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A Numerical Simulation of Self Consistent Dynamo Using A New GPU-based 3D MHD Solver

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Introduction & Motivation

- A long lasting challenge in astrophysics is to identify the mechanism of the generation of magnetic field in the stars and galaxies.
- There exists several theoretical models and some experimental models to explain the same.
- It is well known that the generation of large-scale magnetic field in the stars or in the galaxies are mostly due to mean field dynamo [1].
- In this context it is also observed small scale magnetic field generation via turbulent fluctuation dynamo.
- For these kind of dynamo study the important thing is to identify a class of suitable flow profile. Here, we consider the well known chaotic velocity profile i.e. Arnold–Beltrami–Childress [ABC] flow which is known to support various Dynamo study [3,4].
- It is interesting to explore various dynamo model starting from Induction, Kinematic and Self-consistent via numerical simulation with the presence of ABC flow.
- For this kind of large physics problems, a scalable code is essential. We have very recently upgraded an OpenMP MHD solver MHD3D[2] to a Multi-Node Multi-GPU architecture (GMHD3D) developed at IPR. We used this GMHD3D code for our simulation here.
- We first benchmark our newly developed solver and use it to study various dynamo action, details of which is presented here.

Governing Equations

The following equations govern the dynamics of MagnetoHydroDynamic plasma (Conservative form).

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \tag{1}$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left[\rho \vec{u} \otimes \vec{u} + \left(P + \frac{B^2}{2} \right) \mathbf{I} - \vec{B} \otimes \vec{B} \right] = \mu \nabla^2 \vec{u} + \vec{f} \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \cdot \left(\vec{u} \otimes \vec{B} - \vec{B} \otimes \vec{u} \right) = \eta \nabla^2 \vec{B} \tag{3}$$

$$P = C_s^2 \rho \tag{4}$$

$\vec{u}$, $\vec{B}$, P represent the velocity, magnetic and pressure fields, μ and η the kinematic viscosity and magnetic diffusivity. The initial density is $\rho = 1$. The dimensionless numbers are defined as: $R_e = \frac{u_0 L}{\mu}$, $R_m = \frac{u_0 L}{\eta}$, Magnetic Prandtl number ($P_m = \frac{\mu}{\eta} = \frac{R_m}{R_e}$). The maximum fluid speed is u_0 and the Alfven speed is $V_A = \frac{u_0}{M_A}$, where M_A is the Alfven Mach number. Similarly the sound speed is $C_s = \frac{u_0}{M_s}$, where M_s is the sonic Mach number. Time is normalized as $t = t_0 * t'$, $t_0 = \frac{L}{V_A}$

About code and its solution techniques

- A three dimensional GPU based magnetohydrodynamic solver G-MHD3D has been developed in house at Institute for Plasma Research (IPR). It uses Pseudo-spectral technique with AccFFT libraries and Adams-Bashforth as time updater. We benchmark few recently published work by this newly developed code.

Initial Condition

- As mentioned earlier we have used Arnold–Beltrami–Childress [ABC] flow as an initial flow profile and the magnetic filed is kept constant along all three spatial direction.
- We drive the flow with the same ABC force field which looks like:

$$f_x = A \sin(k_f z) + C \cos(k_f y)$$
$$f_y = B \sin(k_f x) + A \cos(k_f z)$$
$$f_z = C \sin(k_f y) + B \cos(k_f x)$$

where $A = B = C$ are real constants and k_f is the forcing wave number.

Different Dynamo Models

- Induction Dynamo: The phenomenon of magnetic energy growing exponentially with time for a statistically steady flow, where the velocity field is held fixed in time, is called, Induction dynamo action [Only equation 3 (magenta color) evolves].
- Kinematic Dynamo: Dynamo action can also be studied by evolving velocity field according to Navier-Stokes equation and magnetic field according to induction equation. But magnetic coupling between these two fields i.e. $\vec{J} \times \vec{B}$ is neglected [All four set of (1,2,3,4) equations evolve without red colored terms in equation (2)].
- Self-Consistent Dynamo: A Self-Consistent dynamo represents a situation where the magnetic energy grows exponentially for a plasma where the plasma itself evolves in time. Hence the velocity field is not externally imposed like a kinematic dynamo, rather it has a dynamical nature. The time evolution of the velocity field is generally governed by the Navier-Stokes equation including the magnetic feedback on the velocity field i.e. $\vec{J} \times \vec{B}$ is included (Full set of equations (1),(2),(3),(4) evolve).
- We will present the numerical experiment of all these situation in the coming section.

Induction Dynamo

- Here for this study we will only evolve equation (3). The velocity field is taken as ABC Flow as stated earlier.
- First we study the effect of magnetic resistivity η through the magnetic Reynold's number (R_m) that has been widely studied earlier. We observe that by increasing magnetic Reynold's number, dynamo growth rate increased and reproduced the previous results by Galloway et al. [3] using our code.
- Then we fixed the R_m at 450 and change the forcing scale k_f from 1, 2, 4, 8, 16 and observed that the growth rate of magnetic energy is increased as k_f is increased.

Induction Dynamo [Effect of Resistivity, Forcing scale, Alfven Speed]

- The Alfven speed can be defined as $V_A = \frac{u_0}{M_A}$, now if $M_A < 1$; $V_A < u_0$ and the plasma is called Sub-Alfvenic. Similarly the reverse case if $M_A > 1$, the plasma is Super-Alfvenic. For Induction dynamo problem, the growth rate of magnetic energy is seen to be unbiased on the magnitude of M_A . The growth rate for $M_A = 0.1, 1, 10, 100, 1000$ are found to be identical.

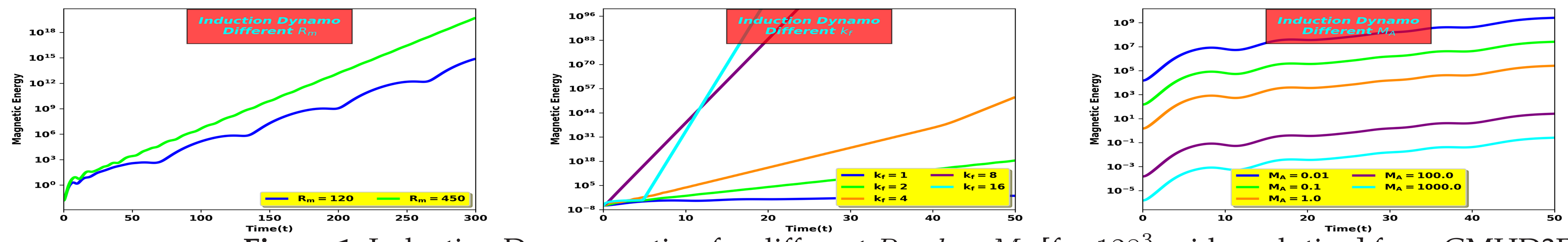

Figure 1: Induction Dynamo action for different R_m, k_f , M_a [for 128^3 grid resolution] from GMHD3D code.

Kinematic Dynamo Action in the presence of subgrided model of viscosity

- Very recently, Sadek et al. [4] studied turbulent dynamo in the presence of chaotic ABC flow, forced at different forcing wave number (k_f) using a sub-grided turbulence model in Kinematic regime [Red terms in equation (2) ignored, rest of the equations are evolving].
- The motivation for choosing this kind of special viscosity model which looks like $\mu(k,t) = 0.27[1 + 3.58(k/k_c)^8]\sqrt{\frac{E_k(k_c,t)}{k_c}}$ is to mimic high Reynolds number flow that require large grid resolution. We also use the same viscosity model and notice that there exists an optimal forcing length scale at $k_f = 4$ for which critical R_m for dynamo onset occurs also observed by Sadek et al. [4] earlier.

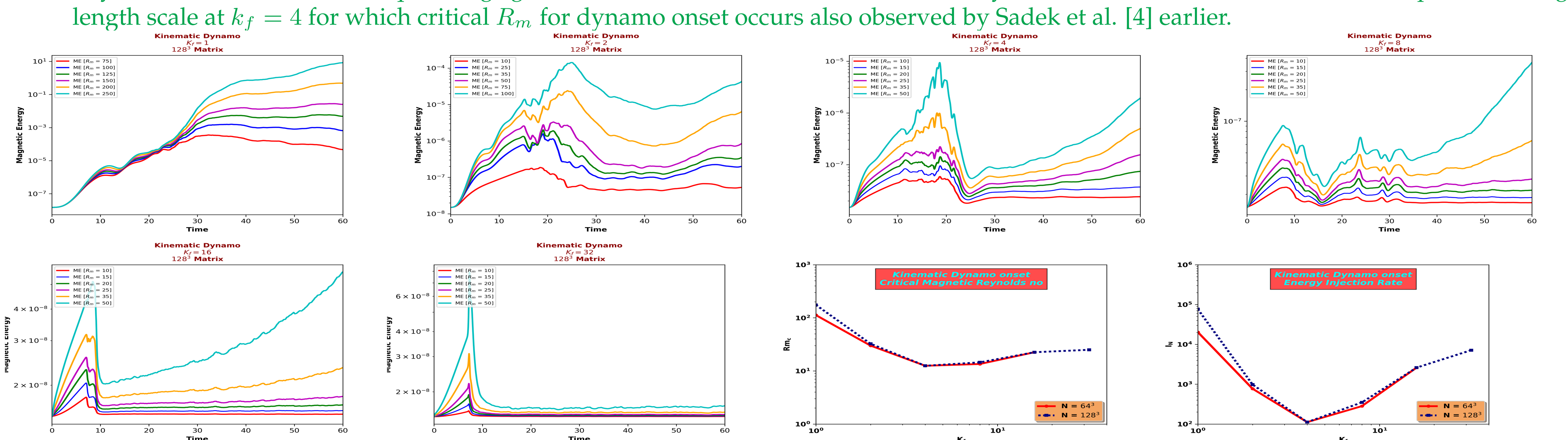

Figure 2: Kinematic Dynamo onset at low magnetic Prandtl number ($P_m < 1$) in the presence of subgrided model of viscosity [for 128^3 grid resolution] from GMHD3D code. These results are exactly matching with Sadek et al. [4]

- For further demonstration we calculate the energy injection rate (I_N) $\propto R_m^3 \times k_f$ following Sadek et al. and reproduce that minimum energy injection rate required for $k_f = 4$ to achieve the dynamo.

Self-Consistent Dynamo with constant viscosity

- As discussed earlier by Self-Consistent we mean that the velocity field and magnetic field both are dynamic in nature along with that there is magnetic feedback on the velocity field via Lorentz force. So we evolve equation (1), (2), (3), (4) for this.
- First we vary u_0 keeping the magnitude of $A = B = C = 0.1$ for all the cases. We run our simulation for $u_0 = 0.1, 0.2, 0.3, 0.4, 0.5$ and observe the trend of dynamo action is identical for all values of u_0.
- Next we vary the magnitudes of $A = 0.1, 0.2, 0.3$ keeping $A = B = C$ and $u_0 = 0.1$ and notice faster growth of dynamo with higher values of forcing via the magnitudes of A, B and C.

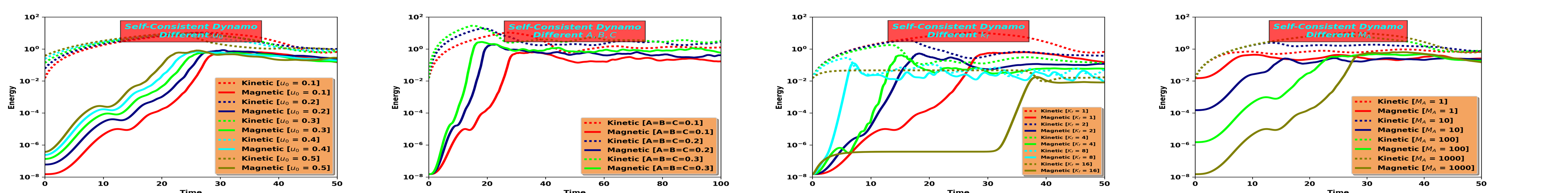

Figure 3: Self-Consistent Dynamo action for different u_0, $A = B = C$, k_f , M_a [for 128^3 grid resolution] from GMHD3D code.

- Then we change the length scale of forcing (k_f) on the velocity field keeping $u_0 = 0.1$, $A = B = C = 0.1$, $R_e = R_m = 450$, $M_s = 0.1$ and $M_a = 1000$ as fixed parameters. We observed, the growth rate of magnetic energy increases while that of kinetic energy decreases as k_f is increased as same as Induction Dynamo.
- Finally we change the Alfven Mach number (M_a) of the plasma, keeping $u_0 = 0.1$, $A = B = C = 0.1$, $R_e = R_m = 450$, $M_s = 0.1$ and $k_f = 1$. Now by changing the value of M_a means we are changing the value of B_0 as we start from a lower value of B_0, the growth rate of the magnetic energy increases. The situation is almost similar to the Induction dynamo action with a significant difference. For Induction dynamo there was no saturation of magnetic energy. On the other hand, in driven self-consistent dynamo, there is a saturation value of the magnetic energy due to magnetic feedback.

Self-Consistent Dynamo Action in the presence of subgrided model of viscosity

- Now we will follow the same way as previous section but along with that we will incorporate the magnetic feedback to the velocity field i.e. we will now consider the red color terms in equation (2).
- We observe that including magnetic feedback to the velocity field does not change the optimal scale for dynamo onset, i.e. we can say that for Self-consistent dynamo model in the presence of subgrided viscosity optimal scale doesn't change. We confirmed the same from I_N also.

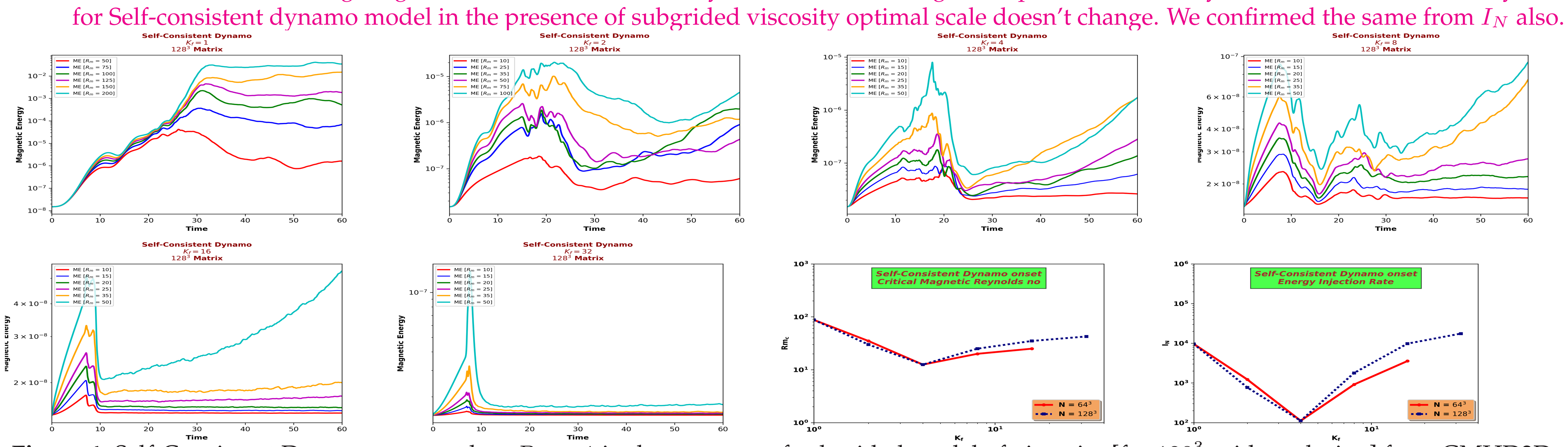

Figure 4: Self-Consistent Dynamo onset at low $P_m < 1$ in the presence of subgrided model of viscosity [for 128^3 grid resolution] from GMHD3D.

Iso-B surface

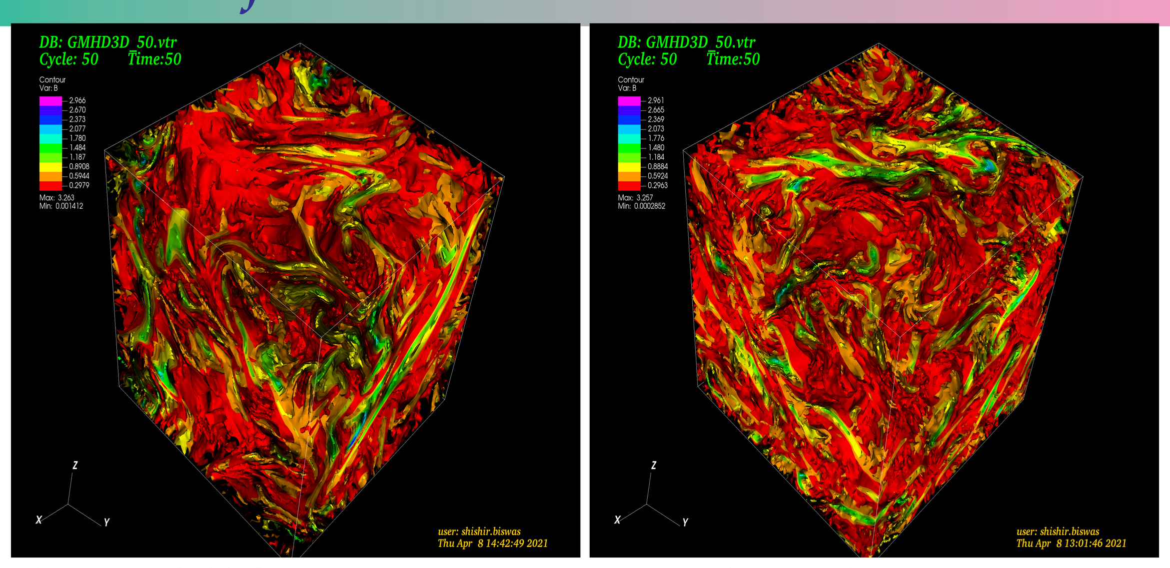

Figure 5: Self-Consistent Dynamo iso-B surface with constant viscosity [left] vs subgrided viscosity [right] model[for 128^3 matrix] at Time 50.0 from GMHD3D code.

Acknowledgement

- All the work [simulation and visualization] reported here are performed on **Antya** cluster at IPR.

Conclusion & Future Work

- We discussed about different dynamo models (Induction, Kinematic & Self-Consistent) in the presence of constant viscosity as well as subgrided model viscosity. For different parameter dependency Self-Consistent dynamo is seen to saturate [due to magnetic feedback] where the Induction dynamo is not.
- With subgrided viscosity model we reproduce the result of Sadek et al. [4] the optimal scale ($k_f = 4$) for Kinematic dynamo onset.
- We also show the optimal scale ($k_f = 4$) does not change by including magnetic feedback to the velocity field. As we are dealing with low P_m regime so the effect of $\vec{J} \times \vec{B}$ is not showing any significant change to the velocity field may be this is the possible reason for getting the same optimal scale.
- All the results shown here are at 128^3 grid resolution. Further high resolution study (256^3, 512^3..) will be done later.

References

[1] R. Moll, J. P. Graham, J. Pratt, et al. The Astrophysical Journal 736, 36 (2011).

[2]Turbulence, flows and Magnetic field generation in plasmas using a Magnetohydrodynamic Model -Phd Thesis (june 2019) by R. Mukherjee [IPR].

[3]D. Galloway and U. Frisch, Geophysical & Astrophysical FluidDynamics 36, 53 (1986).

[4]M Sadek, A Alexakis, and S Fauve, Physical Review Letters, 116(7):074501, 2016.