# INVERSE BREMSSTRAHLUNG ABSORPTION IN MAGNETIZED FUSION PLASMA

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**Abstract**

Laser Magneto-Inertial fusion (MIF) is a developed approach for the thermonuclear fusion. It consists in applying to the laser inertial fusion plasma a strong magnetic field whose the role is to limit the diffusion of the formed plasma during the impact of an intense laser pulse with a target containing the thermonuclear fuel, as well as the confinement of produced alpha particles by the fusion reaction. This permits to reduce the energy losses and even improves the compression conditions.
In this paper, the electron-ion (e-i) inverse bremsstrahlung absorption (IBA) of the laser energy in magnetized plasma in the frame of the MIF is analytically studied. We have considered a cylindrical scheme where the laser wave, circularly polarized, propagates in the direction of magnetic field (parallel mode).
The interaction of the laser pulse with the magnetized plasma is described by the Fokker-Planck (FP) equation with a Landau collision term. In order to resolve the FP equation, we have considered two scales of time evolutions for the distribution function: a fast scale time evolution following the laser wave time variation and a slow hydrodynamic scale time. The electron velocities distribution, developed on the spherical harmonics is analytically calculated.
The e-i IBA, $\langle\vec{E}.\vec{j}\rangle$, is analytically calculated using the found expression of the distribution function. It is explicitly expressed as function the parameters of plasma, laser pulse and the applied magnetic field.
The numerical analysis of model equation shows the variation of the IB absorption with the different physical parameters. We point out in this context that the absorption is affected by the magnetic field and this depends to the polarization of the laser wave, increasing for the left polarization and decreasing for the right polarization.
Practical scaling laws are established for the MIF scheme.
The obtained results in this study permit to optimize the laser and plasma parameters in order to obtain good efficiency of the IB absorption in MIF experiments

## 1. INTRODUCTION

Magneto-inertial fusion (MIF) [1-6] is an approach to fusion which combines aspects of magnetic confinement fusion (MCF) [7,8] and inertial confinement fusion (ICF) [9,10]. In the first aspect, a dense ($\sim 10^{21} m^{-3}$), high temperature ($\sim 20keV$) plasma is stored in a given volume under the influence of a magnetic field pressure countermining the plasma pressure. The second aspect is based on heating the thermonuclear fuel to a high temperature for a period of time comparable to the characteristic time scale of the hydrodynamic expansion of the plasma using powerful sources of energy (Laser, heavy ions beam...).

The main ideas of the MIF have been known for nearly four decades [2,3]. The concept of this approach involves applying a strong magnetic field to the inertial fusion target and freezing magnetic flux in the hot spot or embedding magnetic flux in a target plasma bounded by a conducting shell, called a liner, serving as a magnetic flux conserver [3]. In a manner similar to conventional inertial fusion, the hot spot or the conducting shell (liner) is imploded and the magnetic flux is compressed with it, and thus the intensity of the magnetic field is increased. During compression of the plasma, the thermal diffusivity is significantly reduced by the intense applied magnetic field, and therefore the plasma is easily heated to thermonuclear-fusion temperatures. In addition, the very high magnetic field, formed in the hot spot or in the fusion plasma, traps the alpha particles, produced by the thermonuclear reaction, which permits to boost the alpha energy disposition in the fusion plasma.

The energy absorption has a greater significance in fusion experiments as it directly contributes to the plasma heating in order to achieved the Lawson criteria [11,12].

In the laser fusion plasma, the absorption is achieved by several mechanism including inverse bremsstrahlung absorption (IBA) [13-16], resonance absorption [17,18] and anomalous absorption [19].

The most important absorption mechanism in the laser MIF experiments is the IBA, in which the radiation is absorbed by electrons when they scatter off the coulomb field of ions. In the magnetic fusion devices, the electromagnetic energy can be absorbed by cyclotron resonance mechanism. The non-linear anomalous absorption mechanism, which will be the object of future publication, has not taken into account in this work.

In this paper we are interested to the study of the IBA of laser energy by magnetized plasma in the MIF frame through a theoretical development, and we use the derived formula to calculate the absorption in magnetic fusion (MF) plasma heated by microwave.

This paper is organized as follows. The equations of the $l^{\text{th}}$ component of high and low frequency electronic distribution function expended on the spherical harmonics, are calculated from the Fokker-Planck equation in the presence of a magnetic field [20]. In next section, the absorption is explicitly calculated using the results of previous section. Then, Scaling laws are given for the IBA in Laser MIF and in MF plasma heated by microwave [21]. Finally, an interpretation and conclusion of the obtained results are given.

## 2. ELECTRON VELOCITIES DISTRIBUTION IN MAGNETIZED LASER FUSION PLASMA

In order to formulate the propagation of a laser pulse through magnetized collisional plasma in non-relativistic regime, we consider an inhomogeneous plasma heated by a circularly polarized laser wave in the presence of an axial magnetic field. It is judicious to use the Fokker-Planck equation to describe magnetized collisional plasma. Following the notation of Braginskii [22], the electrons Fokker-Planck equation (FP) is presented as:

$$\frac{\partial f}{\partial t}+\vec{v}.\frac{\partial f}{\partial \vec{r}}-\frac{e}{m_e}\left(\vec{E}+\vec{v}\times\left(\vec{B}+\overrightarrow{B_0}\right)\right).\frac{\partial f}{\partial \vec{v}}=C_{ei}(f), \tag{1}$$

where $(\vec{E},\vec{B})$ denotes the laser electromagnetic field,

, $\overrightarrow{\mathrm{B}_0}$ is the applied magnetic field along the wave propagation.

$f$ , $e$ and $m_e$ are the electron distribution function, the electron charge and the electron mass, respectively.

The right-hand side of equation (1) represents the Landau electron-ion collision operator [23]:

$$C_{ei}(f_e)=\alpha\frac{\partial}{\partial \vec{v}}.\left[\left(\frac{v^2\overleftrightarrow{I}-\vec{v}:\vec{v}}{v^3}\right).\frac{\partial f}{\partial \vec{v}}\right], \tag{2}$$

where $\alpha=\frac{Zn_e e^4}{8\pi\varepsilon_0 m_e}ln\Lambda$, with $Z$ being the ion charge number, $n_e$ is the electron density, $\varepsilon_0$ is the electric permittivity of free space, and $ln\Lambda$ is the coulomb logarithm.

We have neglected the electron-electron collision term for high-order order anisotropic components of distribution function ( $C_{ee}\ll C_{ei}$), which corresponds to the Lorentz-plasma approximation (high Z limit).

In this paper we consider a parallel circularly polarized laser wave propagating in the direction of an applied magnetic field ($z$) and oscillating in the perpendicular plane. Usually, this geometry is used in MIF experiments [19, 24, 27]:

$$\vec{E}(t,z)=Re\left[\frac{E(z)}{\sqrt{2}}\left(\overrightarrow{u_x}\pm j\overrightarrow{u_y}\right)exp(-j\omega t)\right], \tag{3}$$

$$\vec{B}(t,z)=Re\left[\frac{B(z)}{\sqrt{2}}\left(j\overrightarrow{u_x}\mp\overrightarrow{u_y}\right)exp(-j\omega t)\right], \tag{4}$$

and

$$\overrightarrow{B_0}=B_0\overrightarrow{u_z}, \tag{5}$$

where $\left(\overrightarrow{u_x},\overrightarrow{u_y},\overrightarrow{u_z}\right)$ are the unit vectors and $\omega$ is the laser wave frequency.

The distribution function is supposed to constitute of two contributions: a fast oscillation distribution, $f^{(h)}$, which follows the laser wave time variation, $\sim exp(-j\omega t)$, and a quasi-static distribution, $f^{(s)}$, which has a slow variation as the plasma hydrodynamic parameters [25-28], hence:

$$f(\vec{v},t)=f^{(s)}(\vec{v},t)+Real(f^{(h)}(\vec{v},t)). \tag{6}$$

Note that the indices ($s$) and ($h$) respectively refer to the low and high frequency time scales, and will be used throughout this work.

Substituting equations (3) - (6) into equation (1) and separating the time scale orders, we obtain a system of two coupled equations, the high-frequency kinetic equation and a quasi-static kinetic equations:

$$\frac{\partial f^{(h)}}{\partial t} + \vec{v}.\frac{\partial f^{(h)}}{\partial \vec{r}} - \frac{e}{m_e}\frac{E}{\sqrt{2}} exp(-j\omega t)\left(\frac{\partial f^{(s)}}{\partial v_x} \pm j\frac{\partial f^{(s)}}{\partial v_y}\right) - \frac{e}{m_e}\frac{B}{\sqrt{2}} exp(-j\omega t)\left(\mp v_z\frac{\partial f^{(s)}}{\partial v_x} + jv_z\frac{\partial f^{(s)}}{\partial v_y} - \left(jv_y \pm v_x\right)\frac{\partial f^{(s)}}{\partial v_z}\right) - \Omega_{ce}\left(v_y\frac{\partial f^{(h)}}{\partial v_x} - v_x\frac{\partial f^{(h)}}{\partial v_y}\right) = C_{ei}\left(f^{(h)}\right), \tag{7}$$

$$\frac{\partial f^{(s)}}{\partial t} + \vec{v}.\frac{\partial f^{(s)}}{\partial \vec{r}} - \frac{e}{m_e}\frac{1}{\sqrt{2}}\langle E\ exp(-j\omega t)\left(\frac{\partial f^{(h)}}{\partial v_x} \pm j\frac{\partial f^{(h)}}{\partial v_y}\right)\rangle_T - \frac{e}{m_e}\frac{1}{\sqrt{2}}\langle B\ exp(-j\omega t)\left(\pm v_z\frac{\partial f^{(h)}}{\partial v_x} + jv_z\frac{\partial f^{(h)}}{\partial v_y} - \left(jv_y \pm v_x\right)\frac{\partial f^{(h)}}{\partial v_z}\right)\rangle_T - \Omega_{ce}\left(v_y\frac{\partial f^{(s)}}{\partial v_x} - v_x\frac{\partial f^{(s)}}{\partial v_y}\right) = C_{ei}\left(f^{(s)}\right). \tag{8}$$

The symbol $< >_T$ in equations (8) denotes the average over the laser wave cycle time $T = \frac{2\pi}{\omega}$ and $\Omega_{ce} = \frac{eB_0}{m_e}$ is the electronic cyclotron frequency.

The two coupled equations (7) and (8) constitute the basic equations in the present work.

We introduce the spherical coordinates $(v, \mu = \frac{v_z}{v}, \varphi = arctan\frac{v_x}{v_y})$, which are justified by the fact that the Landau collision operator only affects the angular part of the function and has the spherical harmonics as own functions [27].

Equations (7) and (8) become:

$$\frac{\partial f^{(h)}}{\partial t} + v\mu\frac{\partial f^{(h)}}{\partial z} - \frac{e}{m_e}\frac{E}{\sqrt{2}} exp(-j\omega t \pm j\varphi)\left(\sqrt{1-\mu^2}\frac{\partial}{\partial v} - \frac{\mu\sqrt{1-\mu^2}}{v}\frac{\partial}{\partial \mu}\right) f^{(s)}\frac{e}{m_e}\frac{B}{\sqrt{2}} exp(-j\omega t \pm j\varphi)\left(\sqrt{1-\mu^2}\frac{\partial f^{(s)}}{\partial \mu}\right) + \Omega_{ce}\frac{\partial f^{(h)}}{\partial \varphi} = C_{ei}\left(f^{(h)}\right), \tag{9}$$

$$\frac{\partial f^{(s)}}{\partial t} + v\mu\frac{\partial f^{(s)}}{\partial z} - \frac{e}{m_e}\langle\frac{E}{\sqrt{2}} exp(-j\omega t \pm j\varphi)\left(\sqrt{1-\mu^2}\frac{\partial}{\partial v} - \frac{\mu\sqrt{1-\mu^2}}{v}\frac{\partial}{\partial \mu} - \frac{1}{v\sqrt{1-\mu^2}}\right) f^{(h)}\rangle_T \pm \frac{e}{m_e}\langle\frac{B}{\sqrt{2}} exp(-j\omega t \pm j\varphi)\left(\left[\sqrt{1-\mu^2}\frac{\partial}{\partial \mu} + \mu\frac{1}{\sqrt{1-\mu^2}}\right] f^{(h)}\right)\rangle_T = C_{ei}\left(f^{(s)}\right). \tag{10}$$

Equations (9) and (10) imply that $f^{(h)}$ is proportional to $exp(\pm j\varphi)$ and $f^{(s)}$ is independent of $\varphi$. This allows us to expand $f^{(h)}$ using associated Legendre polynomials $P_l^{\pm 1}(\mu)$ and $f^{(s)}$ using Legendre polynomials $P_l(\mu)$ [29, 30], thus:

$$f^{(h)}(\vec{v}, t) = \sum_{l=0}^{\infty} f_l^{(h)}(v, \varphi, t)\, P_l^{\pm 1}(\mu), \tag{11}$$

$$f^{(s)}(\vec{v}, t) = \sum_{l=0}^{\infty} f_l^{(s)}(v, t)\, P_l(\mu). \tag{12}$$

From equation (9), and using recurrence relations between $P_l$ and $P_l^{\pm 1}$, the expression for the *l*[th] components of $f^{(h)}$ is given by:

$$f_l^{(h)} = \frac{e}{m_e}\left(\frac{exp(-j(\omega t \mp \varphi))}{-j(\omega \mp \Omega_{ce}) + l(l+1)\frac{\nu}{v^3}}\right)\left[\frac{E}{\sqrt{2}}\left(\frac{1}{2l+3}\left(\frac{1}{v^{l+1}}\frac{\partial}{\partial v}\left(v^{l+1}\right) + \frac{1}{v}\right) f_{l+1}^{(s)} - \frac{1}{2l-1}\left(v^l\frac{\partial}{\partial v}\left(\frac{1}{v^l}\right) + \frac{1}{v}\right) f_{l-1}^{(s)}\right) \pm B f_l^{(s)}\right]. \tag{13}$$

Additionally, the *l*[th] components of $f^{(s)}$ is given by:

$$\frac{\partial f_l^{(s)}}{\partial t} + v\left(\frac{l}{2l-1}\right)\frac{\partial f_{l-1}^{(s)}}{\partial z} + v\left(\frac{l+1}{2l+3}\right)\frac{\partial f_{l+1}^{(s)}}{\partial z} - \frac{e}{m_e}\langle\frac{E}{\sqrt{2}}\exp\left(-j(\omega t \mp \varphi)\right)\left(\frac{l(l-1)}{2l-1}\left[v^l\frac{\partial}{\partial v}\left(\frac{1}{v^l}\right)+\frac{1}{v}\right]f_{l-1}^{(h)} - \frac{(l+1)(l+2)}{2l+3}\left[\frac{1}{v^{l+1}}\frac{\partial}{\partial v}\left(v^{l+1}\right)+\frac{1}{v}\right]f_{l+1}^{(h)} + \frac{2}{v}\sum_{i=0}^{i\leq\frac{l-1}{2}}(2l+1)\,f_{l+2i+1}^{(h)}\right)\rangle_T - \langle\frac{B}{\sqrt{2}}exp(-j(\omega t \mp \varphi))\left((l+1)lf_l^{(h)} - 2\sum_{i=0}^{i\leq\frac{l-1}{2}}l\,f_{l+2i}^{(h)} - 2\sum_{i=0}^{i\leq\frac{l-1}{2}}(l+1)\,f_{l+2i+2}^{(h)}\right)\rangle_T = -l(l+1)\frac{\nu}{v^3}f_l^{(s)}. \tag{14}$$

The set of equations (13) and (14) allows us to compute all components of the distribution function for magnetized plasma heated by a laser pulse. These equations actually provide us with more information on the plasma than equation (23) of reference [27], by including the magnetic field, and give more information than equation (4) of reference [25], by including the higher anisotropic distribution. Note that at the zeroth order, we recover, for isotropic distribution function, the equation (4) of reference [25]. By ignoring the distortion term in this equation the solution is Maxwellian and it is a super Maxwellian [27] by taking into account the distortion of the isotropic distribution (Langdon effect).

## 3. **Electron-ion inverse bremsstrahlung absorption**

The time-average power dissipated per unit volume by the electron-ion inverse bremsstrahlung mechanism is [25]:

$$\Gamma = \langle\vec{E}.\vec{J}\rangle_T, \tag{15}$$

where $\vec{J}$ is the current density defined by the electronic distribution, $f$, which takes into account the laser field, the applied magnetic field and the electron-ion collisions, so :

$$\vec{J} = -e\int\vec{v}fd^3\vec{v}, \tag{16}$$

$\vec{J}$ contains a quasi static part, $\overrightarrow{J^{(s)}}$ , and a high-frequency part, $\overrightarrow{J^{(h)}}$, which oscillates at the laser frequency, $\omega$, thus:

$$\Gamma = \langle\vec{E}.\overrightarrow{J^{(h)}}\rangle_T + \langle\vec{E}.\overrightarrow{J^{(s)}}\rangle_T\ . \tag{17}$$

The contribution of the static component vanishes since it averages to zero on the laser cycle time, and thus we are only left with the contribution of the high frequency distribution. Using Eqs. (13)-(17), the absorption $\Gamma$ can be expressed as follows:

$$\Gamma = \frac{8\sqrt{3}}{15(2\pi)^{\frac{3}{2}}}m_e\tilde{I}\frac{\omega^2 n_e}{(\omega\mp\Omega_{ce})^2}\nu_{ei}\,v_{th}^2 G(a), \tag{18}$$

where $G(a) = G_{1,4}^{4,1}\left(\frac{a}{27}\middle|\begin{matrix}0\\0,\frac{1}{3},\frac{2}{3},1\end{matrix}\right)$ is the Meijer function [29] with argument $a = \left(\frac{\nu_{ei}}{\sqrt{2}(\omega\mp\Omega_{ce})}\right)^2$.
$\tilde{I} = \frac{v_0^2}{v_{th}^2}$ is a normalized laser intensity, with $v_0 = \frac{e|E|}{m_e\omega}$ being the peak oscillating-electron velocity in the (*hf*) laser electric field, $\nu_{ei} = \frac{v_{th}\sqrt{2}\,n_e e^4 ln\Lambda}{12\pi\sqrt{\pi}\varepsilon_0^2 T_e^2}$ is the *electron-ion* collision frequency, $v_{th} = \sqrt{\frac{T_e}{m_e}}$ is the electron-thermal velocity, whereas $T_e$ is the electron temperature.

We present in Figure 1 the absorption as function of $a$ for typical parameters of plasmas and laser pulse.

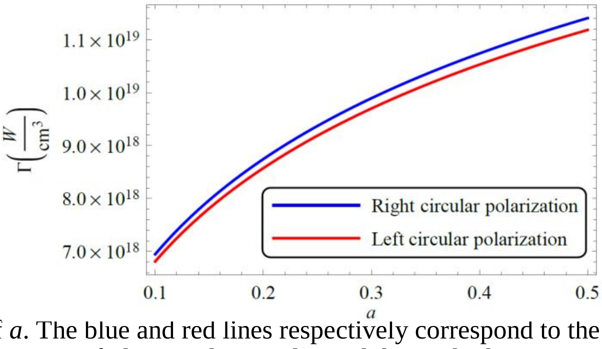


**Fig. 1:** Plot of $\Gamma$ as function of $a$. The blue and red lines respectively correspond to the Right and Left-circular polarizations. The typical parameters of plasmas, laser pulse and the applied magnetic field, respectively, are: $n_e = 10^{19} cm^{-3}$, $I = 10^{15} \frac{W}{cm^2}$, $\lambda = 1.06\ \mu m$ and $B_0 = 10\ T$.

## 4. Scaling laws.

The fusion plasma is created by an intense laser pulse. Consequently, the plasma parameters are given by the laser field parameters. In reference [31], the temperature of the corona is deduced by calculating the energy balance in the critical layer, where the plasma frequency, $\omega_p = \sqrt{\frac{n_e e^2}{m_e \varepsilon_0}}$ , is equal to the laser frequency $(\omega_p = \omega)$.
The critical density , $n_c$ , is directly given by the laser wave length, thus:

$$n_c = 1.11 \times 10^{21} \lambda^{-2}. \tag{19}$$

The electron temperature , $T_e$ , is given by the laser wave length $\lambda$ and the absorbed laser intensity $I_a$, as:

$$T_e = 4.3 \times 10^{-8}\ \lambda^{\frac{4}{3}}\ I_a^{\frac{2}{3}}. \tag{20}$$

Here, $I_a = AI$ is the absorbed laser intensity, with $I$ being the laser pulse intensity expressed by $w/cm^2$, and $A$ is the absorption coefficient. In the case of total absorption, we can consider that $I_a = I$ and $A = 1$.
The critical density $n_c$ , the laser wave length $\lambda$ and the electron temperature $T_e$ in the critical layer are respectively expressed by: $cm^{-3}$, $\mu m$ and $keV$.
Using equations (18), (19) and (20) the absorption for laser MIF plasma can be expressed as:

$$\Gamma = \frac{1.8\times10^{22}\lambda^{-4}}{(10720.7\mp\lambda\, B_0)^2} \times \left(1 - \frac{1.6\times10^{28} I^{-2}\lambda^{-6}}{(10720.7\mp\lambda\, B_0)^2}\right), \tag{21}$$

where $\Gamma, \lambda, I$ and $B_0$ are respectively expressed by $\frac{W}{cm^3}$, $\mu m$, $\frac{W}{cm^2}$ and $T$.

We illustrate in Figure 2 the absorption as function of the laser wavelength using the typical parameters for laser intensity $I = 10^{16} \frac{W}{cm^2}$ and applied magnetic field $B_0 = 10\ T$.

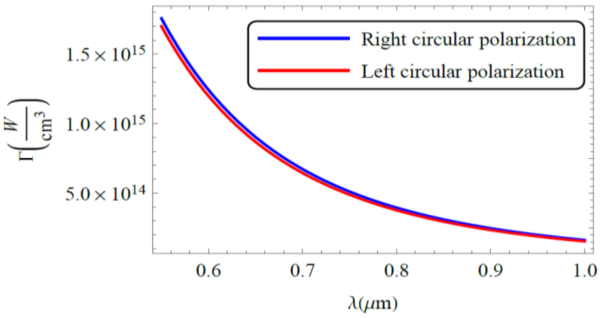


**Fig. 2:** Plot of $\Gamma$ as function of the laser wavelength $\lambda$. The blue and red lines respectively correspond to the Right and Left circular polarizations. The parameters of laser and the applied magnetic field are: $I = 10^{16} \frac{W}{cm^2}$ and $B_0 = 10\ T$ .

This figure shows that absorption decrease with the laser wavelengths, which can be interpreted by the decrease of the critical density as the laser wave length increase $(n_c \sim \lambda^{-2})$ and therefore the decrease in collision frequency $(\nu_{ei} \sim n_c)$, what is implied by the decrease in absorption $\Gamma$.
We show in Figure 3 the effect of the applied magnetic field $B_0$ on the absorption $\Gamma$.
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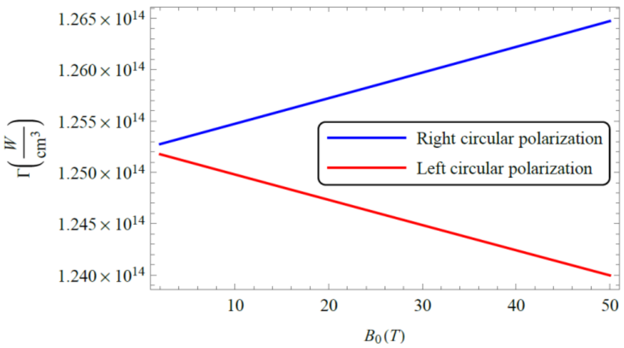


**Fig. 3:** Plot of $\Gamma$ as function of the applied magnetic field $B_0$. The blue and red lines respectively correspond to the Right and Left circular polarizations. The parameters of laser are: $I = 10^{16}\,\frac{W}{cm^2}$ and $\lambda = 1.06\mu m$.

We see from Figure 3 the effect of the applied magnetic field $B_0$ on the absorption $\Gamma$ in the game of $B_0 < 50\,T$. This figure shows that the IB absorption is slowly increasing with $B_0$, which is explained by the fact that in the case of parallel circularly polarized laser wave, the direction of electron motion due to laser field is the same as the direction of cyclotron motion due to the magnetic field, so the pulsation of the electrons increases $(\omega + \Omega_{ce})$. This results increase in the absorption.

## 5. Discussion and conclusion.

– In this paper we studied the *e-i* inverse bremsstrahlung absorption for a magnetized plasma in the frame the MIF. We explicitly calculated the absorption as a function of the laser pulse parameter. We have shown that the magnetic field affects the absorption and that its effect depends on polarization direction. We found that the absorption is actually slightly larger for right polarization and is slightly smaller for left one (figure 1,2,3). We derived useful scaling laws for IBA in a magnetized inertial fusion plasma. In this study we restricted ourselves to the linear approach of IBA. This study allows us to optimize the laser pulse parameters in order to obtain an efficient absorption.

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