulations
  gray: unstable region predicted by theory





Theory Explains Previous Observations on DIII-D

- DIII-D low field experiments ($v_0/v_A \approx 1.5$) observed cntr-modes with $\omega/\omega_{ci} \approx 0.6$
  - Tentatively identified as CAEs, with unexplained $k_\perp \rho_{\perp b} < 0.8$
  - Density scaling also not Alfvénic, conflicting with dispersion relation[9]



[9] W.W. Heidbrink *et al.* Nucl. Fusion **46**, 324 (2006)

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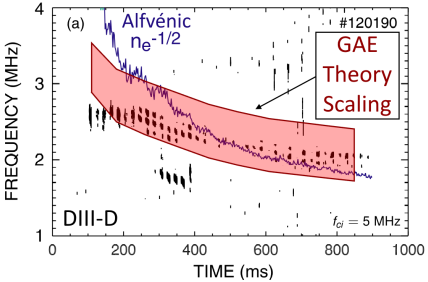
  - Tentatively identified as CAEs, with unexplained $k_\perp \rho_{\perp b} < 0.8$
  - Density scaling also not Alfvénic, conflicting with dispersion relation[9]
- Theory predicts narrow range of unstable GAEs: $\frac{1}{1+v_0/v_A} < \frac{\omega}{\omega_{ci}} < \frac{1}{1+v_0/v_A(1-\lambda_0)^{3/4}}$
  - Compatible with any value of $k_\perp \rho_{\perp b}$
  - Weaker density scaling agrees with data
- Theory predicts higher frequencies for larger λ_0
- Unstable CAEs would require much higher than observed frequencies

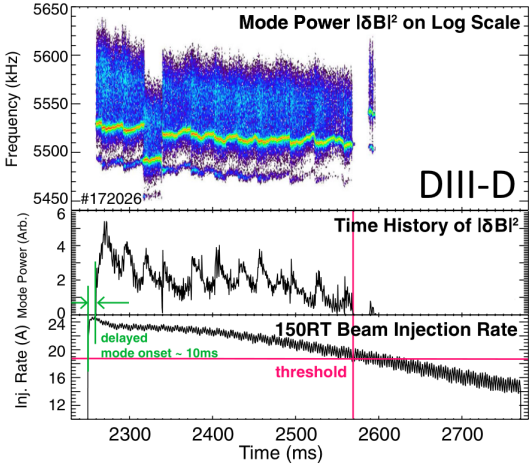


- High frequency observations in DIII-D are more consistent with GAEs

[9] W.W. Heidbrink *et al.* Nucl. Fusion **46**, 324 (2006)

GAE Stabilized with Beam Density Ramp in DIII-D

- cntr-GAE driven by sub-Alfvénic beam ($v_0/v_A = 0.8$) in DIII-D
- Beam density ramped down at constant voltage – mode vanishes![10]
  - Variable beam perveance[11]
- Mode identified with dispersion, resonance, and instability condition
  - Theory predicts unstable GAEs have $0.5 < \omega/\omega_{ci} < 0.8$
    - Observed mode: $\omega/\omega_{ci} = 0.58$
- HYM simulations confirmed unstable GAEs with $|n| = 22 - 24$, $m \approx 3 - 4$



[10] S.X. Tang *et al.* Phys. Rev. Lett. **126**, 155001 (2021)
[11] D.C. Pace *et al.* Phys. Plasmas **25**, 056109 (2018)

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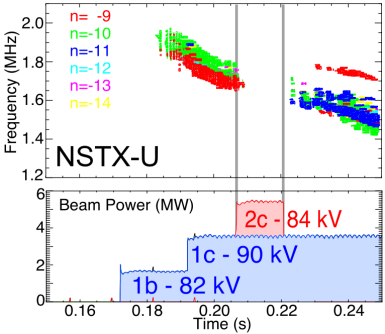
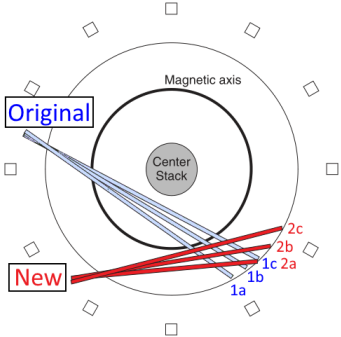
GAE Suppression Discovered on NSTX-U



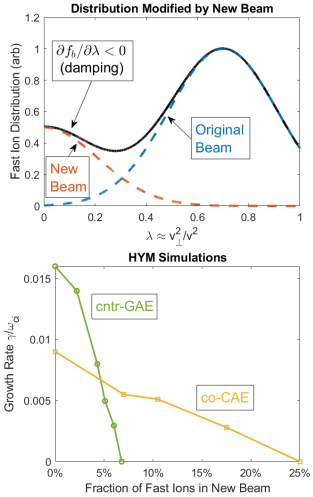


- NSTX-U found robust suppression of cntr-GAEs with addition of new off-axis/tangential beams[12]

[12] E.D. Fredrickson *et al.* Phys. Rev. Lett. **118**, 265001 (2017)

Analytic Theory Explains cntr-GAE Stabilization on NSTX-U

- Tangential injection flips sign of $\partial f_b/\partial\lambda \rightarrow$ damping
- **Stabilization**: damping from new beam balances drive from original beam
  - 7% of fast ions in new beam predicted for complete stabilization of cntr-GAEs
    - Very close to experiment and HYM simulations[13]
- Surprisingly, simulations show that tangential injection also stabilizes co-CAEs
  - Requires $\sim$ 25% fast ions in new beam
  - Theory predicts that very perpendicular injection should also stabilize co-CAEs, challenging to verify



[13] E.V. Belova *et al.* Phys. Plasmas **26**, 092507 (2019)

Stabilization Techniques for Studying Electron Transport

1. Add a new beam in a different geometry (increase damping from $\partial f_b/\partial\lambda$)
   - To suppress **cntr**-GAEs/CAEs, add a more tangential beam
     - explains NSTX-U GAE suppression observations
   - To suppress **co**-CAEs, add a very tangential <u>or</u> perpendicular beam
     - driven by $\partial f_b/\partial\lambda < 0$, opposite condition for cntr-CAEs/GAEs
     - near marginal stability, large radiative damping is sensitive to beam distribution
   - To suppress **either**, counter-inject a new beam
     - Accesses new resonance for same mode, with opposite contribution to drive
2. Add a new beam at a different voltage without changing geometry
   - Adding a beam at a lower voltage should suppress **co**-CAEs
3. Add resonant particles which are stabilizing ($\lambda > \lambda_0$ for cntr-GAEs)
   - Can be achieved by lengthening the tail of the distribution – RF heating?

Summary

- CAEs/GAEs were investigated in NSTX(-U) with hybrid simulations and theory
- A simple analytic theory of sub-cyclotron Alfvén eigenmode instability has been developed for realistic NBI distributions
  - **Perpendicular injection**: drives cntr-propagating CAEs/GAEs and damps co-modes
  - **Tangential injection**: damps cntr-CAEs/GAEs and can drive or damp co-modes
  - Explains experimental observations and simulations of CAE/GAE excitation and stabilization in multiple devices (NSTX, NSTX-U, DIII-D)
- **Impact**: theory for control of CAEs/GAEs will enable investigation of their role in electron energy transport and help identify transport mechanisms
- **Future Applications**: (1) project to ITER (α distribution, multiple ion species), (2) try similar approach to interpret ion cyclotron emission (ICE), and (3) model sub-cyclotron instabilities driven by runaway electrons[14]

[14] C. Liu *et al.* Nucl. Fusion **61**, 036011 (2021)

Backup Slides

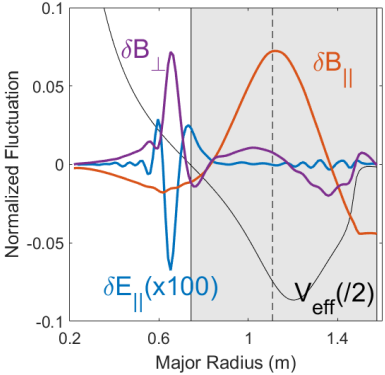
Compressional Alfvén Eigenmodes (CAE)

- Ideal magnetosonic mode in toroidal geometry
  - Compressional polarization
  - In uniform, low β limit, $\omega = kv_A$
- Localized by 2D wave equation

$$\left[\nabla_\perp^2 - V_{\text{eff}}(r,\theta)\right]\delta B_\parallel = 0$$

$$V_{\text{eff}}(r,\theta) = k_\parallel^2 - \frac{\omega^2}{v_A^2} \approx \left(\frac{n}{R}\right)^2 - \left(\frac{\omega}{v_A}\right)^2$$

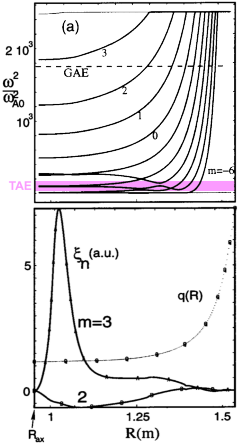
- $V_{\text{eff}} = 0$ coincides with Alfvén resonance, where CAE couples to kinetic Alfvén wave



$n = 4$ CAE calculated by HYM. $\delta B_\parallel$ corresponds to the CAE. Coherent $\delta B_\perp$, $\delta E_\parallel$ structures show the KAW.

Global Alfvén Eigenmodes (GAE)

- Discrete shear Alfvén eigenmode solutions may exist below minimum of Alfvén continuum
  - Approximate dispersion $\omega \leq \left[k_{\parallel}(r)v_A(r)\right]_{min}$
  - Weakly damped due to separation from continuum
- Dominant shear polarization: $\delta B_{\perp} \gg \delta B_{\parallel}$
  - In NSTX conditions, also have large compressional component $\delta B_{\parallel} \approx \delta B_{\perp}$ near edge
- CAEs/GAEs routinely observed in NSTX with $|n| = 3 - 12$ and $\omega/\omega_{ci} \approx 0.1 - 1.2$
  - ICE with $\omega > \omega_{ci}$ also present



GAE calculated by NOVA

HYM Physics Model

Fluid Thermal Plasma

$$\rho \frac{d\boldsymbol{V}}{dt} = -\nabla P + (\boldsymbol{J} - \boldsymbol{J_b}) \times \boldsymbol{B} - e n_b (\boldsymbol{E} - \eta \delta \boldsymbol{J}) + \mu \Delta \boldsymbol{V}$$

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta \delta \boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\mu_0 \boldsymbol{J} = \nabla \times \boldsymbol{B}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{V})$$

$$\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = 0$$

Kinetic Fast Ions

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{q_i}{m_i}(\boldsymbol{E} - \eta \delta \boldsymbol{J} + \boldsymbol{v} \times \boldsymbol{B})$$

δf Scheme

$$F = F_0(\mathcal{E}, \mu, p_\phi) + \delta f(t)$$

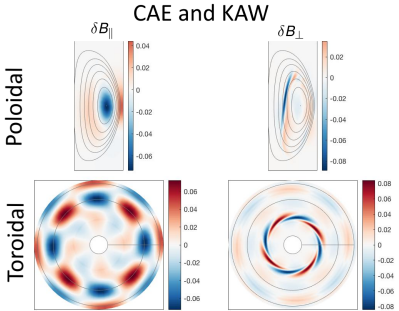
$$w \equiv \delta f / F$$

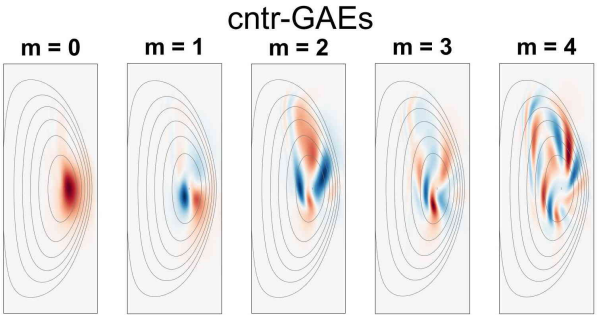
$$\frac{dw}{dt} = -(1 - w)\frac{d \ln F_0}{dt}$$

- ρ, $\boldsymbol{V}$, P are plasma mass density, velocity, and pressure
- n_b, $\boldsymbol{J_b}$ are beam ion density and current ($n_b \ll n_e$, though $\boldsymbol{J_b} \approx \boldsymbol{J_{th}}$)

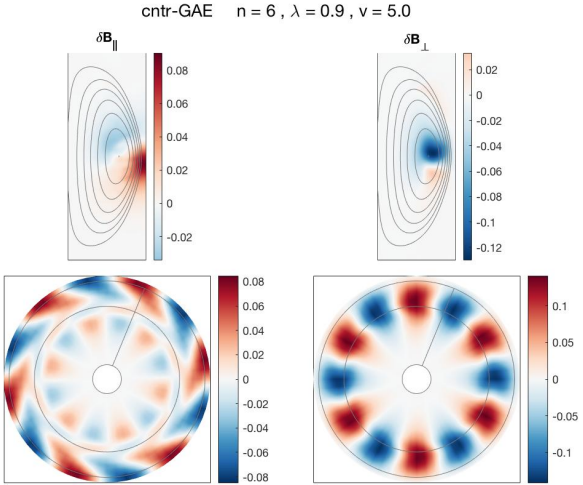
HYM Provides Input for Energy Transport Calculations

- Theories of CAE/GAE-induced electron energy transport require assumptions about the mode properties (frequency, amplitude, polarization, structure, *etc.*)
- HYM simulations generate realistic mode structures, beyond ideal MHD

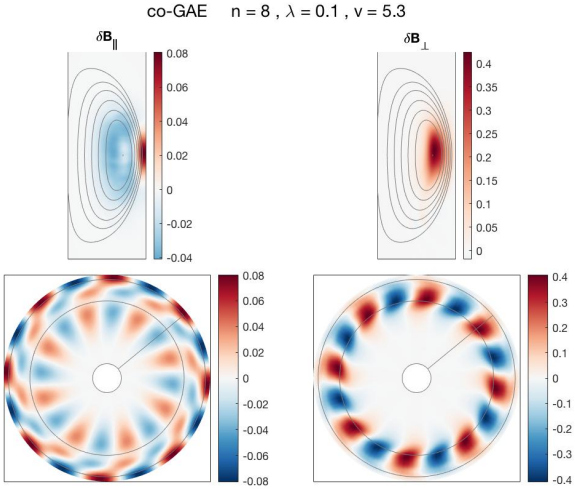




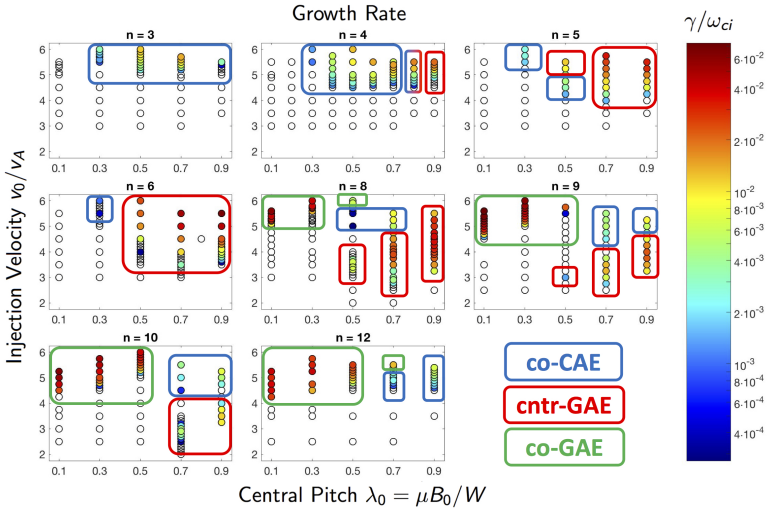
cntr-GAE Mode Structure from HYM



co-GAE Mode Structure from HYM



Linear Simulation Stability Results



Theory Applied to Understand Experiments and Simulations

- Local linear growth rate derived for realistic NBI distribution
  - Uncovers new instability regime – necessary to explain GAE excitation in NSTX-U
- Goal: simple expressions for fast ion drive depending on
  1. fast ion distribution parameters $(\lambda_0, v_0/v_A)$
  2. mode parameters $(\omega/\omega_{ci}, k_{\parallel}/k_{\perp})$
- Approach: restrict to 2D velocity space to avoid assumptions about equilibrium profiles, mode structure, particle orbits, *etc.*
  - Does not include contribution from $\partial f_0/\partial p_\phi$
- Provides upper bound on net growth rate, since neglecting bulk damping sources
  - Reminder: $\gamma_{\text{net}} = \gamma_{\text{EP}} - \gamma_{\text{th,damp}}$

Growth Rate Calculated for Anisotropic Beam Distribution

- For a beam-like distribution, with $x \equiv v_\perp^2/v^2$, $u \equiv v^2/v_0^2 = v_{\parallel,\mathrm{res}}^2/v_0^2(1-x)$

$$\frac{\gamma}{\omega_{ci}} \propto -\sum_\ell \int_0^{1-v_{\parallel,\mathrm{res}}^2/v_0^2} \underbrace{\frac{x}{(1-x)^2} \overbrace{\mathscr{J}_\ell^m\left(\frac{k_\perp v_{\parallel,\mathrm{res}}}{\omega_{ci}}\sqrt{\frac{x}{1-x}}\right)}^{\text{FLR terms}} \overbrace{\frac{e^{-(x-\lambda_0\langle\bar{\omega}_{ci}\rangle)^2/\Delta\lambda^2\langle\bar{\omega}_{ci}\rangle^2}}{1+(4u)^{3/2}}}^{\text{fast ion distribution}}}_{\text{non-negative}} \times$$

$$\left[\underbrace{\frac{2}{\Delta\lambda^2\langle\bar{\omega}_{ci}\rangle^2}\left(x-\lambda_0\langle\bar{\omega}_{ci}\rangle\right)\left(\frac{\ell}{\bar{\omega}}-x\right)}_{\substack{\partial f_b/\partial\lambda \text{ drive/damping} \\ (\text{has sign of } x-\lambda_0\langle\bar{\omega}_{ci}\rangle)}} + \underbrace{\frac{3}{2}\left(1-\frac{1}{1+(4u)^{3/2}}\right)}_{\substack{\partial f_b/\partial\mathcal{E} \text{ damping} \\ (\text{negligible for } \ell\neq 0)}}\right] dx$$

- Can integrate numerically, but further analytic progress requires approximation

Approximations Necessary to Derive Instability Conditions

1. Growth rate dominated by anisotropy for $\ell \neq 0$ resonances $\left(\frac{\omega_{ci}}{\omega}\frac{\partial f_0}{\partial \lambda} \gg \frac{v}{2}\frac{\partial f_0}{\partial v}\right)$
2. "Wide beam approximation" for $\Delta x \approx 0.3$ $\left(\frac{d}{dx}e^{-(x-x_0)^2/\Delta x^2} \approx -2(x-x_0)/\Delta x^2\right)$
3. Small (or large) $k_\perp \rho_{\perp b}$ expansion of finite Larmor radius Bessel function terms
4. Neglect slowing down velocity dependence (weak dependence)

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Define $\eta = v_{\parallel,\mathrm{res}}^2/v_0^2$, then for $k_\perp \rho_{\perp b} \lesssim 1$ and $\ell = 1$, the growth rate is proportional to

$$\gamma \overset{\propto}{\sim} -\int_0^{1-\eta} \frac{x(x-x_0)}{(1-x)^2}dx > 0 \longrightarrow x_0 > \frac{1-\eta^2+2\eta\log\eta}{1-\eta+\eta\log\eta}$$

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$$\Longrightarrow v_0 < \frac{v_{\parallel,\mathrm{res}}}{(1-x_0)^{3/4}}$$

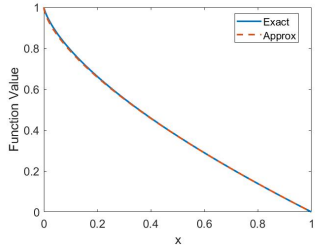
Serendipitous Approximations

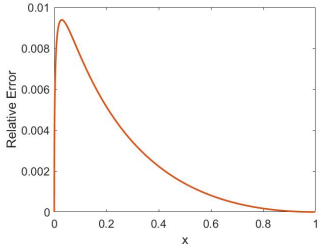
- Where did this approximation come from?

$$f(x) = \frac{1 - x^2 + 2x \log x}{1 - x + x \log x} \approx 1 - x^{2/3}$$

Accurate on $0 < x < 1$ to within 1%!

- Assume $f(x) \approx 1 - x^p$
  - Preserves smoothness, convexity, and monotonicity
  - $f(0) = 1$ and $f^{(n)}(0) \to (-1)^n \infty$ for $0 < p < 1$
  - $f(1) = 0$ and match $f'(1) = -p$
- Correct boundary behavior + sufficiently smooth function $\longrightarrow$ accurate global approximation
  - Same procedure used many times in this work



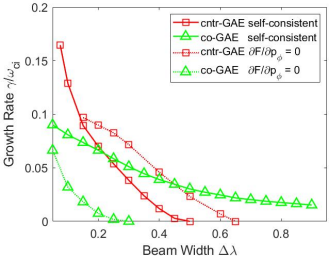


Gradients in p_ϕ Destabilize Co-Propagating Modes

- Local theory analysis neglected $\partial f_0/\partial p_\phi$
- Effect can be determined heuristically by comparing to nonlocal theory[15]

$$\gamma \propto \int d\Gamma \left[\left(\frac{\ell}{\bar{\omega}} - \lambda \right) \frac{\partial f_0}{\partial \lambda} + \mathcal{E} \frac{\partial f_0}{\partial \mathcal{E}} + \frac{n}{\bar{\omega}} \frac{\mathcal{E}}{\omega_{ci}} \frac{\partial f_0}{\partial p_\phi} \right]$$

- For non-hollow distributions, $\partial f_0/\partial p_\phi > 0$ $\rightarrow$ sign of n determines contribution
  - co-modes are **driven**, cntr-modes are **damped**
- HYM simulations that artificially remove $\partial f_0/\partial p_\phi$ contribution confirm its effect for co- vs cntr-GAEs



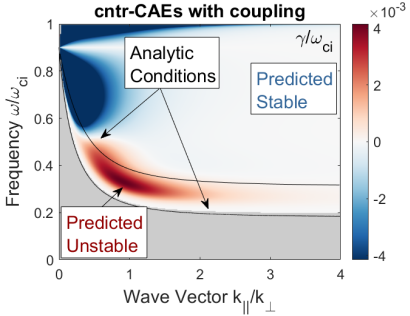
[15] A.N. Kaufman *et al.* Phys. Fluids **15**, 1063 (1972)

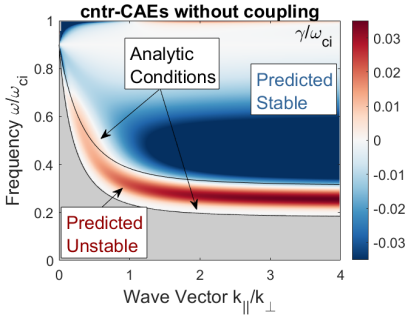
CAE/GAE Coupling Can Alter Most Unstable Modes

- Including two fluid effects, dispersions are coupled, modifying the polarization

$$\left[1-\frac{k_{\|}^2 v_A^2}{\omega^2}\left(1-\frac{\omega^2}{\omega_{ci}^2}\right)\right]\left[1-\frac{k^2 v_A^2}{\omega^2}\left(1-\frac{\omega^2}{\omega_{ci}^2}\right)\right]=\frac{\omega^2}{\omega_{ci}^2}$$

  - Changes growth rate, most unstable parts of spectrum
    - Most important for cntr-CAEs, also co-CAEs at smaller v_0/v_A
- $\ell = 0$ co-GAE can not exist without this coupling



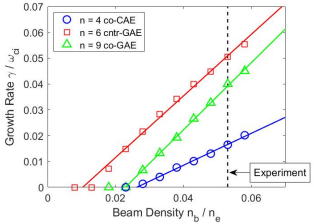


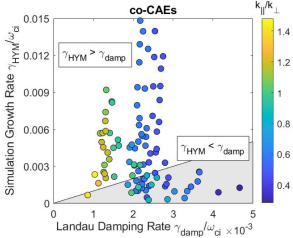
Method for Inferring $k_{\|}/k_{\perp}$ for CAEs

- Measured quantities: ω, k_{ϕ}, and v_A
- Approximate CAE dispersion relation $\omega \approx k v_A \longrightarrow$ infer k
- Assume $k_{\|} \approx k_{\phi} = n/R$ (motivated by mode structure in simulations)
- Derive $k_{\perp} = \sqrt{k^2 - k_{\|}^2}$ from the above quantities
- Construct $k_{\|}/k_{\perp}$, which the CAE instability condition depends on
  - Dispersion: $\omega \approx k v_A$ + resonance: $\omega = k_{\|} v_{\|,\mathrm{res}} \longrightarrow v_{\|,\mathrm{res}}/v_A \approx k/k_{\|}$
  - $\ell = 0$ co-CAE instability requires $v_{\|,\mathrm{res}}$ sufficiently small $\longrightarrow$ lower bound on $k_{\|}/k_{\perp}$
  - Upper bound on $k_{\|}/k_{\perp}$ is derived heuristically, based on largest radial wavelength that will fit within the minor radius

Continuum/Radiative Damping is Dominant in Simulations

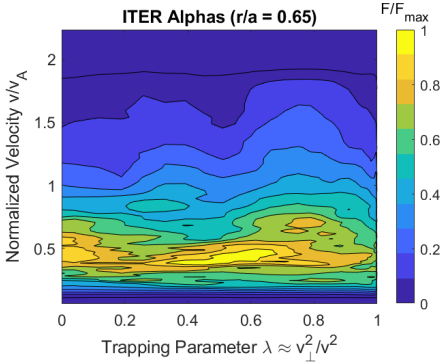
- Beam density scan in simulations shows $\gamma_{\text{damp}}/\gamma_{\text{drive}} \approx 20 - 60\%$
- Attributed to continuum/radiative damping since it is insensitive to viscosity and resistivity
- Electron damping (absent in simulations) calculated analytically for unstable modes
  - GAE electron damping rates are very small $\gamma_{\text{damp}}/\gamma_{\text{drive}} \sim 1\%$
  - CAE electron damping could be large enough to stabilize some modes near marginal stability





CAEs/GAEs May Be Present in Burning Plasmas

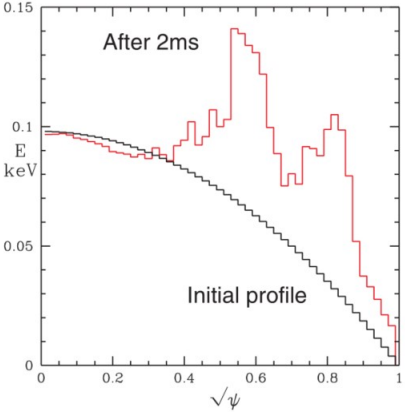
- ITER will have super-Alfvénic NBI and alpha particles ($v_0/v_A = 1.5 - 2$)
- Anisotropy of alphas near the edge could destabilize cntr-GAEs/CAEs
  - Similar to NSTX(-U) beam parameters
- ITER NBI distribution has $\lambda_0 = 0.3 - 0.8$ depending on radius
  - Could be either destabilizing or stabilizing (NSTX-U multi-beam suppression)



- **Open question**: if the modes are excited, will the anomalous electron transport also be present in ITER or is it unique to spherical tokamaks?

CAE/GAE-Induced Ion Heating Was Also Explored on NSTX

- Anomalously high $T_i > T_e$ was observed in some NBI-dominated NSTX discharges[16]
- Proof-of-principle stochastic heating of ions by CAEs shown in test particle simulations[17]
- Subsequent experimental analysis[18] found CAE to thermal ion power transfer to be insufficient to explain the surplus $T_i - T_e$
- Not yet fully resolved



[16] D.A. Gates *et al.* Phys. Rev. Lett. **87**, 205003 (2001)
[17] N.N. Gorelenkov *et al.* Nucl. Fusion **43**, 228 (2003)
[18] E.D. Fredrickson *et al.* Phys. Plasmas **9**, 2069 (2002)

Open Questions

1. Which transport mechanism is the dominant cause of anomalous flat T_e profiles?
2. Will CAEs/GAEs be unstable in ITER? Will they induce anomalous transport?
3. What is the dominant mechanism for co-CAE stabilization by tangential injection?
4. Can the analytic stability boundaries be generalized to $\omega \gg \omega_{ci}$ in order to interpret ion cyclotron emission (ICE)?