**Introduction & Motivation**

A long lasting challenge in astrophysics is to identify the magnetic field generation of magnetic field in the stars and galaxies. There exists several theoretical models and some experimental models to explain the same. It is well known that the generation of large-scale magnetic field in the stars or in the galaxies are mostly due to mean field dynamo effect. In this context it is also observed small scale magnetic field generation via turbulent fluctuation dynamo.

For these kind of dynamo study the important thing is to identify a class of suitable flow profile. Here we consider the well known chaotic velocity profile i.e. Arnold-Beltrami-Childress [ABC] flow which is known to support various Dynamo study [3-4]. It is interesting to explore various dynamo model starting from Induction, Kinematic and Self-consistent via numerical simulation approach.

For this kind of large physics problems, a scalable code is essential. We have very recently upgraded an OpenMP MHD solver MHD3D [2] to a Multi-Node Multi-GPU architecture (GMHD3D) developed at IPR. We used this GMHD3D code for our simulation here.

We first benchmark our newly developed solver and use it to study various dynamo action, details of which is presented here.

**Governing Equations**

The following equations govern the dynamics of Magnetohydrodynamic plasma (Conservative form).

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \\
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p + \rho \mathbf{g} = -\nabla \Phi + \nabla \times \mathbf{B} \\
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \eta \nabla^2 \mathbf{B} \\
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) + \frac{\partial}{\partial x} (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla (\rho u_x) - \eta \frac{\partial^2 \mathbf{B}}{\partial x^2} \\
\mathbf{F} = \eta \frac{\partial^2 \mathbf{B}}{\partial x^2}
\]

\( S, \eta \) represent the velocity, magnetic and pressure fields, \( \rho \) and \( \eta \) the kinematic viscosity and magnetic diffusivity. The initial density is denoted by \( \rho = \rho_0 \). The magnetic field \( \mathbf{B} \) is constant along all three spatial direction.

The maximum fluid speed is \( u_0 \) and the Alfvén speed is \( V_A \) where \( M_A = \frac{u_0}{V_A} \).

**About code and its solution techniques**

- A three dimensional GPU based magnetohydrodynamic solver GMHD3D has been developed at IPR [2]. It uses Pseudo-spectral technique with AccFFT libraries and Adams-Bashforth as time updater. We benchmarked our solver and found good agreement by this newly developed code.

**Initial Condition**

- As mentioned earlier we have used Arnold-Beltrami-Childress [ABC] flow as an initial flow profile and the magnetic field is kept constant along all three spatial direction. We drive the flow with the same ABC flow field which looks like:

\[
\mathbf{u} = (4A_0 \sin \eta t + 3 A_1 \cos \eta t) + (A_0 \sin \eta t + 3 A_1 \cos \eta t)
\]

where \( A_0 = C_0 \), \( A_1 = -3 C_0 \), \( f_0 = B_0 \), \( f_1 = 0 \) and \( \eta = \frac{2}{3} \).

**Different Dynamo Models**

- **Induction Dynamo**: The phenomenon of magnetic energy growing exponentially with time for a statistically steady flow, where the velocity field is held fixed in time, is called Induction dynamo action [Only equation (3) (magenta color) evolves].

- **Kinematic Dynamo**: Dynamo action can also be studied by evolving velocity field according to Navier-Stokes equation and magnetic field according to induction equation [Conservative form]. But magnetic coupling between these two fields i.e. \( \mathbf{J} \) is neglected [All four set of (1,2,3,4) equations evolve without red colored terms in equation (2)].

- **Self-Consistent Dynamo**: A Self-Consistent dynamo represents a situation where the magnetic energy grows exponentially for a plasma where the plasma itself evolves in time. Hence the velocity field is not externally imposed like a kinematic dynamo, rather it has a dynamical nature. The time evolution of the velocity field is generally governed by the Navier-Stokes equation including the magnetic feedback on the velocity field i.e. \( \mathbf{J} \) is included [all set of equations (1,2,3,4) (4) evolves].

We will present the numerical experiment of all these situation in the presence of subgrid model of viscosity.

**Induction Dynamo**

- Here for this study we will only evolve equation (3). The velocity field is taken as ABC flow as stated earlier.

First we study the effect of magnetic resistivity \( \eta \) through the magnetic Reynold’s number \( Re = \frac{V_A^2}{\eta} \) which has been widely studied earlier. We observe that by increasing magnetic Reynold’s number, dynamo growth rate increased and reproduced the previous result by Galloway et al. [3] using our code.

Then we fixed the fluid at 450 and change the forcing scale \( k_f \) from 1, 2, 4, 8, 16 and observed that the growth rate of magnetic energy increases with increase of forcing scale.

**Kinematic Dynamo**

- Very recently, Sadek et al. [4] studied turbulent dynamo in the presence of chaotic ABC flow, forced at different forcing wave number \( k_f \) using a subgrid turbulence model in Kinematic regime [Red term in equation (2) ignored, rest of the equations are evolving].

The motivation for choosing this kind of special viscosity model which looks like \( \eta (k, \lambda) = 0.271 \lambda + 3.58 \lambda (k/k_m)^{2.5} \) is to mimic high Reynolds number flow that require large grid resolution. We also use the same viscosity model and notice that there exists an optimal forcing wave scale at \( k_f = 4 \) for which critical \( Re = 50 \) for dynamo onset also observed by Sadek et al. [4] earlier.

**Self-Consistent Dynamo with constant viscosity**

- As discussed earlier by Self-Consistent we mean that the velocity field and magnetic field both are dynamic in nature along with that there is magnetic feedback on the velocity field via Lorentz force. So we evolve equation (1), (2), (3), (4) for this.

First we vary \( Re \) keeping the magnitude of \( A = B = C = 0 \) for all the cases. We run our simulation for \( \eta_0 = 0.1, 0.2, 0.3, 0.4, 0.5 \) and observe the trend of dynamo action is identical for all values of \( \eta_0 \).

Next we vary the magnitude of \( A = 0.1, 0.2, 0.3 \) keeping \( A = B = C = 0 \) and \( \eta_0 = 0.1 \). We follow the same way as previous section but along with that we will incorporate the magnetic feedback to the velocity field i.e. now we will consider the red color terms in equation (2).

We observe that the magnetic feedback to the velocity field does not change the optimal scale for dynamo onset, i.e. we can say that for Self-Consistent model in the presence of subgrid model of viscosity optimal scale doesn’t change. We confirmed the same from \( f_0 \) also.

**Induction Dynamo Action in the presence of subgrid model of viscosity**

- Now we will follow the same way as previous section but along with that we will incorporate the magnetic feedback to the velocity field i.e. we will now consider the red color terms in equation (2).

We observe that the magnetic feedback to the velocity field does not change the optimal scale for dynamo onset, i.e. we can say that for Self-Consistent model in the presence of subgrid model of viscosity optimal scale doesn’t change. We confirmed the same from \( f_0 \) also.

**Induction Dynamo Action**

- The Alfven speed can be defined as \( V_A = \sqrt{\frac{B^2}{\mu_0 \rho}} \) now if \( M_A = 1 \), \( V_A = u_0 \) and the plasma is called Sub-Alfvenic. Similarly the reverse case if \( M_A > 1 \), the plasma is Super-Alfvenic. For Induction dynamo action, the growth rate of magnetic energy is seen to be unbounded on the magnitude of \( M_A \). The growth rate for \( M_A = 0.1, 1, 10, 100 \) are found to be identical.

**Kinematic Dynamo Action**

- We will present the numerical experiment of all these situation in the presence of subgrid model of viscosity [for 128^3 grid resolution] from GMHD3D code.

**Self-Consistent Dynamo Action**

- Now we will follow the same way as previous section but along with that we will incorporate the magnetic feedback to the velocity field i.e. we will now consider the red color terms in equation (2).

We observe that the magnetic feedback to the velocity field does not change the optimal scale for dynamo onset, i.e. we can say that for Self-Consistent model in the presence of subgrid model of viscosity optimal scale doesn’t change. We confirmed the same from \( f_0 \) also.

**Induction Dynamo in Iso-B surface**

- We discussed about different dynamo models (Induction, Kinematic & Self-Consistent) in the presence of constant viscosity as well as subgrid model of viscosity. For different parameter dependency Self-Consistent dynamo is seen to saturate [due to magnetic feedback] where the Induction dynamo is not.

- With subgrid viscosity model we reproduce the result of Sadek et al. [4] for the optimal scale \( k_f = 4 \) for Kinematic dynamo case.

- We also show the optimal scale \( k_f = 4 \) does not change by including magnetic feedback to the velocity field. As we are dealing with low \( Re \) regime so the effect of \( J \) is not showing any significant change to the velocity field may be this is the possible reason for getting the same optimal scale.

- All the results shown here are at 226^3 grid resolution. Further high resolution study is ongoing.

Reference