STUDY OF LOW N KINETIC BALLOONING MODE IN SPHERICAL TOKAMAK

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Abstract

An investigation on the onset of low n kinetic ballooning mode (KBM) in a plasma environment relevant to the spherical reactors is carried out. The low n modes can be quite global in nature and can set an upper limit on the achievable $\beta$, a critical parameter for better performance and energy production in spherical reactors. The gyrokinetic simulations for a set of numerically generated equilibrium with increasing equilibrium $\beta$ show that global low n KBM gets destabilized when $\beta$ increases and the longest wavelength KBM is only slightly weaker than the most unstable mode. These high $\beta$ equilibria are generally associated with the presence of a large number of energetic ions. The addition of the energetic particle effect in these studies reveals that the low n branch of the KBM can be destabilized at $\beta$ values lower compared to the case without the energetic ions. These modes in the presence of energetic ions tend to peak towards an even further longer wavelength and hence more global.

1. INTRODUCTION

The efficiency of a future fusion reactor will strongly depend on the amount of $\beta$ these machines can achieve and sustain for a longer period. While drift waves might lead to gradual and benign deterioration of the confinement the global MHD mode can stop the plasma abruptly. The kinetic ballooning modes (KBM) [1-4] are such a class of MHD modes modified by the presence of kinetic effects. In a high $\beta$ reactor scale machine, these modes become unstable at longer wavelengths and may put an upper bound on achievable plasma $\beta$. These high $\beta$ plasmas are invariably highly populated with energetic ions due to the various auxiliary heating schemes. These energetic ions are found to interact with underlying instabilities [5-7]. These energetic ions may also interact with the long-wavelength KBMs and alter the stability scenario further. In the present work, we aim to study the low n KBM with and without the energetic ions.

A series of reactor-relevant equilibrium is generated using CHEASE [9,10,11] and a stability analysis is carried out using the flux tube version of the gyrokinetic code GENE [12-14]. Unstable KBMs are observed at a lower toroidal number of long-wavelength regime as the equilibrium $\beta$ is increased. Comparison with MHD calculations is carried out. The introduction of energetic ions is observed to produce the unstable low n KBM at a comparatively lower $\beta$ compared to the case when there are no energetic ions. The KBM peaks even further towards the lower n, making them more global in nature.

2. GENERATION OF EQUILIBRIUM AND INPUT PARAMETERS

A series of eight equilibria with varying $\beta$ encompassing the threshold of electrostatic to electromagnetic transition is generated using the CHEASE code. For a given flux surface shape, pressure, and toroidal current profile the code solves the Grad-Shafranov equation. In the present study, the different equilibria are produced by changing the pressure profile. The plasma surface shape, $\beta$ profiles are shown in Fig.1. The motivation of generating these equilibria is to self-consistently include the effect of changing equilibrium $\beta$ on plasma surface shape and other quantities such as safety factor, shear etc. These equilibria are then used to extract the Miller parameters appropriate for the gyrokinetic code GENE for the study of the instability associated with the equilibria. The Miller parameters for $r/a=0.5$ where the gyrokinetic analysis will be carried out is given in Table 1. Note that in the gyrokinetic simulations we consider the electromagnetic perturbations including compressional component. The equilibrium Er effect and collisions are neglected. Note that the ideal MHD ballooning calculation of mode growth rate solving the equations in Greene and Chance, [15] modified to take into account the inertia effects shows that the highest $\beta$ equilibrium supports the most unstable mode in the region with maximum pressure.
gradient. The presence of weaker shear near the core region allows the lower $\beta$ equilibrium also to be unstable. The threshold for electrostatic to electromagnetic transition occurs near $\beta = 8.1\%$.

TABLE 1. PARAMETERS FOR GYROKINETIC SIMULATIONS

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>1.3%</th>
<th>8.1%</th>
<th>9.6%</th>
<th>11.1%</th>
<th>12.1%</th>
<th>12.9%</th>
<th>13.7%</th>
<th>14.1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>1.538</td>
<td>1.508</td>
<td>1.529</td>
<td>1.584</td>
<td>1.624</td>
<td>1.704</td>
<td>1.818</td>
<td>2.031</td>
</tr>
<tr>
<td>$s$</td>
<td>0.814</td>
<td>1.045</td>
<td>1.106</td>
<td>1.206</td>
<td>1.264</td>
<td>1.360</td>
<td>1.467</td>
<td>1.629</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.306</td>
<td>1.32</td>
<td>1.332</td>
<td>1.354</td>
<td>1.373</td>
<td>1.399</td>
<td>1.440</td>
<td>1.505</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.096</td>
<td>0.114</td>
<td>0.120</td>
<td>0.132</td>
<td>0.141</td>
<td>0.155</td>
<td>0.175</td>
<td>0.208</td>
</tr>
<tr>
<td>$dR/dr$</td>
<td>-0.098</td>
<td>-0.266</td>
<td>-0.300</td>
<td>-0.349</td>
<td>-0.380</td>
<td>-0.420</td>
<td>-0.467</td>
<td>-0.525</td>
</tr>
<tr>
<td>$\beta$(axis)</td>
<td>3.04%</td>
<td>24.26%</td>
<td>29.84%</td>
<td>38.35%</td>
<td>44.44%</td>
<td>52.44%</td>
<td>63.26%</td>
<td>78.13%</td>
</tr>
</tbody>
</table>

**FIG. 1.** Left: Shape of the last closed flux surface relevant to MAST and Right: $\beta$ profiles for different equilibria. The values in the legend represent total $\beta$ values at the location $s = r/a = 0.5$

3. LINEAR GYROKINETIC SIMULATIONS

In this section, we present the linear two species gyrokinetic analysis of KBM instability. We use the flux tube version of the GENE code. The Miller parameters corresponding to the global equilibria generated using CHEASE are used as the input of the gyrokinetic simulations. To simplify the analysis, we consider two cases. In the first set of analysis, we keep the temperature gradient flat and the free energy is provided by the density gradient. In the second case, we set the density profile flat and the free energy source lies in the temperature gradient. This is possible for the KBM mode as the KBM instability solely depends upon the pressure gradient and does not distinguish between whether the gradient comes from the density or temperature profile. Figure 2 shows the real frequency (left panel) and growth rate (right panel) for the equilibria considered. The real frequency increases with $\beta$. The real frequency increases with $k_x$ as well for the lower $k_x$ region which transitions to another mode as $k_x$ increases. The lowest $\beta$ case exhibits mode frequency in the electron diamagnetic direction for which TEM [16,17] or universal mode [18] might be a plausible candidate. The growth rate increases as the equilibrium $\beta$ increases. The strongest growth rate is observed for the highest $\beta$ equilibrium. The growth rates peak near $k_y = 0.2$, however, the growth rates as $k_y \to 0$ are still closer to the peak growth rate. The growth rate decreases as the $\beta$ decreases, the high $\beta$ long-wavelength modes are long-wavelength kinetic ballooning mode which first appears for $\beta = 11.1\%$ and becomes more and more dominant as $\beta$ increases further. The nature of the long-wavelength
KBM is of ideal MHD character which is verified by calculating $E_i$ which is very small. Also, a comparison of the real frequency with diamagnetic drift stabilized MHD calculations [1,2] shows that the real frequency scales as $\omega_{pi}/2$. In the second set of analysis, we consider temperature gradient-driven KBM by considering flat density. The results are shown in Fig. 3. The qualitative nature of the modes remains the same as discussed above.

![FIG. 2. Real frequencies (left) and growth rates (right) versus $k_y\rho_s$ for the density gradient driven case, for two-species simulations.]

However, due to the presence of temperature gradient we also observe other instabilities which are driven by temperature gradient. In the higher $\beta$ equilibria, these modes are identified to exhibit tearing parity like the microtearing mode [19-24] while in the lowest $\beta$ equilibrium the modes are identified to be ITG and then TEM for the higher $k_y$ values [25,26].

![FIG. 3. Real frequencies (left) and growth rates (right) versus $k_y\rho_s$ for the temperature gradient driven case, for two-species simulations.]

4. LINEAR GYROKINETIC SIMULATIONS WITH ENERGETIC IONS

So far we have neglected the contribution of energetic ions. In the present section, we discuss the effect of energetic ions on the density gradient driven and temperature gradient driven KBMs discussed above. The energetic ions temperature is chosen to be 25 times the background plasma, density 2% of the background electron density leading to energetic ion $\beta$ about 20% of the total $\beta$. The results for the density gradient-driven KBM is shown in Fig. 4. The real frequencies increase with decreasing $\beta$ in contrast to the cases without energetic ions. Also, the real frequencies are much higher than no energetic ion cases for a given wave number. The reversal in the gradient of the magnetic field observed in the high $\beta$ cases might be a plausible explanation for this kind of observation. For growth rates, with increasing $\beta$ the growth rates increase. Also, the long-wavelength KBM is observed at even lower $\beta$ in the presence of energetic ions which otherwise stable for this branch of KBM.
peak in the growth rates shifts towards an even longer wavelength making the mode more global. The effect of energetic ions on the temperature gradient-driven KBM is shown in Fig. 5. The general trend remains the same.

![Graph 1](image1.png)

**FIG. 4.** Real frequency (a) and growth rates (b) versus binormal wavenumber of density gradient driven modes in the presence of energetic ions with $\beta_f/\beta = 20\%$

![Graph 2](image2.png)

**FIG. 5.** Real frequency (a) and growth rates (b) versus binormal wavenumber of temperature gradient driven modes in the presence of energetic ions with $\beta_f/\beta = 20\%$

Estimation of kinetic effects due to the energetic ions can be carried out by evaluating the relative phases of the electrostatic potential and pressure. For the cases, without the energetic ions, the fluid phase relationships are recovered. However, in the presence of energetic ions, though background electrons and ions maintain the phase relations, the energetic ions advanced in phase in the positive direction. These observations can be explained based on energetic ion precession frequency which is much higher than the background plasma and the number of trapped energetic ions is also very high for the tight aspect ratio tokamak such as the one considered in the present simulations.

We also simulated the effect of energetic ion temperature ratio compared to the background plasma, for both density gradient driven and temperature gradient driven cases. The real frequencies of the mode increase in both cases as the temperature ratio increases. The growth rates also increase at $k_y \to 0$ with the temperature ratio. Also, when the temperature ratio is sufficiently high one observes a sharp separation between KBM and other modes.

5. **SUMMARY AND CONCLUSIONS**

We have presented the gyrokinetic simulations using GENE for high $\beta$ spherical reactor relevant plasma parameters produced using the CHEASE code. The focus of the present study is the long-wavelength KBM mode. Considering two different cases in terms of free energy source we show that the long-wavelength KBM can be unstable in both density gradient driven and temperature gradient driven scenarios. We have compared the results with ideal MHD calculations and observed that these long-wavelength KBM modes exhibit ideal MHD character.
We observe that the long-wavelength KBM becomes more unstable as $\beta$ increases and has quite global nature. The growth rate at $k_r \to 0$ is only slightly lower than the peak growth rates. As $\beta$ decreases, we see other electrostatic modes such as ITG/TEM. For higher $\beta$ cases at a shorter wavelength, we have observed tearing parity modes as well.

We have also studied the effect of energetic ions. The energetic ions render the mode even more global and more localized in the long-wavelength regime. Both real frequency and growth rates of the KBM increase in the presence of energetic ions. The effect of higher temperature of the energetic ions is found to increase the growth rates at longer wavelengths and create a sharp separation between the long-wavelength branch of the KBM and other modes. It might be emphasized that due to the long wavelength nature of the KBM without and with energetic ions the modes may be very global in the existing spherical tokamak which necessitates a global analysis [27, 28,29] to capture relevant physics.

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**REFERENCES**