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Compact equations for 3D plasma equilibrium

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A system of equations describing the static three-dimensional equilibrium of plasma in a magnetic field with toroidal magnetic surfaces is obtained.

The problem of finding the equilibrium plasma configurations in a magnetic field, **B**, is among the first-priority problems in plasma physics and its applications. The main tool for the calculation of axisymmetric equilibrium magnetic configurations with nested magnetic surfaces $\Psi(\mathbf{r})=$ const: $\mathbf{B}\cdot\nabla\Psi=0$ (**r** is the radius-vector) is the Grad–Shafranov equation (GSE) [1, 2]:

$$\Delta^*\Psi+4\pi r^2\frac{dp(\Psi)}{d\Psi}+\frac{1}{2}\frac{dF^2(\Psi)}{d\Psi}=0. \qquad (1)$$

Here $\Delta^*=r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2}$

is the Grad-Shafranov operator written in cylindrical coordinate system. Plasma pressure, p, is known to be a function of Ψ for any static equilibrium. For the axisymmetric equilibria ($\partial/\partial\varphi=0$) function F, which characterizes poloidal current, appears to be function of Ψ too.

Equation (1) is usually solved for Ψ at given dependences $p(\Psi)$ and $F(\Psi)$ and under the assumption that one of the surfaces $\Psi=$ const is a boundary, i.e., a zero pressure surface. An alternative way for the calculation of equilibrium with the GSE is the assignment of the flux Ψ on a fixed surface of a given shape considered as a boundary magnetic surface. It is clear that in this case there is no guarantee that the system of magnetic surfaces is nested in the entire volume inside the boundary surface.

The compact form of GSE is a result of axial symmetry and of the mixed representation of the magnetic field, **B**=$[\nabla\Psi\times\nabla\varphi]+F(\Psi)\nabla\varphi$.

The advantage of such representation is in the automatic satisfying the solinoidality (div**B** $=0$) and magnetic surface ($\mathbf{B}\cdot\nabla\Psi=0$) conditions. However, it works only in the case of axial symmetry. That explains the attempts [3, 4] to represent **B** by using the so-called reference vectors instead of $\nabla\varphi$. This approach allows one to write the equilibrium equation in a relatively simple form, at that, the entire difficulty of the further analysis lies on finding the reference vectors, which are not initially given and, in their turn, depend on $\Psi(\mathbf{r})$.

In this paper we use again the mixed representation to describe three-dimensional plasma equilibrium in a magnetic field with toroidal magnetic surfaces. To satisfy the magnetic surface condition, $\mathbf{B}\cdot\nabla\Psi=0$, in the 3D case with $\partial\Psi/\partial\varphi\neq 0$ we compensate the component along $\nabla\Psi$ by adding a corresponding summand: **B**=$\gamma[\nabla\Psi\times\nabla\varphi]+F\nabla\varphi-\alpha F\nabla\Psi$, (2)

where

$$\alpha=\frac{1}{|\nabla\Psi|^2}\frac{1}{r^2}\frac{\partial\Psi}{\partial\varphi},\quad |\nabla\Psi|^2=\left(\frac{\partial\Psi}{\partial r}\right)^2+\left(\frac{\partial\Psi}{\partial z}\right)^2+\frac{1}{r^2}\left(\frac{\partial\Psi}{\partial\varphi}\right)^2.$$

In Eq. (2) we have also provided for extra freedom in the poloidal magnetic field by a coefficient γ, which is not supposed to be a function of Ψ. By doing so, we describe the magnetic field with magnetic surfaces of arbitrary topology because Eq. (2) yields a decomposition of vector **B** in three non-complanar directions: $[\nabla\Psi\times\nabla\varphi]$, $\nabla\varphi$, and $\nabla\Psi$.

Three functions Ψ, γ, F should be determined from the common equations: the projection of the force balance equation, $\nabla p=[\mathrm{rot}\mathbf{B}\times\mathbf{B}]/4\pi$, on the direction of $\nabla\Psi$, the solenoidality condition, div**B** $=0$, and the condition that the lines of the current lie on magnetic surfaces, $\mathrm{rot}\mathbf{B}\cdot\nabla\Psi=0$. The final system of equations has the form [5]:

$$\gamma^2\Delta^*\Psi+\gamma(\nabla_0\Psi\cdot\nabla\gamma)+4\pi r^2\frac{dp}{d\Psi}+\frac{F}{|\nabla\Psi|^2}(\nabla_0\Psi\cdot\nabla F)+ \quad (1)$$

$$+\frac{1}{\alpha|\nabla\Psi|^2}\left(\mathbf{B}\cdot\nabla\left(F\alpha\frac{\partial\Psi}{\partial\varphi}\right)-\frac{F^2}{2r^2}\frac{\partial}{\partial\varphi}\left(\alpha\frac{\partial\Psi}{\partial\varphi}\right)\right)=0; \quad (3) \quad (2)$$

$$\mathbf{B}\cdot\nabla\frac{\gamma|\nabla\Psi|^2}{F}=[\nabla\Psi\times\nabla\varphi]\cdot\nabla\left(\frac{B^2r^2}{F^\star}\right); \quad (4) \quad (3)$$

$$\mathbf{B}\cdot\nabla F^\star=4\pi\frac{\partial}{\partial\varphi}\left(p+\frac{B^2}{8\pi}\right). \quad (5) \quad (4)$$

Function $F^\star=F\left(1-\alpha\partial\Psi/\partial\varphi\right)$ in Eqs. (4), (5) plays the same role as F did in axisymmetric case: it is proportional to the toroidal component of the magnetic field, $B_\varphi=F^\star/r$. The "axisymmetric" nabla operator is also used, $\nabla_0=\nabla-\nabla\varphi(\partial/\partial\varphi)$. Equation (3) is the analogue of the GSE in non-axisymmetric case. Equations (4) and (5) have the form of magnetic differential equations (MDEs), i.e., equations of the type $\mathbf{B}\cdot\nabla f=s$, which play an important role in the theory of plasma equilibrium in a magnetic field [6, 7]. It immediately follows from Eq. (5) that if the total pressure of the plasma and the magnetic field depends on the toroidal angle, the function $F^\star=rB_\varphi$ is inhomogeneous on magnetic surface, i.e. $F^\star\neq F^\star(\Psi)$. In the degenerate case of purely poloidal magnetic field ($F=0$), instead of Eq. (4) one should use condition $\gamma=\gamma(\Psi,\varphi)$ following from div$\mathbf{B}=0$, and Eq. (5) implies axial symmetry of the total pressure, $\partial(p+B^2/8\pi)/\partial\varphi=0$, which coincides with the results of Ref. [8].

For axisymmetric case, $\partial/\partial\varphi=0$, Eqs. (4), (5) transfer into conditions $F=F(\Psi)$, $\gamma=\gamma(\Psi)$, and Eq. (3) results in GSE (1).

The productivity of the suggested approach can be demonstrated for the case of "weak"axial asymmetry. In this case the sought functions Ψ, F, and γ can be represented as combinations of basic (axisymmetric) parts and small additives periodical on φ:

$$\begin{aligned}\Psi(r,\,\varphi,\,z)&=\Psi_0(r,\,z)+\varepsilon\Psi_1(r,\,\varphi,\,z),\\ F(r,\,\varphi,\,z)&=F_0(\Psi_0)+\varepsilon F_1(r,\,\varphi,\,z),\\ \gamma(r,\,\varphi,\,z)&=1+\varepsilon\gamma_1(r,\,\varphi,\,z),\qquad\varepsilon\ll1.\end{aligned}$$

with basic solution, Ψ_0, satisfying GSE (1). The further technique of solving the equilibrium equations consists of choosing Ψ_0 and subsequent calculation from Eqs. (3)–(5) the functions Ψ_1, F_1, and γ_1 depending on the toroidal angle. Example of the three-dimensional magnetic surface system obtained in the described manner with the considered in Ref. [9] basic solution similar to the Hill vortex is

$$\Psi=\frac{\Psi_a}{R^4}\left(2R^2r^2-r^4-4k^2r^2z^2\;-4\varepsilon R\cos\varphi\left(R^2r-r^3-2k^2z^2r\right)\right).\,(6) \quad (5)$$

The three-dimensional view of the magnetic surface $\Psi=0.2\Psi_a$ described by Eq. (6) at $\varepsilon=0.3$, $k=0.8$ is shown in Fig. 1.

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