

# Cold-hot coupled waves in a flowing magnetized plasma

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## ABSTRACT

- Nonlinear coupling between cold (fluid) and hot waves in a *time-varying plasma flow* is analyzed with the Vlasov equation.
- The kinetic analysis presents a generalized dispersion relation that involves resonances depending on cold and hot wave dispersions.
- Both the analytic solution and fully kinetic particle-in-cell (PIC) simulations demonstrate that the wave spectrum is nonlinearly determined by the wavenumbers of linearly independent waves.

## BACKGROUND

- Rapid plasma transports are frequently observed in a variety of situations such as sawtooth crashes, tearing modes [2], the burst of edge-localized modes [3], ionospheric flows, and the explosion of coronal loops.
- These events involve time-varying plasma flows  $\mathbf{u}_{s0} = \int \mathbf{v} f_{s0} \mathbf{v} / n_{s0} = \bar{\mathbf{u}}_{s0} + \tilde{\mathbf{u}}_{s0}(t)$  (species  $s$  with a distribution function  $f_s = f_{s0} + f_{s1}$ ), then EM waves are excited by  $\tilde{\mathbf{J}}_0 = \sum_s q_s n_{s0} \tilde{\mathbf{u}}_{s0}$ .
- This work investigates the nonlinear wave-wave coupling, respectively, induced by  $\tilde{\mathbf{J}}_0$  and  $\mathbf{J}_1 = \sum_s q_s \int \mathbf{v} f_{s1} \mathbf{v}$ . A generalized dispersion relation is derived and the resulting wave spectra are studied.

## METHODS & IMPLEMENTATION

### (a) Kinetic analysis

- Zeroth order distribution function:

$$f_{s0}(\mathbf{v}, \mathbf{x}, t) = \frac{n_{s0}}{\pi^{3/2} v_{Ts\perp}^2 v_{Ts\parallel}} \exp\left(-\frac{(v_{\perp} - \mathbf{u}_{s0\perp})^2}{v_{Ts\perp}^2} - \frac{(v_{\parallel} - \mathbf{u}_{s0\parallel})^2}{v_{Ts\parallel}^2}\right) \quad (1)$$

†  $v_T$ : thermal speed

- First order distribution function (integration of the linearized Vlasov Eq.):

$$\frac{df_{s1}}{dt} = -\frac{q_s}{m_s} (\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1) \cdot \frac{\partial f_{s0}}{\partial \mathbf{v}}$$

$$\Rightarrow f_{s1}(\mathbf{v}, \mathbf{x}, t) = -\frac{q_s}{m_s} \int_{t_0}^t (\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1) \cdot \frac{\partial f_{s0}}{\partial \mathbf{v}} dt' \quad (2)$$

- Equation of motion:

$$\frac{d\mathbf{v}(t')}{dt'} = \frac{q_s}{m_s} (\mathbf{E}_0 + \mathbf{v}(t') \times \mathbf{B}_0) \Rightarrow \mathbf{v}(t') = \mathbf{v}^* + \bar{\mathbf{u}}_{s0} + \tilde{\mathbf{u}}_{s0} \quad (3)$$

† bar: time-independent field  
tilde: time-dependent field  $\propto \exp(i\mathbf{k} \cdot \mathbf{x} - i\tilde{\omega}t)$

$\mathbf{v}^* = v_{\perp}(\hat{x} \cos \omega_c t - \hat{y} \sin \omega_c t)$ : Cyclotron motion

$\bar{\mathbf{u}}_{s0}$ : Time-independent flow

$\tilde{\mathbf{u}}_{s0}$ : Time-dependent flow  $\leftarrow$  Cold wave motion  $\frac{\partial \tilde{\mathbf{u}}_{s0}}{\partial t'} = \frac{q_s}{m_s} (\tilde{\mathbf{E}}_0 + \tilde{\mathbf{u}}_{s0} \times \bar{\mathbf{B}}_0)$  (4)

The time-dependent flow  $\tilde{\mathbf{u}}_{s0} = \sum_{l,r} \tilde{u}_{s0lr} \hat{r}$  having the  $l$ th mode  $\tilde{u}_{s0lr} \propto \cos[\tilde{\omega}_l(\tilde{\mathbf{k}}_l t)']$  is determined by the cold wave dispersion relation with the flow wavenumber  $\tilde{\mathbf{k}}$ . A general dispersion relation including the cold-hot coupled wave is obtained by combining  $\mathbf{J}_1 = \sum_s q_s \int \mathbf{v} f_{s1} \mathbf{v}$  (equation (2)) with Maxwell's equations (full analysis and results can be found in [1]).

### (b) Particle-in-cell simulation (1-D periodic boundary condition) [4]

- As an example study, the perpendicular wave ( $k = k_x$ ) spectrum is examined.
- A time-varying flow with a cold wave frequency  $\tilde{\omega}(\tilde{\mathbf{k}})$  is given by the initial EM fields and  $\tilde{\mathbf{u}}_{\text{init}} \propto \cos(\tilde{\mathbf{k}}x)$  satisfying equations (3)-(4).

Case	$\bar{\mathbf{u}}_0$ (constant flow)	$\mathbf{k} \cdot \bar{\mathbf{u}}_0$ (Doppler-shift)	Remarks
(i)	0	0	Static plasma
(ii)	$0.02v_{Te} \hat{y}$	0	Time-varying flow with a single $\tilde{\mathbf{k}}$ : $\tilde{\omega}(\tilde{\mathbf{k}}) = 0.02\omega_{ce}$
(iii)	$0.02v_{Te} \hat{x}$	$k_x \bar{u}_{0x}$	

Table A. Initial conditions with  $\bar{\mathbf{B}}_0 = 2\hat{z}T$ ,  $n_{e,i} = 10^{19} \text{ m}^{-3}$ ,  $T_{e,i} = 1 \text{ keV}$

## OUTCOME

- The kinetic analysis presents a *generalized plasma wave dispersion relation* that involves cold-hot coupled resonances at

$$\omega_{\text{resonant}} = \sum_{l,r} m_{lr} \tilde{\omega}_l + n\omega_{cs} + k_{\parallel} v_{\parallel}, \quad -\infty \leq n, m_{lr} \leq \infty. \quad (5)$$

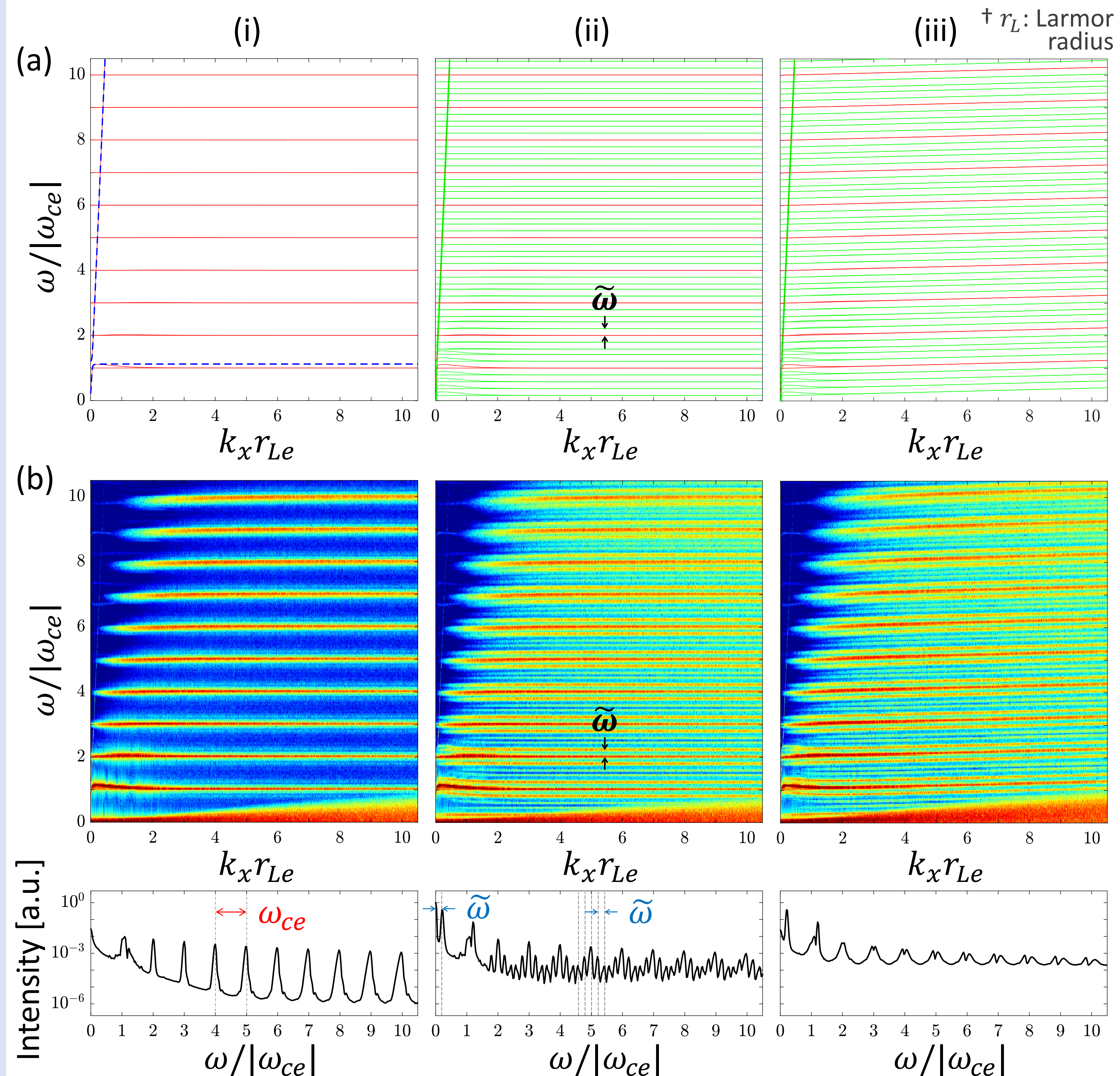


Figure A. Dispersion relations obtained from (a) kinetic analysis and (b) PIC simulations (electric field intensity on a logarithm scale). (i)-(iii) correspond to the cases in Table A. Frequency spectrums of the field intensity from the PIC are plotted in the bottom. In (a), the dispersions of cold (blue, dashed), hot (red only), and cold-hot coupled waves (red & green in (ii, iii)) are shown.

## CONCLUSION

- Nonlinearly coupled harmonic waves are excited by the time-varying plasma flow.
- The spectrum is determined by  $\tilde{\omega}$  via  $\tilde{\mathbf{k}}$  (characteristic length scale of the flow) and  $\mathbf{k} \cdot \bar{\mathbf{u}}_{s0}$  (Doppler effect).
- In experiments, it is expected that the plasma flow (current) results in a broad frequency spectrum by the excitation of cold-hot coupled waves (bottom panel of (ii)) and the Doppler effect (bottom panel of (iii)).
- The theory will be applied to the spectral interpretation of the transport processes in the laboratory and magnetospheric plasmas (e.g., MHD instability, magnetic reconnection, and turbulence).

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