Strong Reversal of Simple Isotope Scaling Laws in Tokamak Edge Turbulence

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General Atomics

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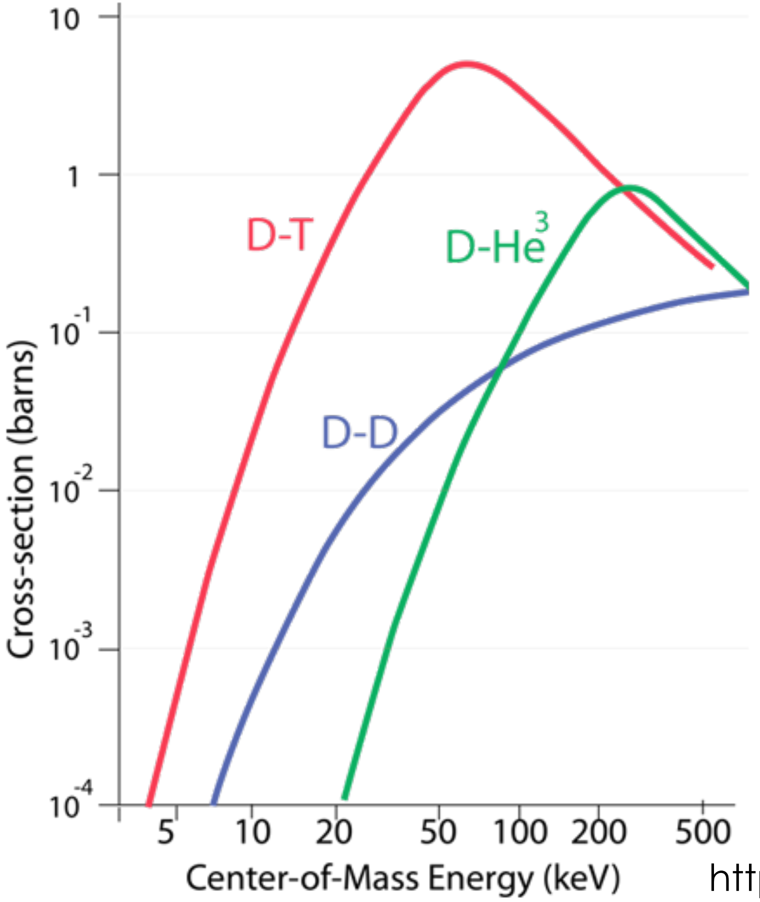
Understanding the scaling of energy confinement time with hydrogenic isotope mass is important in moving toward reactor-relevant DT plasmas.



ITER Operational Phases:
- **H**/He
- **D**
- **50:50 DT**

DT Tokamak Experiments:
- TFTR 1993-1997
- JET DTE1 1997
- JET DTE2 2021

http://hyperphysics.phy-astr.gsu.edu

We have developed a theoretical framework for understanding the hydrogenic isotope mass dependence of turbulent plasma transport.

- Long-standing problem known as the "**isotope effect**"

- Theoretical basis for gyrokinetic isotope mass scaling of the turbulent energy flux (**naive gyroBohm scaling**)

- Transition of theoretical mass scaling from the ion-dominated core to the electron-dominated edge

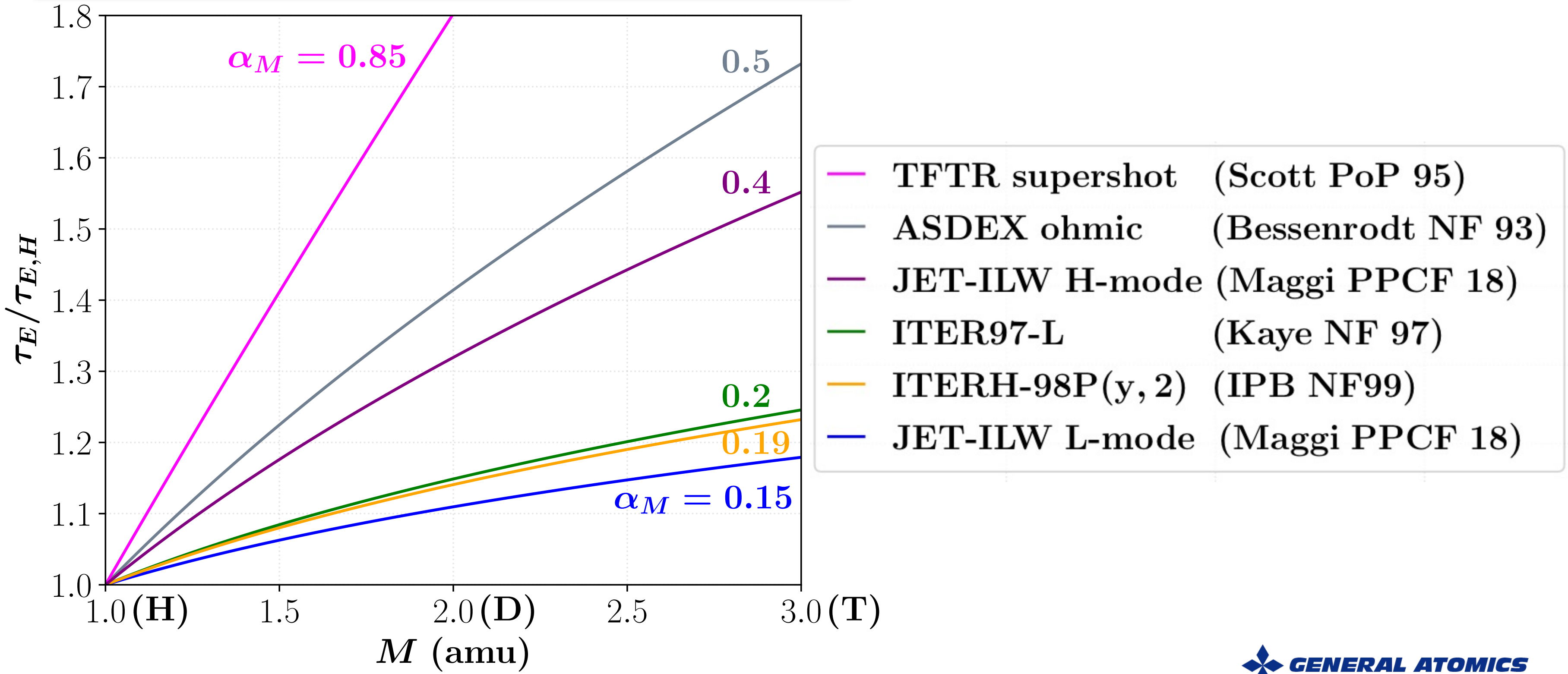
- Role of the **nonadiabatic electron drive** in **reversing naive GB mass scaling**; **New scaling law** for **m_e/m_i dependence** of flux

- Implications for global confinement and L-H power threshold

Experiments generally find an increase in global thermal energy confinement time with increasing hydrogenic isotope mass.

$$\tau_E = C I^{\alpha_I} B^{\alpha_B} \bar{n}^{\alpha_n} P^{\alpha_P} R^{\alpha_R} \kappa^{\alpha_\kappa} \epsilon^{\alpha_\epsilon} S_{cr}^{\alpha_S} M^{\alpha_M}$$

$$\tau_{E,H} < \tau_{E,D} < \tau_{E,DT}$$



GENERAL ATOMICS

Simple gyroBohm-scaling theoretical arguments contradict with experimental observations.

Naive GyroBohm Scaling

$$\chi_i \sim \frac{\Delta x^2}{\Delta t} \sim \frac{\rho_i^2}{(a/\mathrm{v}_{ti})} \Rightarrow \boldsymbol{Q_i = c_0 Q_{GBi}}$$

$$Q_{GBi} = \left(n_0 T_0 \mathrm{v}_{ti} \rho_{*i}^2\right)$$

$$Q_{GBi} = Q_{GBD} \sqrt{\frac{m_i}{m_D}}$$

$$Q_{GBD} = \left(n_0 T_0 \mathrm{v}_{tD} \rho_{*D}^2\right)$$

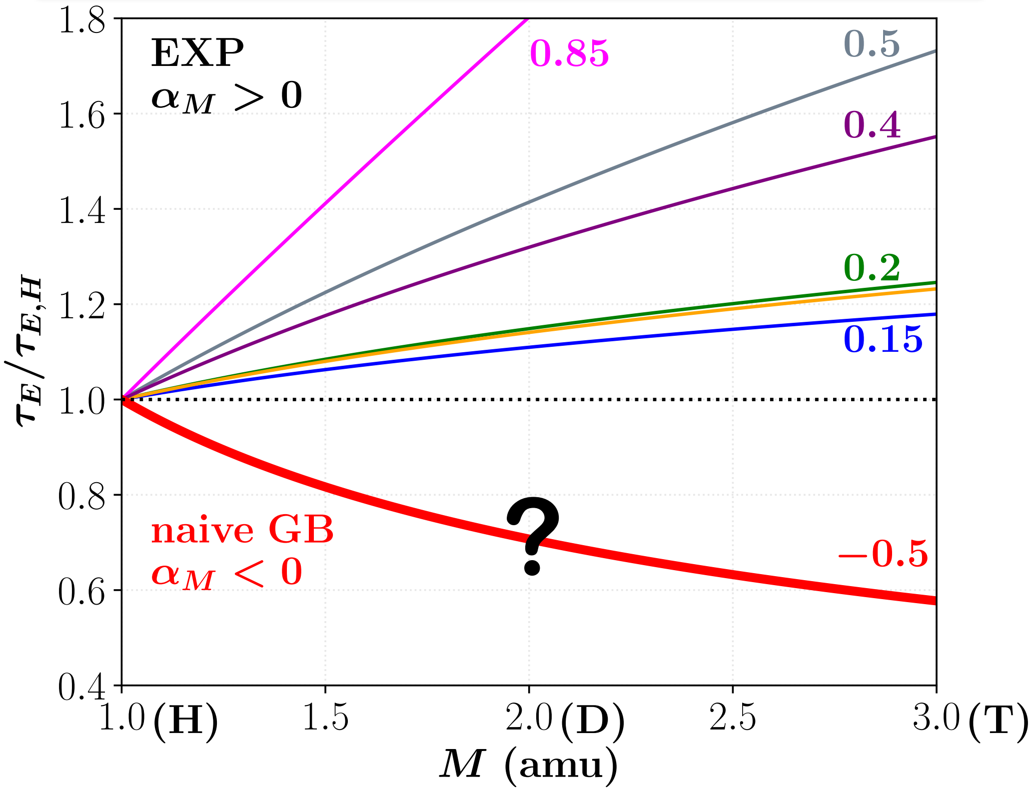
$$Q_i = c_0 Q_{GBD} \sqrt{\frac{m_i}{m_D}}$$

$$\tau_E \sim a^2/\chi_i \qquad \boldsymbol{\tau_E \sim M^{-0.5}}$$

$$Q_H < Q_D < Q_{DT} \Rightarrow \tau_{E,H} > \tau_{E,D} > \tau_{E,DT}$$

Simple gyroBohm-scaling theoretical arguments contradict with experimental observations.

$$\tau_E = C I^{\alpha_I} B^{\alpha_B} \bar{n}^{\alpha_n} P^{\alpha_P} R^{\alpha_R} \kappa^{\alpha_\kappa} \epsilon^{\alpha_\epsilon} S_{cr}^{\alpha_S} M^{\alpha_M}$$



The "isotope effect"

EXP

$$\tau_{E,H} < \tau_{E,D} < \tau_{E,DT}$$

Naive GB

$$\tau_{E,H} > \tau_{E,D} > \tau_{E,DT}$$

Proposed mechanisms that can lead to deviation from naive gyroBohm mass scaling of turbulent ion energy flux

- $\vec{E}\times\vec{B}$ flow shear (Garcia NF 17)
- Electromagnetic fluctuations (Garcia NF 17, Manas NF 19)
- Collisions (Nakata PRL 17, Bonanomi NF 19)
- Impurities (Pusztai PoP 11)
- Fast ions (Garcia NF 18, Bonanomi NF 19)
- Kinetic electrons (Estrada PoP 05, Pusztai PoP 11, Bustos PoP 15)

We present a theoretical framework for the role of the nonadiabatic electron drive in transition of isotopic dependence of turbulent transport from core to edge.

*No GK results with kinetic electrons before 2003

The nonadiabatic electron drive can alter – and even reverse – naive gyroBohm mass scaling.

$$Q_i = c_0 Q_{GBi} = c_0 Q_{GBD}\sqrt{\frac{m_i}{m_D}}$$

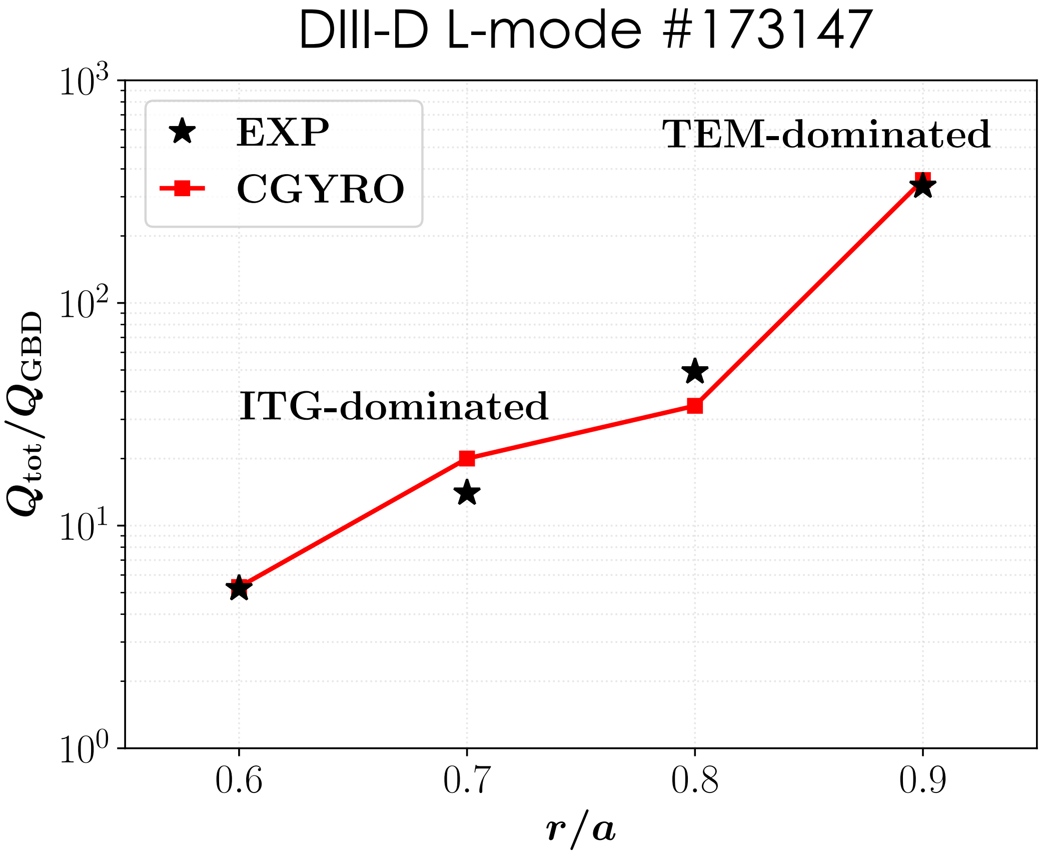
$$Q_H < Q_D < Q_{DT} \qquad \tau_{E,H} > \tau_{E,D} > \tau_{E,DT}$$

↓

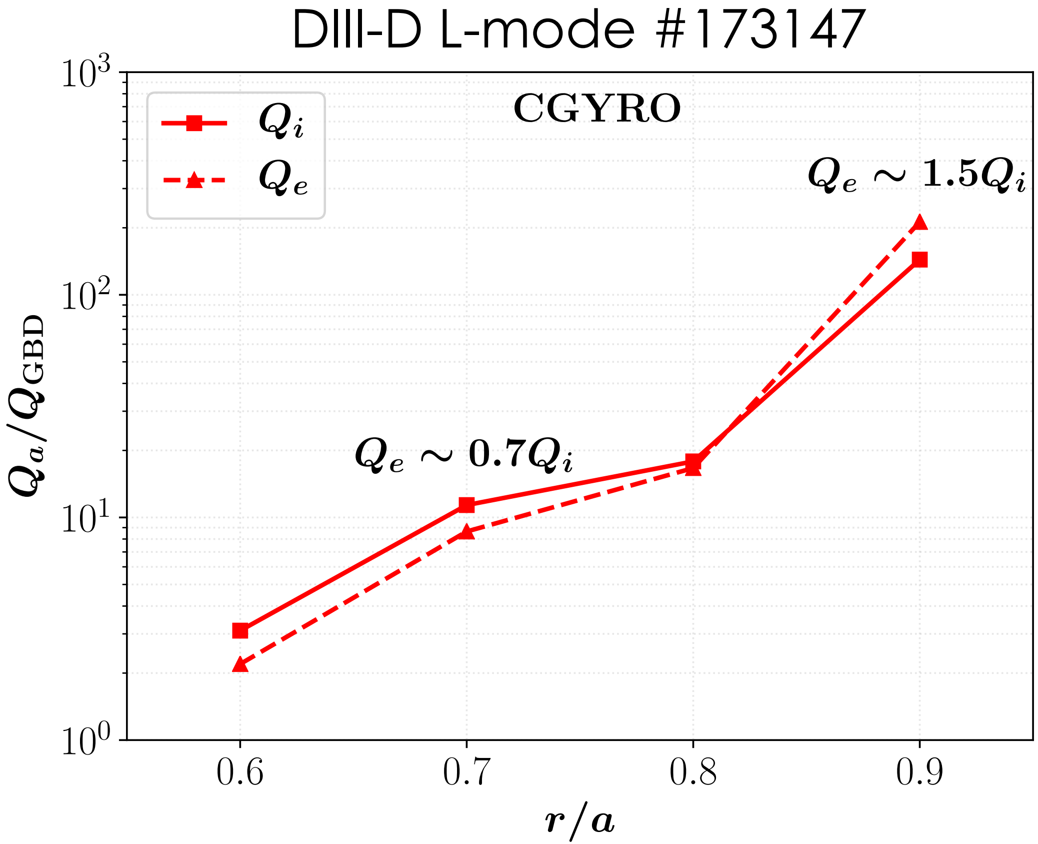
$$Q_i = \tilde{c}_0 \left(\frac{m_e}{m_i}\right) Q_{GBi}$$

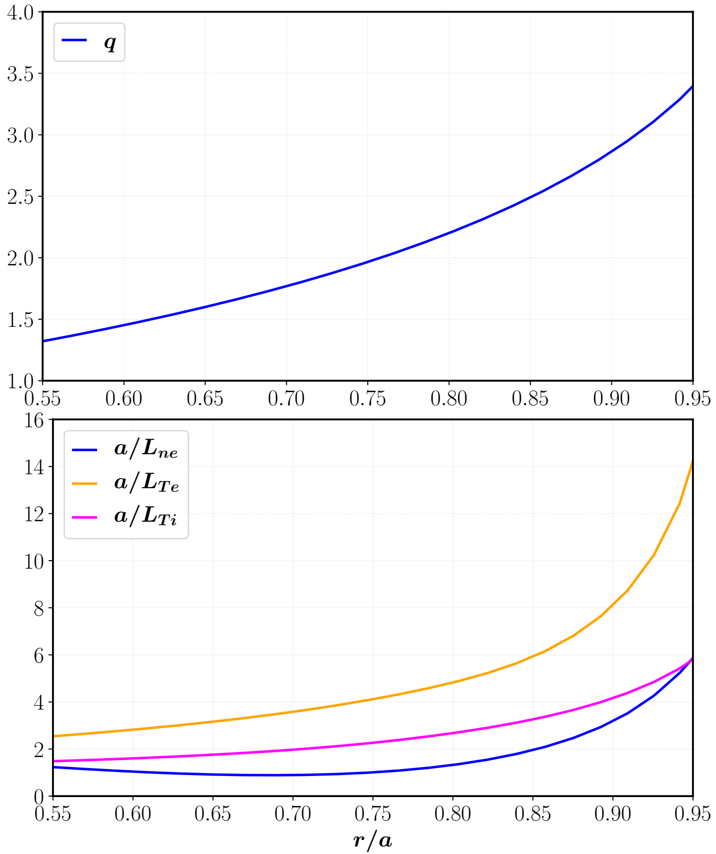
$$Q_H > Q_D > Q_{DT} \qquad \tau_{E,H} < \tau_{E,D} < \tau_{E,DT}$$

CGYRO turbulence simulations match experimental DIII-D power balance (D+e) in the core and the edge.



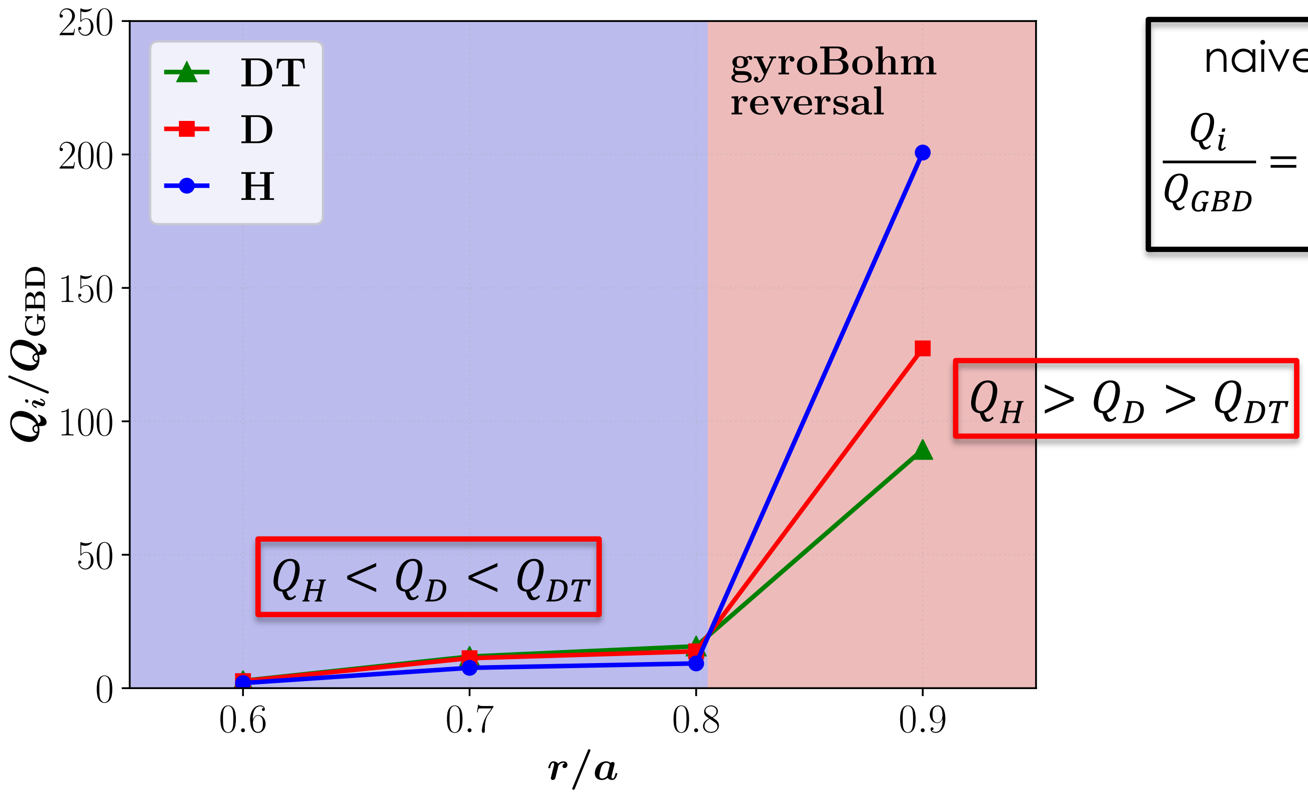
In the edge, the transport becomes explosive due to parameters (large q & profile gradients) that enhance the nonadiabatic electron drive.





GENERAL ATOMICS

A favorable reversal of the naive gyroBohm isotope mass scaling is found in the TEM-dominated edge.

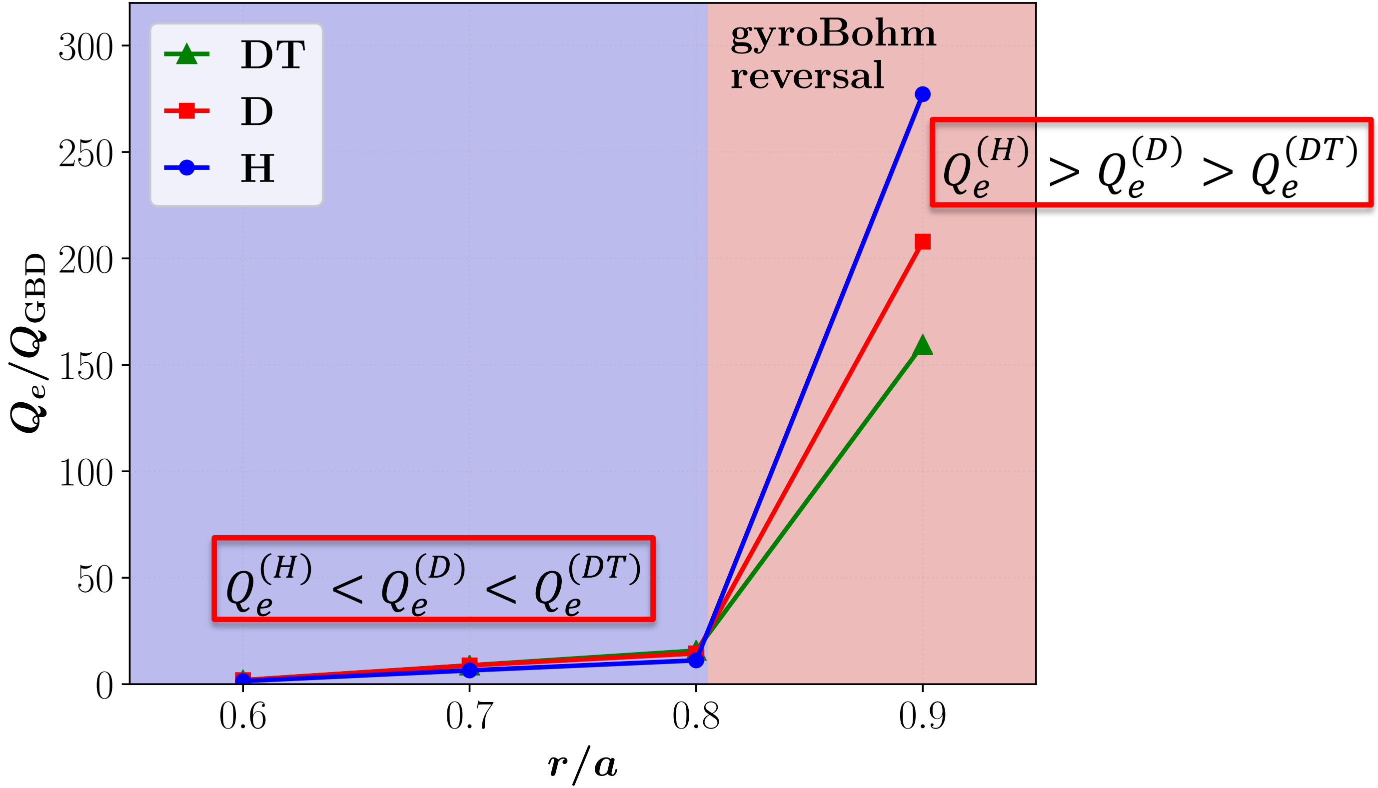


naive GB:

$$\frac{Q_i}{Q_{GBD}} = c_0 \sqrt{\frac{m_i}{m_D}}$$

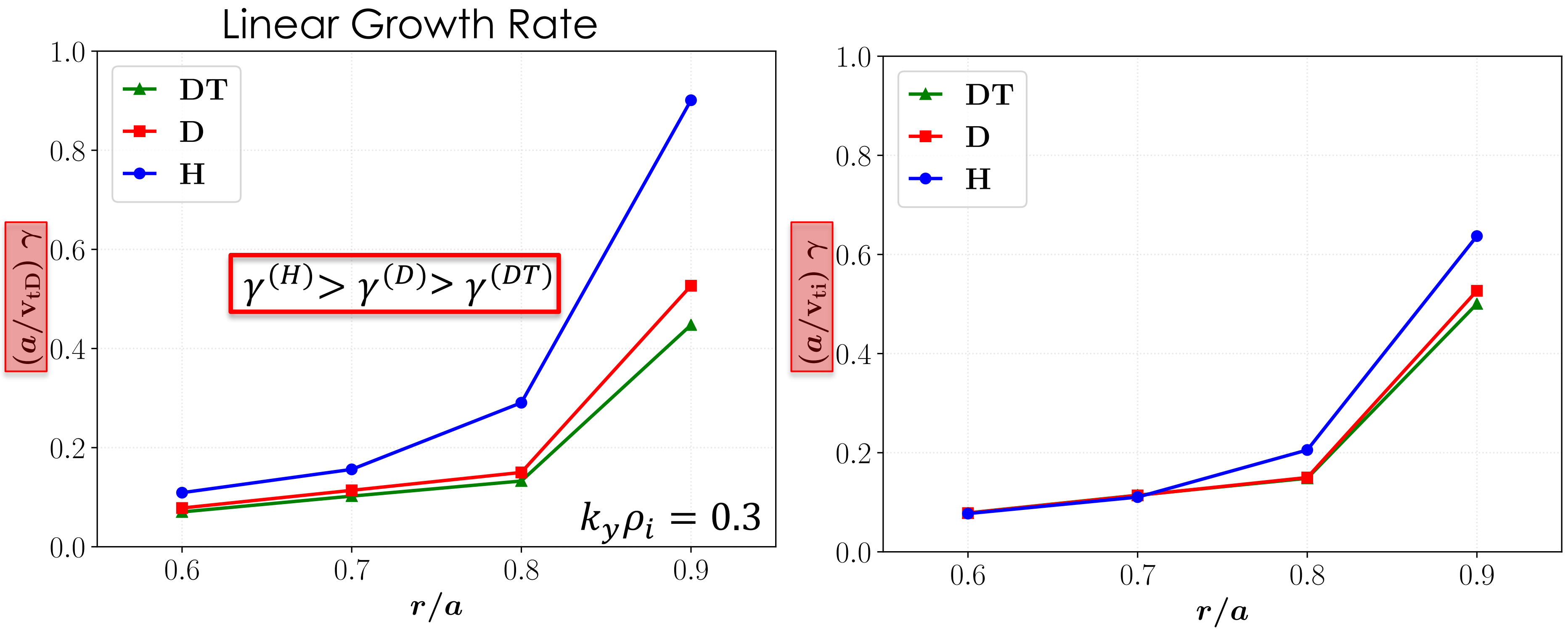
*D corresponds to DIII-D #173147

The electron energy fluxes follow the ion-mass scaling of the ion fluxes.



*D corresponds to DIII-D #173147

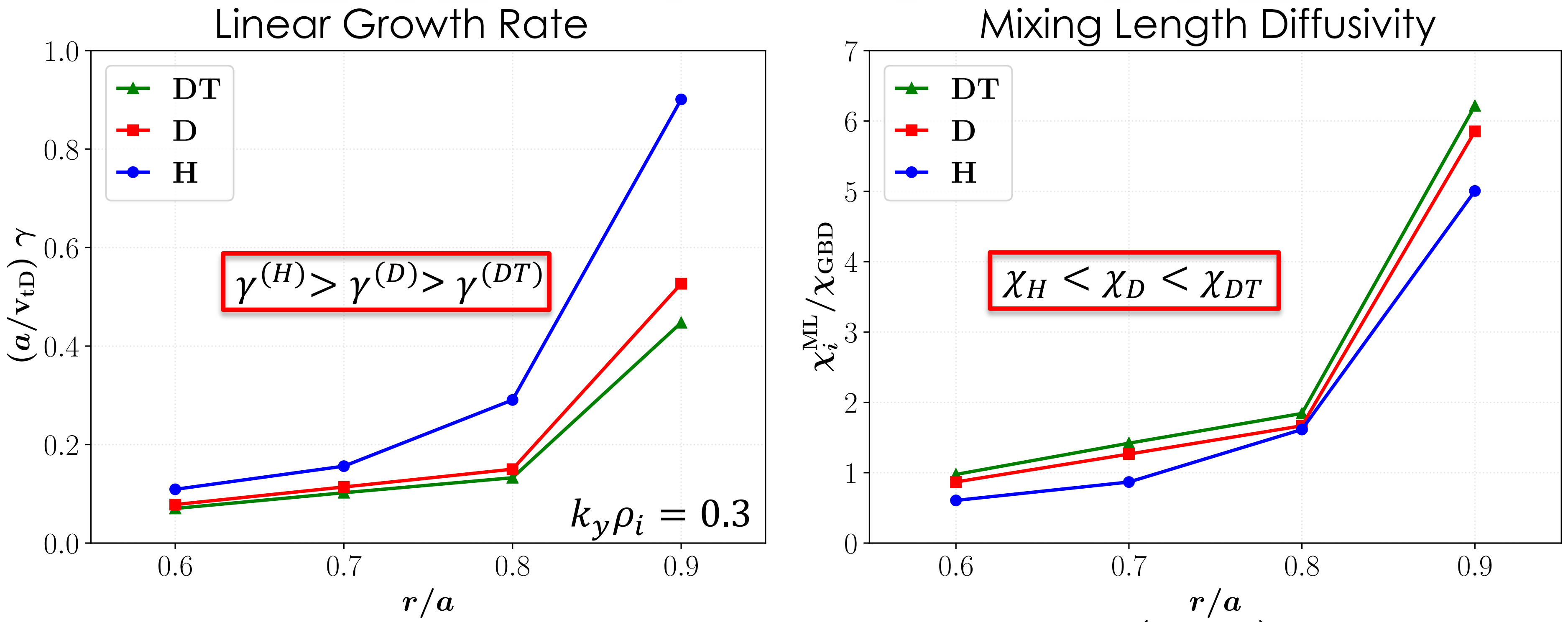
The linear growth rate follows the trend of the intrinsic mass scaling.



Intrinsic Mass Scaling: $\gamma \sim \omega_{*i} \sim k_y \rho_i \frac{\mathrm{v}_{ti}}{L_T} \sim \frac{1}{\sqrt{m_i}} \Rightarrow \left(\frac{a}{\mathrm{v}_{ti}}\right)\gamma = \gamma_0$ independent of ion mass

GENERAL ATOMICS

Mixing length theory is not able to capture the reversal in the ion mass scaling in the edge.



Intrinsic Mass Scaling: $\left(\frac{a}{v_{ti}}\right)\gamma = \gamma_0$ → $\chi_i^{ML} = \frac{\gamma}{k_y^2} \sim \left(\frac{\gamma_0}{k_y^2 \rho_i^2 a}\right) v_{ti} \rho_i^2 \sim \sqrt{m_i}$

GENERAL ATOMICS

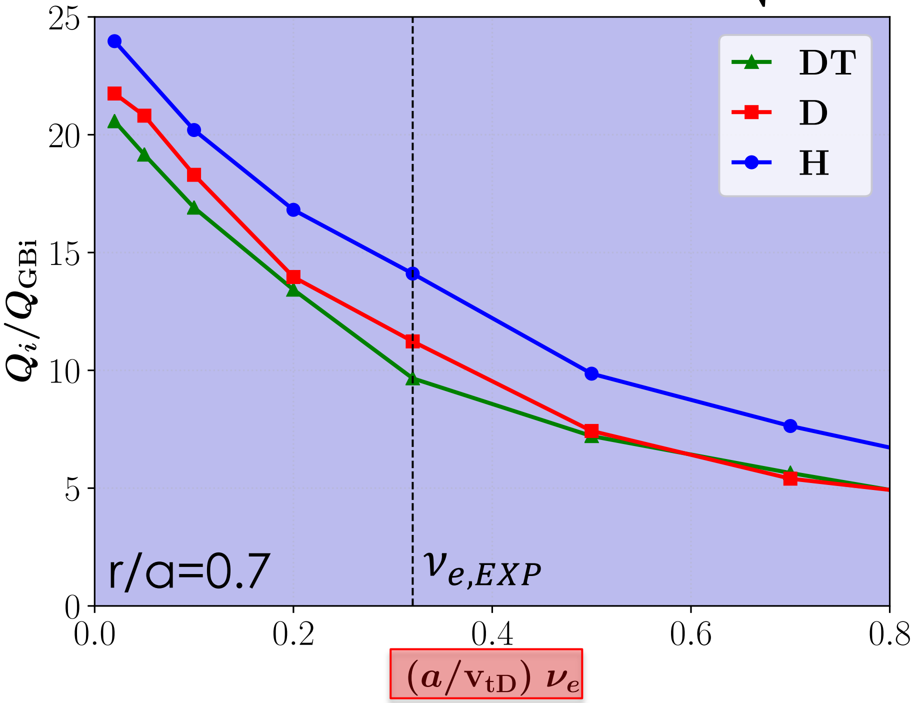
The electron-mass dependence of the turbulent flux enters through several effects.

Electron-ion collisions	Nakata effect	can be rescaled
Plasma rotation	Garcia effect	can be rescaled
Electromagnetic fluctuations	negligible	can be rescaled
Finite electron Larmor radius	negligible	irreducible
Electron parallel motion	dominant	**irreducible**

Lighter isotopes are more weakly stabilized by collisions (in absolute units).

"Nakata Effect" $\frac{\nu_e}{\omega_{ti}} \sim \sqrt{\frac{m_i}{m_e}}$

naive GB: $Q_i = c_0 Q_{GBi}$



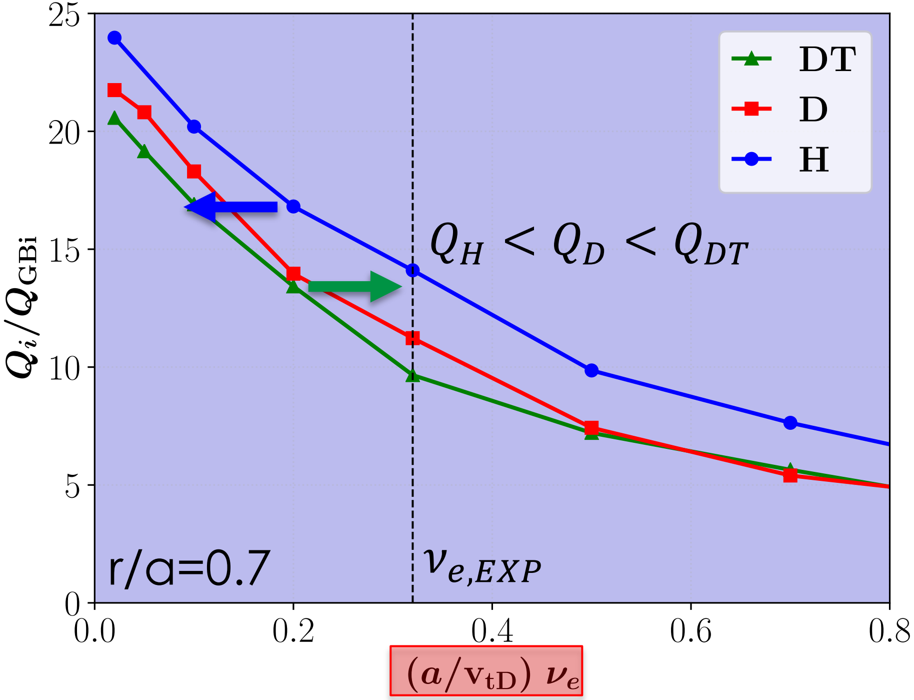
 Nakata et al., PRL 118, 165002 (2017)

GENERAL ATOMICS

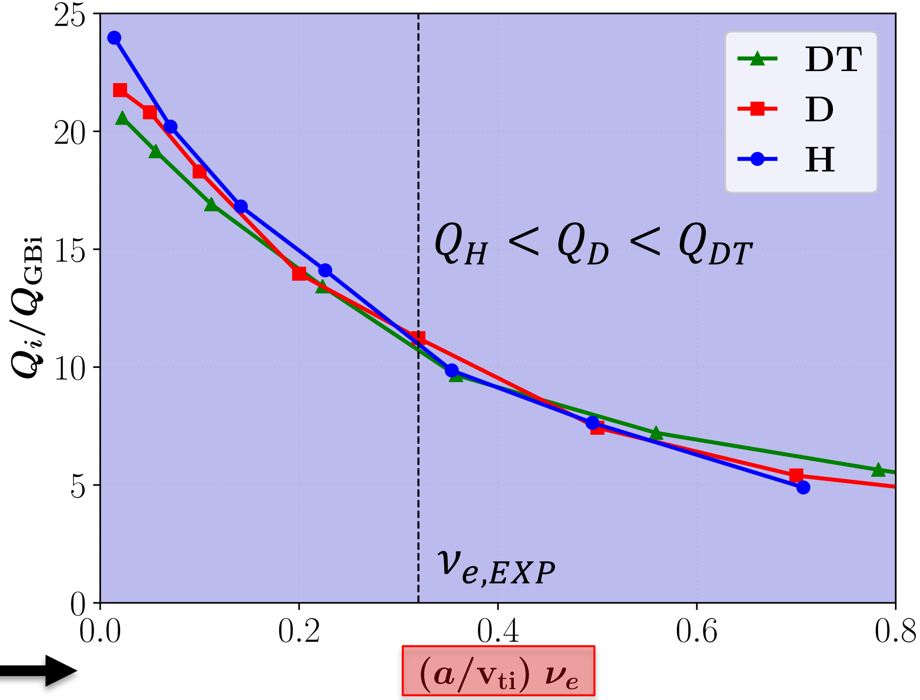
The mass-dependence from collisions can mostly be eliminated by rescaling the collision rate with respect to the main ion time scale.

naive GB: $Q_i = c_0 Q_{GBi}$

"Nakata Effect"



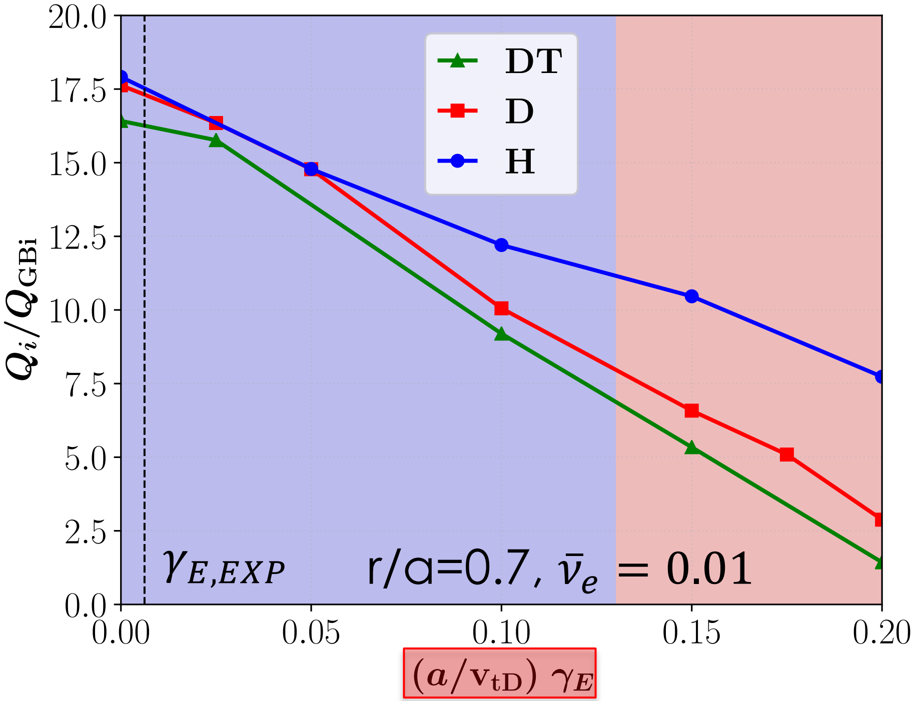
Rescaled



Lighter isotopes are more weakly stabilized by ExB flow shear (in absolute units).

"Garcia Effect" $\frac{\gamma_E}{\gamma_{ITG}} \sim \frac{\omega_0}{v_{ti}} \sim \sqrt{m_i}$

naive GB: $Q_i = c_0 Q_{GBi}$

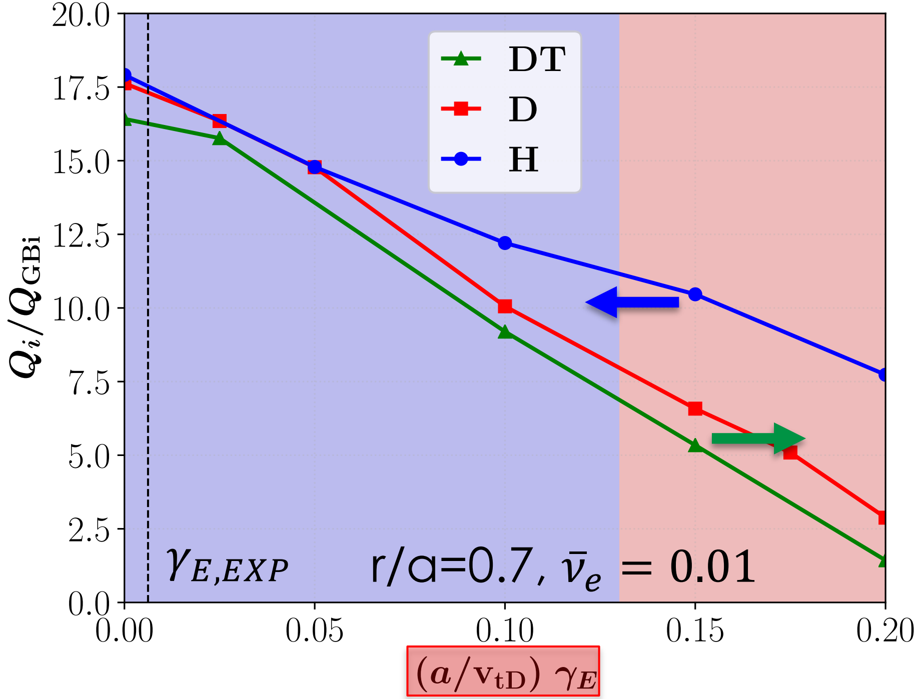


Garcia et al., NF 57, 014007 (2017)

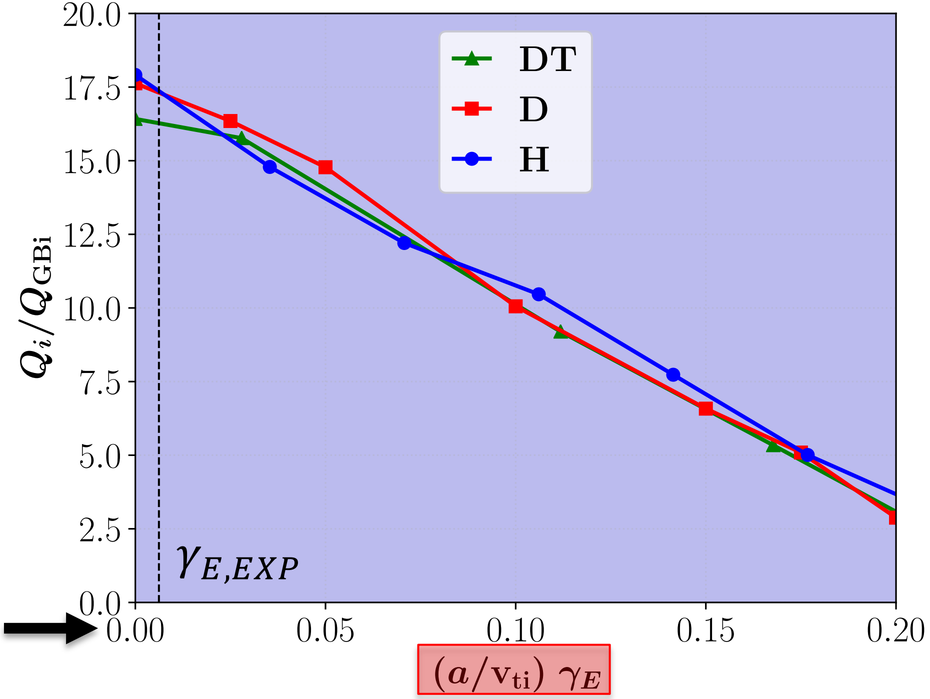
The mass-dependence from rotation can be eliminated by rescaling the rotation rate with respect to the main ion time scale.

naive GB: $Q_i = c_0 Q_{GBi}$

“Garcia Effect”



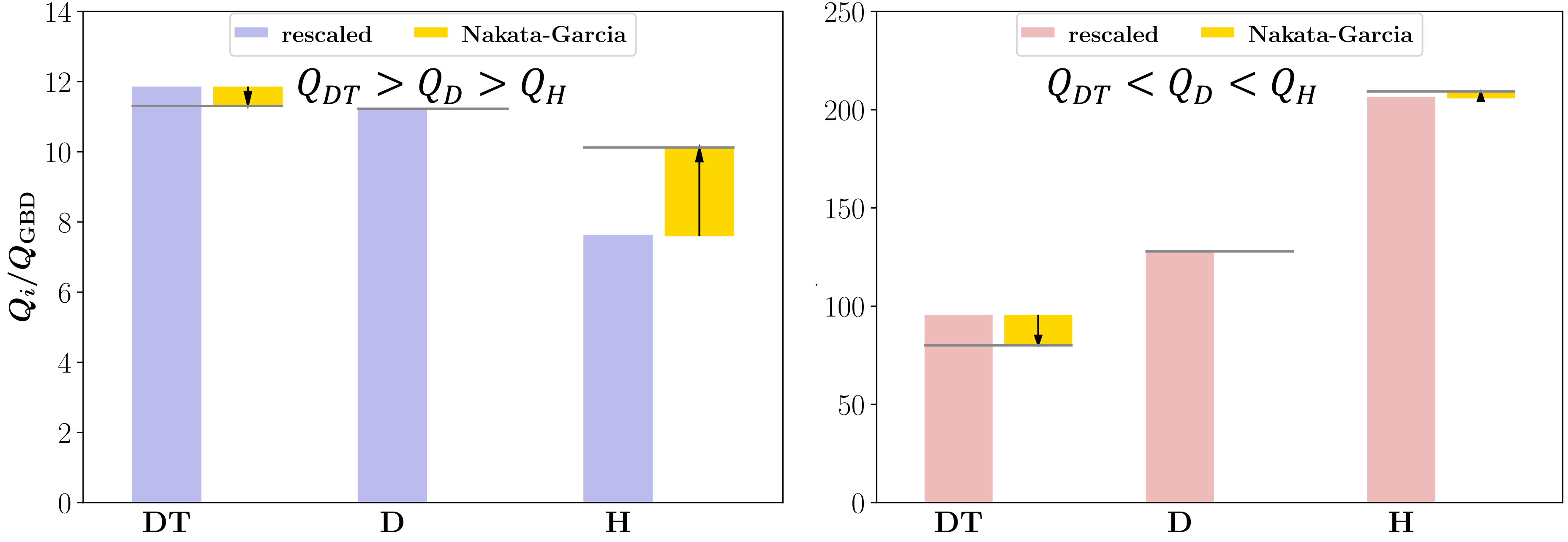
Rescaled



GENERAL ATOMICS

The Nakata and Garcia effects enhance the breaking of naive gyroBohm scaling.

rescaled: $\frac{(\nu_e, \gamma_E)}{v_{ti}/a} = c$ Nakata-Garcia: $\frac{(\nu_e, \gamma_E)}{v_{tD}/a} = c$



r/a=0.7 DIII-D L-mode #173147 r/a=0.9

GENERAL ATOMICS

The dominant electron-mass dependence of the turbulent flux in this case is through the electron parallel motion.

Electron-ion collisions	Nakata effect	can be rescaled
Plasma rotation	Garcia effect	can be rescaled
Electromagnetic fluctuations	negligible	can be rescaled
Finite electron Larmor radius	negligible	irreducible
Electron parallel motion	dominant	**irreducible**

A new isotope-mass scaling law is proposed to describe the electron-to-ion mass ratio dependence of the turbulent energy flux.

Electron mass dependence arises from electron parallel motion:

$$\frac{\partial H_i}{\partial \tau_i} + \frac{u_\parallel(\theta)}{q\mathcal{R}}\frac{\partial H_i}{\partial \theta} = G_i(H_i, \Phi, \boldsymbol{p}) \qquad \frac{\partial H_e}{\partial \tau_i} + \sqrt{\frac{\boldsymbol{m_i}}{\boldsymbol{m_e}}}\frac{u_\parallel(\theta)}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, \boldsymbol{p})$$

$$\boldsymbol{p} = \begin{bmatrix} q, s, \epsilon, \kappa, \ldots & geometry \\ \frac{T_i}{T_e}, \frac{a}{L_{Ti}}, \frac{a}{L_{Te}}, \ldots & profile \\ \frac{a\gamma_E}{\mathrm{v}_{ti}}, \frac{a\gamma_p}{\mathrm{v}_{ti}}, \ldots & rotation \\ \frac{a\nu_{ee}}{\mathrm{v}_{ti}}, \frac{a\nu_{ei}}{\mathrm{v}_{ti}}, \ldots & collisions \end{bmatrix} \qquad \tau_i = \left(\frac{\mathrm{v}_{ti}}{a}\right)t$$

“fixed”: independent of electron-to-ion mass ratio

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Corrections to bounce-averaged limit obtained by expanding in $\frac{\boldsymbol{\omega}}{\boldsymbol{\omega_{be}}}$:

$$H_e = \langle H_e \rangle_b + \varepsilon H_e^{(1)} + \varepsilon^2 H_e^{(2)} + \cdots \qquad \boldsymbol{\varepsilon \doteq q\mathcal{R}\sqrt{m_e/m_i}}$$

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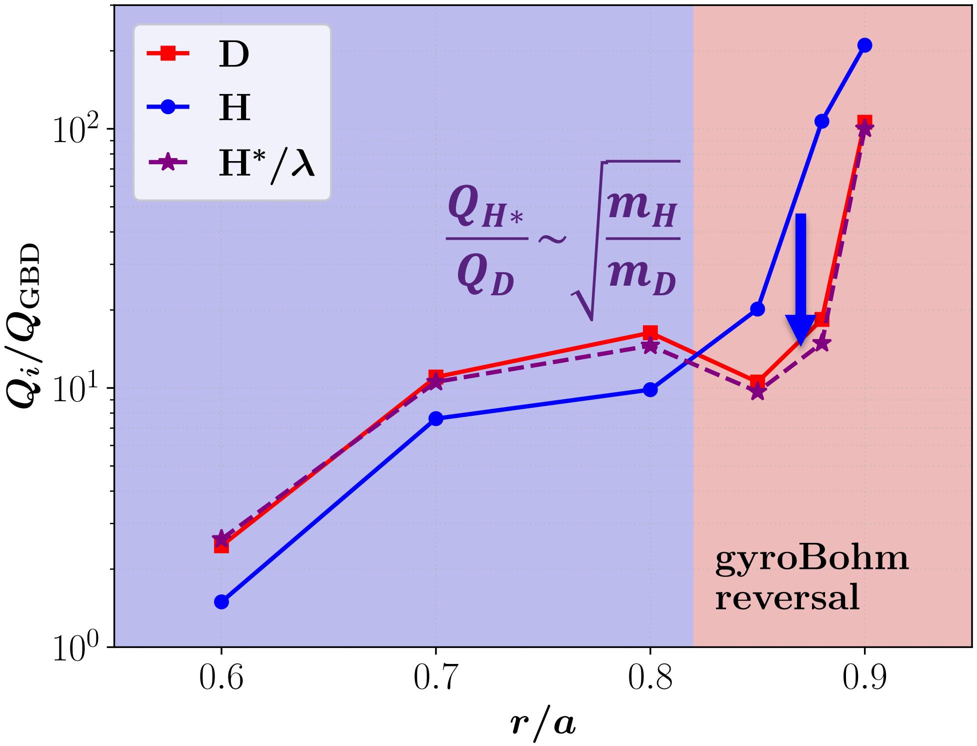
⬇

$$Q_i = Q_{GBi}[c_0(\boldsymbol{p}) + \varepsilon c_1(\boldsymbol{p}) + \varepsilon^2 c_2(\boldsymbol{p}) + \ \ldots]$$

$$\frac{Q_i}{Q_{GBD}} = \boldsymbol{c_0(p)}\sqrt{\frac{\boldsymbol{m_i}}{\boldsymbol{m_D}}} + \boldsymbol{c_1(p)q\mathcal{R}}\sqrt{\frac{\boldsymbol{m_e}}{\boldsymbol{m_D}}} + \boldsymbol{c_2(p)(q\mathcal{R})^2}\frac{\boldsymbol{m_e}}{\sqrt{\boldsymbol{m_i m_D}}}$$

naive gyroBohm | **weak nonadiabatic** | **strong nonadiabatic**

Naive gyroBohm scaling is recovered for hydrogen (relative to deuterium) if the electron parallel response time is shortened.



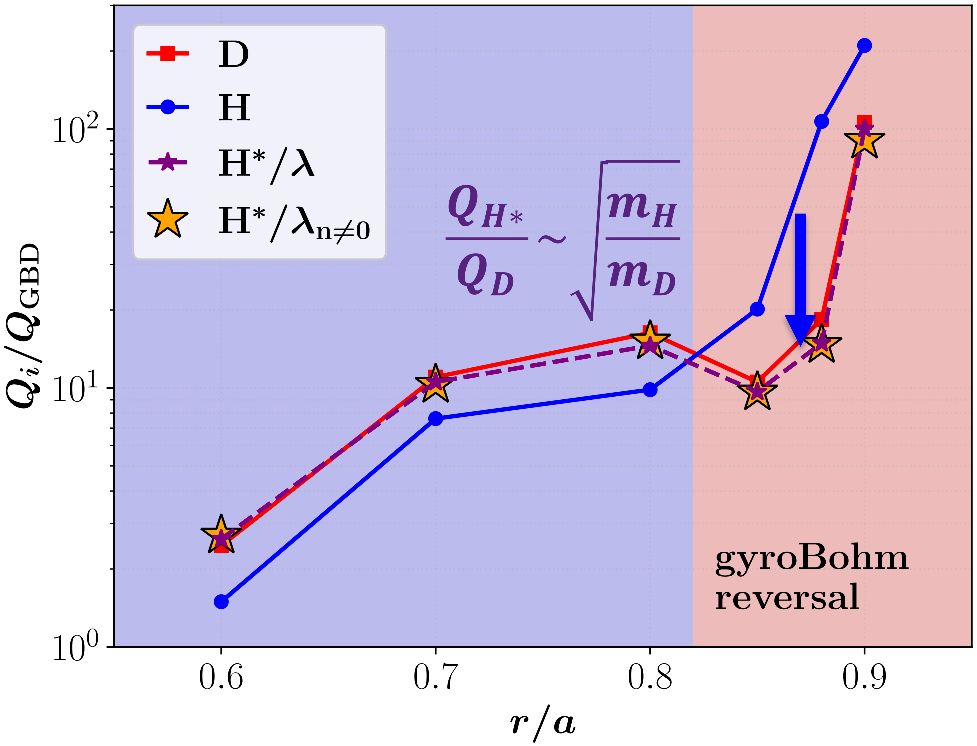
$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

$$\frac{v_{ti}}{a}\tau_e \sim \lambda q\mathcal{R}\frac{v_{ti}}{v_{te}} \sim \lambda q\mathcal{R}\sqrt{\frac{m_e}{m_i}}$$

$$H^*: \lambda = \sqrt{\frac{m_H}{m_D}}$$

GENERAL ATOMICS

The mass-scaling is controlled by non-zonal parallel electron motion.



$$\frac{Q_{H*}}{Q_D} \sim \sqrt{\frac{m_H}{m_D}}$$

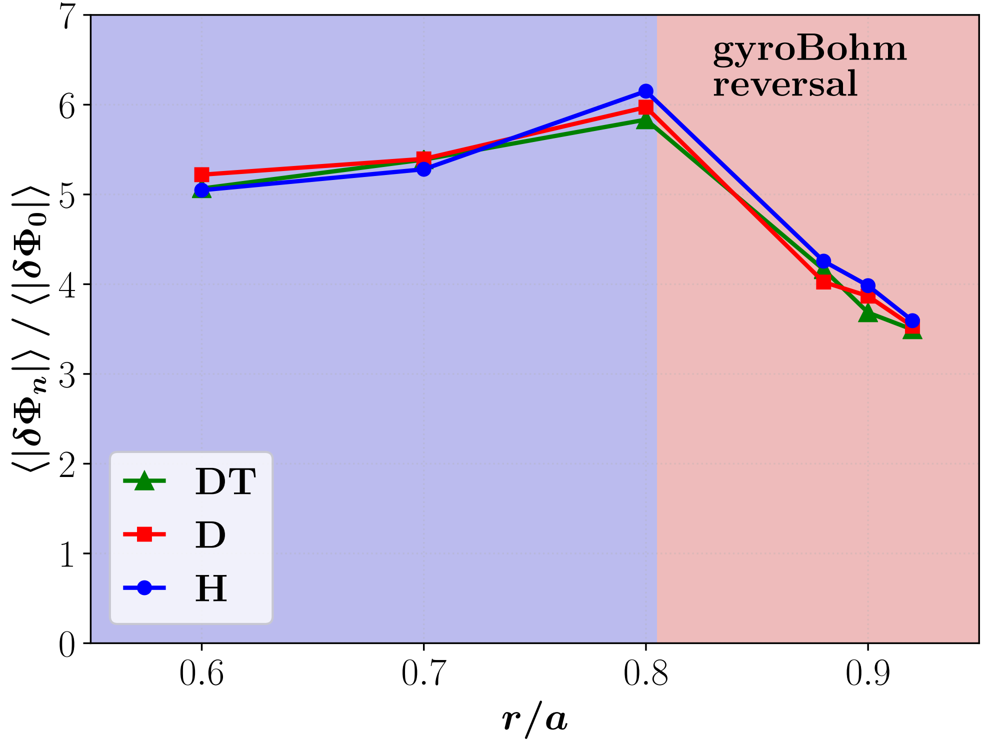
$$\frac{\partial H_e^{n\neq0}}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e^{n\neq0}}{\partial\theta} = G_e(H_e,\Phi,p)$$

$$\frac{v_{ti}}{a}\tau_e \sim \lambda q\mathcal{R}\frac{v_{ti}}{v_{te}} \sim \lambda q\mathcal{R}\sqrt{\frac{m_e}{m_i}}$$

$$H^*: \lambda = \sqrt{\frac{m_H}{m_D}}$$

GENERAL ATOMICS

Zonal flows do not play a significant role in the mass-scaling behavior.



$$\frac{\sum_{n\neq 0}\langle|\delta\Phi_n|\rangle}{\langle|\delta\Phi_{n=0}|\rangle} = c_\phi$$

independent of ion mass

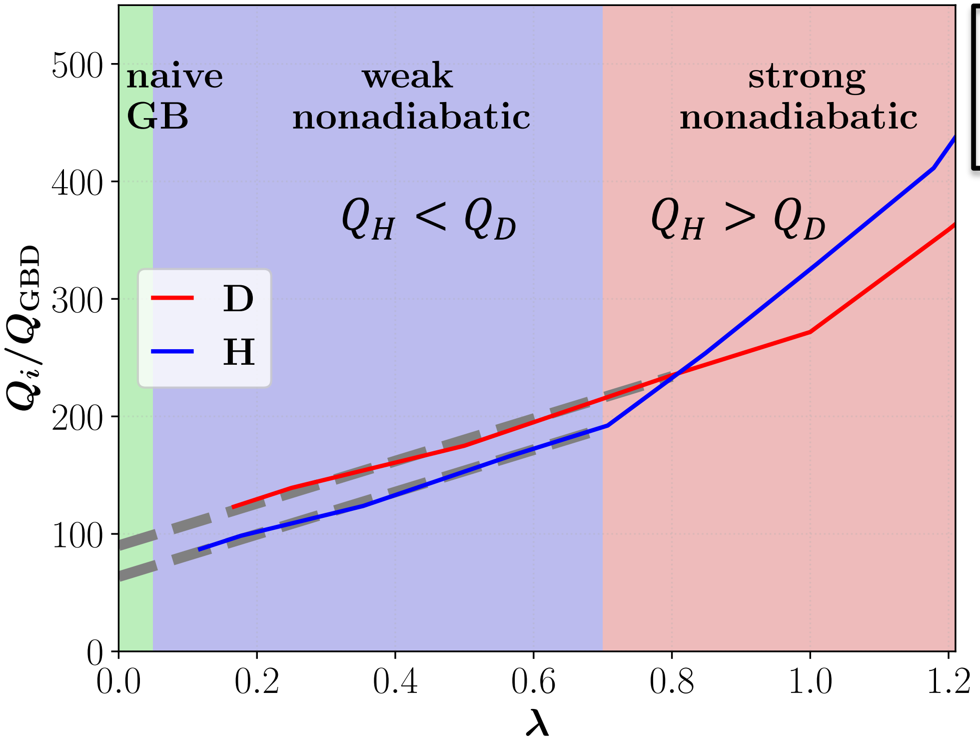
$$\delta\Phi_{n,rms} \doteq \sum_{n\neq 0}\langle|\delta\Phi_n|\rangle$$

$$\delta\Phi_{n,rms} = \rho_{*i} c_{n\phi}\left(\frac{m_e}{m_i}, p\right)$$

GENERAL ATOMICS

The finite electron mass correction dominates the mass scaling in the edge and plays a key role in reversing the GB mass scaling.

DIII-D L-mode #173147, r/a=0.9



$$\frac{Q_i}{Q_{GBD}} = c_0\sqrt{\frac{m_i}{m_D}} + c_1 q\mathcal{R}\sqrt{\frac{m_e}{m_D}} + c_2(q\mathcal{R})^2\frac{m_e}{\sqrt{m_i m_D}}$$

naive GB — weak nonadiabatic — strong nonadiabatic

$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

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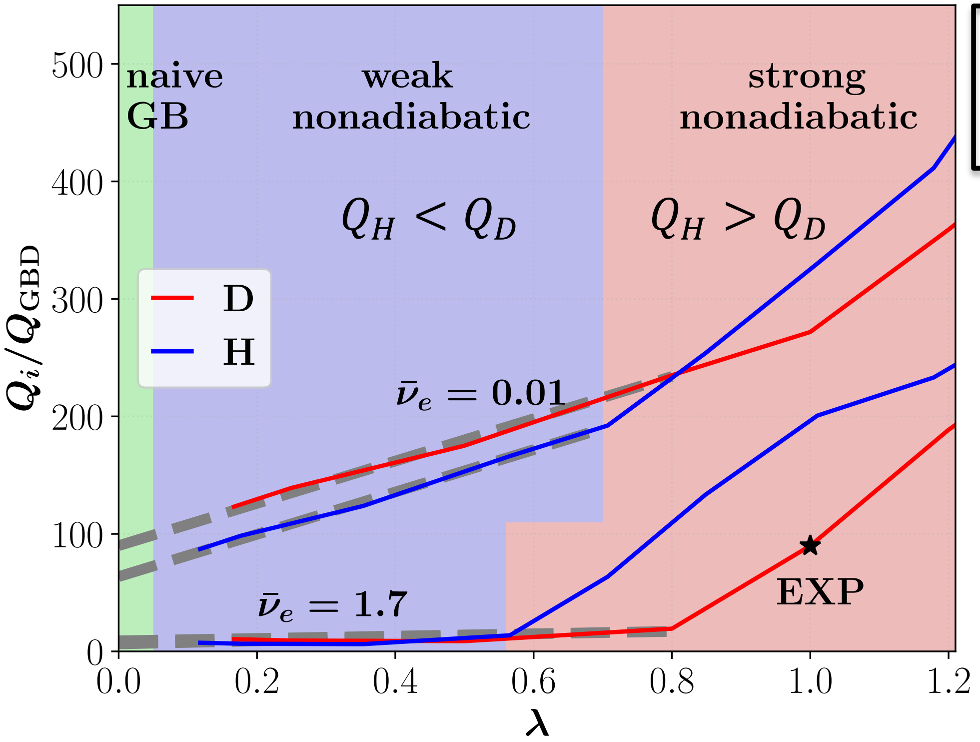
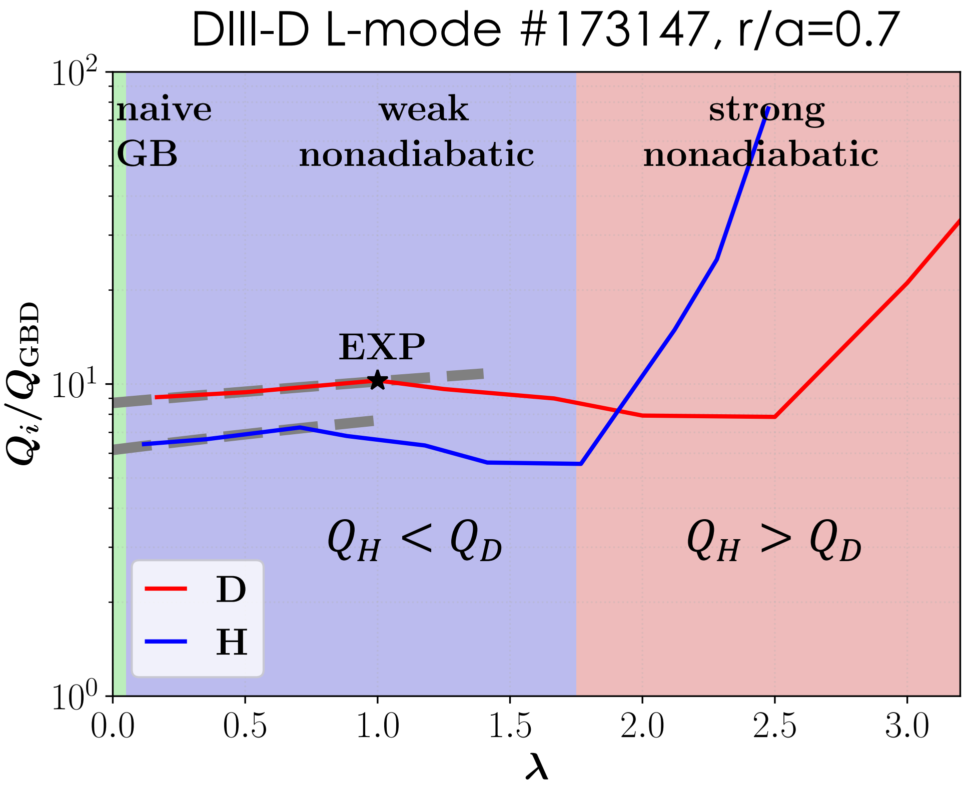


$$\frac{Q_i}{Q_{GBD}} = c_0\sqrt{\frac{m_i}{m_D}} + c_1 q\mathcal{R}\sqrt{\frac{m_e}{m_D}} + c_2 (q\mathcal{R})^2 \frac{m_e}{\sqrt{m_i m_D}}$$

naive GB | weak nonadiabatic | strong nonadiabatic

$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

The finite electron mass correction is only weakly nonadiabatic in the core and thus a reversal of the GB mass scaling is not expected.



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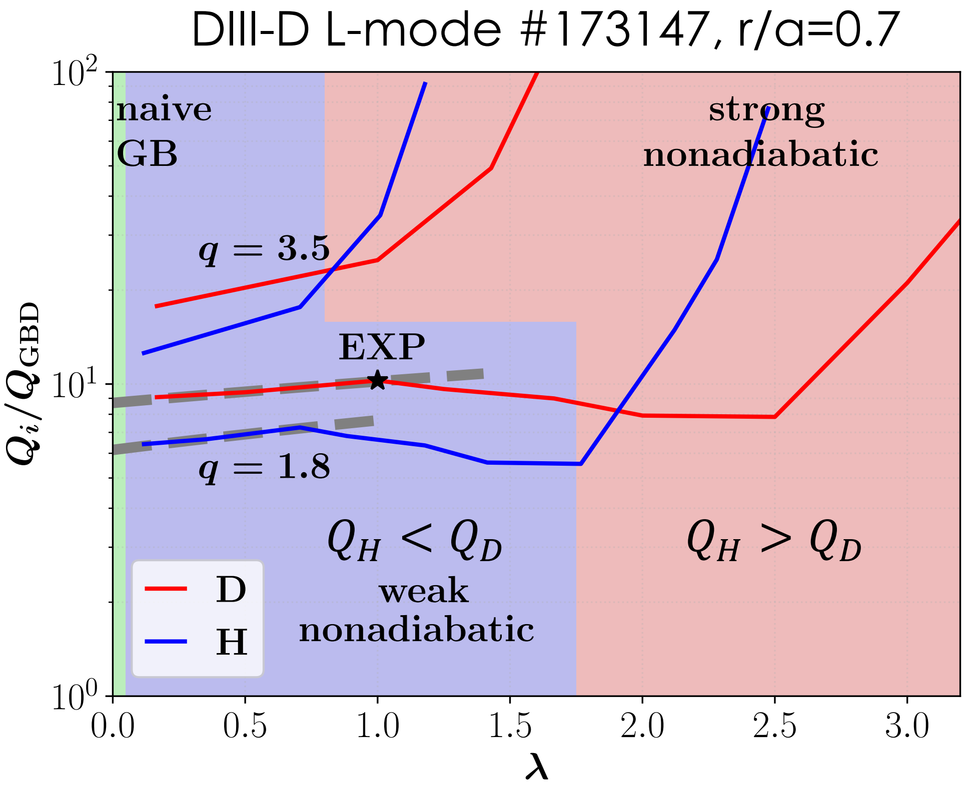
naive GB — weak nonadiabatic — strong nonadiabatic

$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

GENERAL ATOMICS

The nonadiabatic electron drive can dominate the ion mass scaling in the ITG core at high q.



$$\frac{Q_i}{Q_{GBD}} = c_0\sqrt{\frac{m_i}{m_D}} + c_1 q\mathcal{R}\sqrt{\frac{m_e}{m_D}} + c_2 (q\mathcal{R})^2 \frac{m_e}{\sqrt{m_i m_D}}$$

naive GB	weak nonadiabatic	strong nonadiabatic

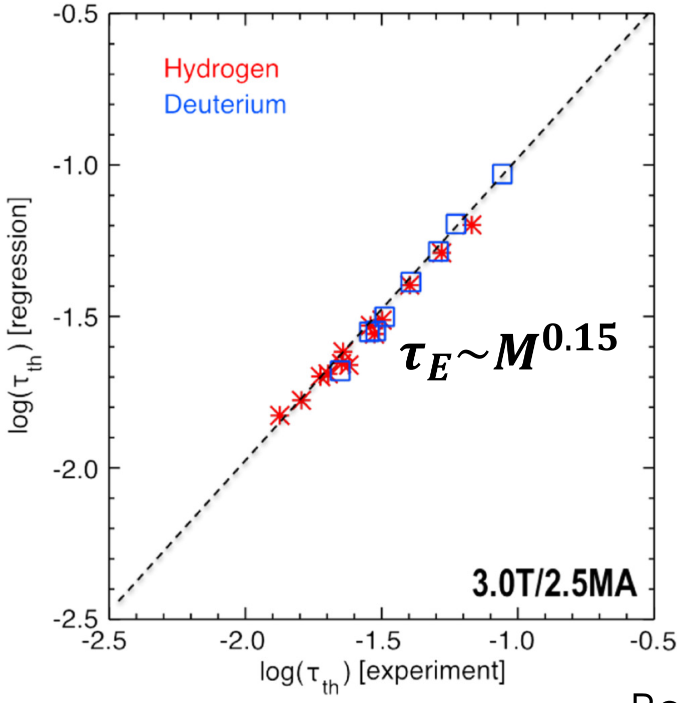
$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

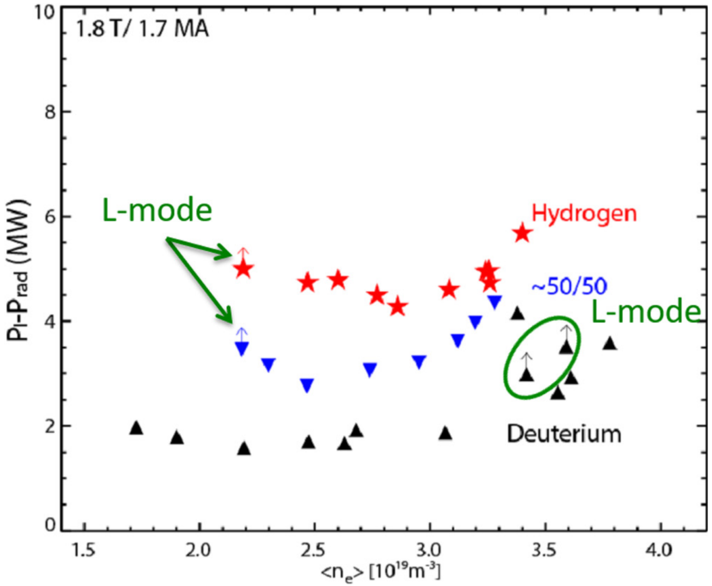
GENERAL ATOMICS

New scaling implies favorable increase in global confinement & lowering the L-H power threshold, in agreement with experimental trends.

$Q_H > Q_D > Q_{DT}$ ➡ $\boldsymbol{\tau_{E,H} < \tau_{E,D} < \tau_{E,DT}}$ ➡ $\boldsymbol{P_{LH,H} > P_{LH,D} > P_{LH,DT}}$

$$\frac{\partial\langle n_i T_i\rangle}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial r}(V'Q_i) = S_{Ti}$$





Results from JET-ILW: Maggi PPCF 2018

GENERAL ATOMICS

Summary

- A **new isotope-mass scaling law** is proposed to describe **electron-to-ion mass ratio dependence** of turbulent energy fluxes in both ion-dominated **core** & electron-dominated **edge** transport regimes.
- Electron-to-ion mass ratio dependence arises from the nonadiabatic response associated with **electron parallel motion**.
- The **nonadiabatic electron drive** strongly regulates the turbulence levels and plays a key role in **altering** – and in the L-mode edge, **reversing** – **naive gyroBohm ion mass scaling**.
- The finite-m_e correction is larger for light ions and higher q $\rightarrow$ weak in the core but **dominates the mass scaling in the edge**.
- More info: E. Belli et al., PRL 125, 015001 (2020)

  E. Belli et al., PoP 26, 082305 (2019)

GENERAL ATOMICS